カイラル感受率とU(1)量子異常

Hidenori Fukaya (Osaka U.)

for JLQCD collaboration

S. Aoki, Y. Aoki, HF, S. Hashimoto, T. Kaneko, C. Rohrhofer, K. Suzuki, in preparation.

JLQCD collaboration

KEKを中心とした大規模格子QCD数値計算の研究グループ。

元々KEKのスパコンを使ってたが、、、

2017年 shutdown（涙）.

HITACHI SR16000

IBM BG/Q

現在はHPCI (革新的ハイパフォーマンス・コンピューティング・インフラ)

筑波大CCSの学際共同利用プログラムなどに応募、
元気にやっています。

Oakforest-PACS at JCAHPC

JLQCD's finite T QCD project

今日お話しする有限温度QCDプロジェクトは
2019年度HPCI利用研究課題優秀成果賞
に選ばれました。

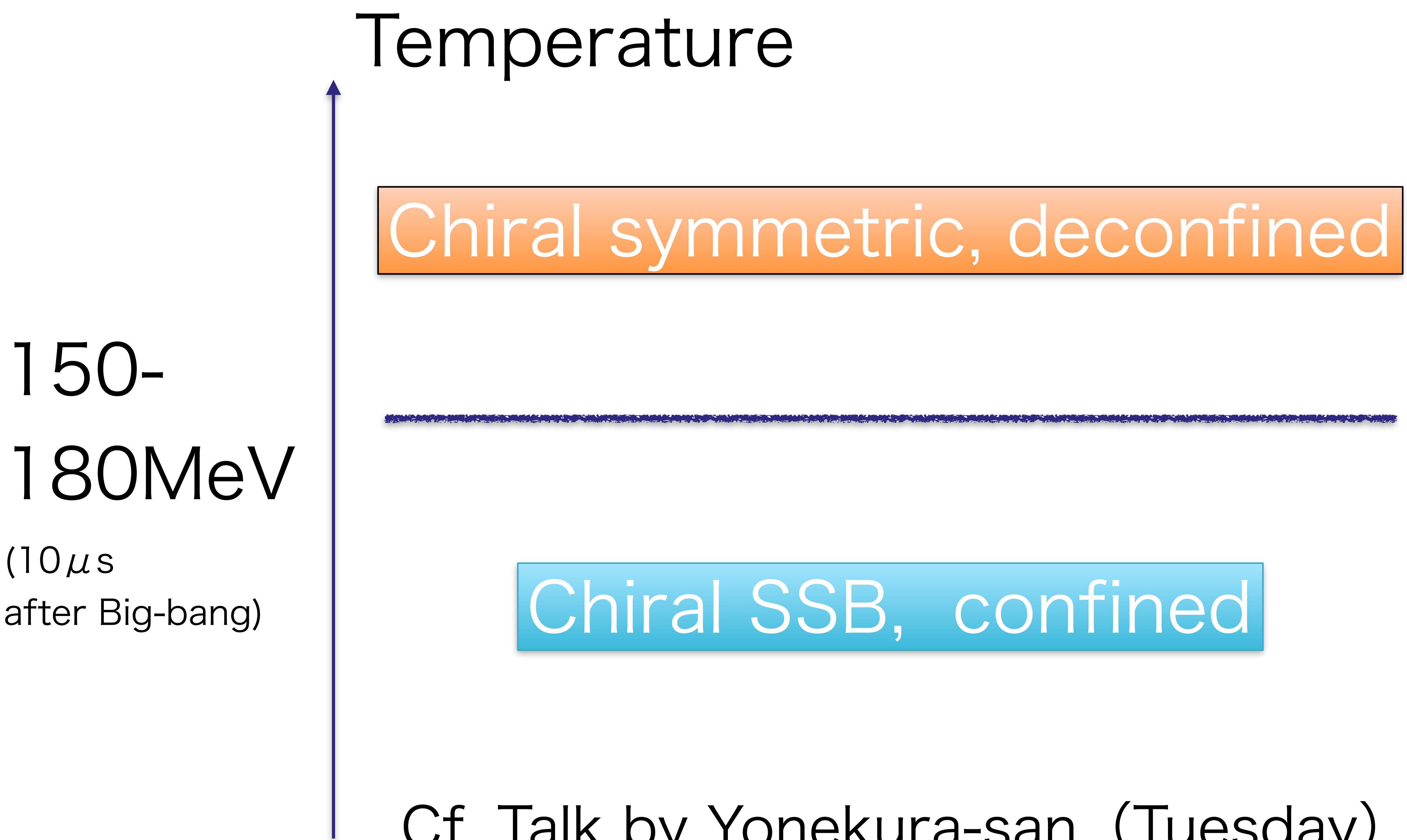
HPCI利用研究課題優秀成果賞受賞課題

令和2年7月2日に開催された成果報告会プログラム委員会での審議の結果、第7回成果報告会におけるHPCI利用研究課題優秀成果賞として下記8課題が選定されました。（課題番号順、敬称略）

分野	課題名(課題番号、課題の種類)	課題代表者(所属)
物質・材料・科学	タイヤ用ゴム材料の大規模分子動力学シミュレーション (hp170063、「京」産業利用課題(実証利用))	角田昌也(住友ゴム工業株式会社)
物理・素粒子・宇宙	ニュートリノレス二重ベータ崩壊の原子核行列要素 (hp180232、「京」一般課題)*	岩田順敬(関西大学)
物理・素粒子・宇宙	低質量星の熱対流と磁場活動の探査 (hp190070、「京」若手人材育成課題)	堀田英之(千葉大学)
物理・素粒子・宇宙	量子色力学の高温相におけるトポロジー励起 (hp190090、「京」以外一般課題)	深谷英則(大阪大学)
エネルギー	二次電池用新規水系電解液の動的挙動解析 (hp190101、「京」一般課題)	山田淳夫(東京大学)
バイオ・ライフ	リアルな細胞膜と膜タンパク質の全原子分子動力学計算 (hp190120、「京」以外一般課題)	篠田恵子(東京大学)
情報・計算機科学	超大規模並列深層学習のための革新的最適化手法の開発 (hp190122、「京」以外一般課題)	横田理央(東京工業大学)
環境・防災・減災	多量の瓦礫・流木を含んだ豪雨災害・津波シミュレーション (hp190130、「京」以外一般課題)	青木尊之(東京工業大学)

https://www.hpci-office.jp/pages/project_report_meeting　より抜粋。

Chiral phase transition



Cf. Talk by Yonekura-san（Tuesday)

What is chiral susceptibility?

QCD partition function

$$Z(m) = \int [dA] \det(D(A) + m)^{N_f} e^{-S_G(A)}$$

chiral condensate

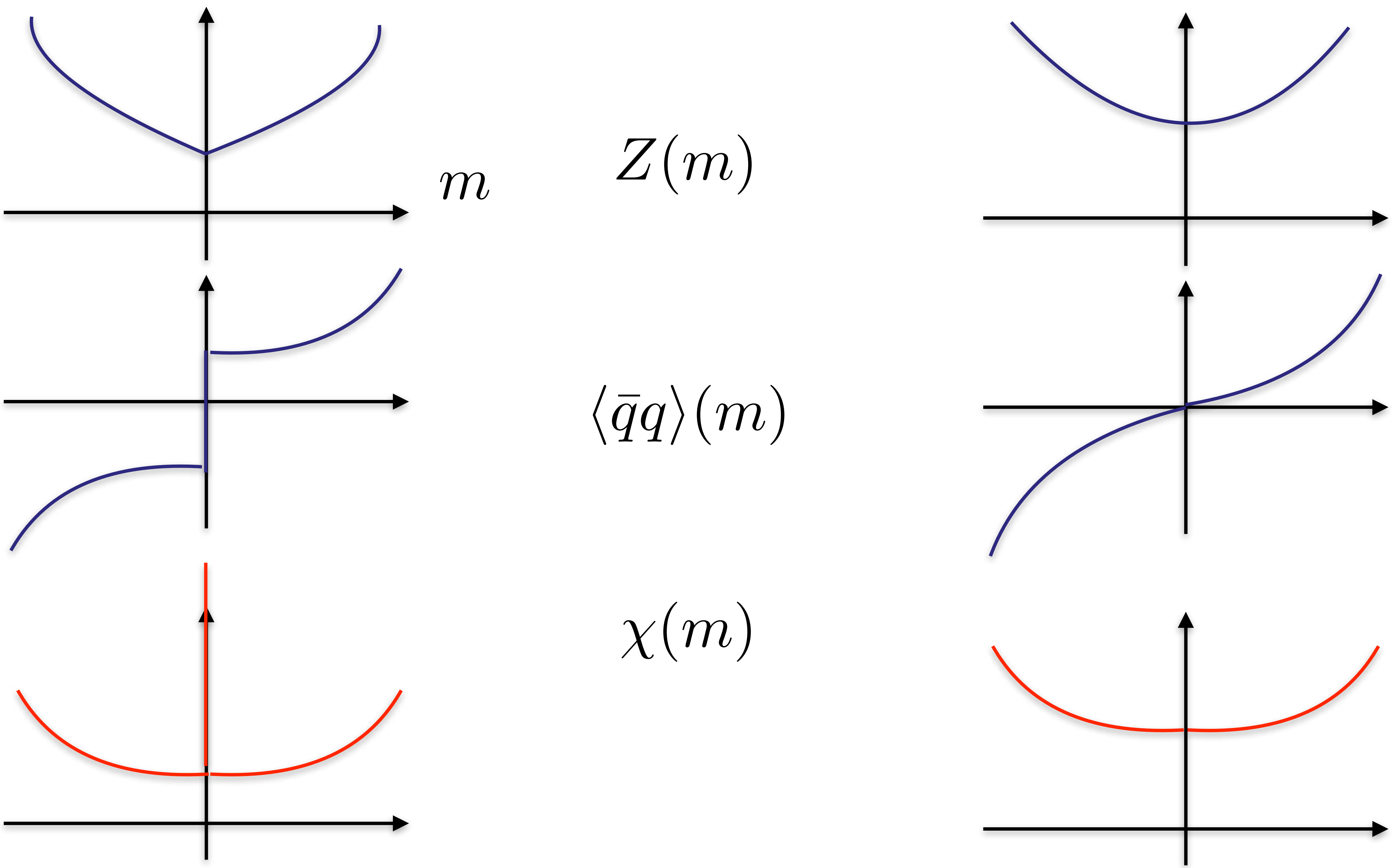
$$-\langle \bar{q}q \rangle = \frac{1}{N_f V} \frac{\partial}{\partial m} \ln Z(m)$$

chiral susceptibility (this talk)

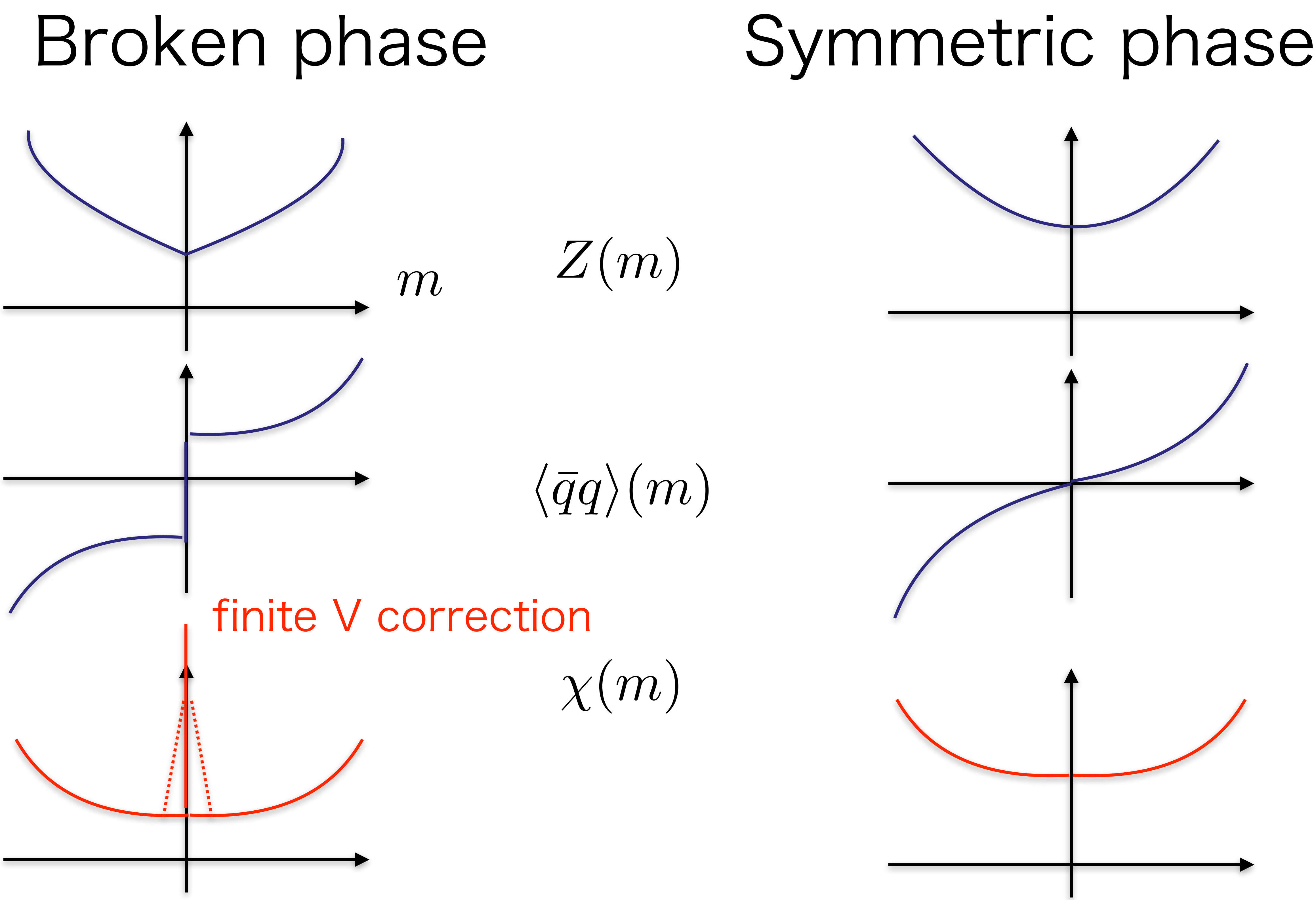
$$\chi(m) = \frac{1}{N_f V} \frac{\partial^2}{\partial m^2} \ln Z(m)$$

What is chiral susceptibility?

Broken phase Symmetric phase



What is chiral susceptibility?



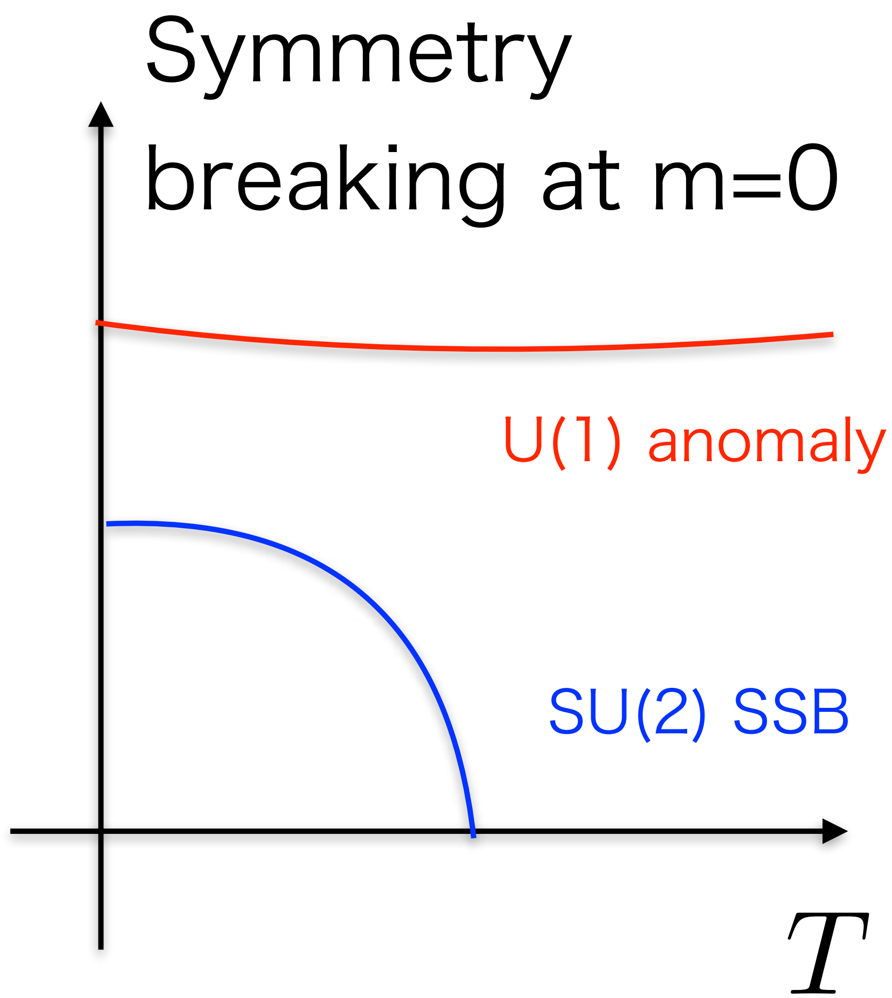
SU(2) or U(1)?

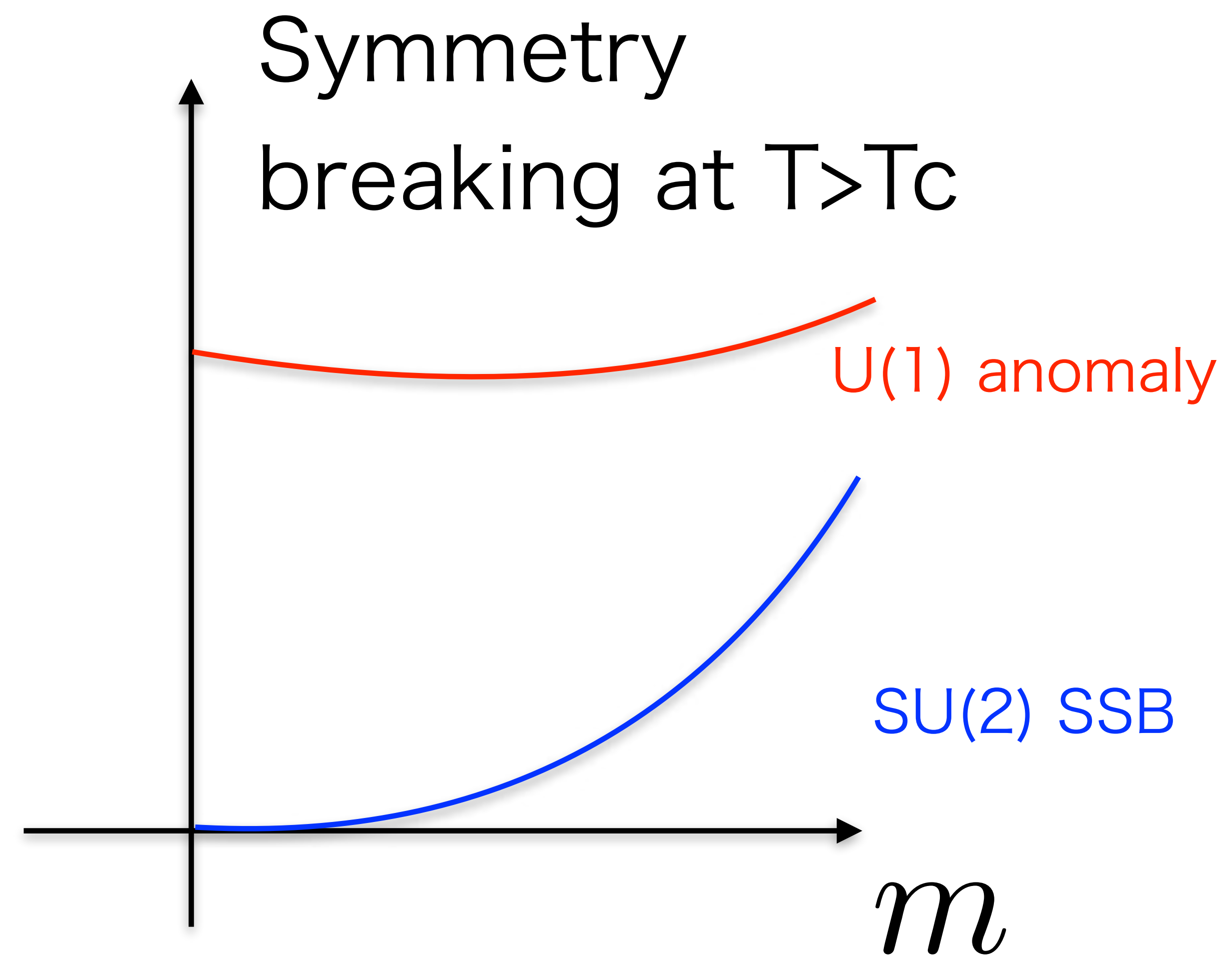
Chiral condensate or chiral susceptibility has been used as an order parameter of $SU(2)_L \times SU(2)_R$ chiral symmetry breaking.

But the condensate also breaks axial U(1)…

Naive answer : axial U(1) is anomalous given at a cut-off scale, and kept broken at all temperatures. We do not expect a strong dependence on T and m.

Naive expectation





T/m dependences of chiral condensate and its m-derivative should reflect SU(2) SSB rather than U(1) anomaly.

This talk
= a counterintuitive result

In this work we show that
the signal of chiral susceptibility is
dominated by axial U(1) anomaly (at T>Tc),
rather than $SU(2)_L \times SU(2)_R$.

Contents

Dirac eigenmode decomposition

$$Z(m) = \int [dA] \det(D(A)+m)^{N_f} e^{-S_G(A)} = \int [dA] \prod_\lambda (i\lambda(A)+m)^{N_f} e^{-S_G(A)}$$

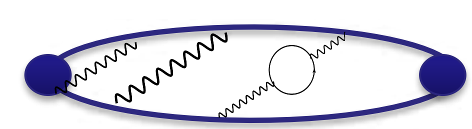
O(100)個ならLatticeで非摂動計算可能。

$$-\langle \bar{q}q \rangle = \frac{1}{N_f V}\frac{\partial}{\partial m}\ln Z(m) = \frac{1}{V}\left\langle \sum_\lambda \frac{1}{i\lambda(A)+m} \right\rangle,$$

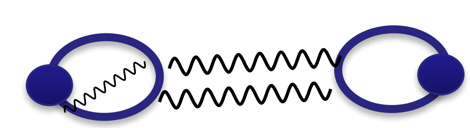
chiral susceptibility

$$\chi(m) = \frac{1}{N_f V}\frac{\partial^2}{\partial m^2}\ln Z(m) = \chi^{con.}(m) + \chi^{dis.}(m),$$

$$\chi^{con.}(m) = -\frac{\partial}{\partial m_{valence}}\langle \bar{q}q \rangle \Big|_{m_{valence}=m}$$



$$\chi^{dis.}(m) = -\frac{\partial}{\partial m_{sea}}\langle \bar{q}q \rangle \Big|_{m_{sea}=m}$$



Connected part

$$\chi^{con.}(m) = -\frac{1}{V}\left\langle \sum_\lambda \frac{1}{(i\lambda(A)+m)^2}\right\rangle$$

$$= -\frac{1}{V}\left\langle \sum_\lambda \frac{2m^2}{(\lambda(A)^2+m^2)^2}\right\rangle + \frac{1}{m}\left[\frac{1}{V}\left\langle \sum_\lambda \frac{m}{\lambda(A)^2+m^2}\right\rangle\right]$$

$$= -\Delta(m) + \frac{-\langle \bar{q}q\rangle}{m},$$

axial U(1) susceptibility!

$$\Delta(m) = \sum_x \left[\langle \pi(x)\pi(0)\rangle - \langle \delta(x)\delta(0)\rangle\right],$$

U(1)$_A$ pair

Namely, connected part is dominated by the U(1) anomaly when condensate is zero.

Disconnected part

$$\chi^{dis.}(m) = \frac{N_f}{V}\left[\left\langle\left(\sum_\lambda \frac{m}{\lambda(A)^2+m^2}\right)^2\right\rangle - \left\langle\sum_\lambda \frac{m}{\lambda(A)^2+m^2}\right\rangle^2\right].$$

$$= \frac{N_f}{V}\left[\frac{\langle N_0^2\rangle - \langle N_0\rangle^2}{m^2} + \frac{2}{m}\left(\left\langle N_0 \sum_{\lambda>0}\frac{2m}{\lambda(A)^2+m^2}\right\rangle - \langle N_0\rangle\left\langle\sum_{\lambda>0}\frac{2m}{\lambda(A)^2+m^2}\right\rangle\right)\right.$$
$$\left.+\left\langle\left(\sum_{\lambda>0}\frac{2m}{\lambda(A)^2+m^2}\right)^2\right\rangle - \left\langle\sum_{\lambda>0}\frac{2m}{\lambda(A)^2+m^2}\right\rangle^2\right].$$

$N_0 = n_+ + n_-$: number of zero modes

From the zero modes, we can separate the instanton number Q (U(1) anomaly effect) .

$$Q = n_+ - n_- = N_0 - 2n_- = 2n_+ - N_0$$

Disconnected part

$$\chi^{dis.}(m) = \frac{N_f}{m^2}\frac{\langle Q^2\rangle - \langle Q\rangle^2}{V}$$

U(1) anomaly effect
(topological susceptibility)

$$+\frac{2N_f}{V}\frac{\langle n_+ n_-\rangle - \langle n_+\rangle\langle n_-\rangle}{m^2}$$

→ empirically small

$$+\frac{N_f}{V}\left[\frac{2}{m}\left(\left\langle N_0 \sum_{\lambda>0}\frac{2m}{\lambda(A)^2+m^2}\right\rangle - \langle N_0\rangle\left\langle \sum_{\lambda>0}\frac{2m}{\lambda(A)^2+m^2}\right\rangle\right)\right.$$

$$\left. + \left\langle\left(\sum_{\lambda>0}\frac{2m}{\lambda(A)^2+m^2}\right)^2\right\rangle - \left\langle \sum_{\lambda>0}\frac{2m}{\lambda(A)^2+m^2}\right\rangle^2\right].$$

Non-zero modes effect
responsible for $SU(2)_L \times SU(2)_R$.

Lattice formulas

With the overlap Dirac operator, we have

$$\chi^{con.lat}(m) = -\Delta^{lat}(m) + \frac{-\langle \bar{q}q \rangle^{lat}}{m},$$

$$\Delta^{lat}(m) = \frac{1}{V(1-m^2)^2} \left\langle \sum_{\mathrm{all}\lambda_m} \frac{2m^2(1-\lambda_m^2)^2}{\lambda_m^4} \right\rangle,$$

$$-\langle \bar{q}q \rangle^{lat} = \frac{1}{V(1-m^2)} \left\langle \sum_{\mathrm{all}\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2} \right\rangle.$$

$$\chi^{dis.lat}(m) = \frac{N_f}{V} \left[\frac{1}{(1-m^2)^2} \left\langle \left(\sum_{\mathrm{all}\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2} \right)^2 \right\rangle - |\langle \bar{q}q \rangle^{lat}|^2 V^2 \right].$$

where $\lambda_m = \text{eigenvalues of } H_m = \gamma_5[(1-m)D_{ov} + m]$

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Chiral condensate and susceptibility are used as a probe for SU(2)xSU(2) SSB but…

✔ 2. Dirac eigenmode decomposition

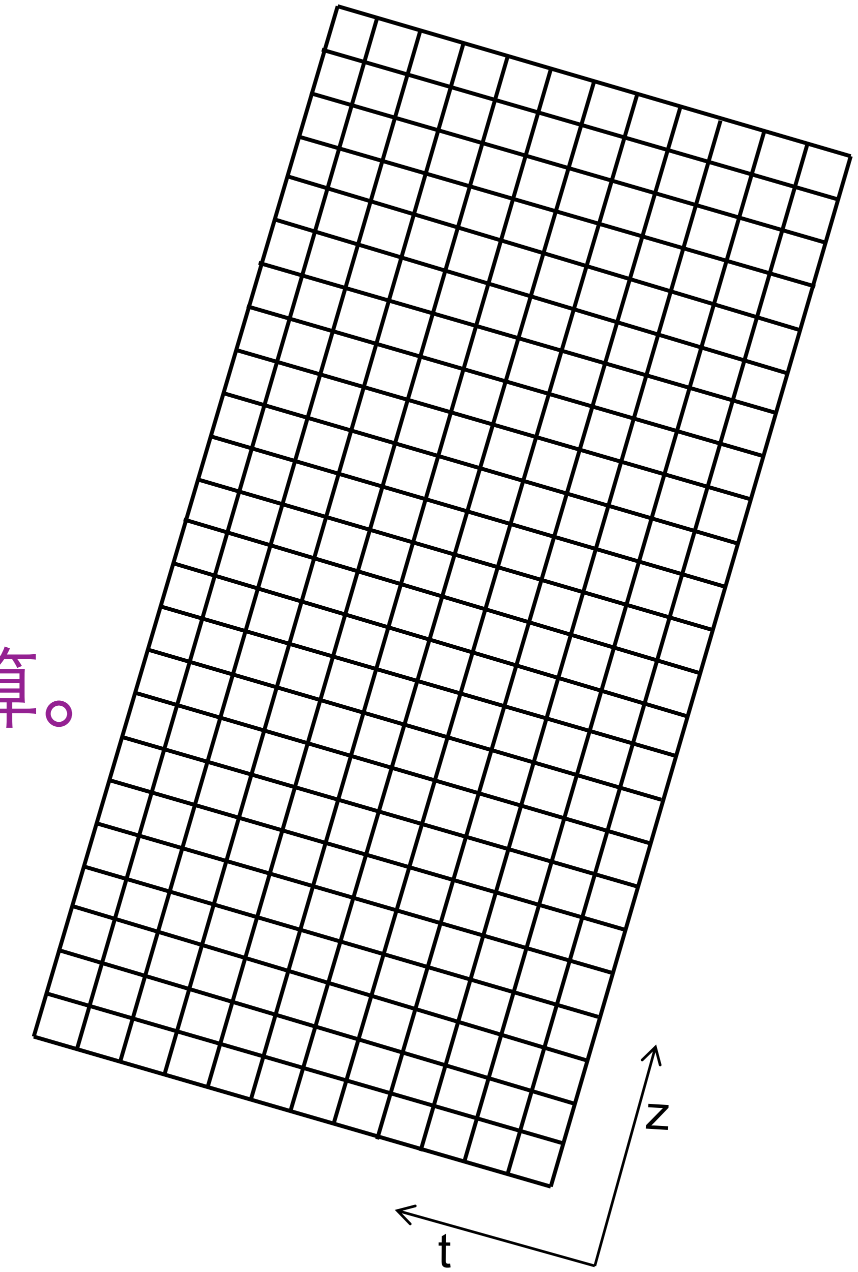
Chiral susceptibility contains purely axial U(1) anomaly effects.

3. Lattice set-up

4. Numerical results

5. Summary

Simulation setup



厳密なカイラル対称性を保つDirac 演算子を
用いた世界初の有限温度QCD大規模数値計算。

1/a = 2.6 GeV (0.074fm)

L=24,32,40,48 [1.8-3.6fm]

Symanzik gauge action

Mobius domain-wall fermions with m_{res}<1MeV
and reweighted overlap fermion.

Quark mass from 3MeV (< phys. pt. ~4MeV) to 30MeV.

T=190, 220, 260, 330 MeV (Lt=8,10,12,14).

Tc is estimated to be around 175MeV (from Polyakov loop).

Overlap & (Mobius) domain-wall

= Edge states of 5D topological insulator

$$D_{\mathrm{ov}}(m) = \left[\frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \,\mathrm{sgn}(H_M)\right]$$

→ perfect chiral sym.

$$H_M = \gamma_5 \frac{2D_W}{2+D_W}$$

$$(\to_{a\to 0} D_{\mathrm{continuum}} + m)$$

numerically violation ~ 1keV

$$D^{4D}_{\mathrm{DW}}(m) = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \frac{1-(T(H_M))^{L_s}}{1+(T(H_M))^{L_s}}$$

→ good chiral sym.

with Ls=16.

numerically violation ~ 1MeV

Violation of chiral symmetry enhanced at finite T

Checking chiral sym. for EACH eigenmode

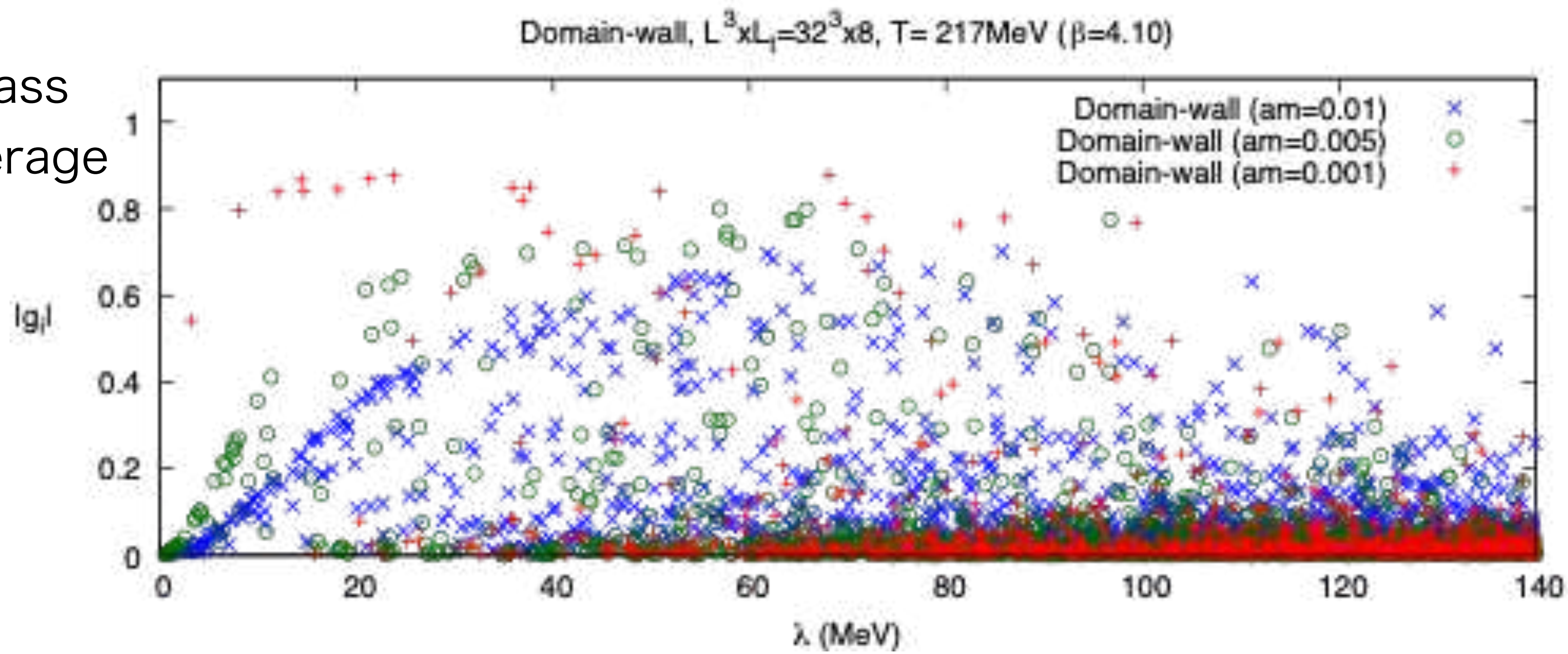
$$g_i = \left(v_i^\dagger, \frac{D\gamma_5 + \gamma_5 D - aRD\gamma_5 D}{\lambda_i} v_i \right)$$

Bad modes appear above Tc for a~0.1fm.

[JLQCD (Cossu et al.) 2015, JLQCD(Tomiya et al.) 2016]

Note: residual mass is (weighted) average of them.

For T=0, gi are consistent with residual mass.



Overlap/domain-wall reweighting

The fermion action can be changed AFTER simulation.

$$\langle O\rangle_{overlap} = \frac{\int dAO[\det D_{\rm ov}(m)]^2 e^{-S_G}}{\int dA[\det D_{\rm ov}(m)]^2 e^{-S_G}}$$

$$= \frac{\int dAOR[\det D_{\rm DW}^{\rm 4D}(m)]^2 e^{-S_G}}{\int dAR[\det D_{\rm DW}^{\rm 4D}(m)]^2 e^{-S_G}}$$

$$= \frac{\langle OR\rangle_{domain-wall}}{\langle R\rangle_{domain-wall}}$$

$$R \equiv \frac{\det[D_{\rm ov}(m)]^2}{\det[D_{\rm DW}^{\rm 4D}(m)]^2}$$

can be numerically computable.

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Nf=2 QCD w/ MDWF and rewegihted overlap. at T=190-330MeV near physical m~4MeV.

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5. Summary

Low-mode approximation

In the eigenvalue summations,

$$\chi^{con.lat}(m) = -\Delta^{lat}(m) + \frac{-\langle\bar{q}q\rangle^{lat}}{m},$$

$$\Delta^{lat}(m) = \frac{1}{V(1-m^2)^2}\left\langle \sum_{\mathrm{all}\lambda_m} \frac{2m^2(1-\lambda_m^2)^2}{\lambda_m^4} \right\rangle,$$

$$-\langle\bar{q}q\rangle^{lat} = \frac{1}{V(1-m^2)}\left\langle \sum_{\mathrm{all}\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2} \right\rangle.$$

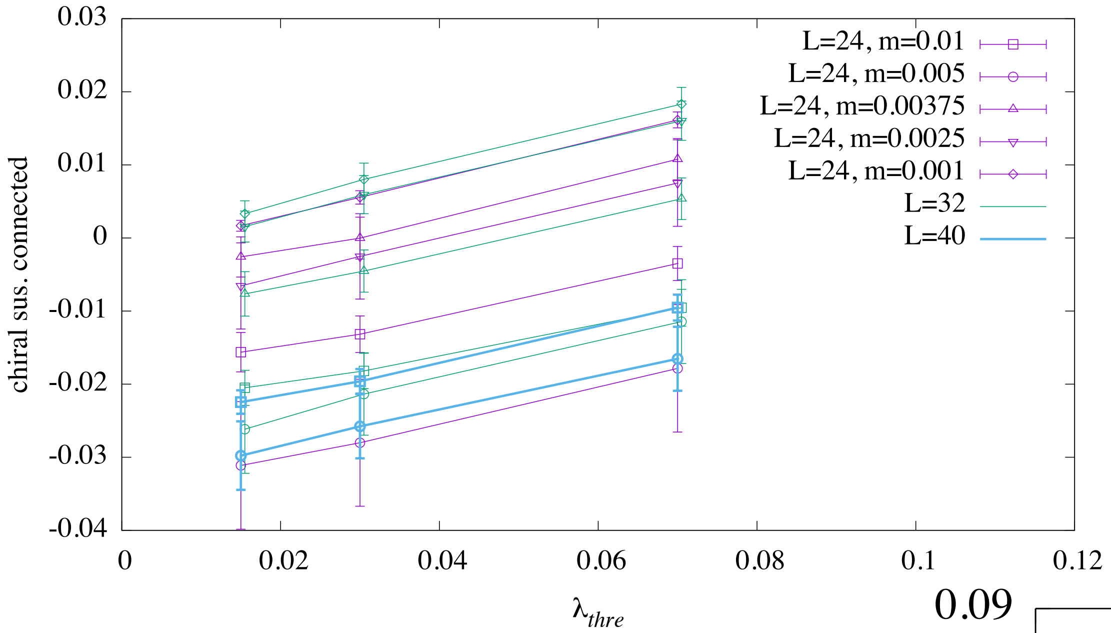
$$\chi^{dis.lat}(m) = \frac{N_f}{V}\left[\frac{1}{(1-m^2)^2}\left\langle \left(\sum_{\mathrm{all}\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2}\right)^2 \right\rangle - |\langle\bar{q}q\rangle^{lat}|^2 V^2\right].$$

where $\lambda_m = \text{eigenvalues of } H_m = \gamma_5[(1-m)D_{ov} + m]$

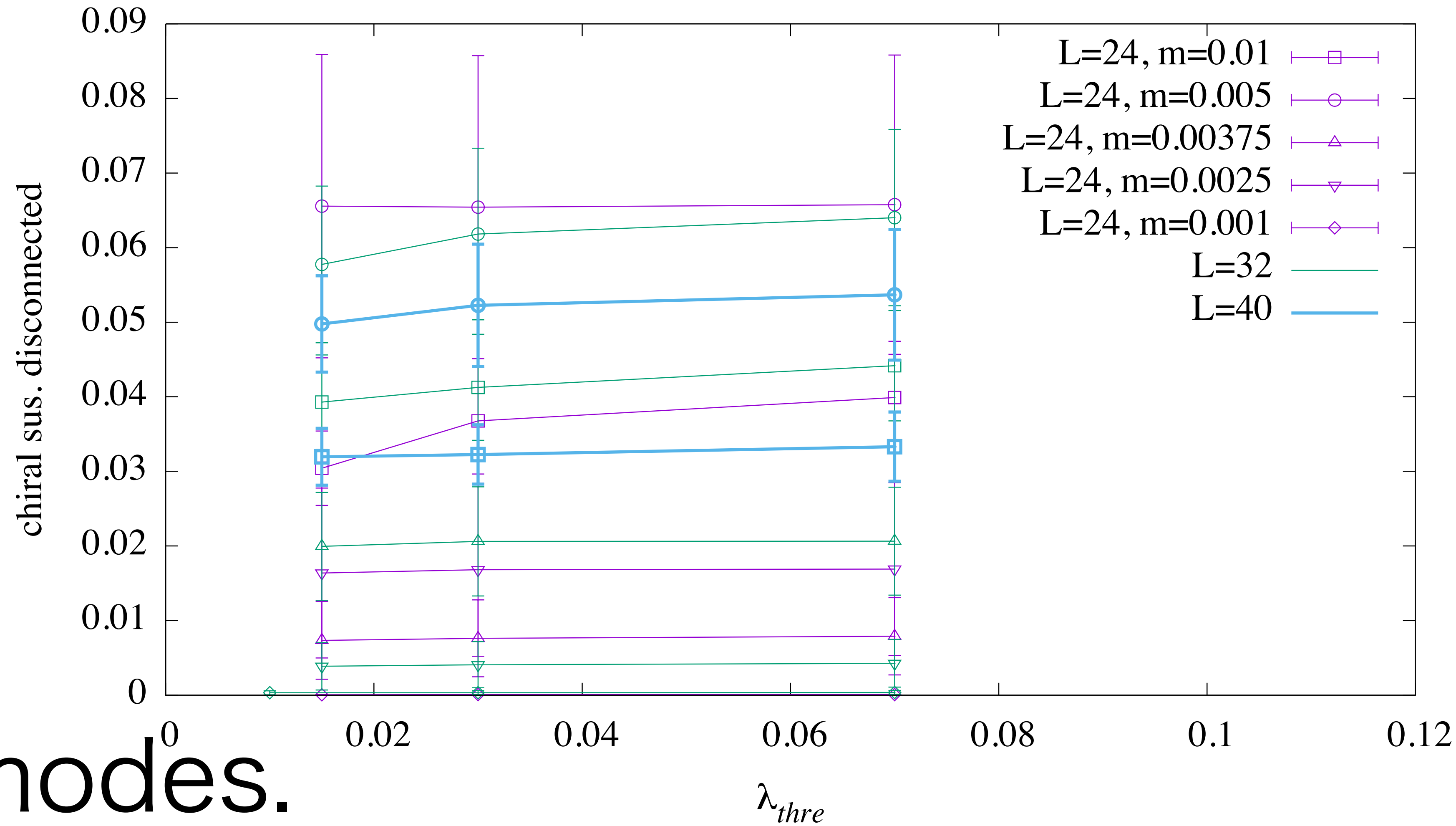
we truncate at 30-40th lowest mode

($\lambda_{\mathrm{threshold}} \sim 150\text{–}300\ \mathrm{MeV}$).

Low mode approximation



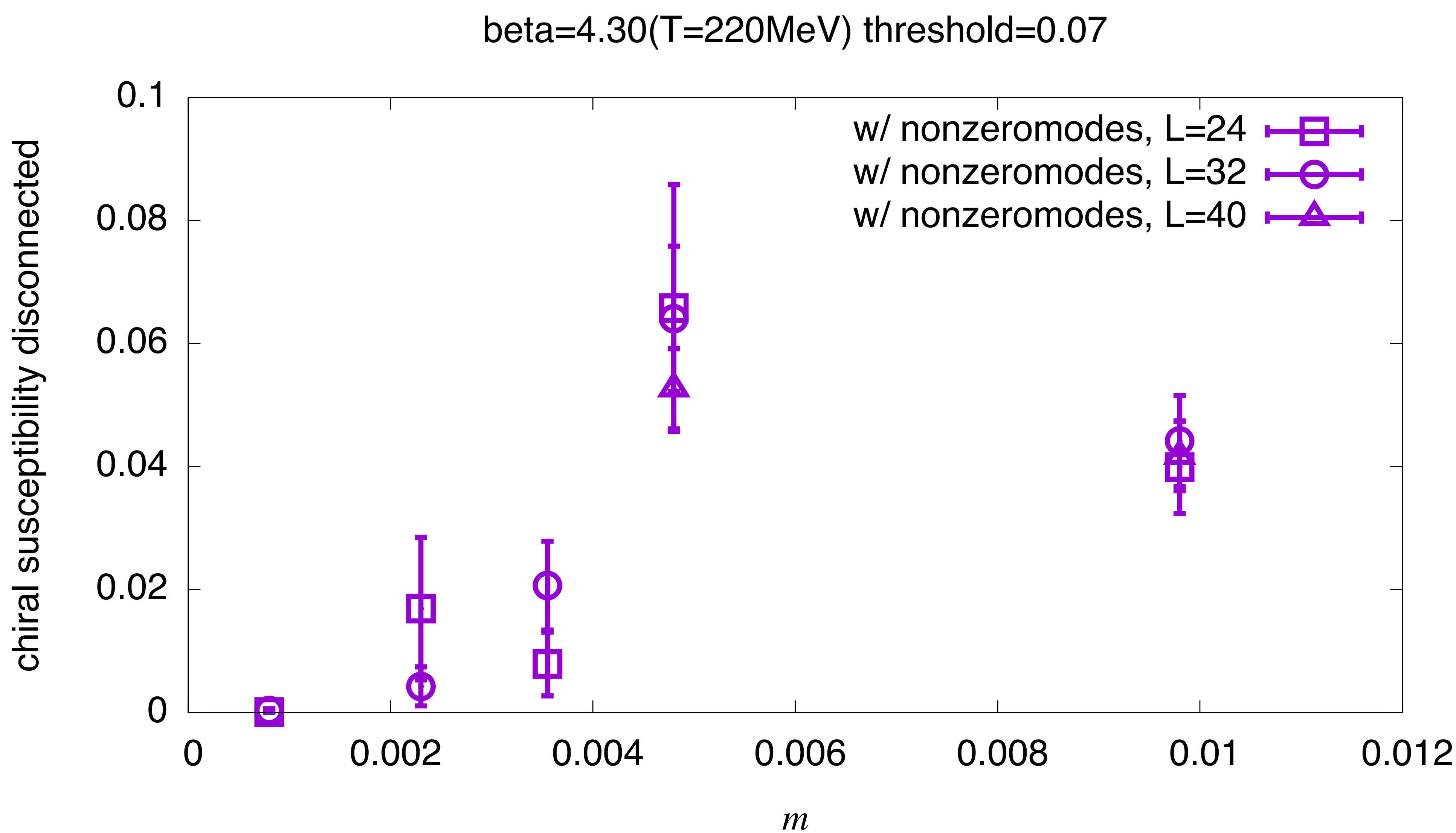
connected
(we do not use)



disconnected part is well described by 30-40 lowest modes.

Disconnected susceptibility at T=220MeV

$$\chi^{dis.}(m) = \frac{N_f}{V}\left[\left\langle\left(\sum_\lambda \frac{m}{\lambda(A)^2+m^2}\right)^2\right\rangle - \left\langle\sum_\lambda \frac{m}{\lambda(A)^2+m^2}\right\rangle^2\right].$$



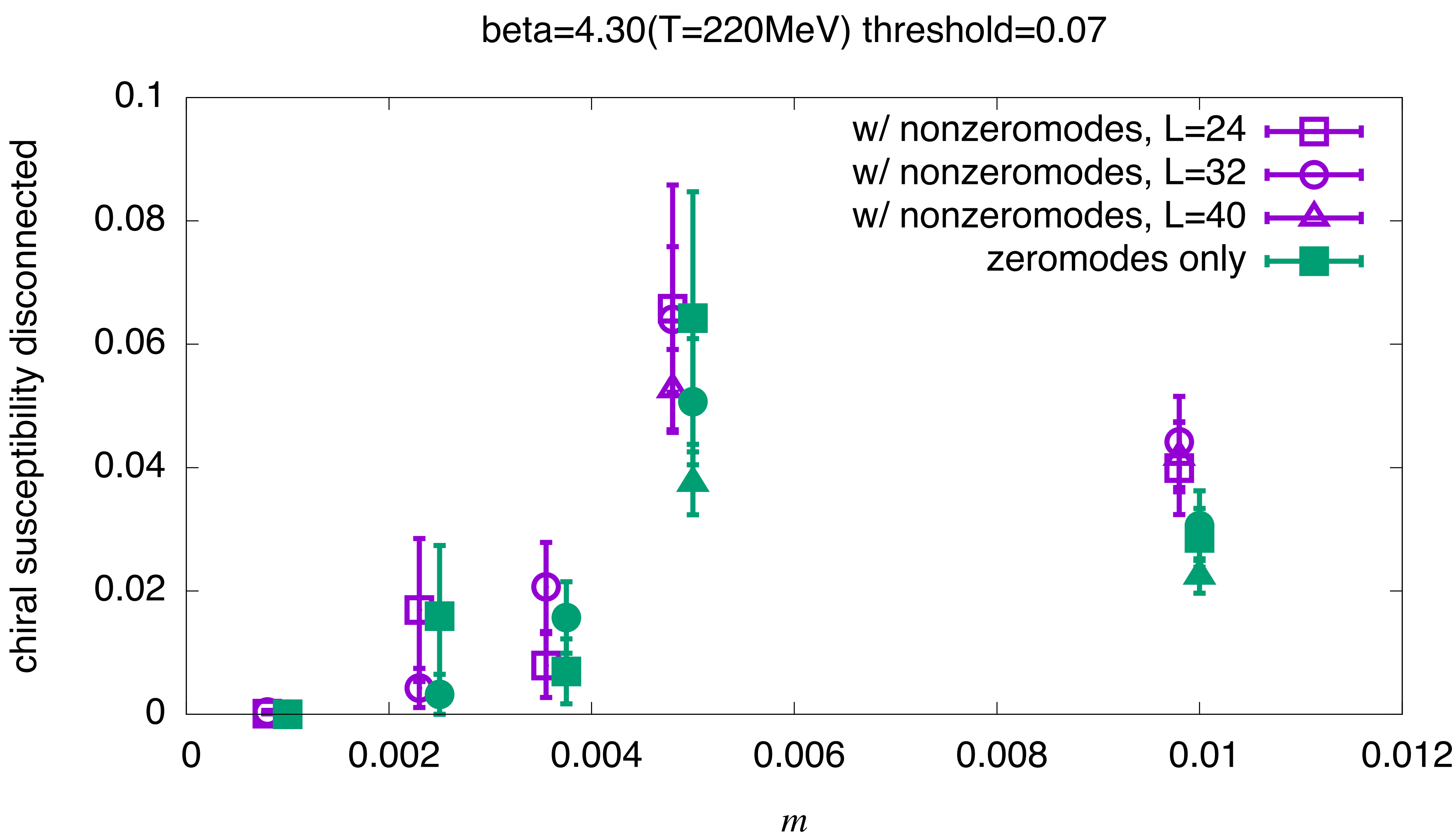
Data on L=1.8-3.6fm lattices are consistent.

Pseudo-peak at m=0.005 (14MeV)?

Disconnected susceptibility at T=220MeV

$$\chi^{dis.}(m) \sim \frac{N_f}{V}\frac{\langle N_0^2\rangle - \langle N_0\rangle^2}{m^2}$$

N_0 : number of zero modes

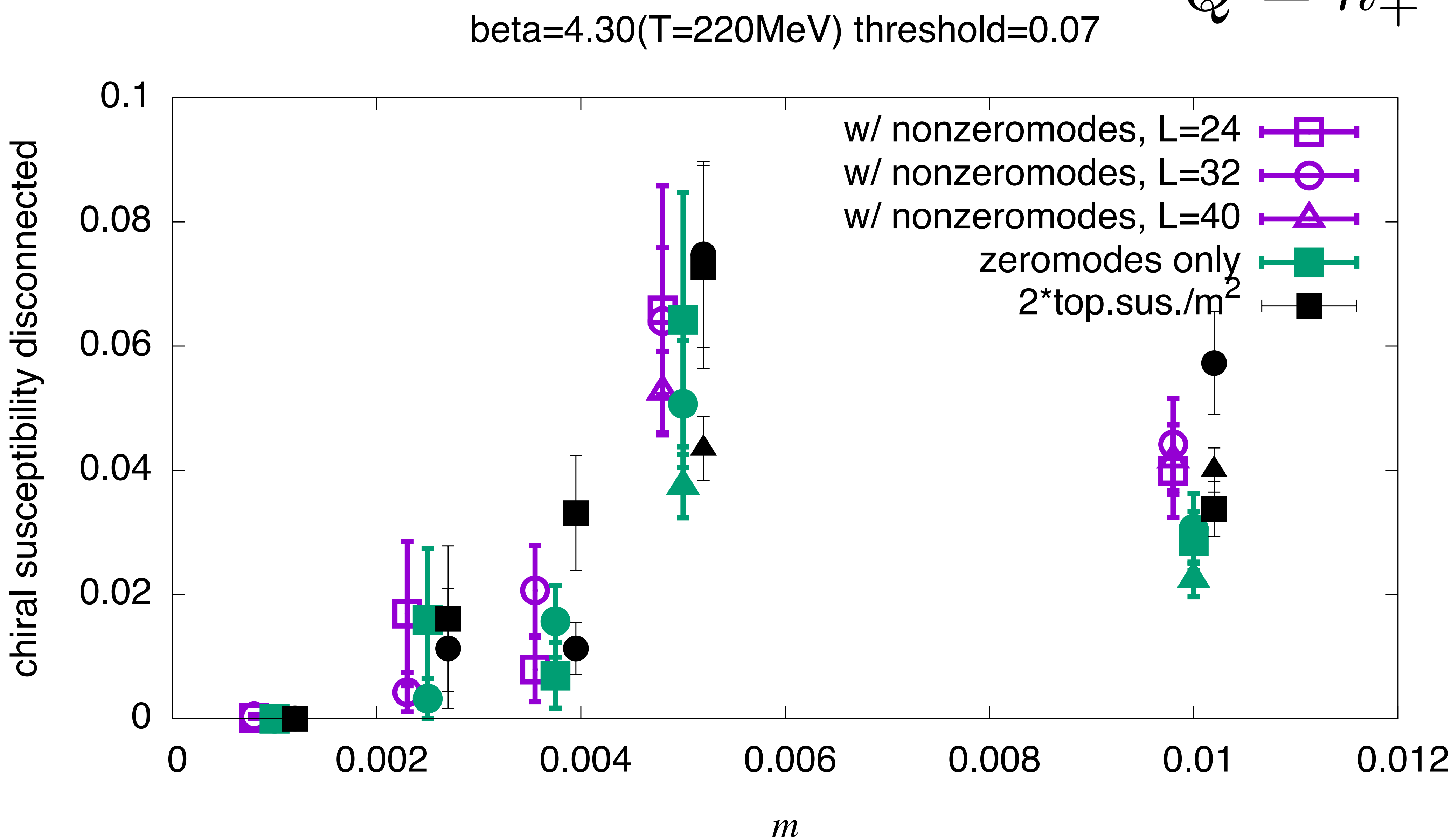


is dominated by chiral zero modes.

Disconnected susceptibility at T=220MeV

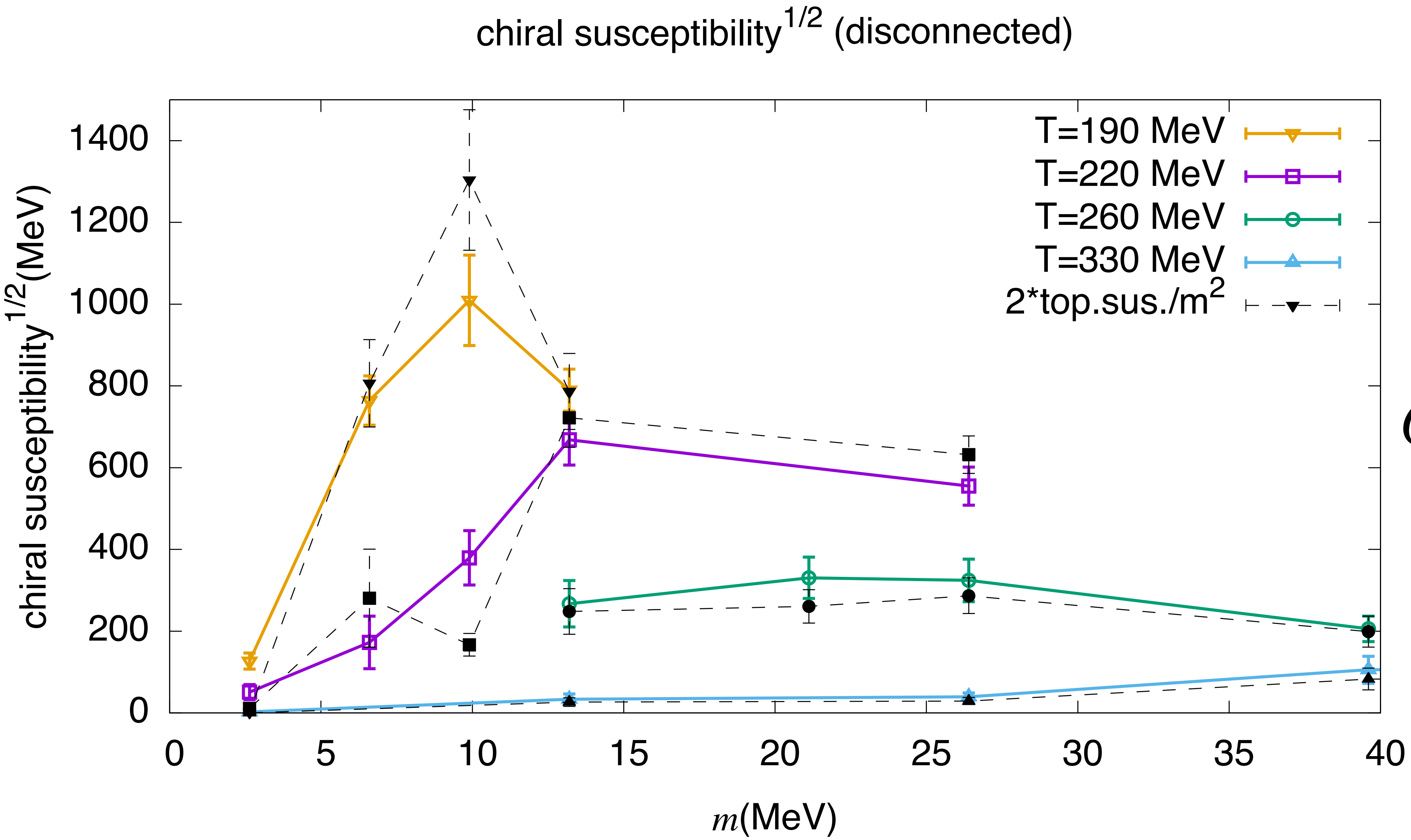
$$\chi^{dis.}(m) \sim \frac{N_f}{V}\frac{\langle N_0^2\rangle - \langle N_0\rangle^2}{m^2} \sim \frac{N_f}{V}\frac{\langle Q^2\rangle - \langle Q\rangle^2}{m^2}$$

$$Q = n_+ - n_-$$



is dominated by U(1) anomaly !

Disconnected part at different T



$$\chi^{dis.}(m)$$

$$\sim \frac{N_f}{V}\frac{\langle Q^2\rangle - \langle Q\rangle^2}{m^2}$$

Q : instanton number

The dominance by topological susceptibility is seen at 4 different temperatures.

Moreover, the chiral limit is consistent with zero.

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✔ 4. Numerical results

Disconnected chiral susceptibility is dominated by axial U(1) anomaly (topological susceptibility).

5. Summary (and discussion)

Summary

1. Chiral condensate/susceptibility are related to both SU(2)xSU(2) and $U(1)_A$.
2. In the Dirac eigenmode decomposition of exactly chiral symmetric Dirac op, we can separate the purely U(1) anomaly effect.
3. Connected part = condensate/m + axial U(1) susceptibility.
4. Disconnected part $\sim \frac{2\langle Q^2\rangle}{m^2 V}$ $\quad Q : \text{instanton number}$
5. Axial U(1) anomaly may play more important role in QCD phase transition than expected.

May not be a surprise …

* A long ago, people tried to explain the SU(2)xSU(2) breaking by instantons=U(1) anomaly

[Polyakov,'t Hooft, Shuryak…] [Cf. talk by Hamada].

$$\chi^{dis.}(m) \sim \frac{N_f}{V}\frac{\langle Q^2\rangle - \langle Q\rangle^2}{m^2}$$

Q : instanton number

may be its revival.

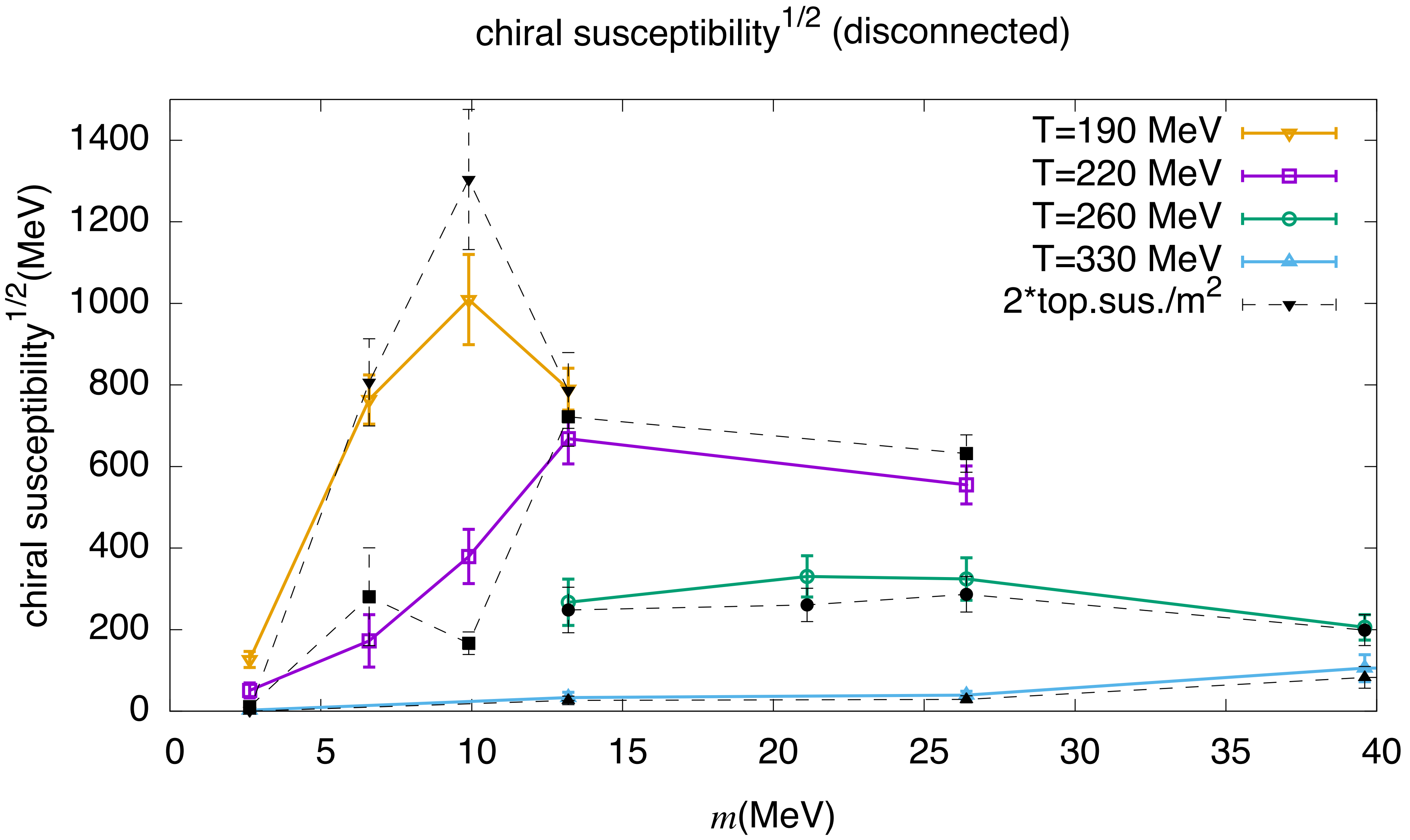
But typical confs with Q≠0 is far from instanton semi-classical "solutions".

* Also, $\chi(m=0) = \mathrm{U}(1)$ anomaly

was also indicated from Ward-Takahashi identities

[LLNL/RBC Collaboration 2013]

What’s still nontrivial?

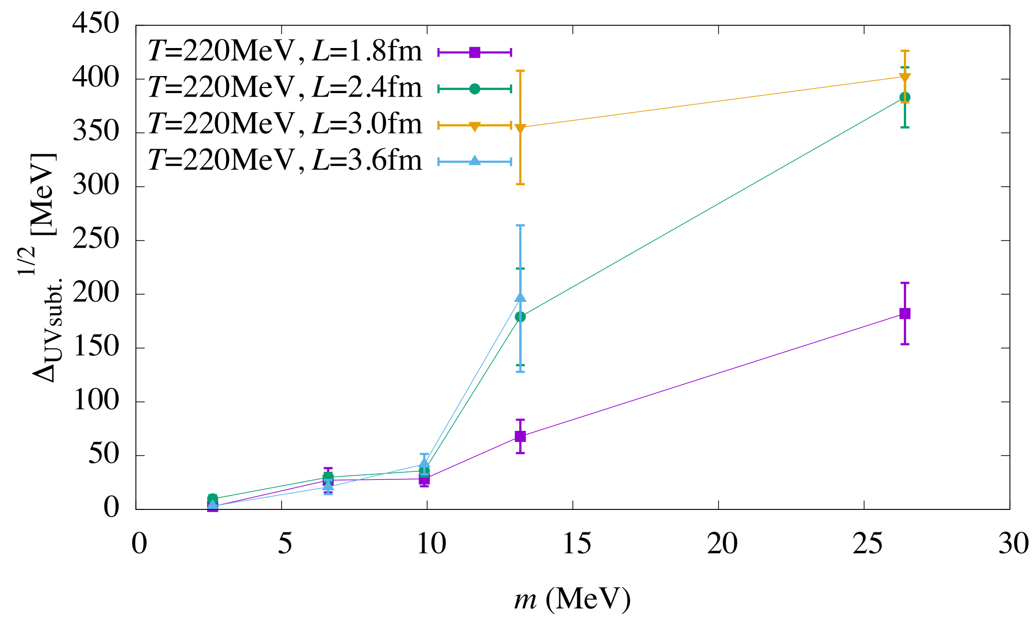


* dominance by topological susceptibility,
* seen at finite m up to 40 MeV,
* suppression in the chiral limit, consistent with zero.

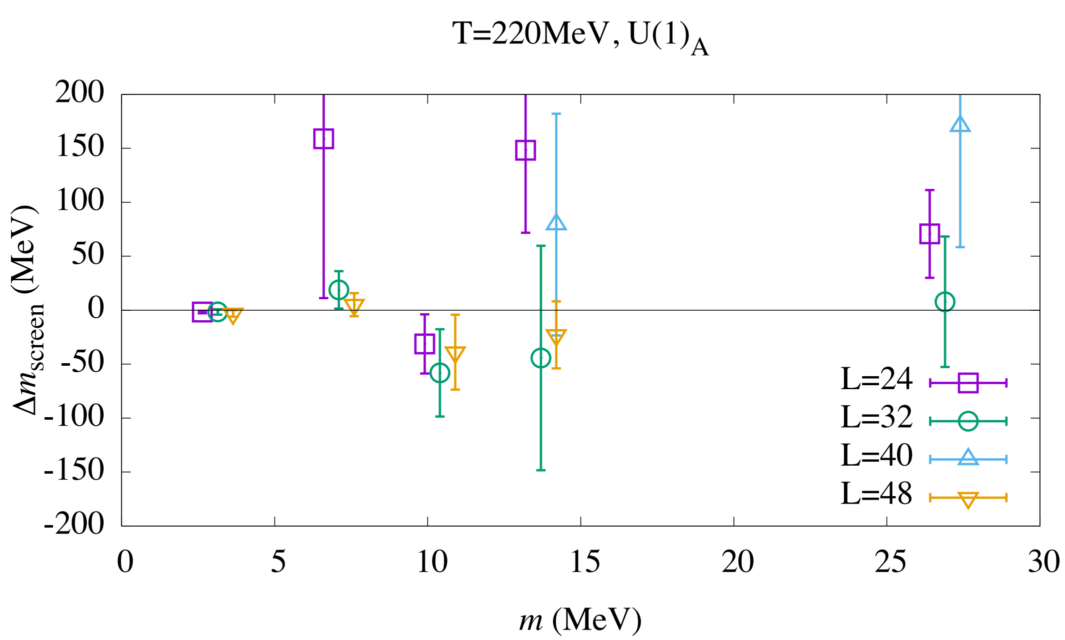
Disappearance of U(1) anomaly

are seen in other observables.

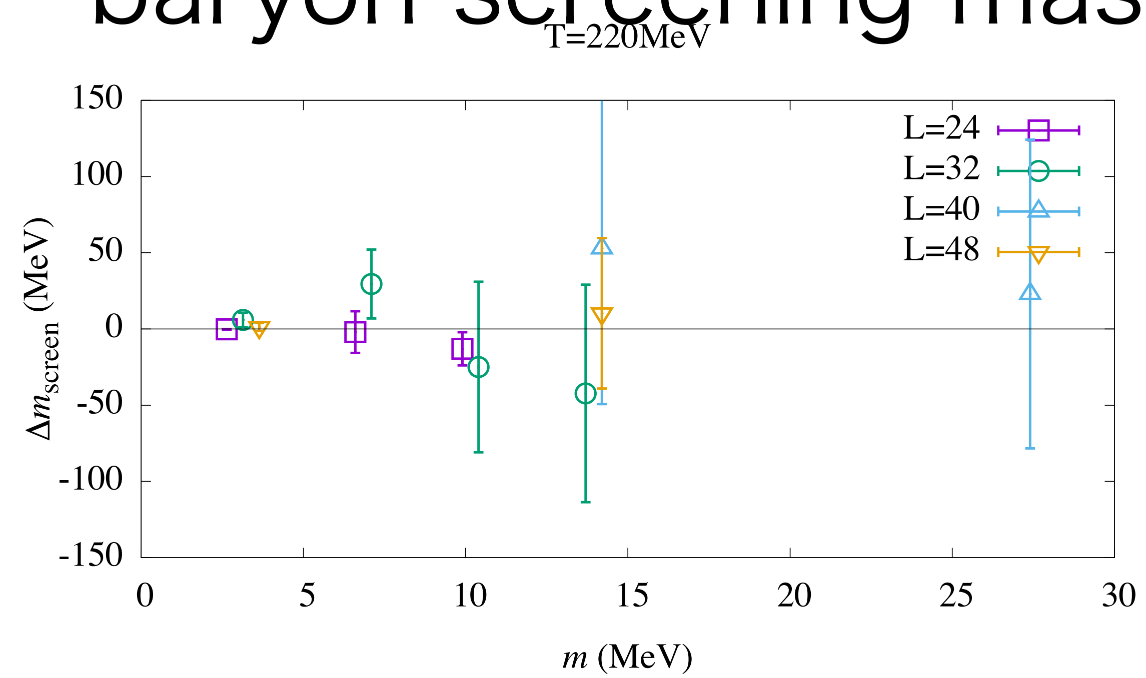
U(1) susceptibility



meson screening mass



baryon screening mass



What if axial U(1) “restored”?

Not only $SU(2)_L x SU(2)_R$ but also $U(1)_A$ may be restored at Tc.

Then, the effective action = SU(2)xSU(2)[or O(4)] linear sigma model needs additional degrees of freedom.

-> effective potential becomes complicated

-> 1st-order transition is favored [Pisarski & Wilczek]

(the same suggestion as Yonekura-san’s but from different point of view.)

What if chiral phase transition is 1st order?

* 1st order region may be spanned to finite quark mass.
* If physical point is not a crossover but 1st order, QCD may explain dark matter (Witten) -> ask Yonekura-san
* Gravitational waves due to QCD bubbles?
* Axion dark matter scenario may be difficult (abundance is too big).

Interesting! But we have not detected its sign.

CAN U(1)ANOMALY DISAPPEAR AT FINITE T? → MANY ANSWERS.

Before 2012

Cohen 1996, 1998 (theory)
Bernard et al. 1996 (staggered)
Chandrasekharan et al. 1998 (staggered)
HotQCD 2011 (staggered)
Ohno et al. 2011 (staggered)
and many others

After 2012

HotQCD 2012 (Domain-wall)
Aoki-F-Taniguchi 2012 (theory)
Ishikawa et al2013, 2014,2017. (Wilson)
JLQCD 2013, 2016 (overlap)
TWQCD 2013 (optimal DW)
LLNL/RBC 2013 (Domain-wall) [may be at higher T]
Pelisseto and Vicari 2013(theory)
Nakayama-Ohtsuki 2015, 2016(CFT)
Sato-Yamada 2015(theory),
Kanazawa & Yamamoto 2015, 2016 (theory)
Dick et al. 2015 (OV in HISQ sea)
Sharma et al. 2015, 2016 (OV in DW sea)
Glozman 2015, 2016 (theory)
Borasnyi et al. 2015 (staggered & OV)
Brandt et al. 2016 (Wilson)
Ejiri et al. 2016 (Wilson)
Azcoiti 2016,2017(theory)
Gomez-Nicola & Ruiz de Elvira 2017 (theory)
Rorhofer et al. 2017(Mobius DW) ……

Red: YES
Blue: NO
Green: Not (directly) answered but related

Take-home message

あなたが $SU(2)_L \times SU(2)_R$ の

order parameterだと思っている

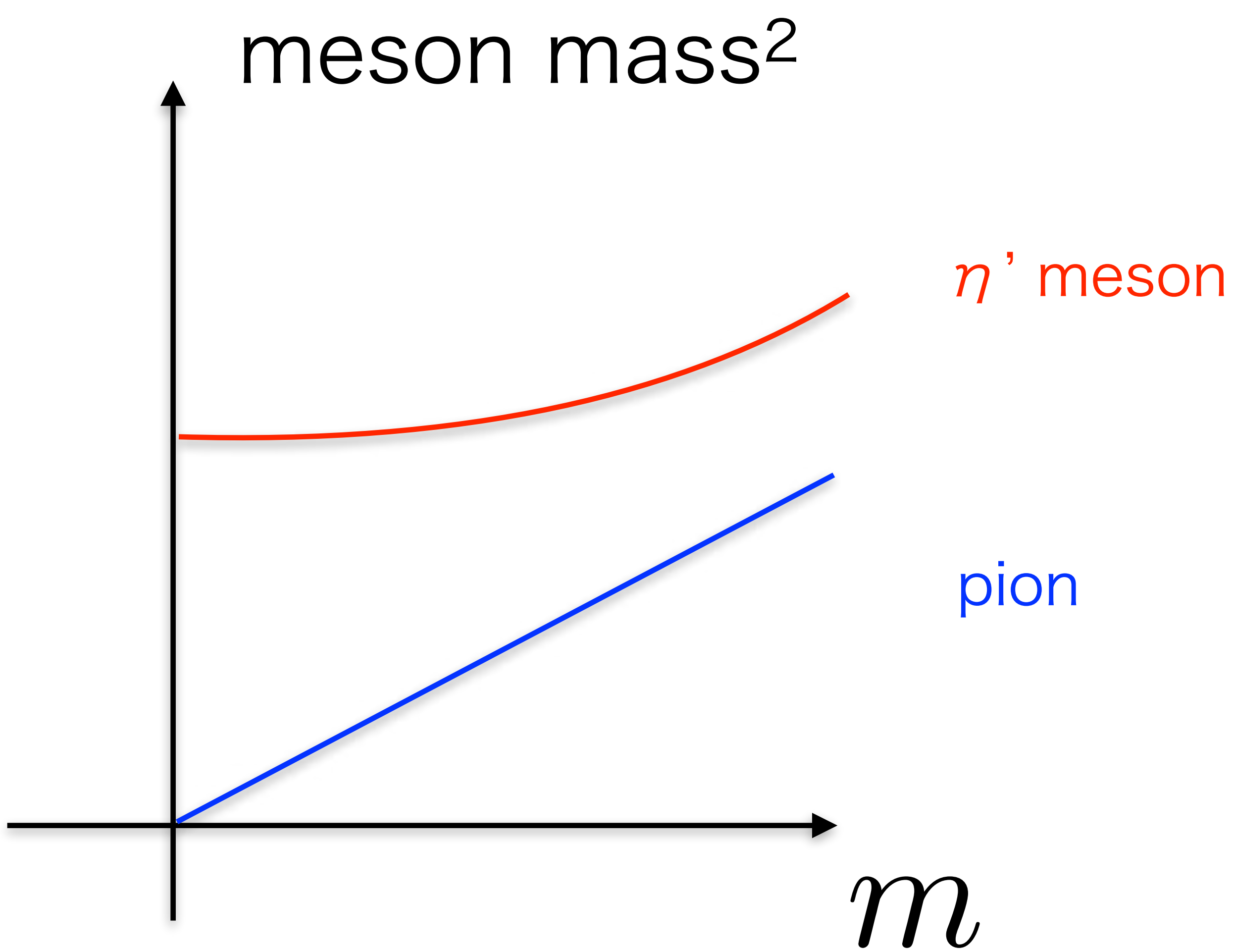
$$\frac{\partial}{\partial m}\langle\bar{\psi}\psi\rangle$$

実は $U(1)_A$ 量子異常まみれ

かもしれません…。

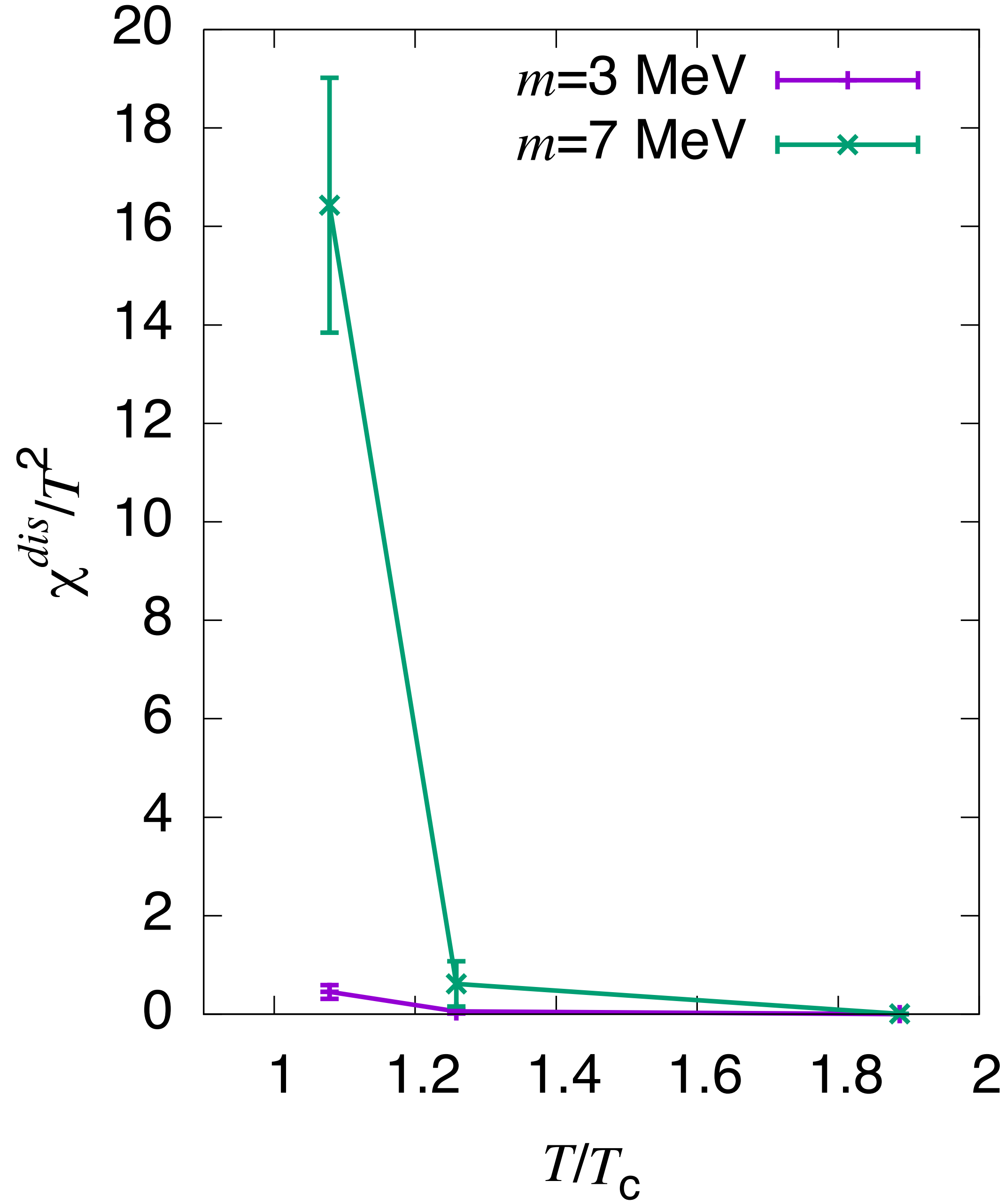
Back-up slides

What we know at T=0



Temperature dependence

Nf=2 JLQCD 2020 preliminary



Cf. Nf=2+1 result by HotQCD 2011