Symmetry and Geometry in generalized HEFT

Yoshiki Uchida

Nagoya University

arXiv : 1904.07618

In collaboration with

Ryo Nagai (ICRR)
Masaharu Tanabashi (Nagoya U.)
Koji Tsumura (Kyoto U.)

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The sequel to the presentation by Nagai-san in PPP2018

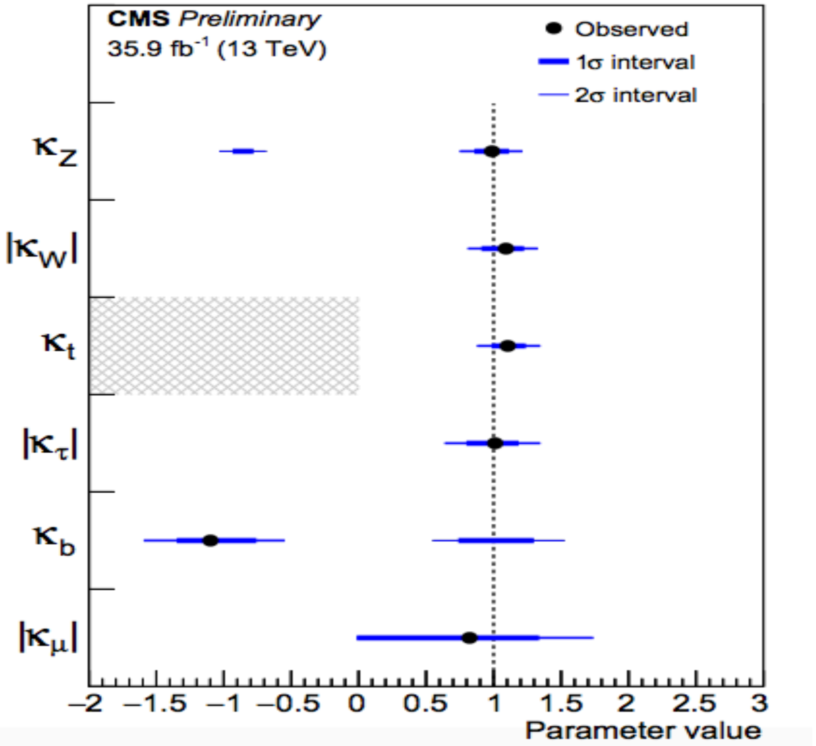
SM is not complete

・Hierarchy problem　　・Dark Matter　　…

Beyond the standard model (BSM) is needed !

・SUSY　　・Composite Higgs　　・Extra dim.　　…

In many BSM, the scalar sector is extended from SM



CMS-PAS-HIG-17-031

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- Hierarchy problem
- Dark Matter
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- SUSY
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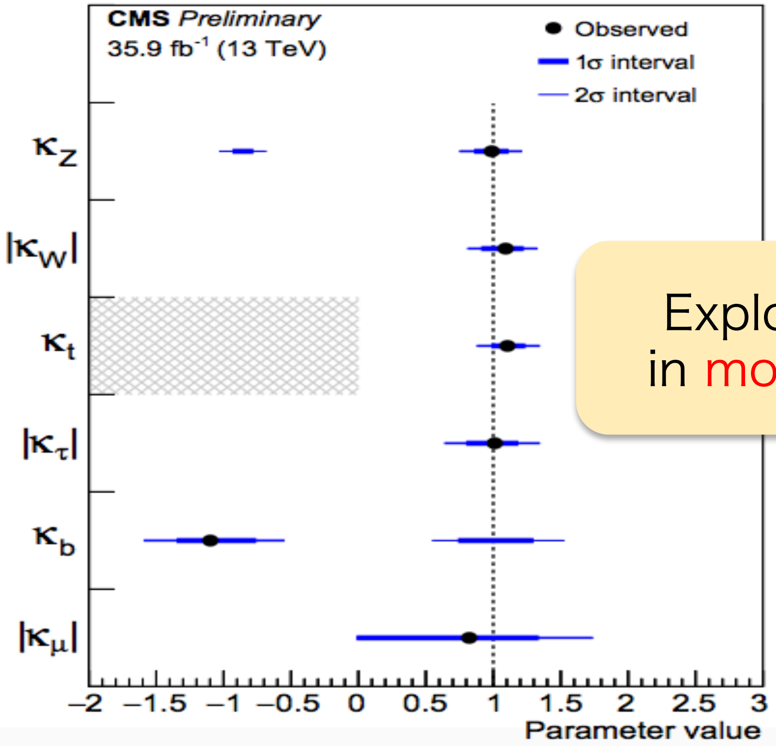
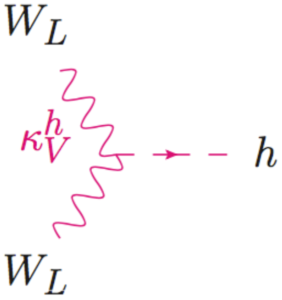


CMS-PAS-HIG-17-031

Our Goal

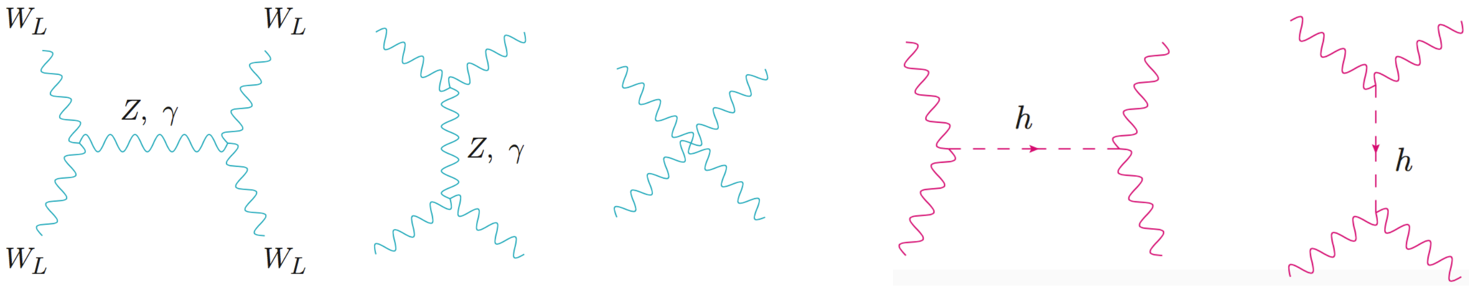
Explore the scalar sector in model-independent way

＜Standard Model＞



I. Higgs unitalize W_L scattering amplitude at tree level (tree level unitarity)

$$\mathcal{M}_{W_L W_L \to W_L W_L} \simeq \frac{s+t}{v^2}\left(1-(\kappa_V^h)^2\right) \quad (\ \kappa_V^h = 1 \ \text{ in SM })$$

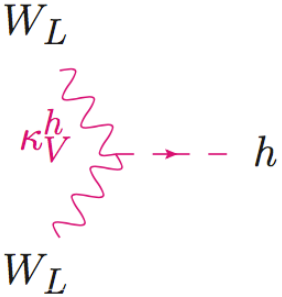


II. Higgs cancels the divergence in oblique corrections

Peskin Takeuchi
Phys. Rev. Lett. 65 (1990) 964

$$S \simeq \frac{1}{12\pi}\left(1-(\kappa_V^h)^2\right)\ln\frac{\Lambda^2}{\mu^2}$$

<Standard Model>



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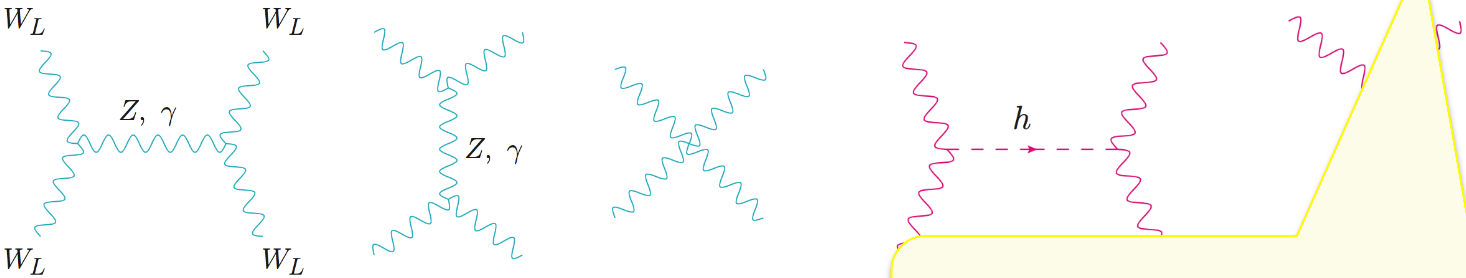
($\kappa_V^h = 1$ in SM)



What will happen
if κ_V^h deviate from 1 ?

II. Higgs cancels the divergence in oblique corrections

Peskin Takeuchi
Phys. Rev. Lett. 65 (1990) 964

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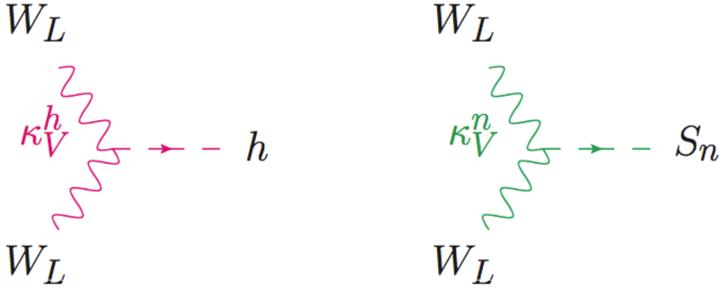
＜Singlet extension＞



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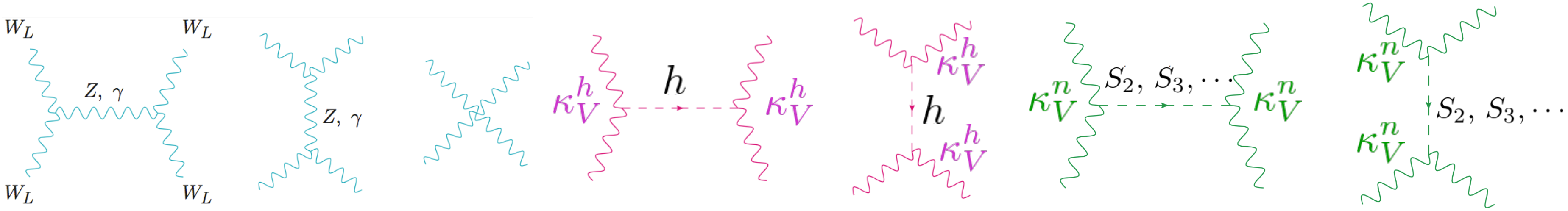
$$\mathcal{M}_{W_L W_L \to W_L W_L} \simeq \frac{s+t}{v^2}\left(1-(\kappa_V^h)^2-\sum_{n=2}^{N}(\kappa_V^n)^2\right)$$



unitarity sum rules $\quad 1-(\kappa_V^h)^2-\sum_{n=2}^{N}(\kappa_V^n)^2=0$

J F Gunion, H E Haber, J Wudka
Phys. Rev. D 43 904 (1991)

II. Higgs cancels the divergence in oblique corrections

$$S \simeq \frac{1}{12\pi}\left(1-(\kappa_V^h)^2-\sum_{n=2}^{N}(\kappa_V^n)^2\right)\ln\frac{\Lambda^2}{\mu^2}$$

finiteness conditions

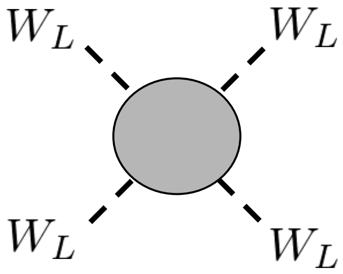
$$1-(\kappa_V^h)^2-\sum_{n=2}^{N}(\kappa_V^n)^2=0$$

Our Goal

Explore the scalar sector in model-independent way

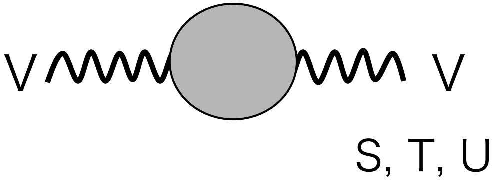
We impose the two conditions on the extended scalar sector

I. tree level unitarity



$$\sim \left(1\underbrace{-(\kappa_V^h)^2-\cdots}_{=0}\right)E^2$$

II. finiteness of oblique parameter (1-loop)



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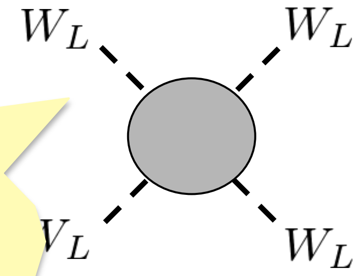
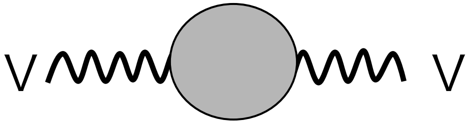


$$\sim \left(\underbrace{1-(\kappa_V^h)^2-\cdots}_{=0}\right)E^2$$

Independent condition ?

II. finiteness f oblique parameter (1-loop)



S, T, U

$$\sim \left(\underbrace{1-(\kappa_V^h)^2-\cdots}_{=0}\right)\ln\Lambda^2$$

Our Goal

Explore the scalar sector in model-independent way

We impose the two conditions on the extended scalar sector

I. tree level unitarity

- singlet extension w custodial sym.

unitarity sum rules

$$1-(\kappa_V^h)^2-\sum_{n=2}^{N}(\kappa_V^n)^2=0$$

Exactly same !

↔

finiteness conditions

$$1-(\kappa_V^h)^2-\sum_{n=2}^{N}(\kappa_V^n)^2=0$$

- singlet extension w/ custodial sym.

unitarity sum rules ⇒ finiteness conditions

R Nagai, M Tanabashi, K Tsumura Phys. Rev. D 91, 034030 (2015)

S, T, U

= 0

Our Goal

Explore the scalar sector in model-independent way

We impose the two conditions on the extended scalar sector

tree level unitarity

Goal

unitarity sum rules ⇒ finiteness conditions

in arbitrary scalar sector ?

S, T, U

= 0

- Introduction

- unitarity vs oblique corrections

- T parameter

- Summary

- Introduction

- unitarity vs oblique corrections

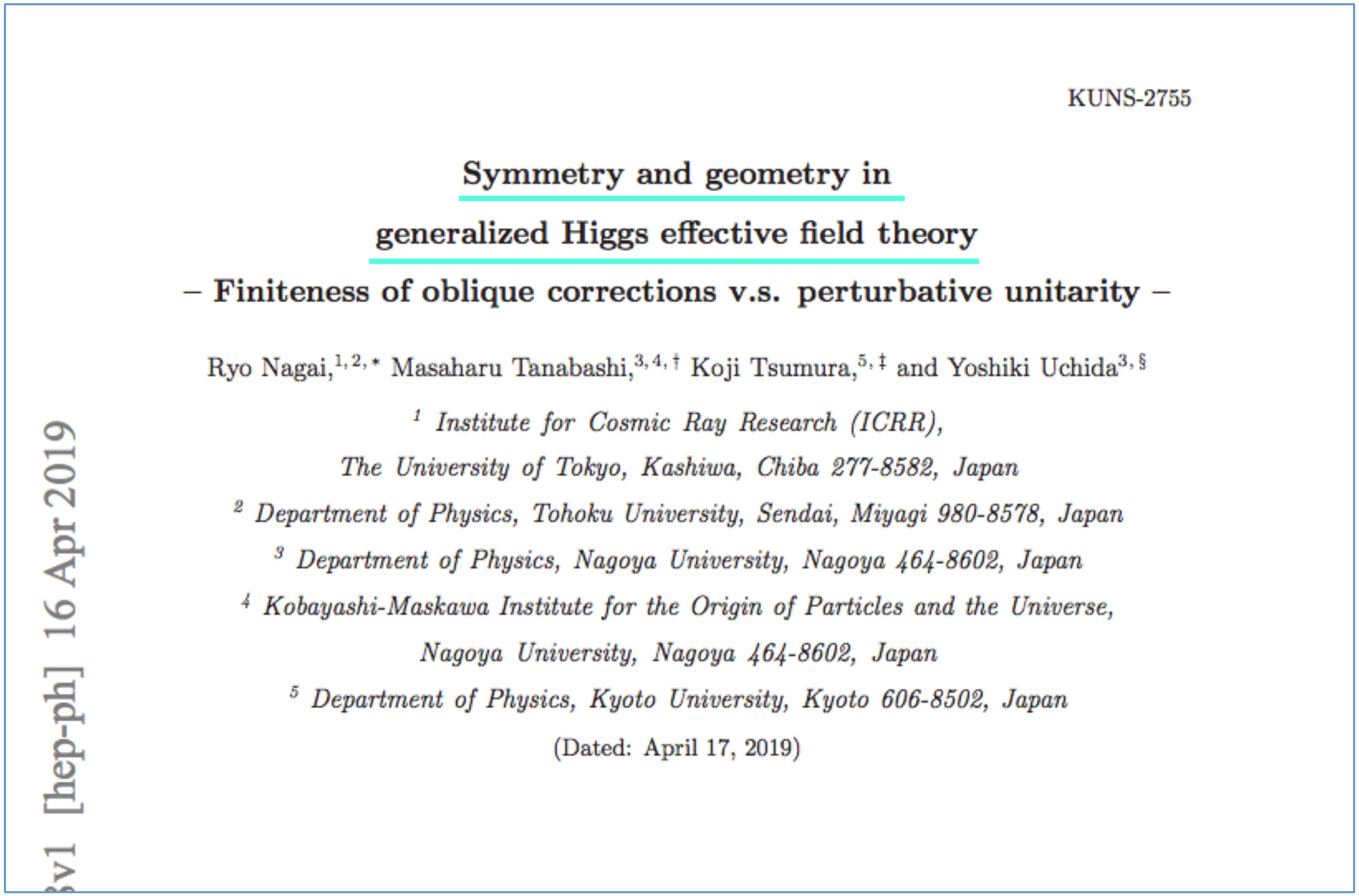
- T parameter

- Summary

unitarity sum rules ⇒ finiteness conditions

in arbitrary scalar sector ?

Approach based on Symmetry and Geometry



KUNS-2755

Symmetry and geometry in generalized Higgs effective field theory

– Finiteness of oblique corrections v.s. perturbative unitarity –

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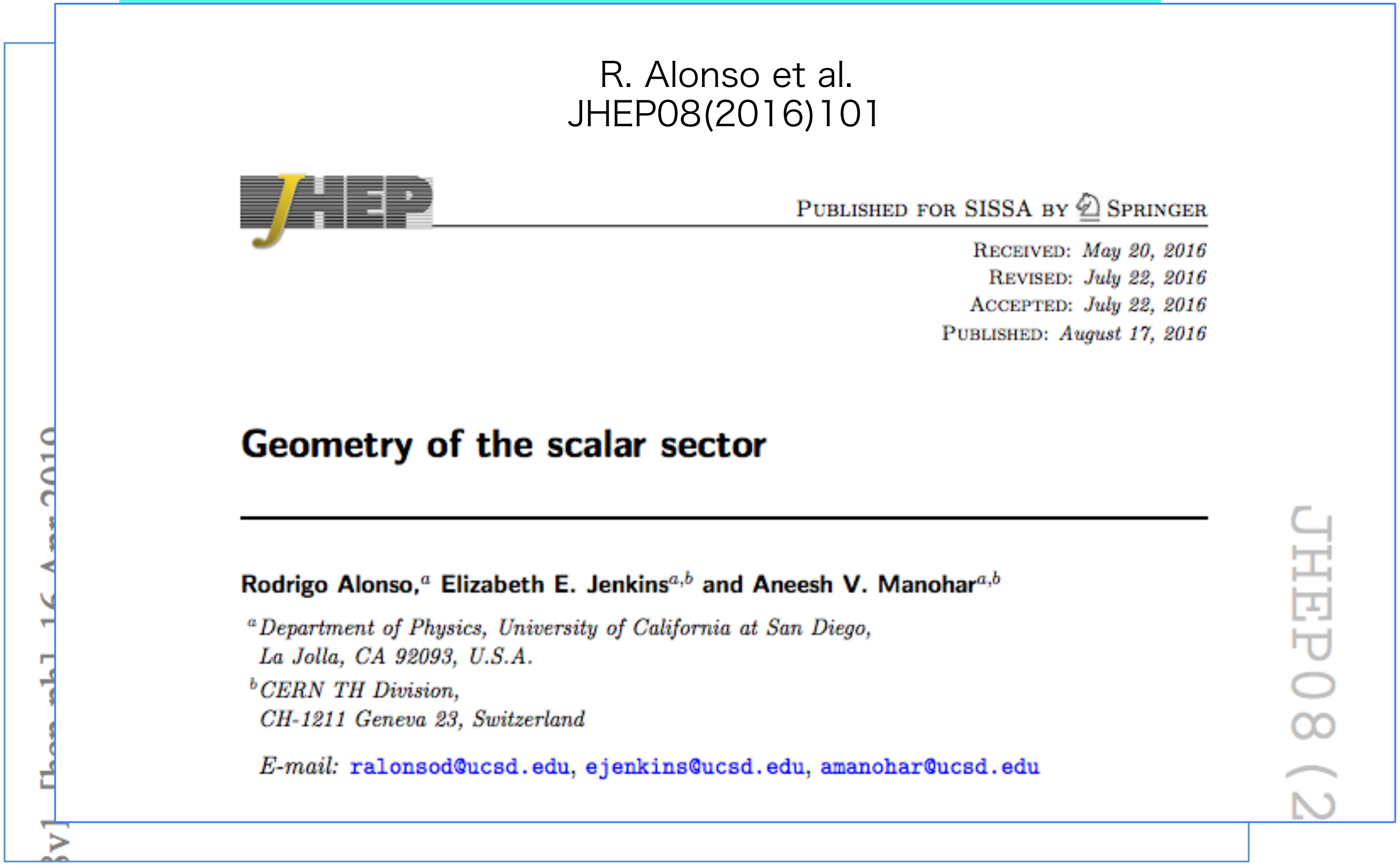
(Dated: April 17, 2019)

unitarity sum rules ⇒ finiteness conditions

in arbitrary scalar sector ?

Approach based on Symmetry and Geometry



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JHEP08(2016)101

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Geometry of the scalar sector

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Effective field theory approach

SM + Singlet scalar

$$\mathcal{L}_{\mathrm{SM+S}} = \frac{v^2}{4}\mathrm{tr}\left[(D_\mu U)^\dagger D^\mu U\right]\left(1 + 2c\frac{h}{v} + 2s\frac{H}{v} + c^2\frac{h^2}{v^2} + s^2\frac{H^2}{v^2} + 2cs\frac{hH}{v^2}\right) + \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu H \partial^\mu H$$

Integrate out H

$$\mathcal{L}_{\mathrm{SM+S}} = \frac{v^2}{4}\mathrm{tr}\left[(D_\mu U)^\dagger D^\mu U\right]\left(1 + \kappa_1\frac{h}{v} + \kappa_2\frac{h^2}{v^2} + \cdots\right) + \frac{1}{2}\partial_\mu h \partial^\mu h$$

New Physics is encoded in the κ_1 , κ_2 $\cdots$

Approach based on Symmetry and Geometry

SM + Singlet scalar

$$\mathcal{L}_{\mathrm{SM+S}} = \frac{v^2}{4}\mathrm{tr}\left[(D_\mu U)^\dagger D^\mu U\right]\left(1 + 2c\frac{h}{v} + 2s\frac{H}{v} + c^2\frac{h^2}{v^2} + s^2\frac{H^2}{v^2} + 2cs\frac{hH}{v^2}\right)$$
$$+ \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu H \partial^\mu H$$

$\Downarrow$ Integrate out H

$$\mathcal{L}_{\mathrm{SM+S}} = \frac{v^2}{4}\mathrm{tr}\left[(D_\mu U)^\dagger D^\mu U\right]\left(1 + \kappa_1\frac{h}{v} + \kappa_2\frac{h^2}{v^2} + \cdots\right) + \frac{1}{2}\partial_\mu h \partial^\mu h$$

Approach based on Symmetry and Geometry

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$$U := e^{i\frac{\pi^a}{v}\tau_a} \qquad\qquad + \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{1}{2}\partial_\mu H\partial^\mu H$$

$$= \frac{1}{2}\partial_\mu(\pi^a \;\; h \;\; H)\begin{pmatrix} F(h,H)(\delta_{ab}+\mathcal{O}(\pi^2)) & & \\ & 1 & \\ & & 1 \end{pmatrix}\partial^\mu\begin{pmatrix}\pi^b \\ h \\ H\end{pmatrix}$$

$$= \frac{1}{2}g_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j \qquad\qquad \phi^i = \pi^a, h, H$$

New Physics is encoded in $g_{ij}(\phi)$ and $V(\phi)$

Approach based on Symmetry and Geometry

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2} g_{ij}(\phi)(D_\mu \phi)^i (D^\mu \phi)^j - V(\phi)$$

Geometry

$$R_{ijkl}(\phi)$$

(Riemann tensor)

Symmetry

$$w^i_a(\phi)\,,\quad y^i(\phi)$$

(Killing vector)

Approach based on Symmetry and Geometry

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$$w_a^i(\phi) \, , \quad y^i(\phi)$$

SU(2)L & U(1)Y Killing vectors,
respectively

(Killing vector)

Approach based on Symmetry and Geometry

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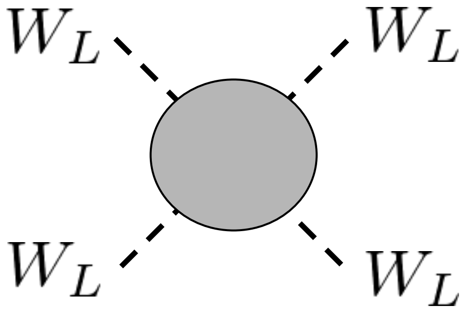
⟷



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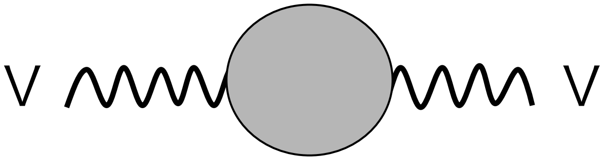

S, T, U

Approach based on Symmetry and Geometry

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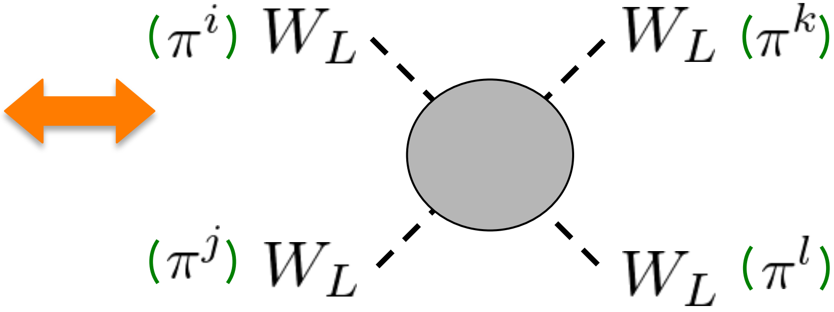

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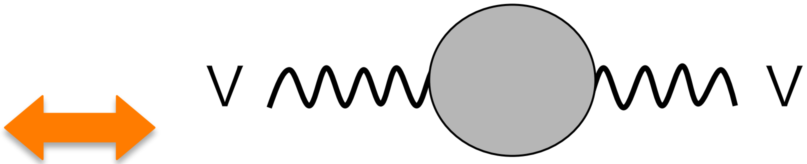

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Approach based on Symmetry and Geometry

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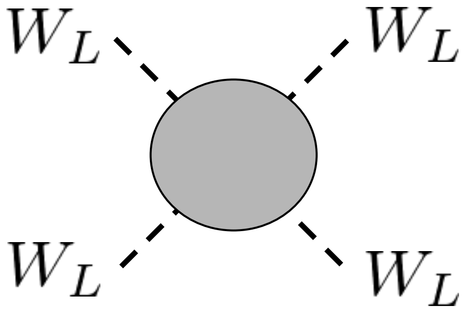

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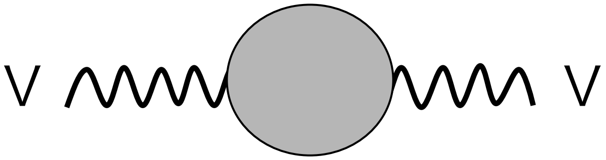

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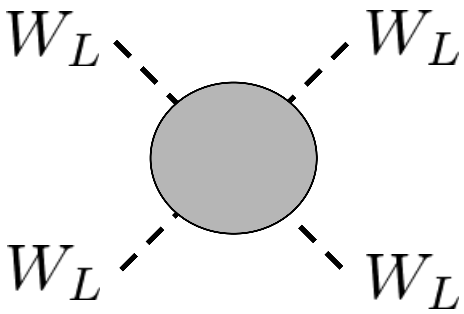

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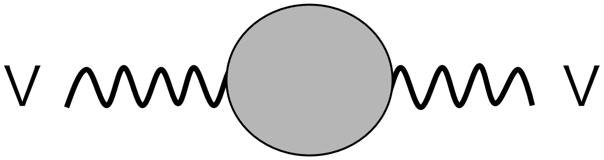

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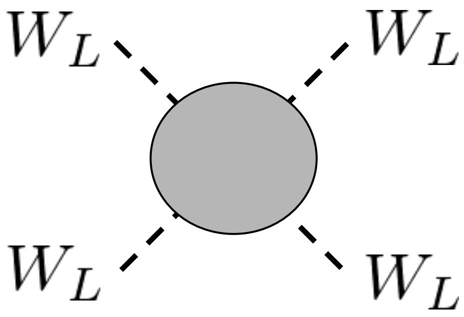


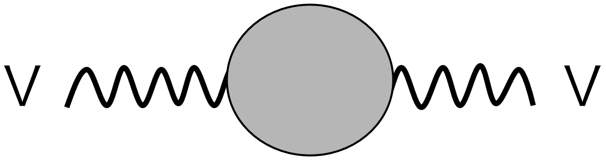
Killing eq.

$$v_{i;j;k} = R^l{}_{kji} v_l$$

Symmetry

$$w^i_a(\phi)\,,\ \ y^i(\phi)$$

(Killing vector)



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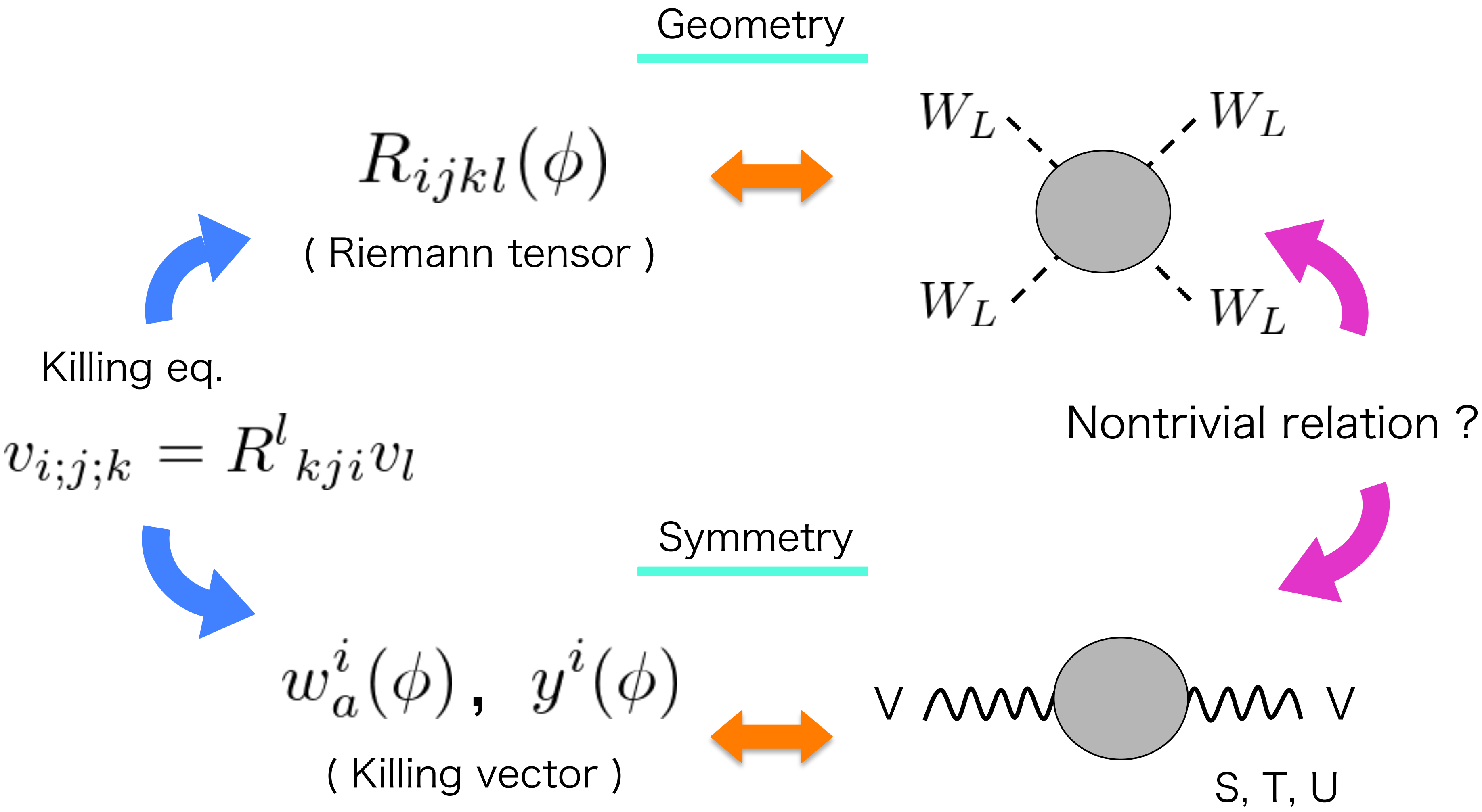

Geometry

$$R_{ijkl}(\phi)$$

(Riemann tensor)

Killing eq.

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Nontrivial relation ?

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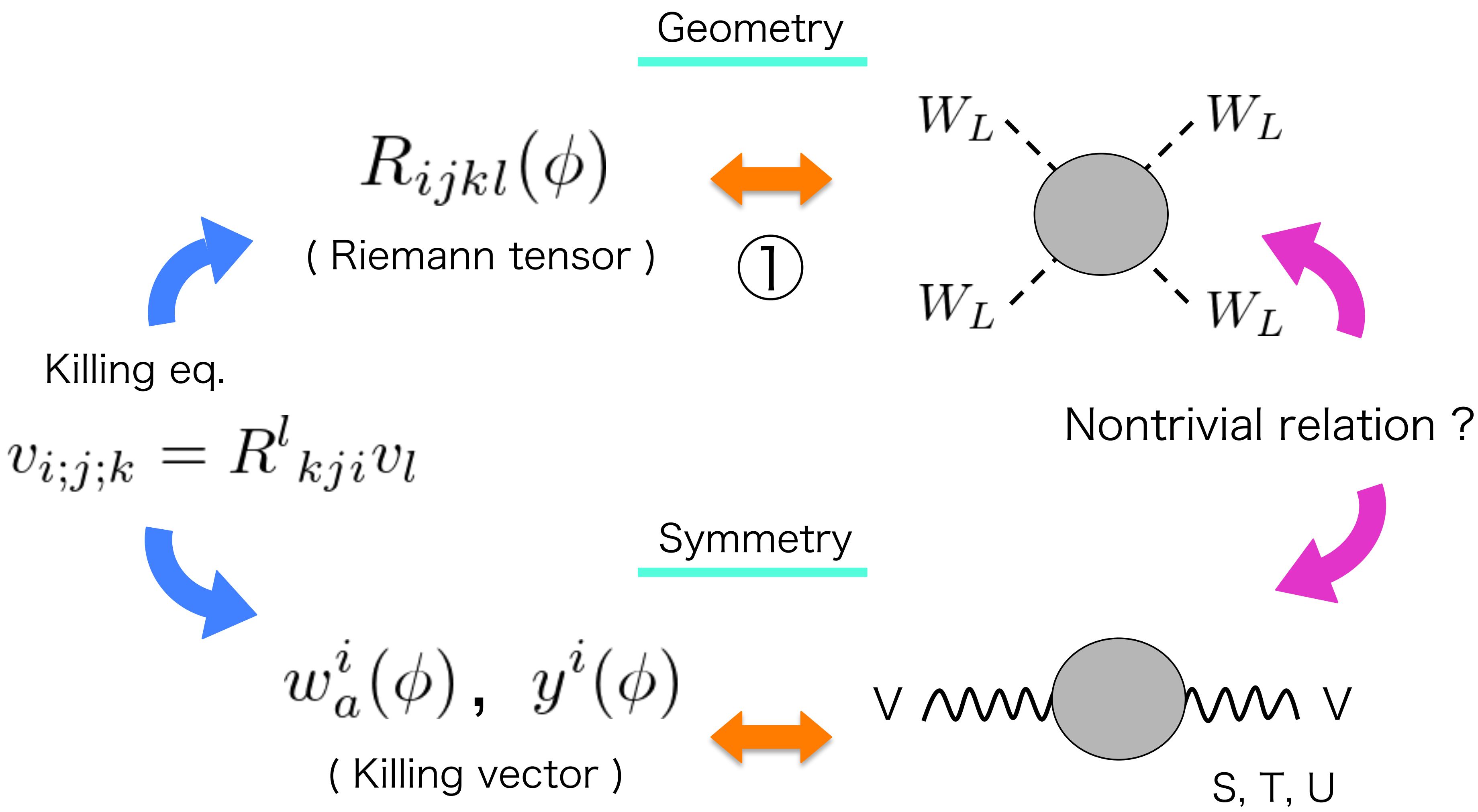

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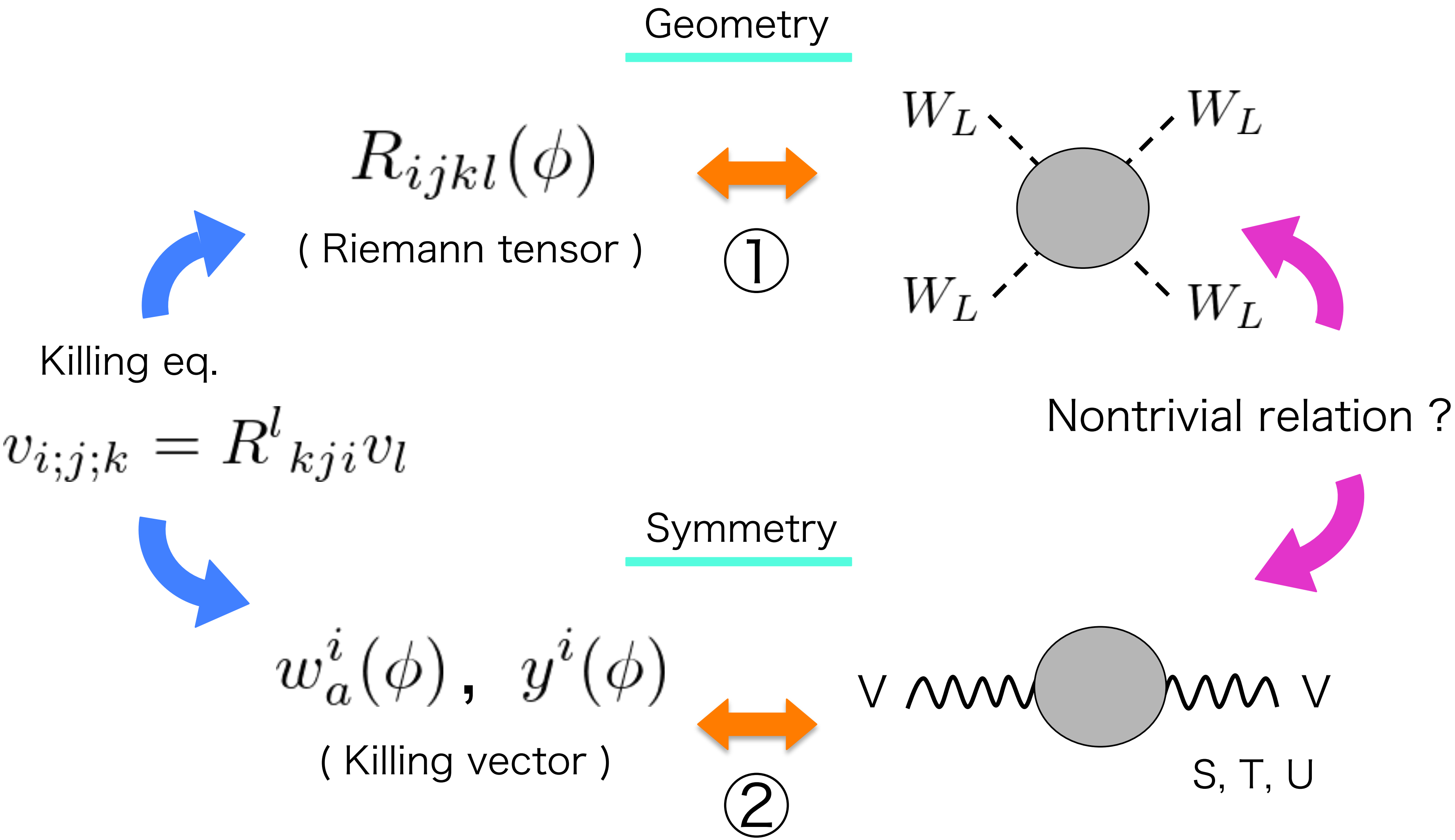

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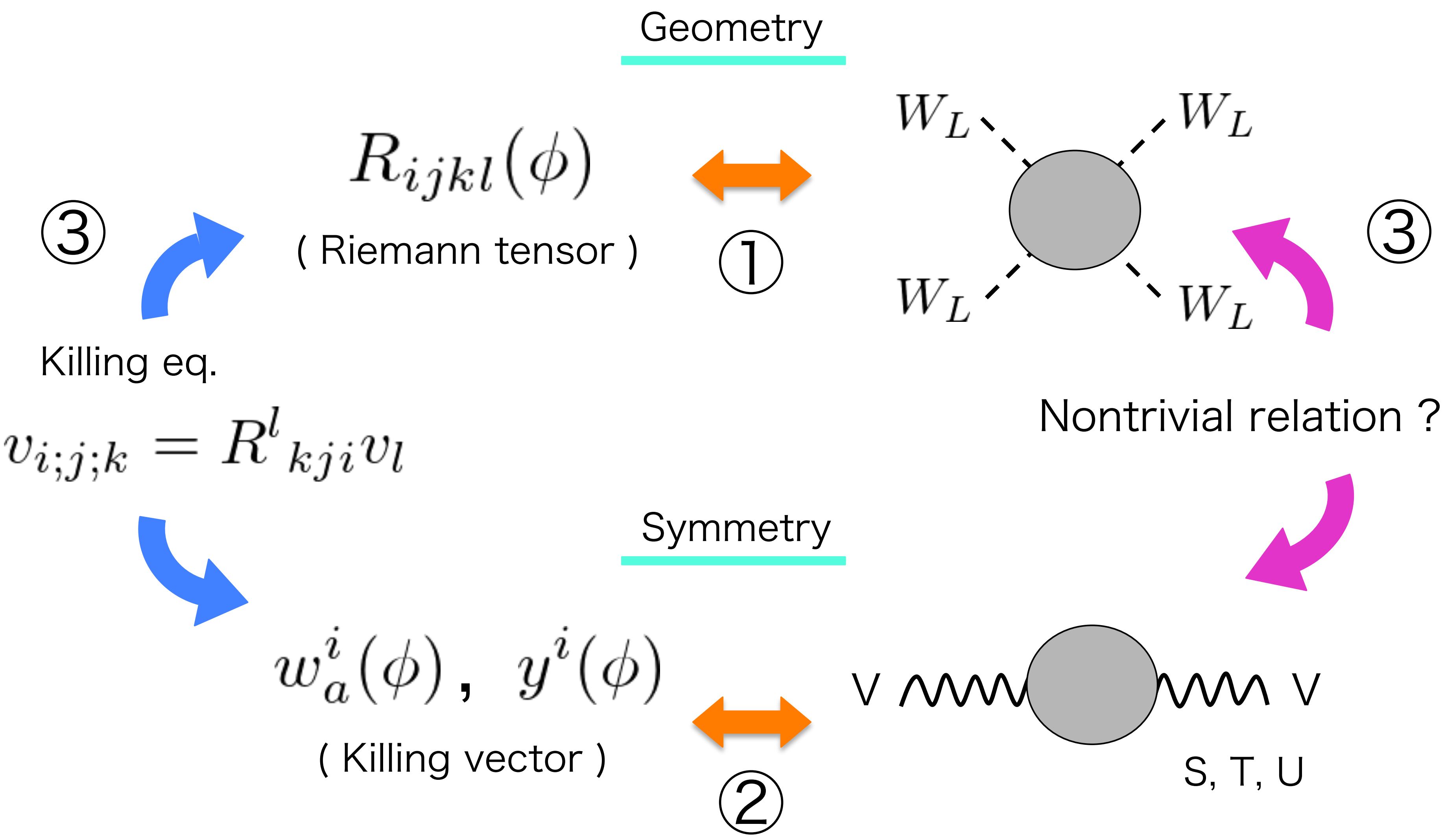
②

S, T, U

Approach based on Symmetry and Geometry



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③

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S, T, U

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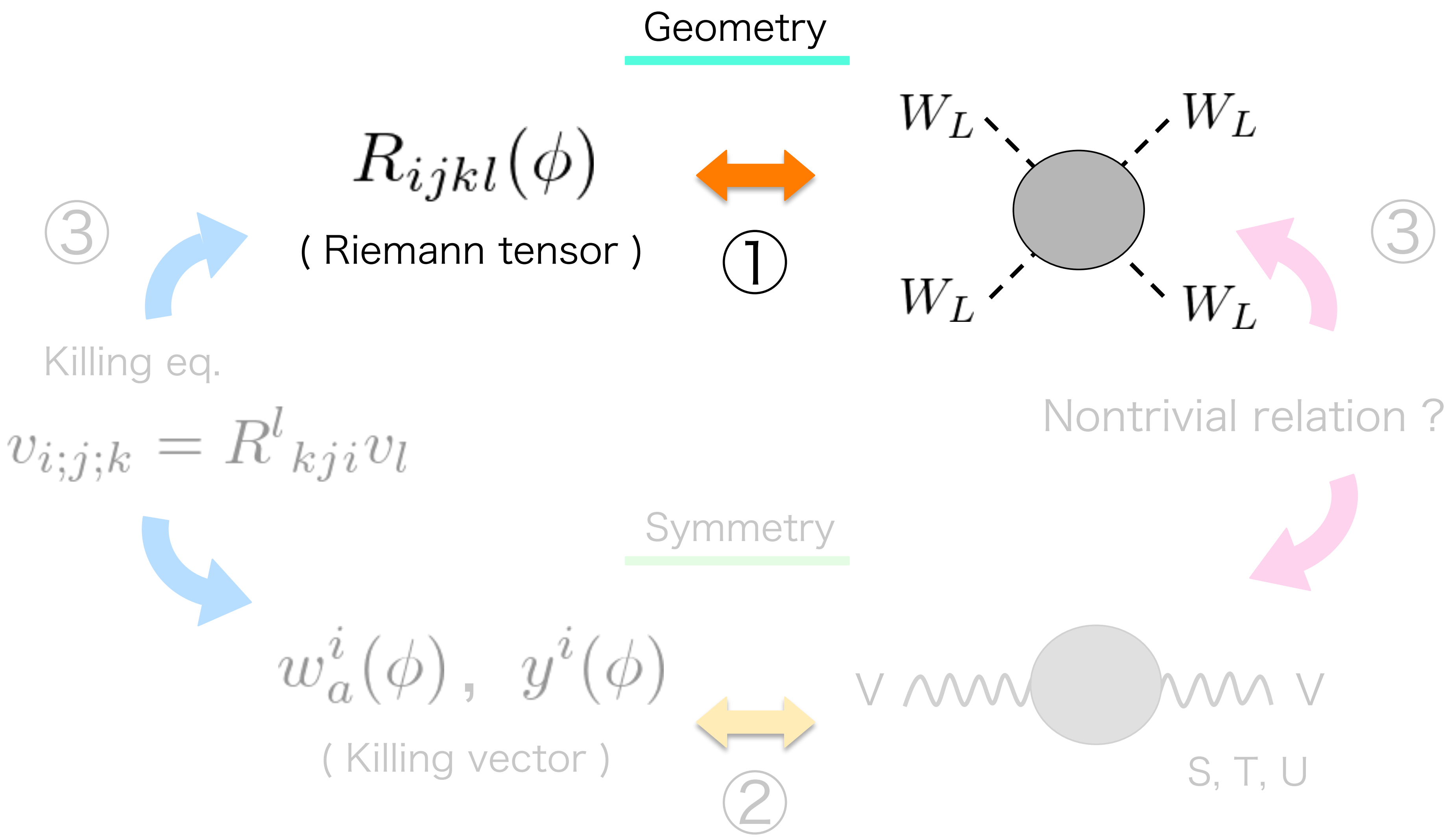

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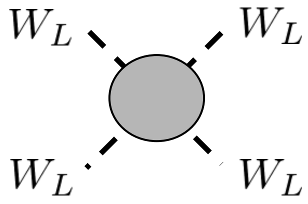
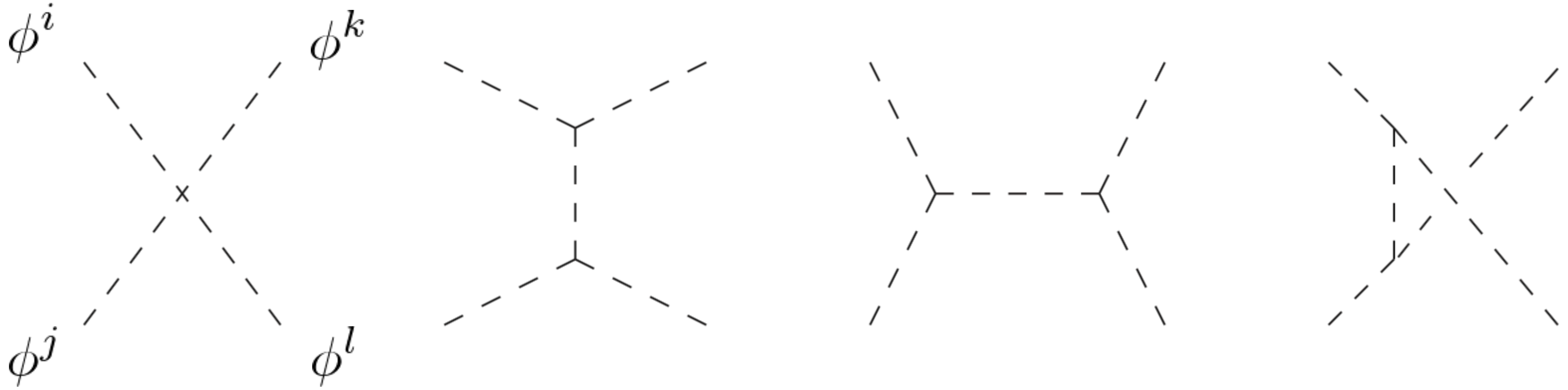
S, T, U

① $R_{ijkl}(\phi)$ ↔

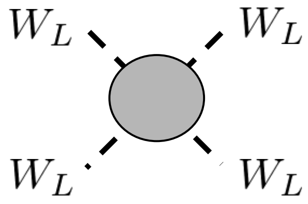


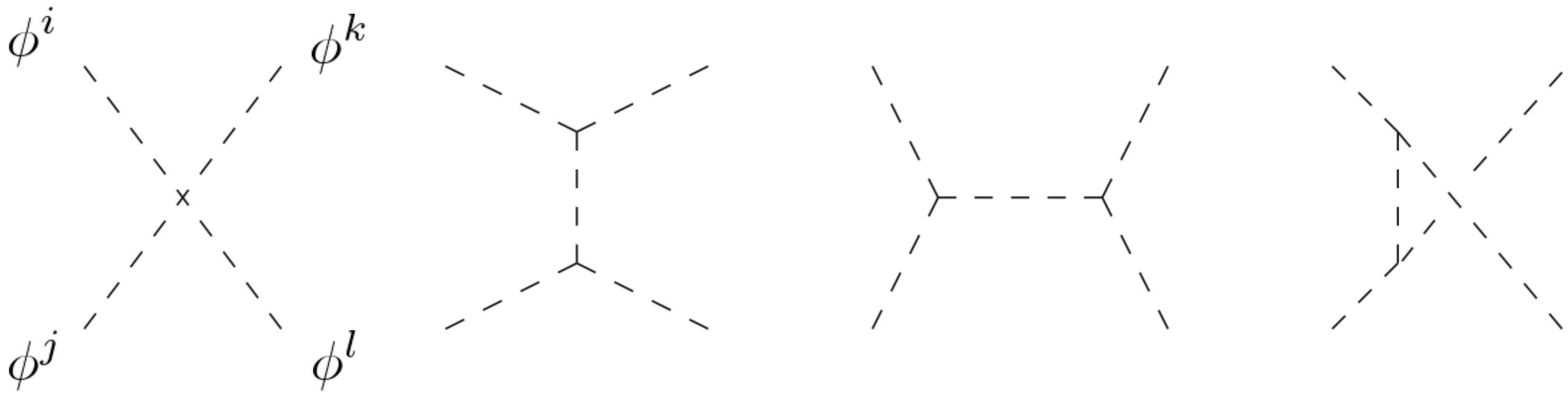


$$\mathcal{M}_{\phi_i\phi_j\to\phi_k\phi_l} \sim \frac{s}{3}(\bar{R}_{iklj}+\bar{R}_{ilkj})+\frac{t}{3}(\bar{R}_{ijlk}+\bar{R}_{iljk})+\frac{u}{3}(\bar{R}_{ijkl}+\bar{R}_{ikjl})$$

4-point perturbative unitarity condition : $\bar{R}_{ijkl}=0$

① $R_{ijkl}(\phi)$ ↔





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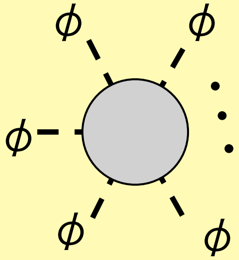
4-point perturbative unitarity condition : $\bar{R}_{ijkl}=0$

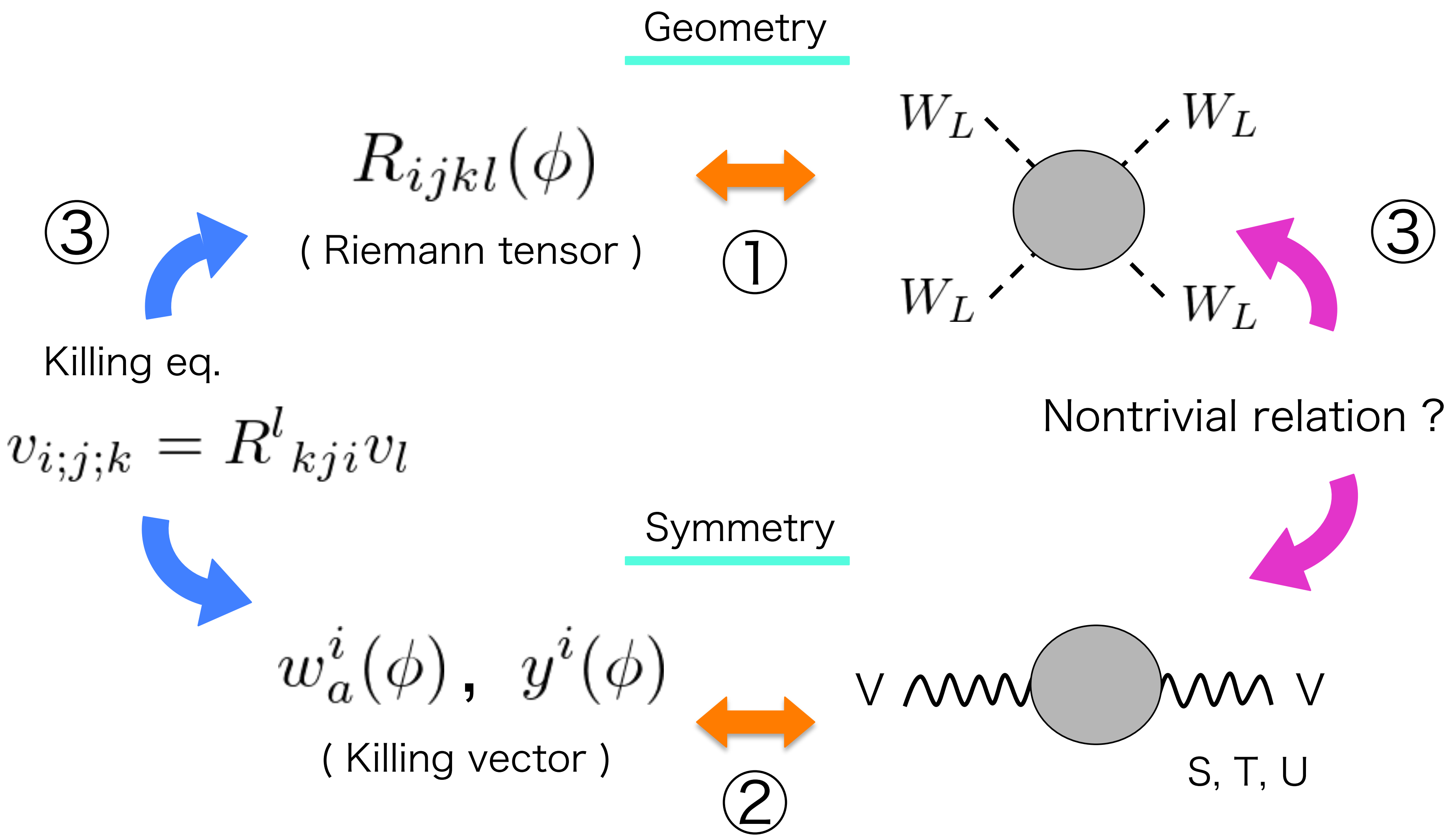


N-point

's unitarity is ensured by

$$\left\{\begin{array}{l} \bar{R}_{i_1i_2i_3i_4}=0 \\ \vdots \\ \bar{R}_{i_1i_2i_3i_4;i_5\cdots i_N}=0 \end{array}\right.$$

Approach based on Symmetry and Geometry



Geometry

$R_{ijkl}(\phi)$

(Riemann tensor)

①

③

Killing eq.

$$v_{i;j;k} = R^{l}{}_{kji} v_l$$

③

Nontrivial relation ?

Symmetry

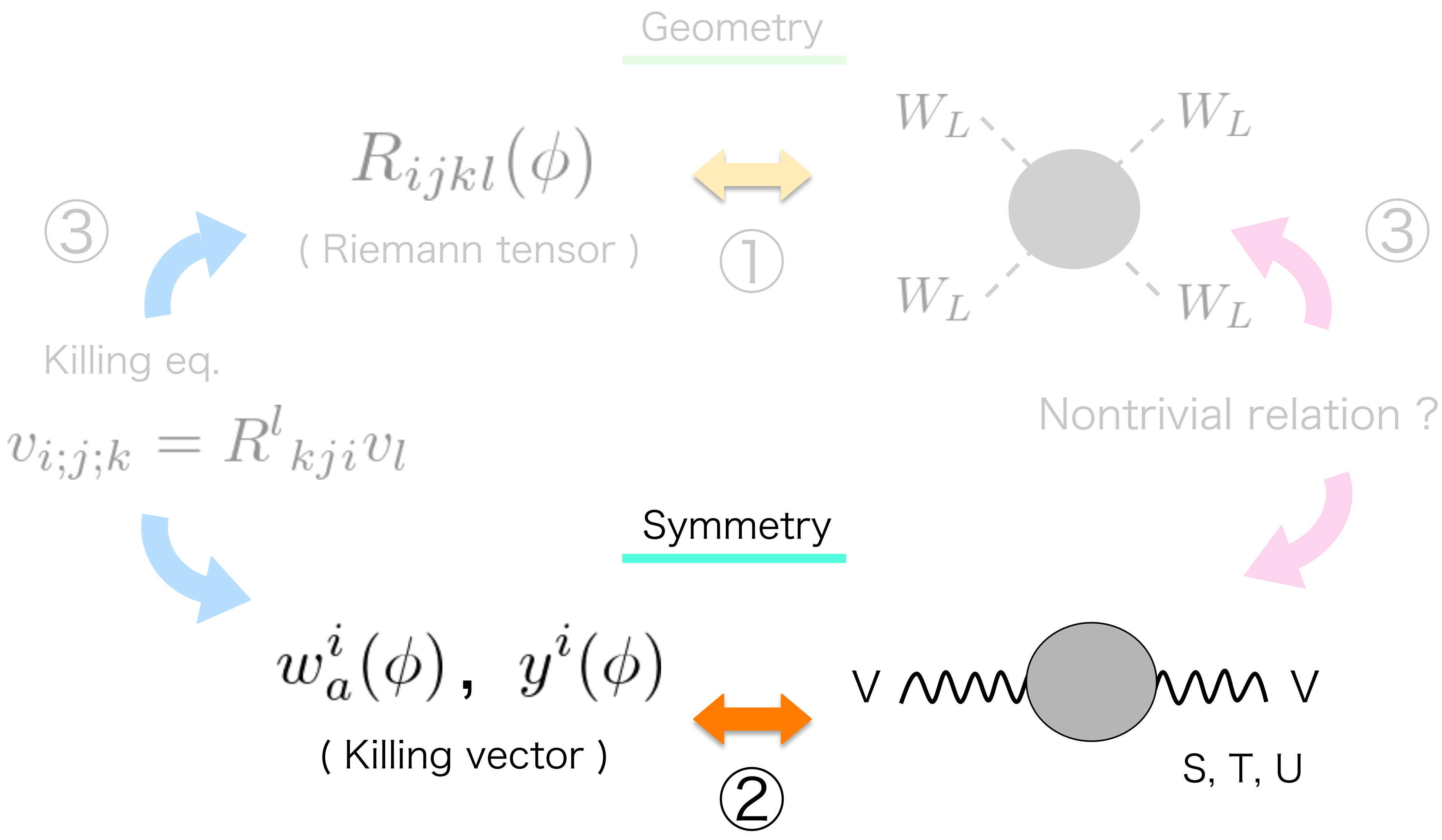
$w^i_a(\phi)\,,\ \ y^i(\phi)$

(Killing vector)

②

S, T, U

Approach based on Symmetry and Geometry



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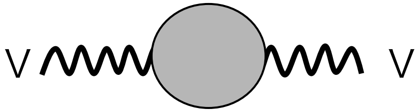
② $w_a^i(\phi)$, $y^i(\phi)$ ↔

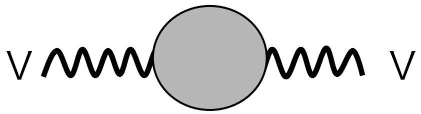


$\mathcal{L}_{\text{scalar}}$ is invariant under

SU(2)L $$\phi'^i = \phi^i + \theta^a w_a^i(\phi)$$ (a = 1,2,3)

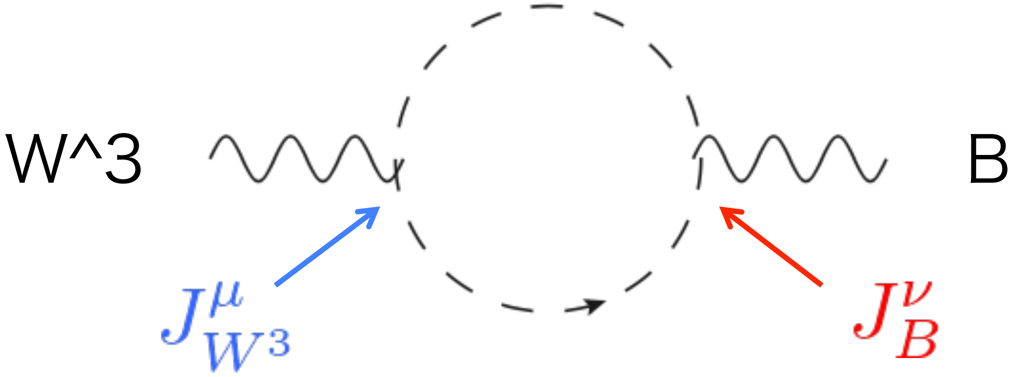
U(1)Y $$\phi'^i = \phi^i + \theta \, y^i(\phi)$$

② $w^i_a(\phi)$, $y^i(\phi)$ ↔



Killing vector $w^i_a(\phi)$, $y^i(\phi)$ $\sim$ conserved current

$$S \propto \frac{d}{dq^2}\int d^4x \left(e^{-iqx} J^{\mu}_{W^3}\, J^{\nu}_{B}\right)\Big|_{g^{\mu\nu}} \propto (\bar{w}^i_a)_{;j}(\bar{y}^j)_{;i}$$



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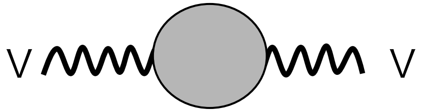

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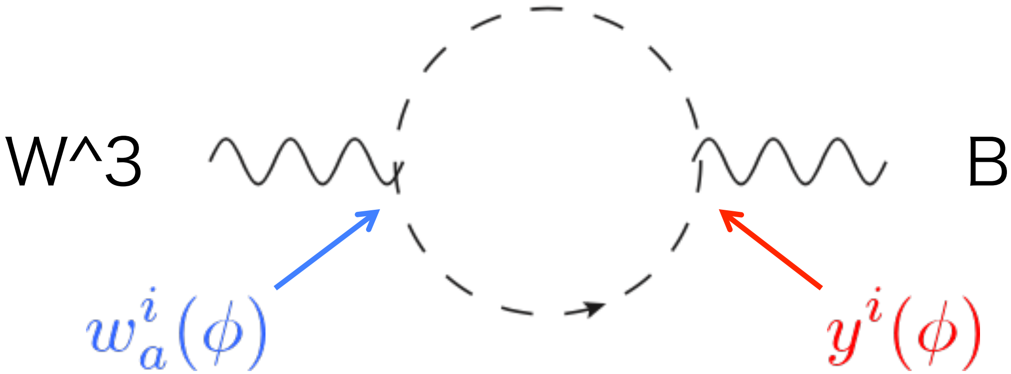

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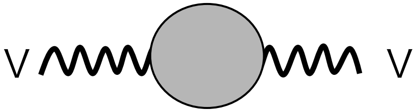

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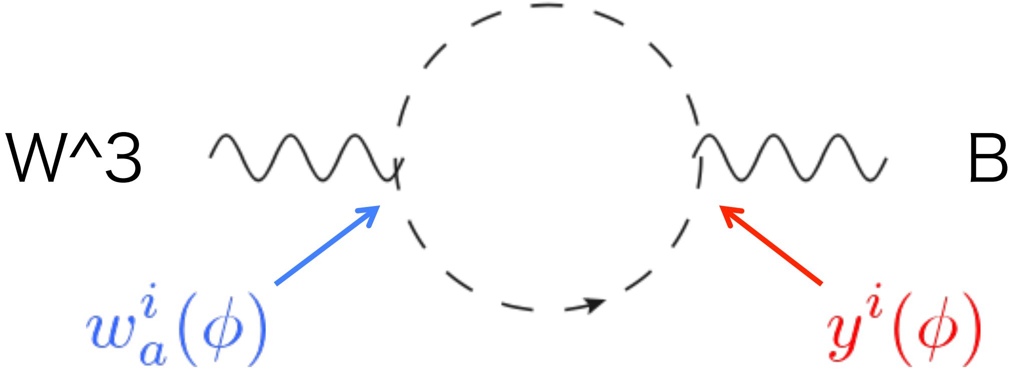
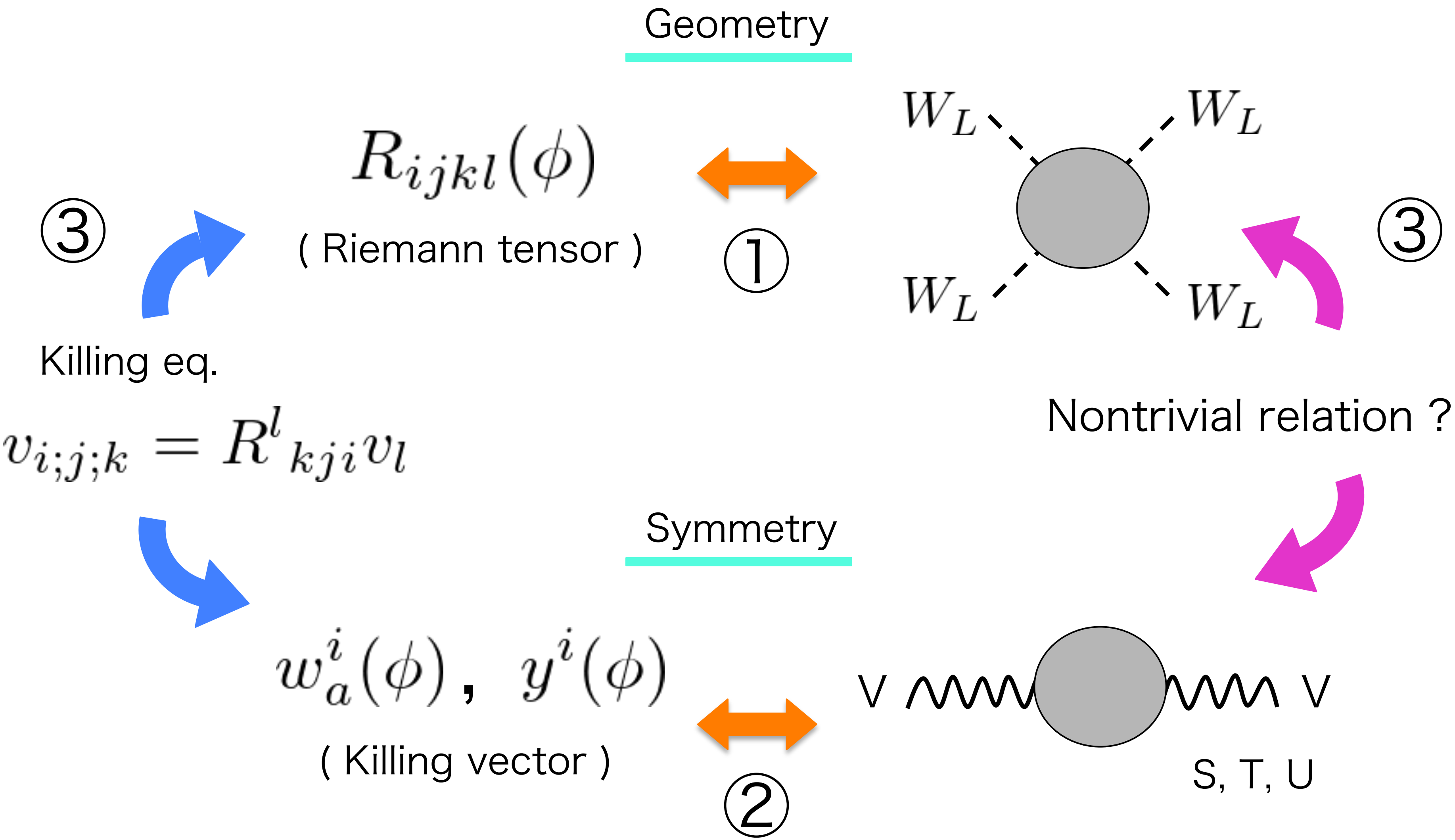


$$\boxed{S_{\rm div} = -\frac{1}{12\pi}(\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i}\ln\frac{\Lambda^2}{\mu^2}}$$

Approach based on Symmetry and Geometry



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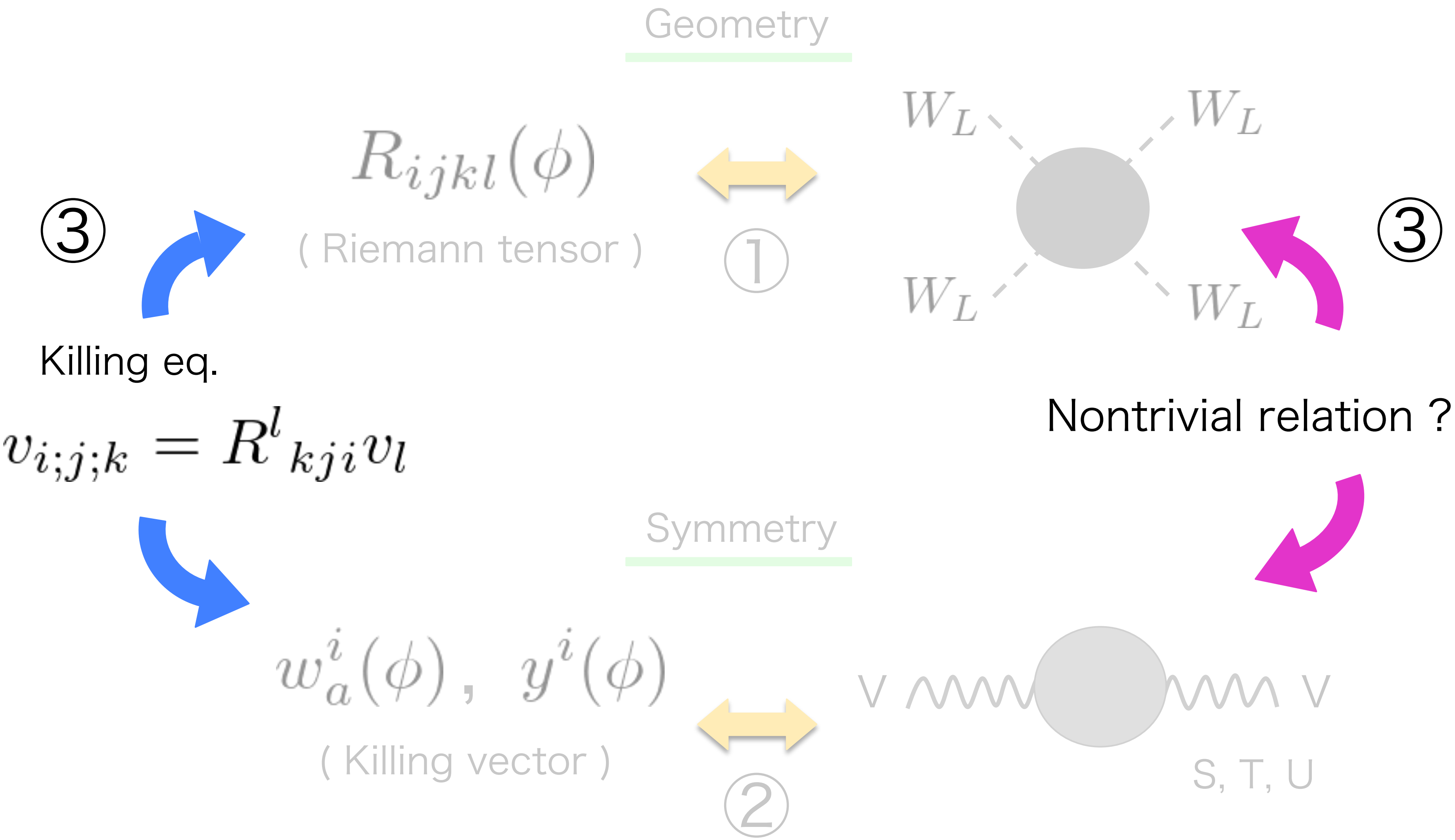

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S, T, U

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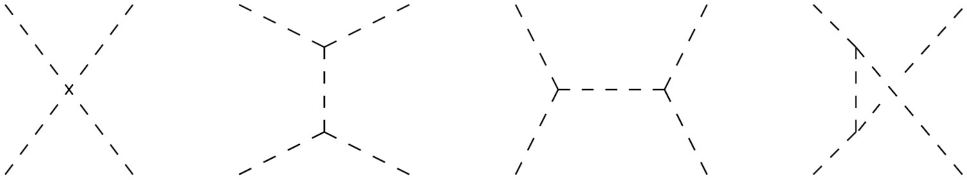
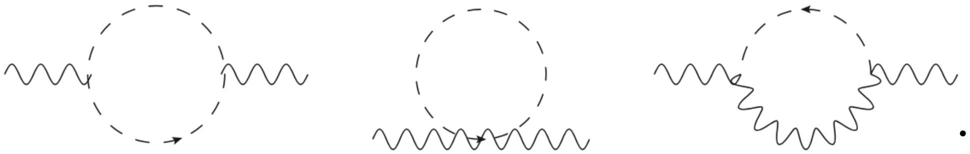
Geometry



pertubative unitary if

$$\bar{R}_{ijkl} = 0$$

Symmetry



...

S is 1-loop finite if

$$(\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i} = 0$$

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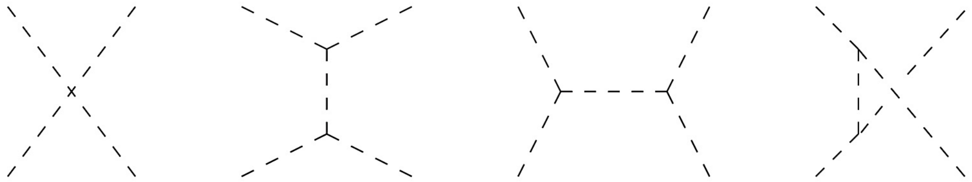
Geometry



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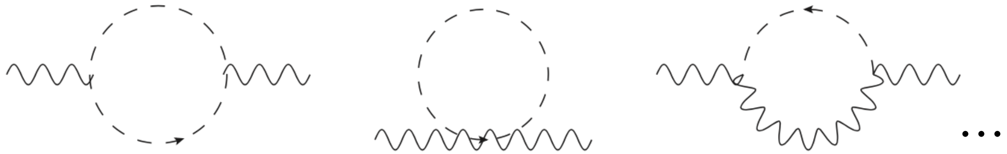

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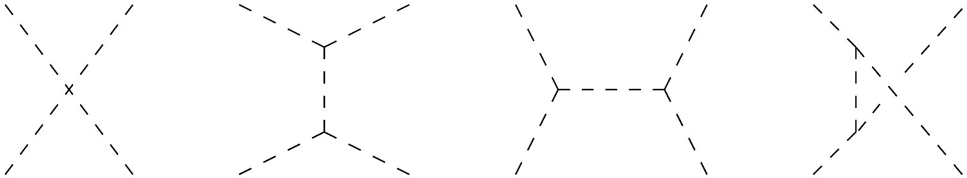
$$R^{l}{}_{kji} v_l = v_{i;j;k}$$

Perturbative unitarity ensures the finiteness of S ?

③ $$v_{i;j;k} = R^{l}{}_{kji} v_l$$

Geometry



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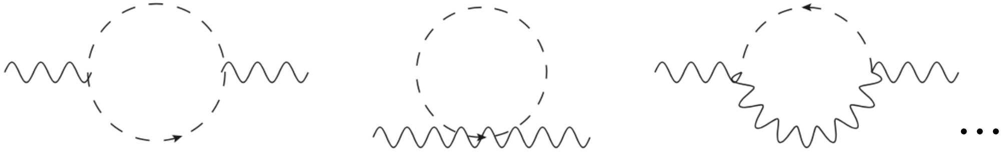

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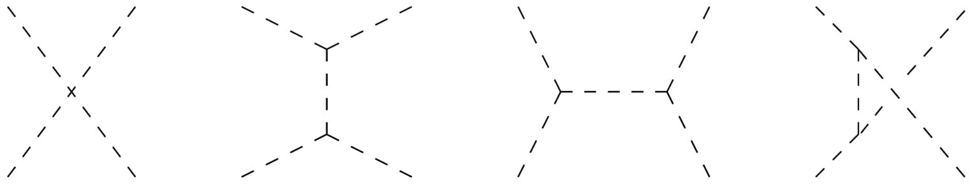
$$R^{l}{}_{kji} v_l = v_{i;j;k}$$

$$+$$

$$[w_a\,, w_b] = \varepsilon_{abc} w_c\ , \quad [w_a\,, y] = 0$$

③ $$v_{i;j;k} = R^l{}_{kji} v_l$$

Geometry



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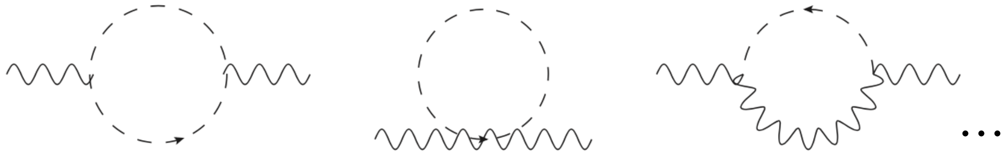


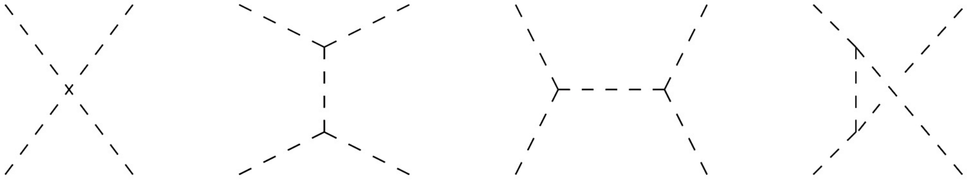
S is 1-loop finite if

$$(\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i} = 0$$

$$(\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i} \propto \epsilon_{3bc}(\bar{w}_c^k)(\bar{w}_3^l)\bar{R}^i{}_{jkl}(\bar{w}_b^j)_{;i} + \epsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}$$

③ $v_{i;j;k} = R^l{}_{kji} v_l$

Geometry



pertubative unitary if

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Symmetry

S is 1-loop finite if

$$(\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i} = 0$$

If perturbarive unitarity is ensured,

$$(\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i} \propto \epsilon_{3bc}(\bar{w}_c^k)(\bar{w}_3^l)\overset{=0}{\boxed{\bar{R}^i{}_{jkl}}}(\bar{w}_b^j)_{;i} + \epsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\overset{=0}{\boxed{\bar{R}^i{}_{jkl}}}(\bar{y}^j)_{;i}$$

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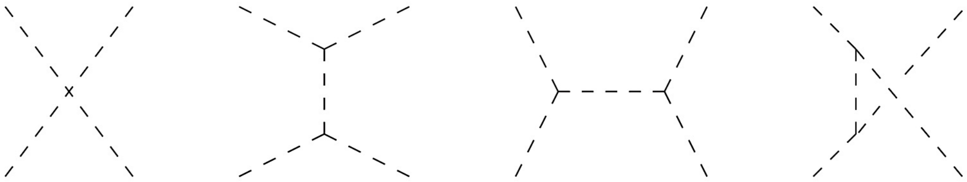
Geometry



pertubative unitary if

$$\bar{R}_{ijkl} = 0$$

Symmetry

S is 1-loop finite if

$$\boxed{(\bar{w}_3^i)_{;j} (\bar{y}^j)_{;i} = 0}$$

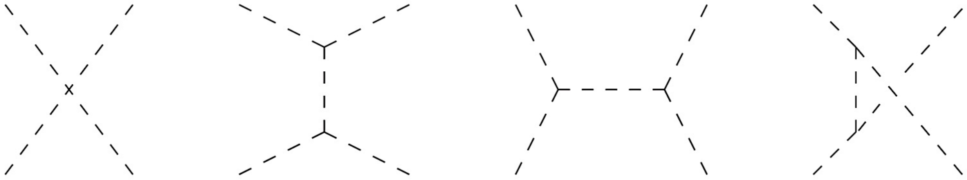
If perturbarive unitarity is ensured,

$$\underset{=0}{\boxed{(\bar{w}_3^i)_{;j} (\bar{y}^j)_{;i}}} \propto \epsilon_{3bc} (\bar{w}_c^k)(\bar{w}_3^l) \bar{R}^i{}_{jkl} (\bar{w}_b^j)_{;i} + \epsilon_{3bc} (\bar{w}_b^k)(\bar{w}_c^l) \bar{R}^i{}_{jkl} (\bar{y}^j)_{;i}$$

Then 1-loop finiteness of S is guaranteed

③ $v_{i;j;k} = R^{l}{}_{kji} v_{l}$

Geometry



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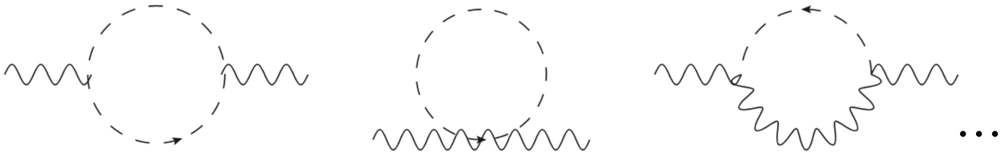

U is 1-loop finite if

$$(\bar{w}_1^i)_{;j}(\bar{w}_1^j)_{;i} - (\bar{w}_3^i)_{;j}(\bar{w}_3^j)_{;i} = 0$$

If perturbarive unitarity is ensured,

$$(\bar{w}_1^i)_{;j}(\bar{w}_1^j)_{;i} - (\bar{w}_3^i)_{;j}(\bar{w}_3^j)_{;i}$$

$$\propto \epsilon_{1bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^{i}{}_{jkl}(\bar{w}_1^j)_{;i} - \epsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^{i}{}_{jkl}(\bar{w}_3^j)_{;i}$$

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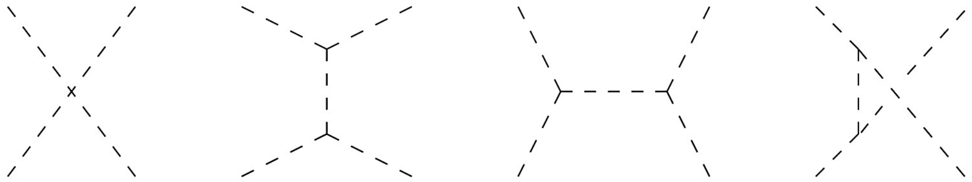

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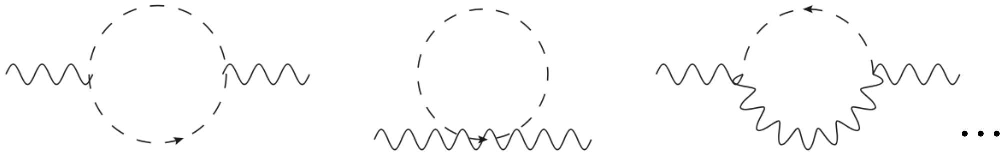
Symmetry



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If perturbarive unitarity is ensured,

$$(\bar{w}_1^i)_{;j}(\bar{w}_1^j)_{;i} - (\bar{w}_3^i)_{;j}(\bar{w}_3^j)_{;i}$$

$$\propto \epsilon_{1bc}(\bar{w}_b^k)(\bar{w}_c^l)\overset{=0}{\boxed{\bar{R}^i{}_{jkl}}}(\bar{w}_1^j)_{;i} - \epsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\overset{=0}{\boxed{\bar{R}^i{}_{jkl}}}(\bar{w}_3^j)_{;i}$$

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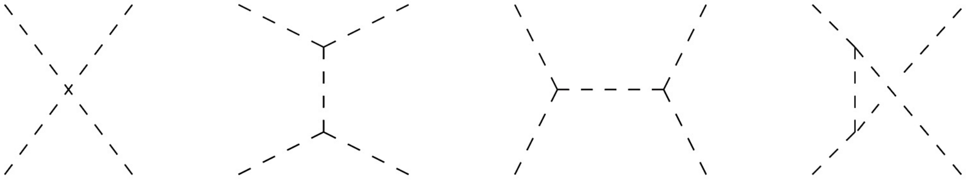
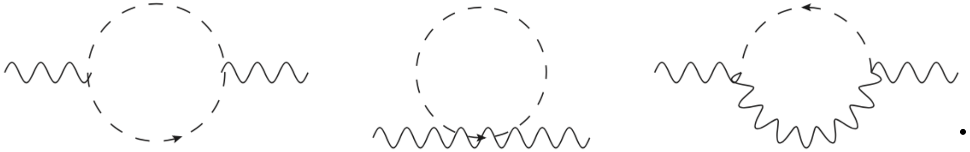


pertubative unitary if

$$\bar{R}_{ijkl} = 0$$

Symmetry



…

U is 1-loop finite if

$$\boxed{(\bar{w}_1^i)_{;j}(\bar{w}_1^j)_{;i} - (\bar{w}_3^i)_{;j}(\bar{w}_3^j)_{;i} = 0}$$

If perturbarive unitarity is ensured,

$$\boxed{(\bar{w}_1^i)_{;j}(\bar{w}_1^j)_{;i} - (\bar{w}_3^i)_{;j}(\bar{w}_3^j)_{;i}}$$

$= 0$

$$\propto \epsilon_{1bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{w}_1^j)_{;i} - \epsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{w}_3^j)_{;i}$$

Then 1-loop finiteness of U is guaranteed

- Introduction
- unitarity vs oblique corrections
- T parameter
- Summary

T parameter

$$\begin{aligned} T_{\rm div} \;\sim\; & \Big[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)\Big] \\ & \times \Big\{ \bar{R}_{ikjl}[\bar{g}^{km}\bar{g}^{ln}\bar{V}_{;mn} + \bar{w}_a^k\bar{w}_a^l + \bar{y}^k\bar{y}^l] \\ & \qquad - 4g_W^2(\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl} - 4g_Y^2(\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl}\Big\} \ln\frac{\Lambda^2}{\mu^2} \end{aligned}$$

T parameter

$$
\begin{aligned}
T_{\rm div} \;\sim\;& \Big[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)\Big] \\
&\times \Big\{ \bar{R}_{ikjl}[\bar{g}^{km}\bar{g}^{ln}\bar{V}_{;mn} + \bar{w}_a^k\bar{w}_a^l + \bar{y}^k\bar{y}^l] \\
&\qquad - 4g_W^2(\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl} - 4g_Y^2(\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl}\Big\}\ln\frac{\Lambda^2}{\mu^2}
\end{aligned}
$$

T parameter

$$T_{\text{div}} \sim \left[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)\right]$$

$$\times \Big\{ \bar{R}_{ikjl}[\bar{g}^{km}\bar{g}^{ln}\bar{V}_{;mn} + \bar{w}_a^k \bar{w}_a^l + \bar{y}^k \bar{y}^l]$$

$$- 4g_W^2 (\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl} - 4g_Y^2 (\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl} \Big\} \ln \frac{\Lambda^2}{\mu^2}$$

$\Downarrow$ if $\bar{R}_{ikjl} = 0$

$$T_{\text{div}} \sim \left[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)\right]$$

$$\times \Big\{ - 4g_W^2 (\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl} - 4g_Y^2 (\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl} \Big\} \ln \frac{\Lambda^2}{\mu^2}$$

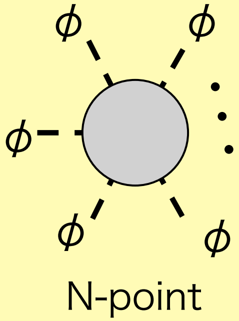
= 0 (SM)

= 0 (2HDM)

≠ 0 (Georgi Machacek Model)

Summary

New !!



N-point

's unitarity is ensured by

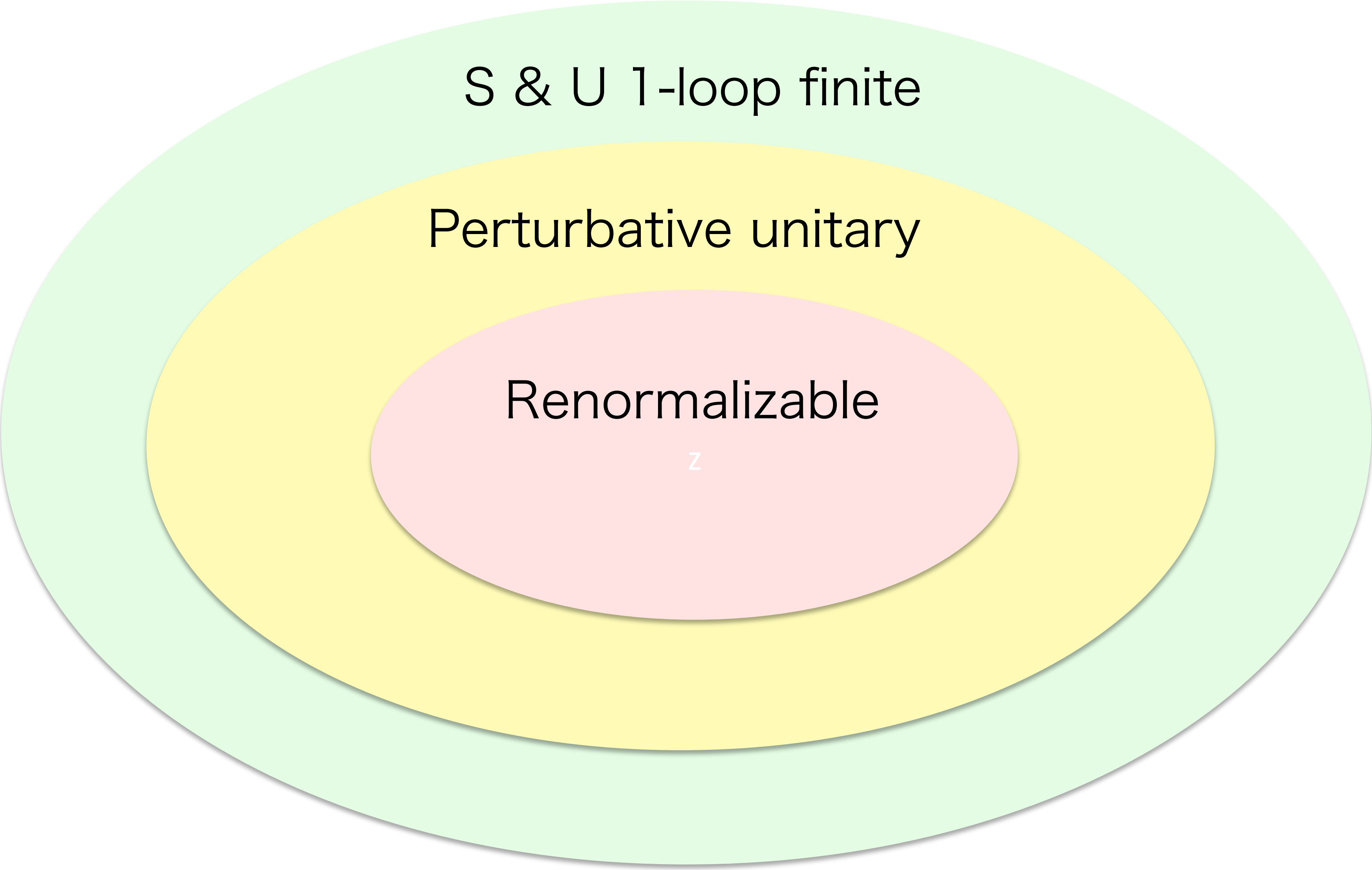
$$\left\{\begin{array}{l} \bar{R}_{i_1 i_2 i_3 i_4} = 0 \\ \vdots \\ \bar{R}_{i_1 i_2 i_3 i_4 ; i_5 \cdots i_N} = 0 \end{array}\right.$$

New !!

$$T_{\text{div}} \sim \left[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j) \right] \times \Big\{ \bar{R}_{ikjl} [\bar{g}^{km} \bar{g}^{ln} \bar{V}_{;mn} + \bar{w}_a^k \bar{w}_a^l + \bar{y}^k \bar{y}^l] - 4 g_W^2 (\bar{w}_a^k)_{;i} (\bar{w}_a^l)_{;j} \bar{g}_{kl} - 4 g_Y^2 (\bar{y}^k)_{;i} (\bar{y}^l)_{;j} \bar{g}_{kl} \Big\} \ln \frac{\Lambda^2}{\mu^2}$$

BACK UP

Oblique correction finiteness v.s. Unitarity v.s. Renoralizability



S & U 1-loop finite

Perturbative unitary

Renormalizable

z

We neglect the effects of $\mathcal{O}(p^4)$ operators

Ex.) aQGC violate tree level unitarity,

But we neglect the tree level effects

from the $\mathcal{O}(p^4)$ operators

SMEFT v.s. HEFT

SMEFT : $\mathcal{L}_{\text{scalar}}$ is written in terms of H

$$\mathcal{L}_{\text{scalar}} = (D_\mu H)^\dagger D^\mu H + \frac{C_1}{\Lambda^2}(H^\dagger D_\mu H)^*(H^\dagger D^\mu H) + \cdots$$

HEFT : $\mathcal{L}_{\text{scalar}}$ is written in terms of π^a (NGBs) and h

$$\mathcal{L}_{\text{scalar}} = \left(1 + \kappa_1 \frac{h}{v} + \kappa_2 \frac{h^2}{v^2} + \cdots\right) \frac{v^2}{4} \mathrm{tr}\left[(D_\mu U)^\dagger D^\mu U\right] - V(h)$$

$$U := e^{i\frac{\pi^a}{v}\tau_a}$$

- HEFT is more general than SMEFT
- New Physics effect is encoded in C_i or κ_i

We know that the Killing eq. connect the geometry to the symmetry

$$v_{i;j;k} = R^{l}{}_{kji} v_l$$

Using commutation relation of the Killing vectors

$$[w_a\,, w_b] = \varepsilon_{abc} w_c$$

$$[w_a\,, y] = 0$$

We get the following expression

$$S_{\rm div} = -\frac{1}{12\pi}\left(\epsilon_{3bc}(\bar{w}_c^k)(\bar{w}_3^l)\bar{R}^i{}_{jkl}(\bar{w}_b^j)_{;i} + \epsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

$$U_{\rm div} = \frac{1}{12\pi}\left(\epsilon_{1bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{w}_1^j)_{;i} - \epsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{w}_3^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

This expression is “ universal ” in any new physics model

Suppose that $\mathcal{L}_{\mathrm{scalar}}$ is invariant under the following transformation

$$\phi'^{i} = \phi^{i} + v^{i}(\phi)$$

$$0 = (v^{k})\, g_{ij,k} + (v^{k})_{,i}\, g_{kj} + (v^{k})_{,j}\, g_{ik}$$

v^{i} : Killing vector

If ϕ^{i} transform linearly, and symmetric transformation consists of $\mathcal{G}$

$$v^{i} = i[T_{\mathcal{G}}]^{i}{}_{j}\phi^{j}$$

EWPT

S and U parameter can be written in terms of EW current,

They can be expressed in terms of Killing vectors of scalar manifold

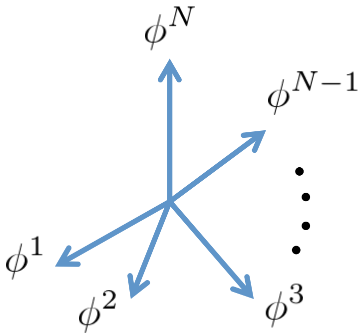
$$S_{\rm div} = -\frac{1}{12\pi}(\bar{w}^i_3)_{;j}(\bar{y}^j)_{;i}\ln\frac{\Lambda^2}{\mu^2}$$

$$U_{\rm div} = \frac{1}{12\pi}\Big((\bar{w}^i_1)_{;j}(\bar{w}^j_1)_{;i} - (\bar{w}^i_3)_{;j}(\bar{w}^j_3)_{;i}\Big)\ln\frac{\Lambda^2}{\mu^2}$$

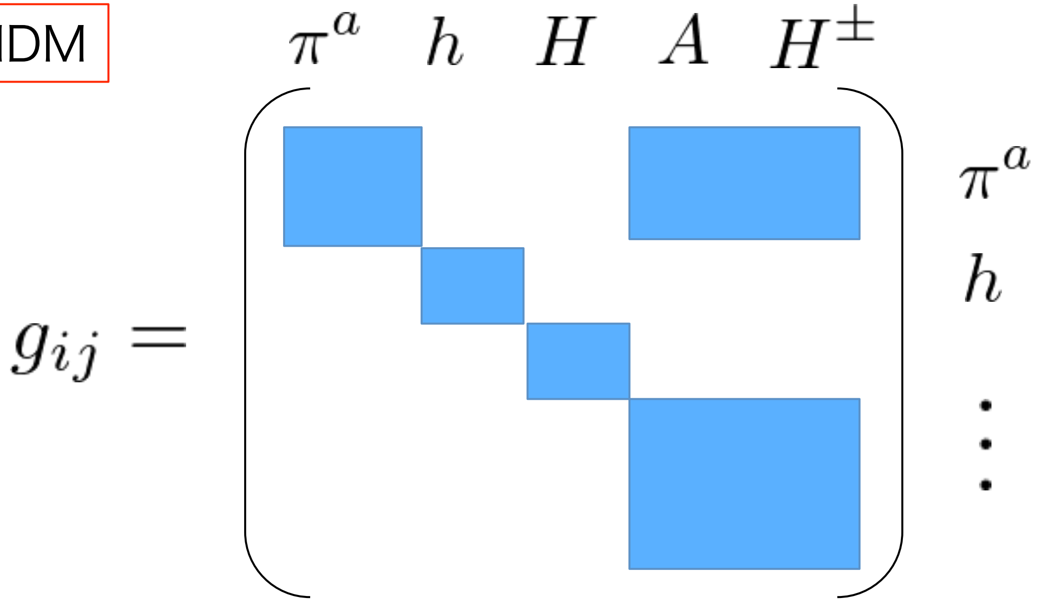
This expression is “ universal ” in any new physics model

- # of indices = # of scalar fields

- form = expression of the scalars

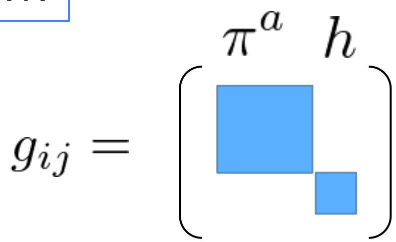


2HDM

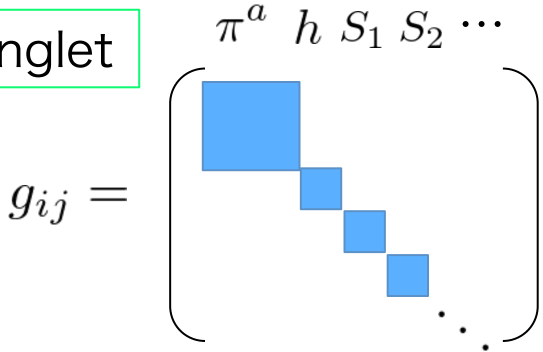


Nagai-san's
slide at PPP2018

SM



SM + Singlet



③ Killing eq.

connect the Geometry and the Symmetry

① ②

$$v_{i;j;k} = R^{l}{}_{kji} v_l$$

v_i and $R^{i}{}_{jkl}$ in SM should satisfy Killing eq.

v_i and $R^{i}{}_{jkl}$ in MCHM should satisfy Killing eq.

v_i and $R^{i}{}_{jkl}$ in SM + Singlet Model should satisfy Killing eq.

Effective field theory approach

SMEFT : $\mathcal{L}_{\text{scalar}}$ is written in terms of H

$$\mathcal{L}_{\text{scalar}} = (D_\mu H)^\dagger D^\mu H + \frac{C_1}{\Lambda^2}(H^\dagger D_\mu H)^*(H^\dagger D^\mu H) + \cdots$$

HEFT : $\mathcal{L}_{\text{scalar}}$ is written in terms of π^a (NGBs) and h

$$\mathcal{L}_{\text{scalar}} = \left(1 + \kappa_1 \frac{h}{v} + \kappa_2 \frac{h^2}{v^2} + \cdots\right) \frac{v^2}{4} \text{tr}\left[(D_\mu U)^\dagger D^\mu U\right] - V(h)$$

$$U := e^{i\frac{\pi^a}{v}\tau_a}$$

- HEFT is more general than SMEFT
- New Physics effect is encoded in C_i or κ_i

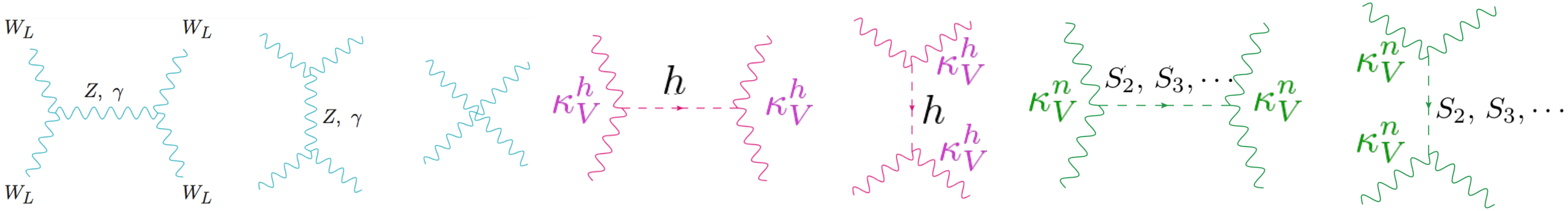
< Singlet extension >

$$\mathcal{L} = \frac{v^2}{4} F(h, S_n) \mathrm{Tr}[(D_\mu U)^\dagger D^\mu U] + \mathcal{L}_{kinetic} - V(h, S_n)$$

$$F(h, S_n) = 1 + 2\kappa_V^h \frac{h}{v} + 2\sum_n \kappa_V^n \frac{S_n}{v} + \cdots \qquad U = \exp\left(i \frac{\pi^a}{v} \frac{\tau^a}{2}\right)$$

I. Higgs unitalize W_L scattering amplitude at tree level (tree level unitarity)

$$\mathcal{M}_{W_L W_L \to W_L W_L} \simeq \frac{s+t}{v^2}\left(1 - (\kappa_V^h)^2 - \sum_{n=2}^{N} (\kappa_V^n)^2\right)$$



II. Higgs cancels the divergence in oblique corrections

Peskin Takeuchi
Phys. Rev. Lett. 65 (1990) 964

$$S \simeq \frac{1}{12\pi}\left(1 - (\kappa_V^h)^2 - \sum_{n=2}^{N} (\kappa_V^n)^2\right) \ln \frac{\Lambda^2}{\mu^2}$$

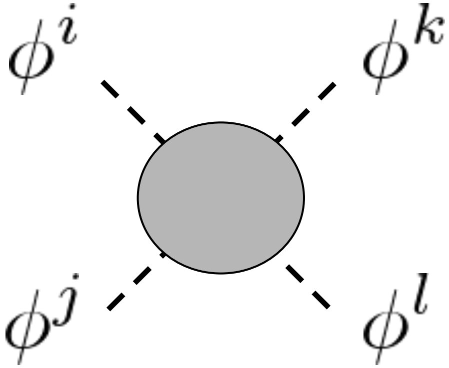
Approach based on Symmetry and Geometry

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2} g_{ij}(\phi)(D_\mu \phi)^i (D^\mu \phi)^j - V(\phi)$$

Geometry

$$R_{ijkl}(\phi)$$

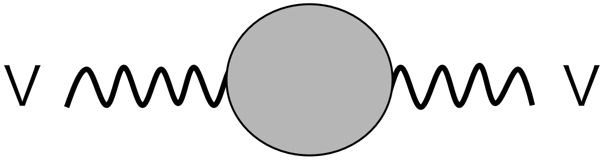
(Riemann tensor)



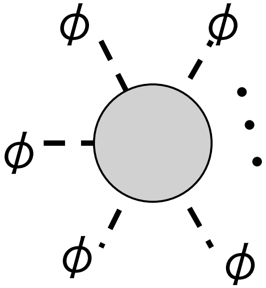
Symmetry

$$w_a^i(\phi) , \quad y^i(\phi)$$

(Killing vector)



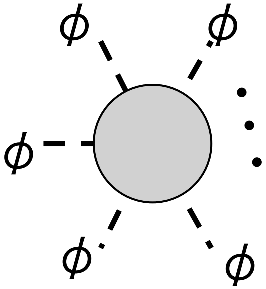
S, T, U



N point amplitude

$$= \quad \bar{R}_{i_1 i_2 i_3 i_4 ; i_5 \cdots i_N} \; s$$

We divide the verification prosedue into two steps



$=$

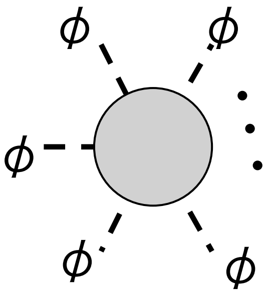
N point amplitude

$$= \bar{R}_{i_1 i_2 i_3 i_4 ; i_5 \cdots i_N} \; s$$

We divide the verification prosedue into two steps

$$g_{ij}(\phi) = \bar{g}_{ij} + \bar{G}_{ijk}\,\varphi^k + \frac{1}{2}\bar{G}_{ijkl}\,\varphi^k\varphi^l + \frac{1}{3!}\bar{G}_{ijklm}\,\varphi^k\varphi^l\varphi^m + \quad \cdots$$

Part 1



$$= -\frac{i}{2}\sum_{m<n} s_{mn}\bar{G}_{(i_m i_n)(i_1 i_2\cdots\check{i}_m\cdots\check{i}_n\cdots i_N)}$$

N point amplitude

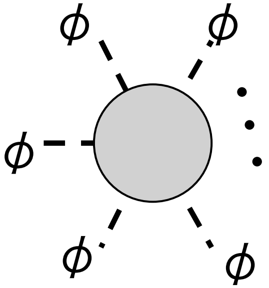
Part 2

$$-\frac{i}{2}\sum_{m<n} s_{mn}\bar{G}_{(i_m i_n)(i_1 i_2\cdots\check{i}_m\cdots\check{i}_n\cdots i_N)} \;=\; \bar{R}_{i_1 i_2 i_3 i_4;i_5\cdots i_N}\; s$$

We divide the verification prosedue into two steps

$$g_{ij}(\phi) = \bar{g}_{ij} + \bar{G}_{ijk}\,\varphi^k + \frac{1}{2}\bar{G}_{ijkl}\,\varphi^k\varphi^l + \frac{1}{3!}\bar{G}_{ijklm}\,\varphi^k\varphi^l\varphi^m + \quad \cdots$$

Part 1



$$= -\frac{i}{2}\sum_{m<n} s_{mn}\bar{G}_{(i_m i_n)(i_1 i_2 \cdots \check{i}_m \cdots \check{i}_n \cdots i_N)}$$

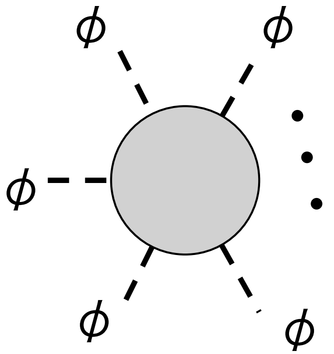
N point amplitude

Part 2

$$-\frac{i}{2}\sum_{m<n} s_{mn}\bar{G}_{(i_m i_n)(i_1 i_2 \cdots \check{i}_m \cdots \check{i}_n \cdots i_N)} = \bar{R}_{i_1 i_2 i_3 i_4 ; i_5 \cdots i_N}\ s$$

Part 1

$$g_{ij}(\phi) = \bar{g}_{ij} + \bar{G}_{ijk}\,\varphi^k + \frac{1}{2}\bar{G}_{ijkl}\,\varphi^k\varphi^l + \frac{1}{3!}\bar{G}_{ijklm}\,\varphi^k\varphi^l\varphi^m + \ \cdots$$



$$= -\frac{i}{2}\sum_{m<n} s_{mn}\bar{G}_{(i_m i_n)(i_1 i_2\cdots\check{i}_m\cdots\check{i}_n\cdots i_N)}$$

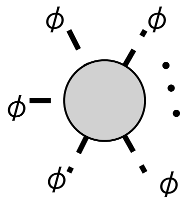
$$s_{mn} := (p_m + p_n)^2$$

N point amplitude

$i_1 i_2 \cdots \check{i}_m \cdots \check{i}_n \cdots i_N$ denote the absence of i_m and i_n

i.e. $i_1 i_2 \cdots i_{m-1}\, i_{m+1} \cdots i_{n-1}\, i_{n+1} \cdots i_N$

In order to verify



N point amplitude

$$= -\frac{i}{2}\sum_{m<n} s_{mn}\bar{G}_{(i_m i_n)(i_1 i_2 \cdots \check{i}_m \cdots \check{i}_n \cdots i_N)}$$

We must confirm that

$$0 = \bar{G}_{(IJ)(KL)} + \bar{G}_{(IK)(JL)} + \bar{G}_{(IL)(JK)},$$

$$0 = \bar{G}_{(IJ)(KLM)} + \bar{G}_{(IK)(JLM)} + \bar{G}_{(IL)(JKM)} + \bar{G}_{(IM)(JKL)},$$

$$0 = \bar{G}_{(IJ)(KLMN)} + \bar{G}_{(IK)(JLMN)} + \bar{G}_{(IL)(JKMN)} + \bar{G}_{(IM)(JKLN)} + \bar{G}_{(IN)(JKLM)},$$

$$\vdots$$

the relation above hold for arbitrary n

U. Muller et al Gen. Rel. Grav. 31 (1999) 1759

Actually,

We can confirm that

$$0 = \bar{G}_{(IJ)(KL)} + \bar{G}_{(IK)(JL)} + \bar{G}_{(IL)(JK)},$$

$$0 = \bar{G}_{(IJ)(KLM)} + \bar{G}_{(IK)(JLM)} + \bar{G}_{(IL)(JKM)} + \bar{G}_{(IM)(JKL)},$$

$$0 = \bar{G}_{(IJ)(KLMN)} + \bar{G}_{(IK)(JLMN)} + \bar{G}_{(IL)(JKMN)} + \bar{G}_{(IM)(JKLN)} + \bar{G}_{(IN)(JKLM)},$$

$$\vdots$$

the relation above hold for arbitrary n

once we admit $\bar{G}_{IJ}$, $\bar{G}_{IJK}$, $\bar{G}_{IJKL}$ $\cdots$ are proportional to

(the derivative of) Riemann tensor.

Actually,

We can confirm that

For example

$$\boxed{0 = \bar{G}_{(IJ)(KL)} + \bar{G}_{(IK)(JL)} + \bar{G}_{(IL)(JK)},}$$

$$0 = \bar{G}_{(IJ)(KLM)} + \bar{G}_{(IK)(JLM)} + \bar{G}_{(IL)(JKM)} + \bar{G}_{(IM)(JKL)},$$

$$0 = \bar{G}_{(IJ)(KLMN)} + \bar{G}_{(IK)(JLMN)} + \bar{G}_{(IL)(JKMN)} + \bar{G}_{(IM)(JKLN)} + \bar{G}_{(IN)(JKLM)},$$

$$\vdots$$

the relation above hold for arbitrary n

once we admit $\bar{G}_{IJ}$, $\bar{G}_{IJK}$, $\bar{G}_{IJKL}$ $\cdots$ are proportional to

(the derivative of) Riemann tensor.

For example

$$\boxed{0 \;=\; \bar{G}_{(IJ)(KL)} + \bar{G}_{(IK)(JL)} + \bar{G}_{(IL)(JK)},}$$

For example

Conclusion :

$$\boxed{0 \;=\; \bar{G}_{(IJ)(KL)} + \bar{G}_{(IK)(JL)} + \bar{G}_{(IL)(JK)},}$$

Verification :

① Write down $\bar{G}_{(IJ)(KL)}$ in terms of $\bar{R}_{IJKL}$

$$\underline{\bar{G}_{(IJ)(KL)}} = \frac{2}{3}\left(\underline{\bar{R}_{IKLJ}} + \underline{\bar{R}_{ILKJ}}\right) \quad \cdots (\ast)$$

symmetrize the indices of $\bar{R}_{IJKL}$

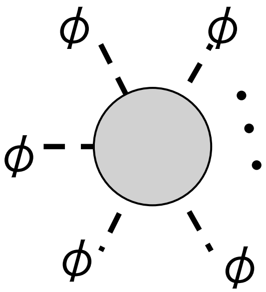
② Write down the relation above in terms of (∗)

$$\bar{G}_{(IJ)(KL)} + \bar{G}_{(IK)(JL)} + \bar{G}_{(IL)(JK)} \underset{\substack{\uparrow \\ (\ast)}}{\propto} \bar{R}_{I(JKL)} \underset{\substack{\uparrow \\ \text{trivial}}}{\equiv} 0$$

We divide the verification prosedue into two steps

$$g_{ij}(\phi) = \bar{g}_{ij} + \bar{G}_{ijk}\,\varphi^k + \frac{1}{2}\bar{G}_{ijkl}\,\varphi^k\varphi^l + \frac{1}{3!}\bar{G}_{ijklm}\,\varphi^k\varphi^l\varphi^m + \quad \cdots$$

Part 1



$$= -\frac{i}{2}\sum_{m<n} s_{mn}\bar{G}_{(i_m i_n)(i_1 i_2\cdots\check{i}_m\cdots\check{i}_n\cdots i_N)}$$

N point amplitude

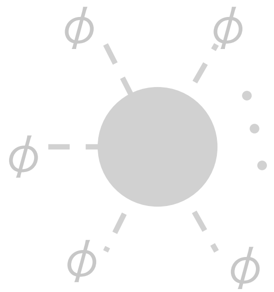
Part 2

$$-\frac{i}{2}\sum_{m<n} s_{mn}\bar{G}_{(i_m i_n)(i_1 i_2\cdots\check{i}_m\cdots\check{i}_n\cdots i_N)} = \bar{R}_{i_1 i_2 i_3 i_4;i_5\cdots i_N}\ s$$

We divide the verification prosedue into two steps

$$g_{ij}(\phi) = \bar{g}_{ij} + \bar{G}_{ijk}\,\varphi^k + \frac{1}{2}\bar{G}_{ijkl}\,\varphi^k\varphi^l + \frac{1}{3!}\bar{G}_{ijklm}\,\varphi^k\varphi^l\varphi^m + \quad \cdots$$

Part 1



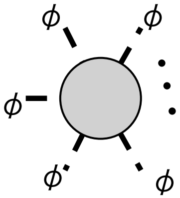
$$= -\frac{i}{2}\sum_{m<n} s_{mn}\bar{G}_{(i_m i_n)(i_1 i_2\cdots\check{i}_m\cdots\check{i}_n\cdots i_N)}$$

N point amplitude

Part 2

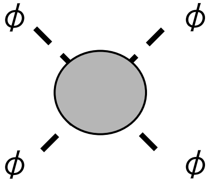
$$-\frac{i}{2}\sum_{m<n} s_{mn}\bar{G}_{(i_m i_n)(i_1 i_2\cdots\check{i}_m\cdots\check{i}_n\cdots i_N)} \;=\; \bar{R}_{i_1 i_2 i_3 i_4; i_5\cdots i_N}\; s$$

In **Part 1** , we verify



N point amplitude

$$= -\frac{i}{2}\sum_{m<n} s_{mn}\bar{G}_{(i_m i_n)(i_1 i_2 \cdots \check{i}_m \cdots \check{i}_n \cdots i_N)}$$

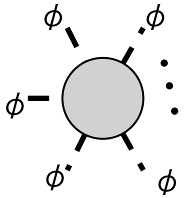


4 point

$$= -\frac{i}{2}\Big\{ s_{12}\bar{G}_{(i_1 i_2)(i_3 i_4)} + s_{13}\bar{G}_{(i_1 i_3)(i_2 i_4)} + s_{14}\bar{G}_{(i_1 i_4)(i_2 i_3)}$$

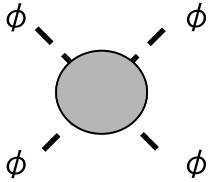
$$+ s_{23}\bar{G}_{(i_2 i_3)(i_1 i_4)} + s_{24}\bar{G}_{(i_2 i_4)(i_1 i_3)} + s_{34}\bar{G}_{(i_3 i_4)(i_1 i_2)} \Big\}$$

In Part 1 , we verify



N point amplitude

$$= -\frac{i}{2}\sum_{m<n} s_{mn}\bar{G}_{(i_m i_n)(i_1 i_2\cdots\check{i}_m\cdots\check{i}_n\cdots i_N)}$$

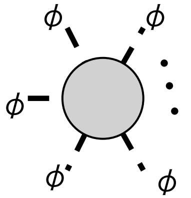


4 point

$$= -\frac{i}{2}\Big\{ s_{12}\bar{G}_{(i_1 i_2)(i_3 i_4)} + s_{13}\bar{G}_{(i_1 i_3)(i_2 i_4)} + s_{14}\bar{G}_{(i_1 i_4)(i_2 i_3)}$$
$$+ s_{23}\bar{G}_{(i_2 i_3)(i_1 i_4)} + s_{24}\bar{G}_{(i_2 i_4)(i_1 i_3)} + s_{34}\bar{G}_{(i_3 i_4)(i_1 i_2)} \Big\}$$

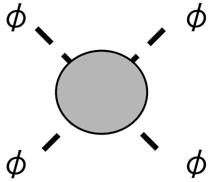
In Part 1 , we also found $\bar{G}_{(IJ)(KL)} = \frac{2}{3}\left(\bar{R}_{IKLJ} + \bar{R}_{ILKJ}\right)$

In **Part 1** , we verify



N point amplitude

$$= -\frac{i}{2}\sum_{m<n} s_{mn}\bar{G}_{(i_m i_n)(i_1 i_2\cdots\check{i}_m\cdots\check{i}_n\cdots i_N)}$$



4 point

$$= -\frac{i}{2}\Big\{ s_{12}\bar{G}_{(i_1 i_2)(i_3 i_4)} + s_{13}\bar{G}_{(i_1 i_3)(i_2 i_4)} + s_{14}\bar{G}_{(i_1 i_4)(i_2 i_3)} + s_{23}\bar{G}_{(i_2 i_3)(i_1 i_4)} + s_{24}\bar{G}_{(i_2 i_4)(i_1 i_3)} + s_{34}\bar{G}_{(i_3 i_4)(i_1 i_2)} \Big\}$$

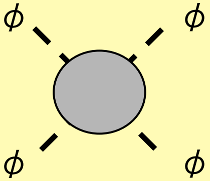
In **Part 1** , we also found $\bar{G}_{(IJ)(KL)} = \frac{2}{3}\left(\bar{R}_{IKLJ} + \bar{R}_{ILKJ}\right)$

Using $s_{12} + s_{13} + s_{23} = 0$

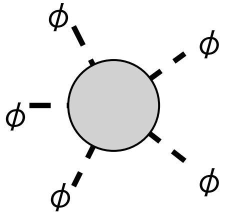
&

$\bar{R}_{i_1 i_2 i_3 i_4} + \bar{R}_{i_1 i_3 i_4 i_2} + \bar{R}_{i_1 i_4 i_2 i_3} = 0$ (Bianchi identity)

we can verify



$\propto s_{12}\bar{R}_{i_1 i_3 i_4 i_2} + s_{13}\bar{R}_{i_1 i_2 i_4 i_3}$



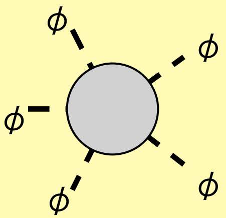
5 point

$$= -\frac{i}{2}\Big\{ + s_{12}\,\bar{G}_{(i_1i_2)(i_3i_4i_5)} + s_{13}\,\bar{G}_{(i_1i_3)(i_2i_4i_5)} + s_{14}\,\bar{G}_{(i_1i_4)(i_2i_3i_5)} + s_{15}\,\bar{G}_{(i_1i_5)(i_2i_3i_4)}$$
$$+ s_{23}\,\bar{G}_{(i_2i_3)(i_1i_4i_5)} + s_{24}\,\bar{G}_{(i_2i_4)(i_1i_3i_5)} + s_{25}\,\bar{G}_{(i_2i_5)(i_1i_3i_4)}$$
$$+ s_{34}\,\bar{G}_{(i_3i_4)(i_1i_2i_5)} + s_{35}\,\bar{G}_{(i_3i_5)(i_1i_2i_4)} + s_{45}\,\bar{G}_{(i_4i_5)(i_1i_2i_3)}\Big\}$$

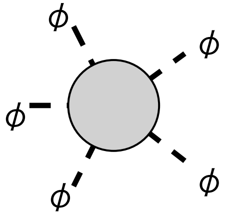
Using $\bar{G}_{(IJ)(KLM)} = \bar{R}_{I(KL)J,M} + \bar{R}_{I(LM)J,K} + \bar{R}_{I(KM)J,L}$

$$s_{12} + s_{13} + s_{14} + s_{15} = 0$$
$$s_{21} + s_{23} + s_{24} + s_{25} = 0$$
$$s_{31} + s_{32} + s_{34} + s_{35} = 0$$
$$s_{41} + s_{42} + s_{43} + s_{45} = 0$$
$$s_{51} + s_{52} + s_{53} + s_{54} = 0$$

we can verify



$$\propto \; s_{12}\Big(\bar{R}_{i_5(i_3i_4)i_2,i_1} + \bar{R}_{i_5(i_3i_4)i_1,i_2}$$
$$+ \bar{R}_{i_5(i_1i_2)i_4,i_3} + \bar{R}_{i_5(i_1i_2)i_3,i_4}\Big)$$
$$+ s_{13}\bar{R}_{i_1i_4i_3i_5,i_2} + s_{14}\bar{R}_{i_1i_3i_4i_5,i_2}$$
$$+ s_{23}\bar{R}_{i_2i_4i_3i_5,i_1} + s_{24}\bar{R}_{i_2i_3i_4i_5,i_1}$$



5 point

$$= -\frac{i}{2}\Big\{ + s_{12}\,\bar{G}_{(i_1i_2)(i_3i_4i_5)} + s_{13}\,\bar{G}_{(i_1i_3)(i_2i_4i_5)} + s_{14}\,\bar{G}_{(i_1i_4)(i_2i_3i_5)} + s_{15}\,\bar{G}_{(i_1i_5)(i_2i_3i_4)}$$
$$+ s_{23}\,\bar{G}_{(i_2i_3)(i_1i_4i_5)} + s_{24}\,\bar{G}_{(i_2i_4)(i_1i_3i_5)} + s_{25}\,\bar{G}_{(i_2i_5)(i_1i_3i_4)}$$

Difficult to write down the general formula for N point amp.

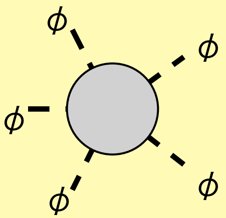
(we must verify case by case)

$$s_{12} + s_{13} + s_{14} + s_{15} = 0$$

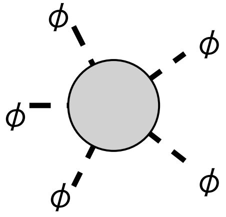
Gram determinant constraints (N $\geq$ 6)

$$s_{51} + s_{52} + s_{53} + s_{54} = 0$$

we can verify



$$\propto \quad s_{12}\Big(\bar{R}_{i_5(i_3i_4)i_2,i_1} + \bar{R}_{i_5(i_3i_4)i_1,i_2} + \bar{R}_{i_5(i_1i_2)i_4,i_3} + \bar{R}_{i_5(i_1i_2)i_3,i_4}\Big) + s_{13}\bar{R}_{i_1i_4i_3i_5,i_2} + s_{14}\bar{R}_{i_1i_3i_4i_5,i_2} + s_{23}\bar{R}_{i_2i_4i_3i_5,i_1} + s_{24}\bar{R}_{i_2i_3i_4i_5,i_1}$$



5 point

$$= -\frac{i}{2}\Big\{ + s_{12}\,\bar{G}_{(i_1i_2)(i_3i_4i_5)} + s_{13}\,\bar{G}_{(i_1i_3)(i_2i_4i_5)} + s_{14}\,\bar{G}_{(i_1i_4)(i_2i_3i_5)} + s_{15}\,\bar{G}_{(i_1i_5)(i_2i_3i_4)}$$

$$+ s_{23}\,\bar{G}_{(i_2i_3)(i_1i_4i_5)} + s_{24}\,\bar{G}_{(i_2i_4)(i_1i_3i_5)} + s_{25}\,\bar{G}_{(i_2i_5)(i_1i_3i_4)}$$

Difficult to write down the general formula for N point amp.

(we must verify case by case)

$$s_{12} + s_{13} + s_{14} + s_{15} = 0$$

Gram determinant constraints (N $\geq$ 6)

$$s_{51} + s_{52} + s_{53} + s_{54} = 0$$

we can verify

$$s_{12}\Big(\bar{R}_{i_5(i_3i_4)i_2,i_1} + \bar{R}_{i_5(i_3i_4)i_1}$$

We come up with a good idea solving problems above !!

$$+ s_{23}R_{i_2i_4i_3i_5,i_1} + s_{24}R_{i_2i_3i_4i_5,i_1}$$

Compute 4-point amp. in following limit

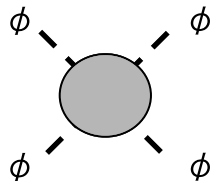
$$s_{12} = s_{34} = -s_{13} = -s_{24} =: s$$

other $s_{mn} = 0$

→ We expect to get the necessary conditions

for the perturbative unitarity

We get, however, the most stringent condition : $\bar{R}_{i_1 i_4 i_2 i_3} = 0$



$$\propto \quad s\,\bar{R}_{i_1 i_4 i_2 i_3}$$

$\bar{R}_{i_1 i_4 i_2 i_3} = 0$ is the necessary and sufficient conditions for the perturbative unitarity

Compute 4-point amp. in following limit

$$s_{12} = s_{34} = -s_{13} = -s_{24} =: s$$

other $s_{mn} = 0$

→ We expect t e necessary conditions

itarity

In 6-point amp., Gram det. constraint is

$$\begin{vmatrix} 0 & s_{12} & s_{13} & s_{14} & s_{15} \\ s_{12} & 0 & s_{23} & s_{24} & s_{25} \\ s_{13} & s_{23} & 0 & s_{34} & s_{35} \\ s_{14} & s_{24} & s_{34} & 0 & s_{25} \\ s_{15} & s_{25} & s_{35} & s_{45} & 0 \end{vmatrix} = 0,$$

→ we must chose appropriate limit consistent with Gram det. constraint

We get

$= 0$

$\bar{R}_{i_1 i_4}$

ns