２カラーQCDの低温高密度領域における物理

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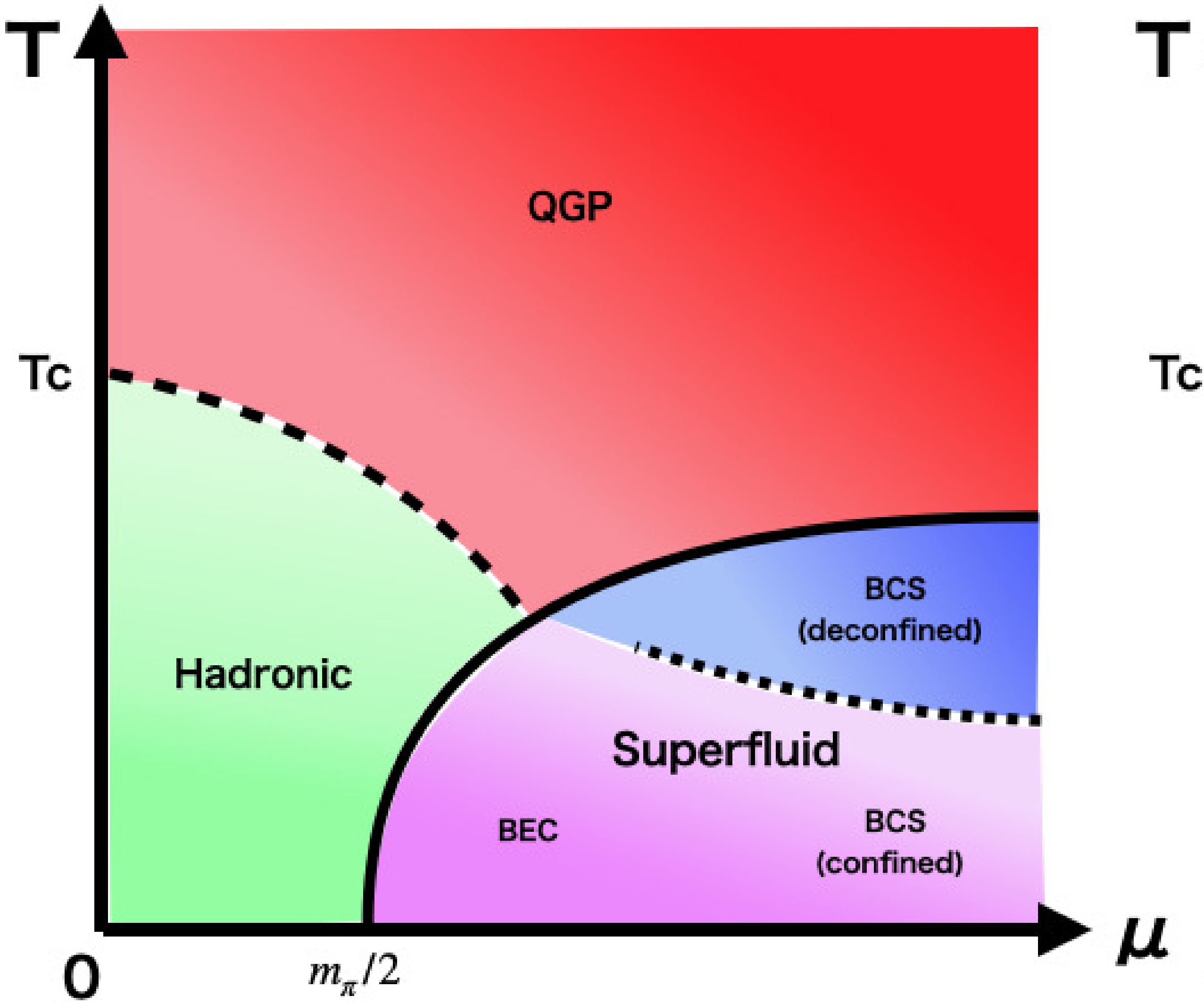
Refs:
(1) K.Iida, EI, T.-G. Lee: JHEP2001(2020)181 (arXiv:1910.07872)
(2) K.Iida, EI, T.-G. Lee:arXiv:2008.06322
(3) T.Furusawa, Y.Tanizaki, EI:PRResearch 2(2020)033253

素粒子物理学の進展2020 , online, 2020/09/01

Reference (3):
Finite-Density Massless Two-Color QCD at Isospin Roberge-Weiss Point and 't Hooft Anomaly
T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

Results by lattice simulations

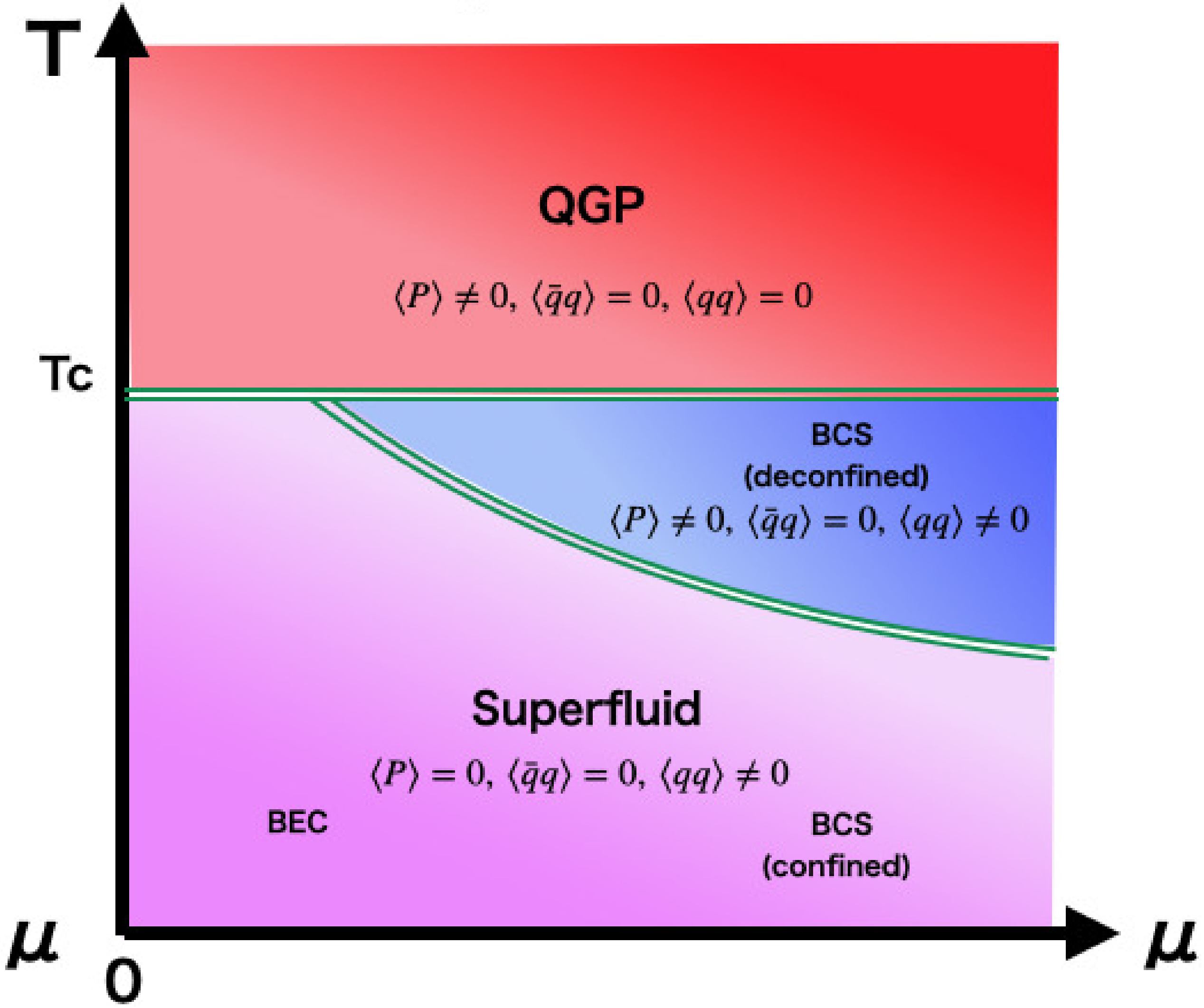
Nf=2 massive QC_2D



Results by anomaly matching

Nf=2 massless QC_2D

at Isospin RW Point



右図については、東工大(西田研)D2の古澤くんへ

Plan of talk

1. Why 2color QCD?
   - Sign problem and numerical-instability problem

2. Criterion of phase diagram
   - spontaneous flavor symmetry breaking in Nc=Nf=2

3. Simulation results
   - Phase diagram at T=0.45Tc, 0.89Tc
   - Topological susceptibility

4. Summary

Action of finite density QCD

Fermion action in continuum limit

$$S_F^{cont.} = \underbrace{\int d^4x\,\bar{\psi}(x)(\gamma_\mu D_\mu + m)\psi(x)}_{\text{QCD}} + \underbrace{\mu\hat{N}}_{\text{Number op.}}$$

$$\hat{N} = \bar{\psi}\gamma^0\psi$$

$\mu\ (=\mu_q)$: quark chemical potential

3-color QCD : $\mu_q = \mu_B/3$ where μ_B is baryon chemical potential

2-color QCD : $\mu_q = \mu_B/2$ where μ_B is baryon chemical potential

動機

少数のクォークや核子の系は大体わかった...！

低温領域：

クォークの閉じ込め、カイラル対称性の破れ、インスタントンの存在

ハドロンの質量をQCDから計算(格子計算で３つのパラメータから多数のハドロンの質量を再現)

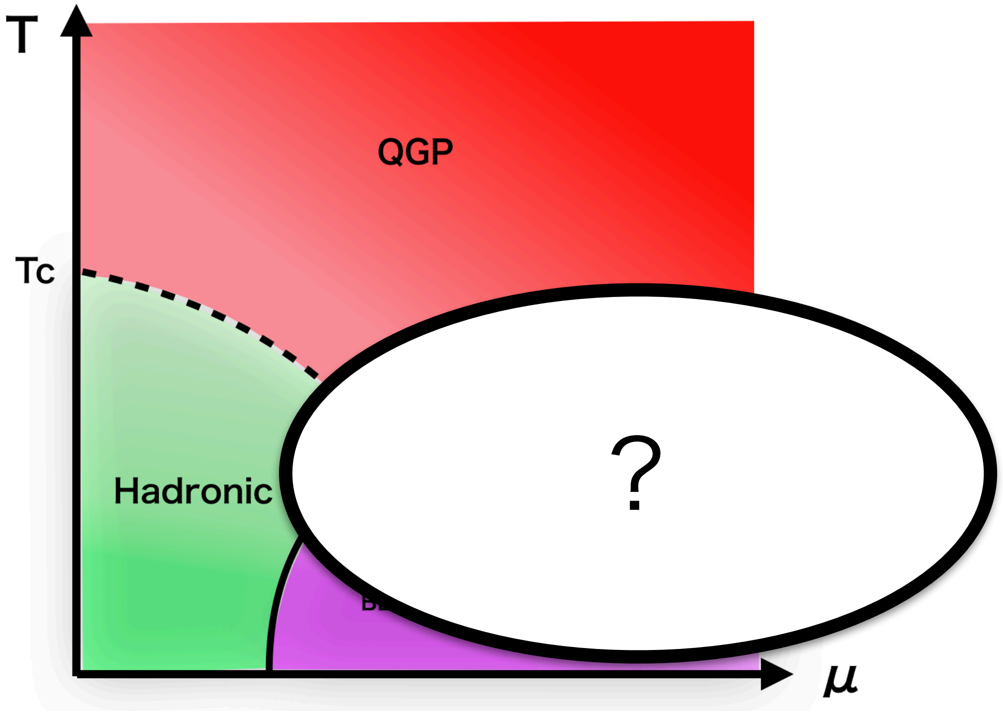
ハドロン間の相互作用(ポテンシャル描像)も格子計算で第一原理計算で求められるようになった(HAL QCD法、Luescher法)

高温領域：

クォーク・グルオンが非閉じ込め(QGP相：格子計算とRHIC実験)

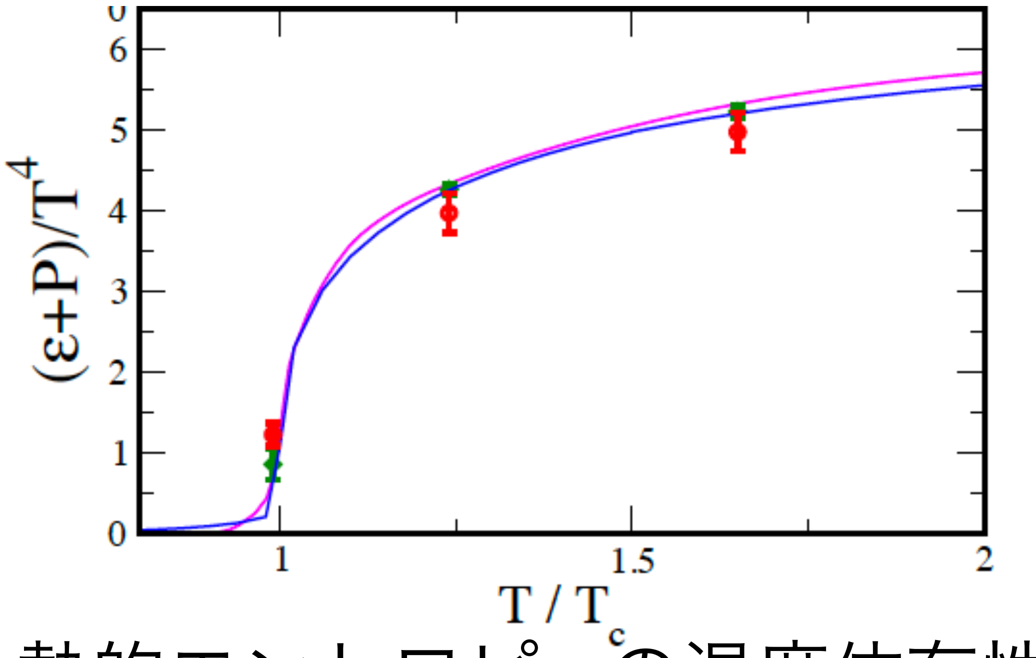
カイラル対称性の回復

状態方程式、輸送係数の温度依存性(格子計算/RHIC実験による完全流体描像)



QCD相図

Asakawa, Hatsuda, EI, Kitazawa, Suzuki : Phys. Rev. D 90, 011501 (2014)



熱的エントロピーの温度依存性

動機

多数のクォークが詰まった有限密度系は….?

実際の物理系は存在するのに、理論的に理解するのは難しい…

LHCb, RHIC (中間密度、高温領域)

中性子星の中 (高密度、低温領域)

LIGO　　NICER

中性子星の中は
どうなっているか

「中性子星の中は
どうなっているか」

日経サイエンス2020年1月号

非常に高密度ではフェルミ縮退圧を下げるため
クォークはボソンを作り凝縮している….？

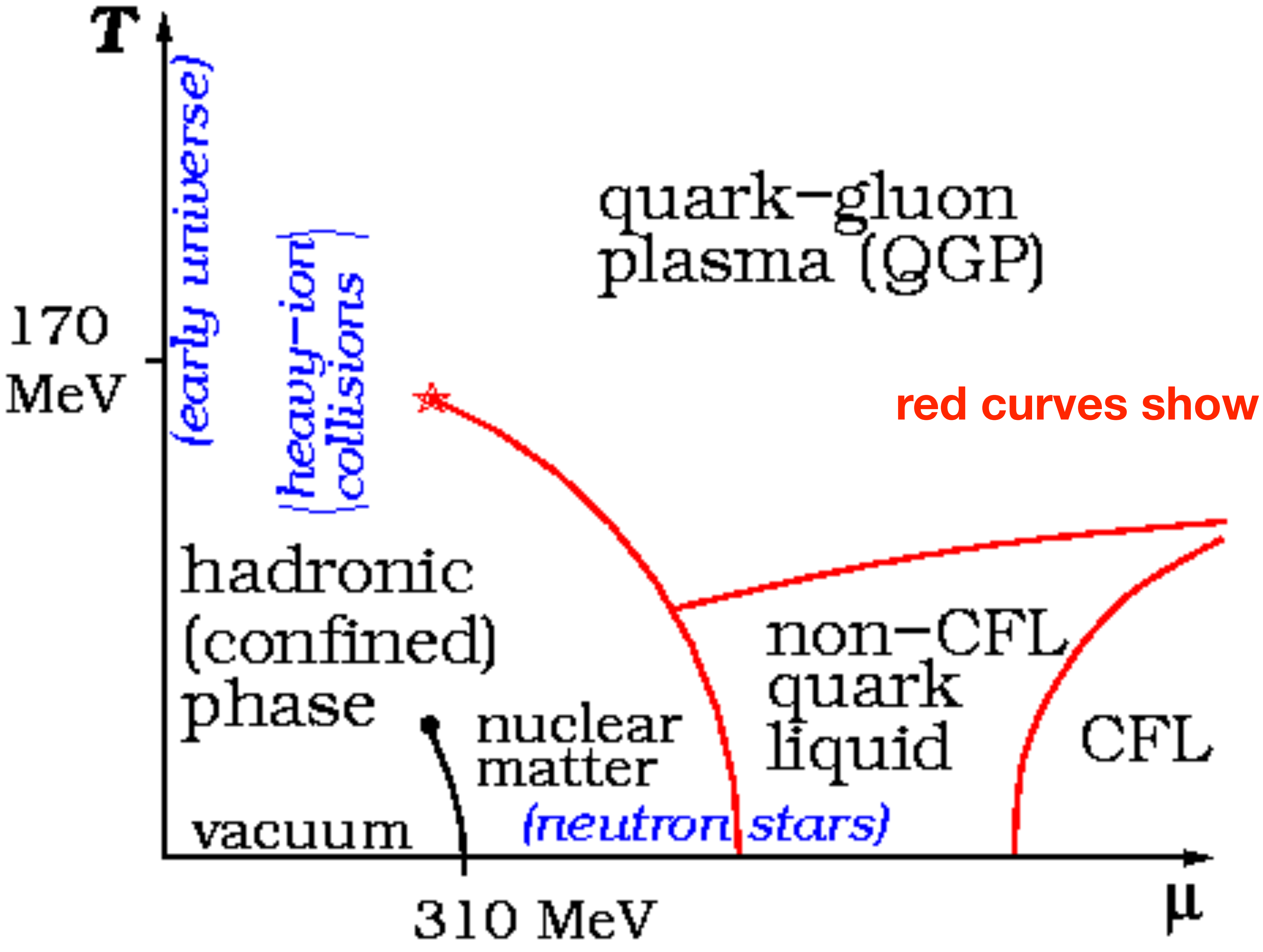
何を知りたいか？

- 温度密度に依存した相図
- インスタントンの有無など非摂動的性質
- 核力、ハドロン質量の密度依存性
- 状態方程式(圧力、内部エネルギー、エントロピー)
- 輸送係数 (粘性、超流動密度)

ゼロ密度QCDで成功した格子計算の有限密度への拡張？

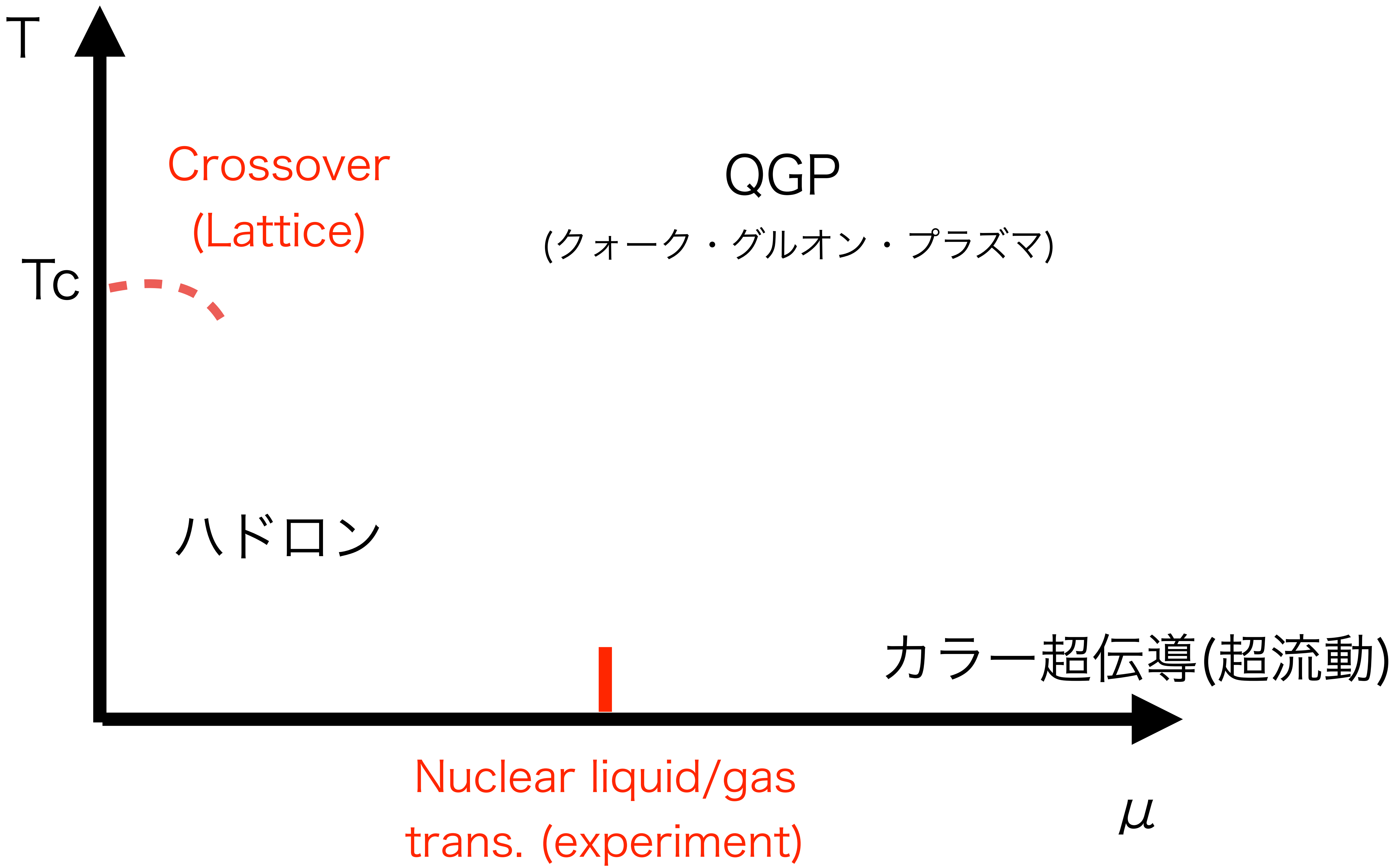
Schematic picture

QCD phase diagram in Wikipedia



red curves show the conjectures

What is really known…



T

Crossover
(Lattice)

QGP
(クォーク・グルオン・プラズマ)

Tc

ハドロン

カラー超伝導(超流動)

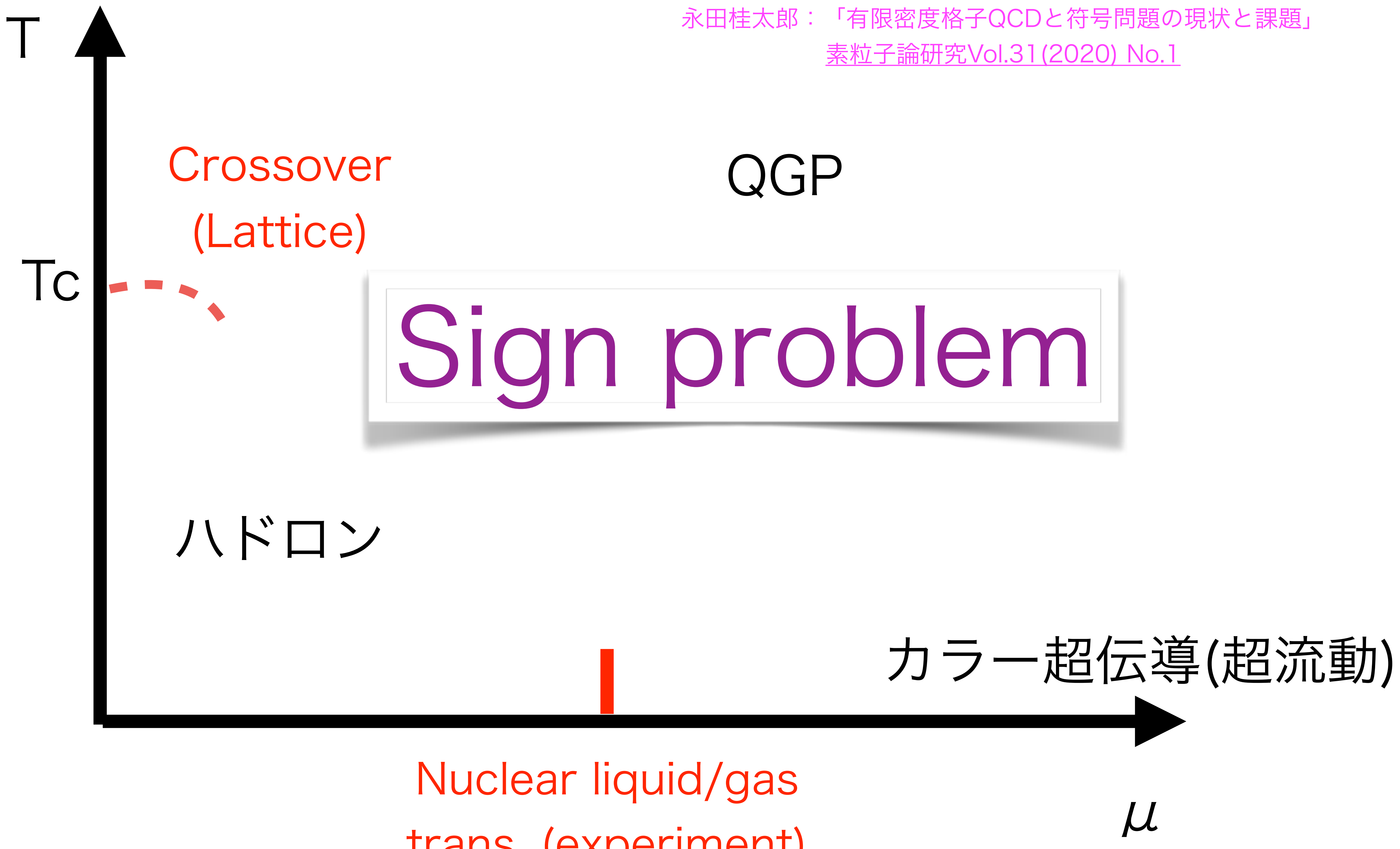
Nuclear liquid/gas
trans. (experiment)

μ

Pochodzalla et al. PRL75 (1995) 1040

What is really known…

永田桂太郎：「有限密度格子QCDと符号問題の現状と課題」
素粒子論研究Vol.31(2020) No.1



T

Crossover
(Lattice)

QGP

Tc

Sign problem

ハドロン

カラー超伝導(超流動)

Nuclear liquid/gas
trans. (experiment)

μ

Pochodzalla et al. PRL75 (1995) 1040

Two problems in finite-density QCD simulations

(1) sign problem

$$\langle \mathcal{O} \rangle = \frac{1}{Z}\int DUD\psi\mathcal{O}\boxed{e^{-S_g-\int\bar{\psi}D\psi}} = \frac{1}{Z}\int DU\mathcal{O}\boxed{(\det D)^{N_f}e^{-S_g}}$$

確率重みとするなら real-positive でないといけない

In zero density($\mu = 0$), $D^\dagger = \gamma_5 D \gamma_5$ ➡ $\det D$ real

In non-zero density($\mu \neq 0$), $\Delta(-\mu)^\dagger = \gamma_5 \Delta(\mu) \gamma_5$ ➡ $\det \Delta(\mu)$ complex

In two-color QCD $\det \Delta(\mu)$ is real (positive or negative),

since the fundamental reps. of SU(2) takes a pseudo-real reps.

(2) numerical instability

in the low-T and high-density regime:

$\mu/m_{PS} \geq 1/2$ in low-T

m_{PS}: pseudo-scalar (pion) mass at $\mu = 0$

Dynamical pair-creation and annihilation frequently occur,

then system becomes unstable

Action with diquark source term

Fermion action in continuum limit

$$S_F^{cont.} = \underbrace{\int d^4x \bar{\psi}(x)(\gamma_\mu D_\mu + m)\psi(x)}_{\text{QCD}} + \underbrace{\mu \hat{N}}_{\text{Number op.}} - \underbrace{\frac{j}{2}(\bar{\psi}_1 K \bar{\psi}_2^T - \psi_2^T K \psi_1)}_{\text{diquark source}}$$

Related works on Nc=2 with even # flavor
Kogut et al. NPB642 (2002)18, **Alles et al.** NPB752 (2006)124,
Hands et al. NPB752 (2006) 124, PRD81 (2010) 091502,, EPJ. A47 (2011) 60, PRD87 (2013) 034507, **Kotov et al**. PRD94 (2016) 114510, JHEP 1803 (2018) 161

The QCD phase diagram appears in the j->0 limit

Fermion action on the lattice

$$\det[\mathcal{M}^\dagger \mathcal{M}]^{1/2} = \det[\Delta^\dagger(\mu)\Delta(\mu) + |\bar{J}|^2]^{1/2} \det[\Delta^\dagger(-\mu)\Delta(-\mu) + |J|^2]^{1/2}$$

j-source lifts the eigenvalue of Dirac op. up

Our strategy

(qualitatively) understand the QCD phase diagram

at low-T and high density

(1) sign problem

Avoid the sign problem (consider 2color 2flavor QCD)

(2) Numerical instability $\mu/m_{PS} \geq 1/2$ in low-T

Introduce the diquark source in the action

cf.) diquark -> π^- in 3-color QCD with isospin chemical

D. H. Rischke, D. T. Son and M. A. Stephanov, Phys. Rev. Lett.87(2001) 062001
D. T. Son and M. A. Stephanov, Phys. Atom. Nucl.64(2001) 83
B. B. Brandt, G. Endrodi and S. Schmalzbauer, Phys. Rev.D 97(2018) 05451

2 color QCD vs 3 color QCD

(少なくとも$\mu = 0$で)定性的には同じ

低温領域：

クォークの閉じ込め、

カイラル対称性の破れ(注: massless 2カラーQCDではU(1)Bが破れてカイラルが回復することが可能)

インスタントンの存在

ハドロンの質量スペクトルの順番

高温領域：

クォーク・グルオンが非閉じ込め

カイラル対称性の回復

状態方程式、輸送係数の温度依存性

定量的にもそんなに違わない…?

(例) pure SU(N) ゲージ理論の

トレースアノマリー($\Delta = (\epsilon - 3p)$)

(コメント)

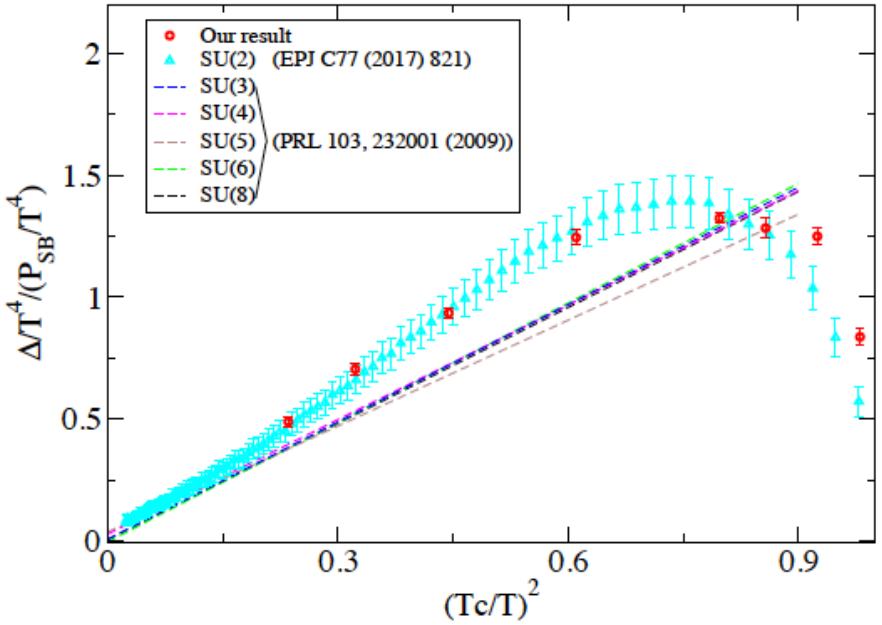
QCD phase diagramの両軸 : T とμ [MeV]

物理スケールはクォーク質量やフレーバー数に強く依存

ユニバーサルには

縦軸：T/Tc　横軸：μ/m_{PS} を使うと良い

T. Hirakida, EI, H. Kouno, PTEP **2019** (2019) 033B01



Plan of talk

1. Why 2color QCD?

   Sign problem and numerical-instability problem

2. Criterion of phase diagram

   spontaneous flavor symmetry breaking in Nc=Nf=2

3. Simulation results

   Phase diagram at T=0.45Tc, 0.89Tc

   Topological susceptibility

4. Summary

What is a good order parameter to see a phase in high density region?

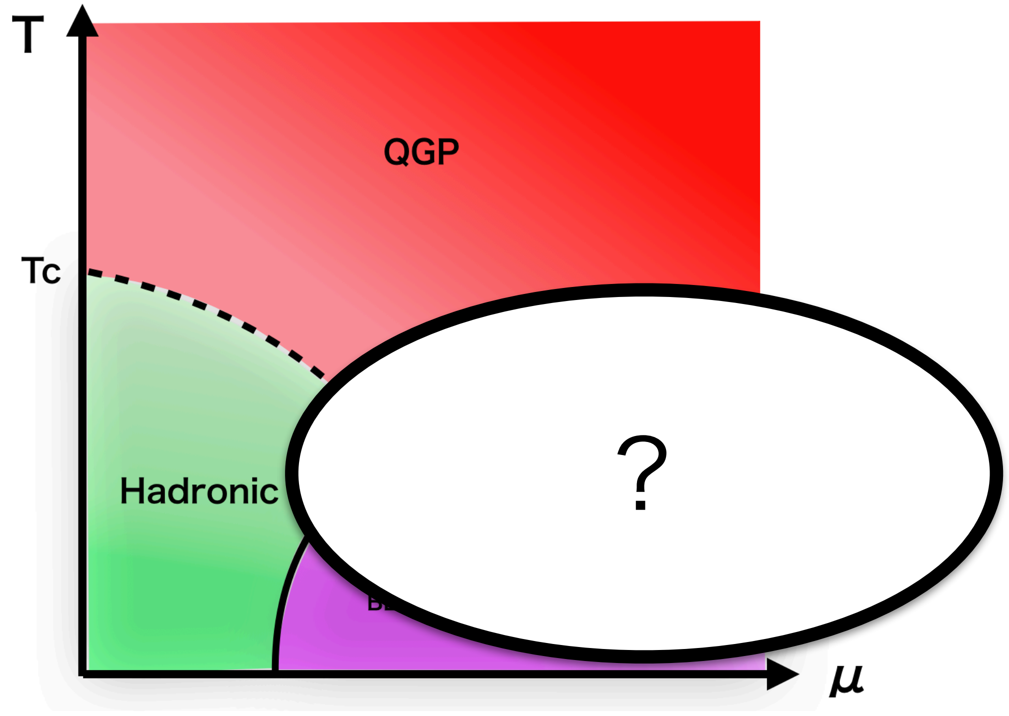
At $\mu = 0$, QCD has two phase transitions

- Confinement (low T)/deconfinement (high T)

 (approximate) order parameter: Polyakov loop

- Chiral symmetry broken (low T)/restoration (high T)

 (approximate) order parameter: chiral condensate



QCD相図

Two phase transition temperatures are (almost) the same (Tc)

Flavor symmetry and its breaking

Nf=2 Nc=3

standard symmetry

$SU(2)_L \times SU(2)_R \times \cancel{U(1)_A} \times U(1)_B$

$m = 0, \mu = 0$

$\Big\downarrow$ **chiral condensation**

$SU(2)_V \times U(1)_B$

Nf=Nc=2

enhanced symmetry

$SU(4)$ $m = 0, \mu = 0$

$\Big\downarrow$ explicit breaking $m > 0$ $\mu > 0$

$SU(2)_V \times U(1)_B$

meson-baryon sym.

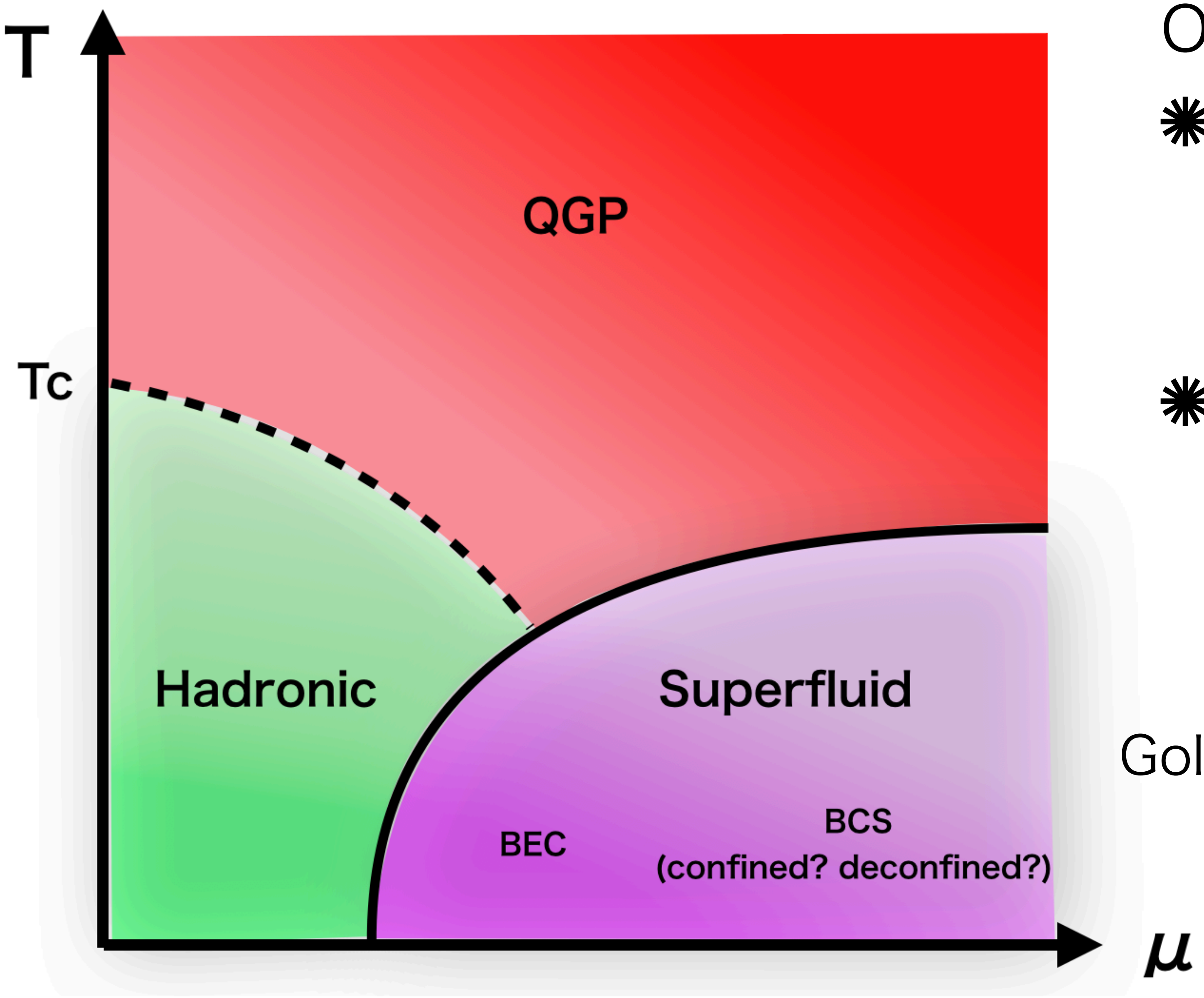
$\Big\downarrow$ **diquark condensation**

$$\psi \to e^{i\alpha}\psi$$
$$\bar{\psi} \to \bar{\psi}e^{-i\alpha}$$

$Sp(1)_V \simeq SU(2)_V$

diquark condensate in finite mu regime plays an alternative role of chiral condensate in zero mu regime.

Expected phase diagram in Two-color QCD



Order parameters

- ✳ Polyakov loop

$\langle |L| \rangle \sim 0$ confined

$\langle |L| \rangle \neq 0$ deconfined

- ✳ (Isoscalar) diquark cond.

$\langle qq \rangle = 0$ no superfluidity

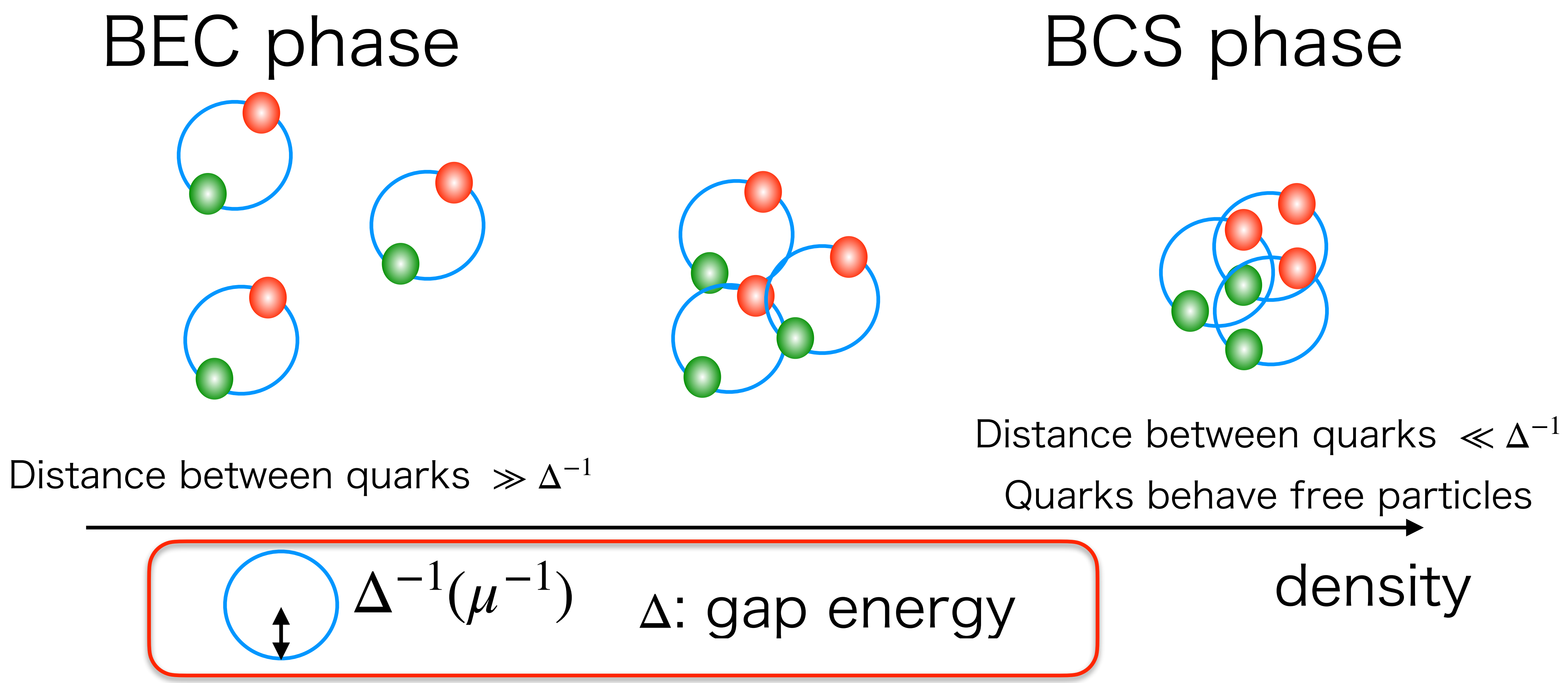
$\langle qq \rangle \neq 0$ superfluidity

Goldstone mode of $U(1)_B$ sym. breaking

$$\psi \to e^{i\alpha}\psi \quad \bar{\psi} \to \bar{\psi}e^{-i\alpha}$$

	Hadronic	QGP	Superfluid	
			BEC	BCS
$\langle \| L \| \rangle$	zero	non-zero		
$\langle qq \rangle$	zero	zero	non-zero	$\propto \mu^2$
$\langle n_q \rangle$				

BEC / BCS crossover in superfluid phase



BEC phase

BCS phase

Distance between quarks $\gg \Delta^{-1}$

Distance between quarks $\ll \Delta^{-1}$

Quarks behave free particles

density

$\Delta^{-1}(\mu^{-1})$ Δ: gap energy

Number density of free particle $n_q^{\mathrm{tree}}(\mu) = \frac{4N_cN_f}{N_s^3N_\tau}\sum_k \frac{i\sin\tilde{k}_0[\sum_i \cos k_i - \frac{1}{2\kappa}]}{[\frac{1}{2\kappa} - \sum_\nu \cos\tilde{k}_\nu]^2 + \sum_\nu \sin^2\tilde{k}_\nu}$

	Hadronic	QGP	Superfluid	
			BEC	BCS
$\langle \lvert L \rvert \rangle$	zero	non-zero		
$\langle qq \rangle$	zero	zero	non-zero	$\propto \mu^2$
$\langle n_q \rangle$			non-zero	$n_q/n_q^{\mathrm{tree}} \approx 1$

Plan of talk

1. Why 2color QCD?

    Sign problem and numerical-instability problem

2. Criterion of phase diagram

    spontaneous flavor symmetry breaking in Nc=Nf=2

3. Simulation results

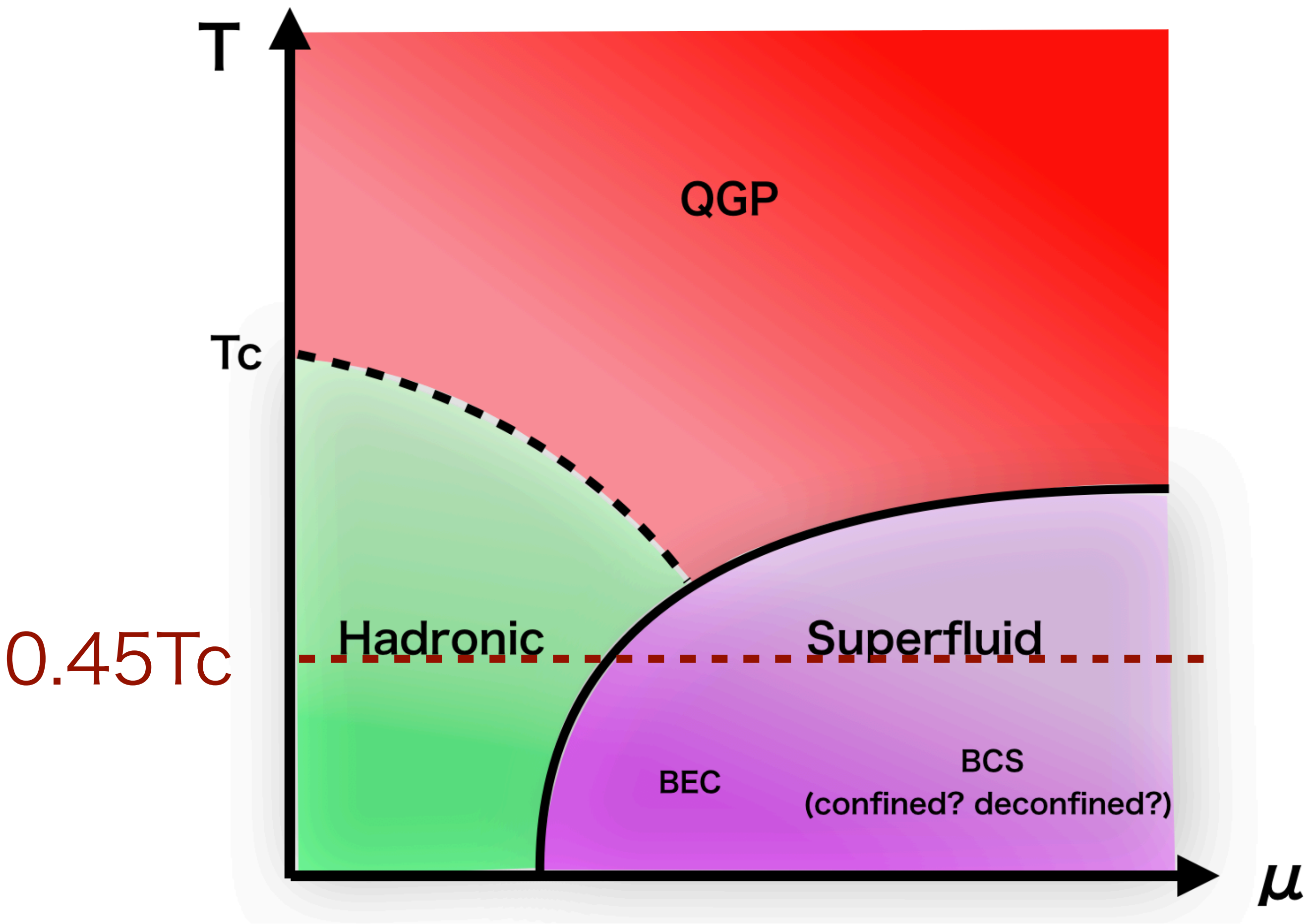
    Phase diagram at T=0.45Tc (~90MeV), 0.89Tc (~180MeV)

    Topological susceptibility

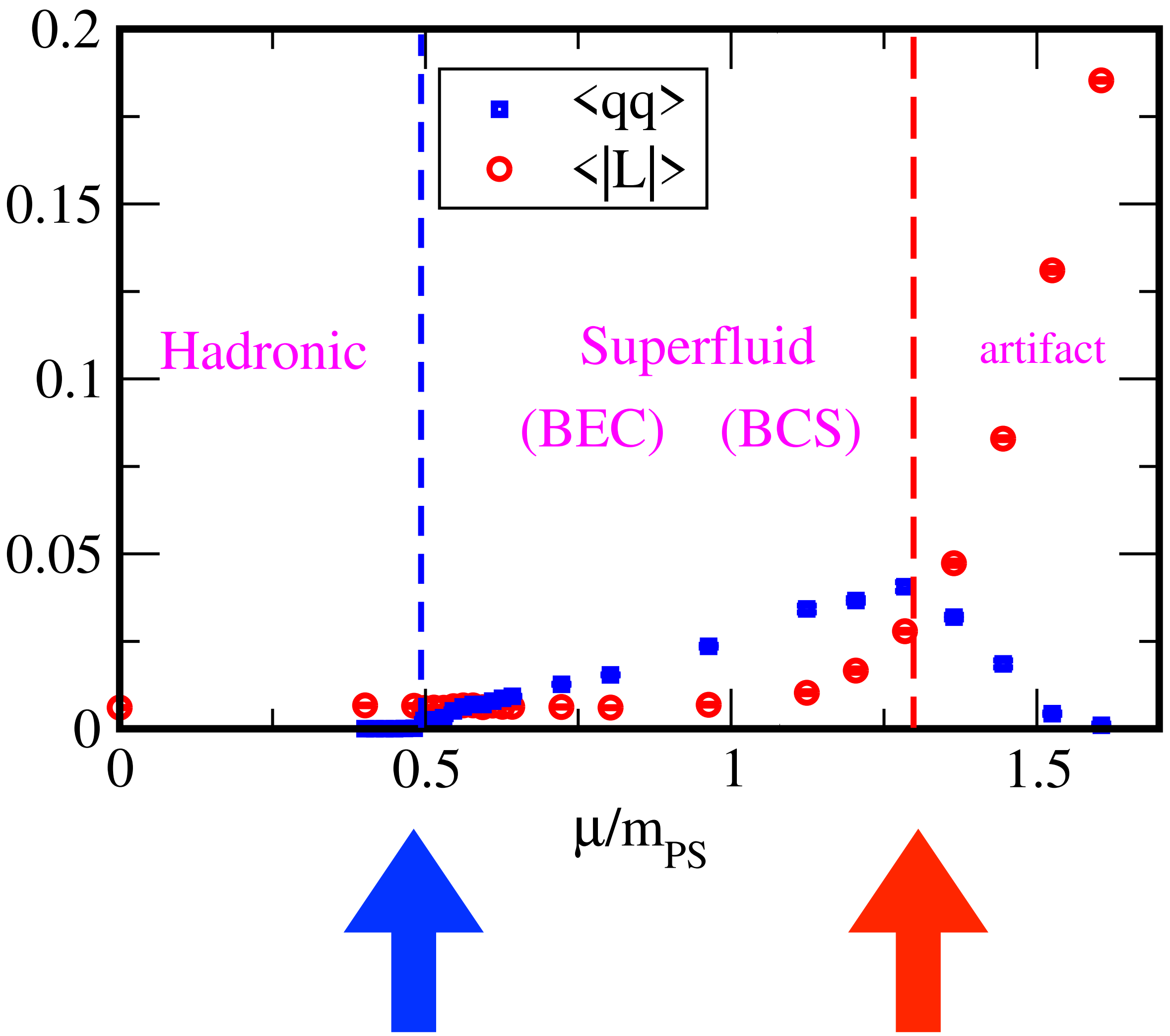
4. Summary

Results

Lattice size: 16^4 :   T=0.45Tc (~ 90MeV)



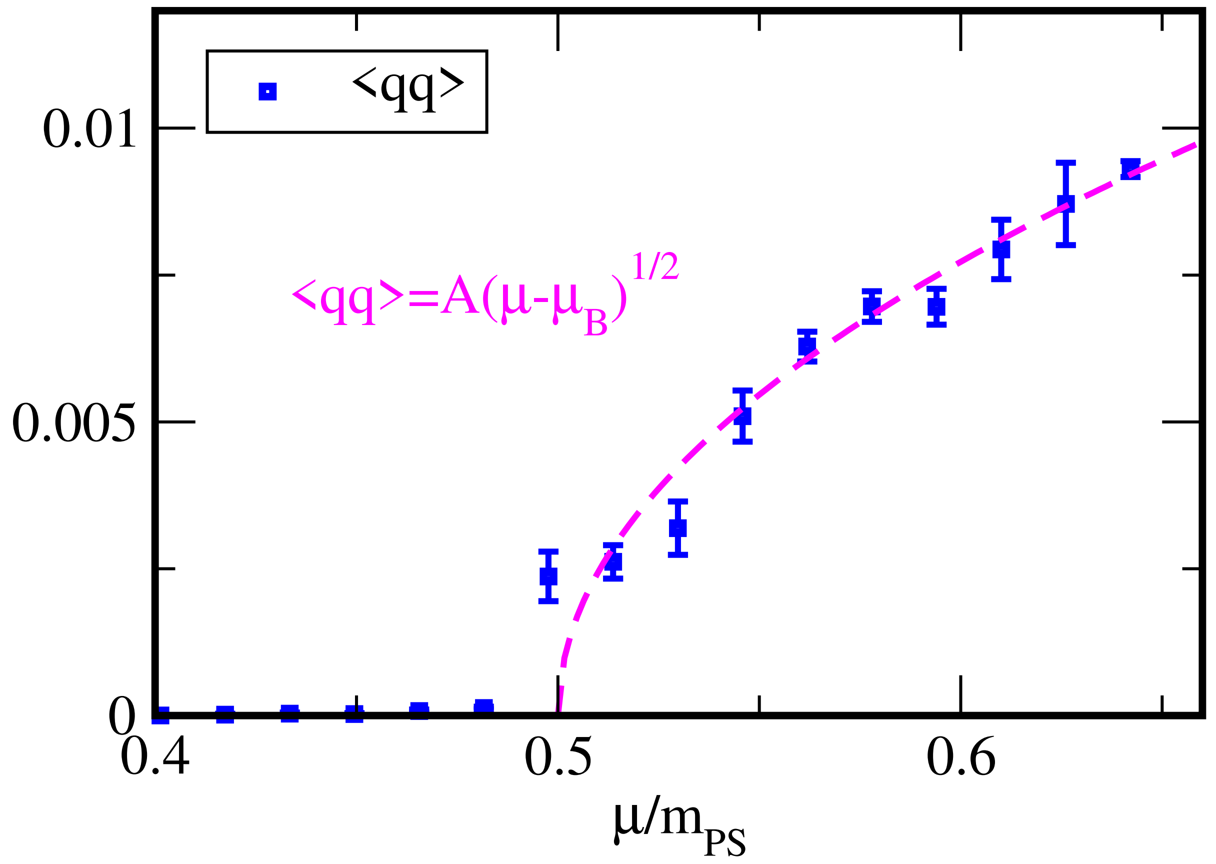
Phase diagram in j=0 limit



$\mu_B/m_{\rm PS} \simeq 0.50$

$\mu/m_{\rm PS} \simeq 1.28$

$(\mu_D/m_{\rm PS} \simeq 1.44)$



Scaling law of order param.

Is consistent with ChPT.

Ref.) Kogut, Stephanov, Toublan, Verbaarshot, Zhitnitsly
NPB 582 (2000) 477

At T=0.45Tc, we find the BCS with confined phase until $\mu \lesssim 1152 MeV$.

Cf.) At $T \simeq 0.25Tc$, ~~there is a contradiction~~

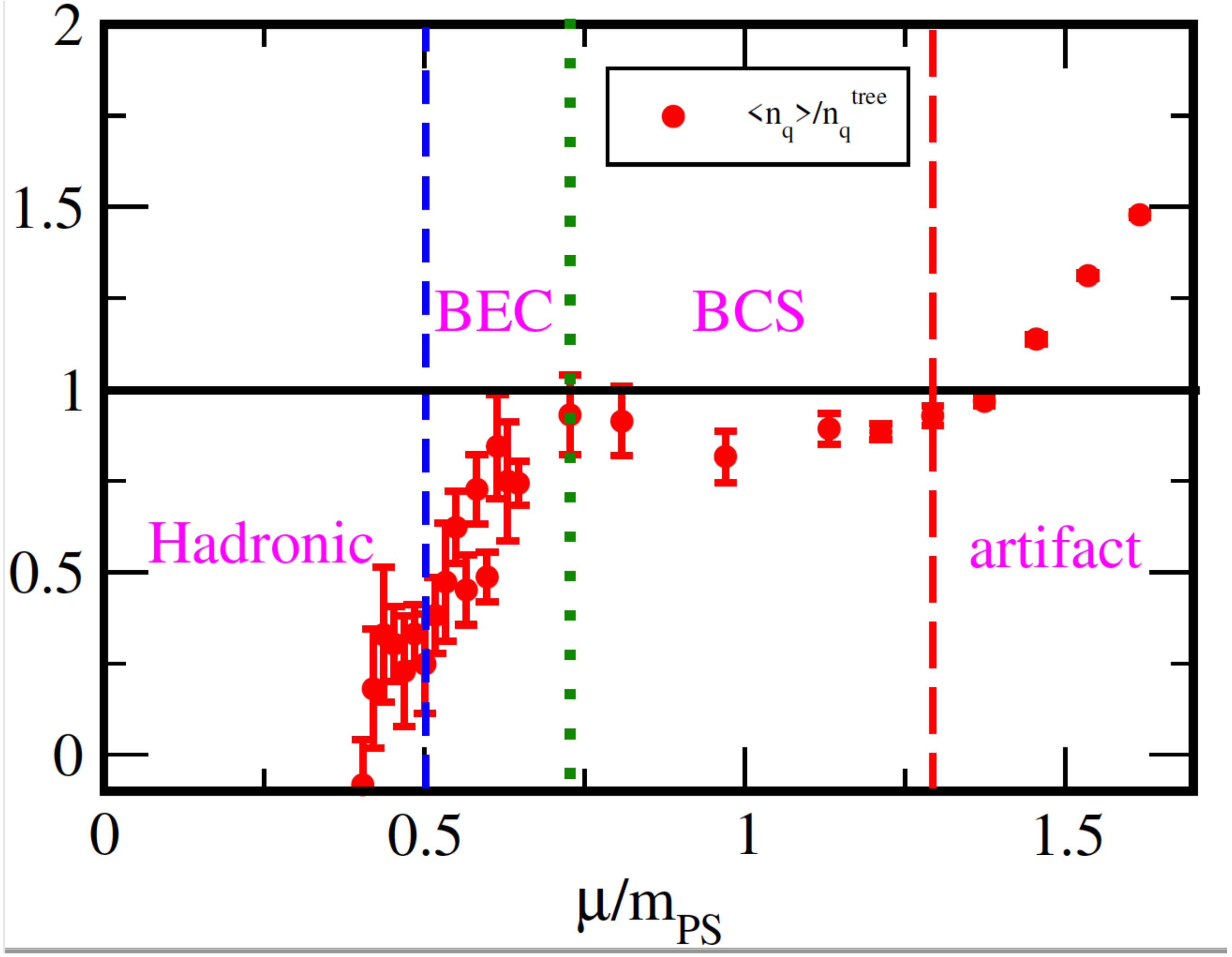
Confined/deconfined transition at $\mu \approx$ 800MeV by Wilson fermion was artifact (Hands, 2011, arXiv:1912.10975)

Cannot find the transition $\mu \lesssim$ 1410MeV by rooted staggered (Kotov, 2016)

quark number density

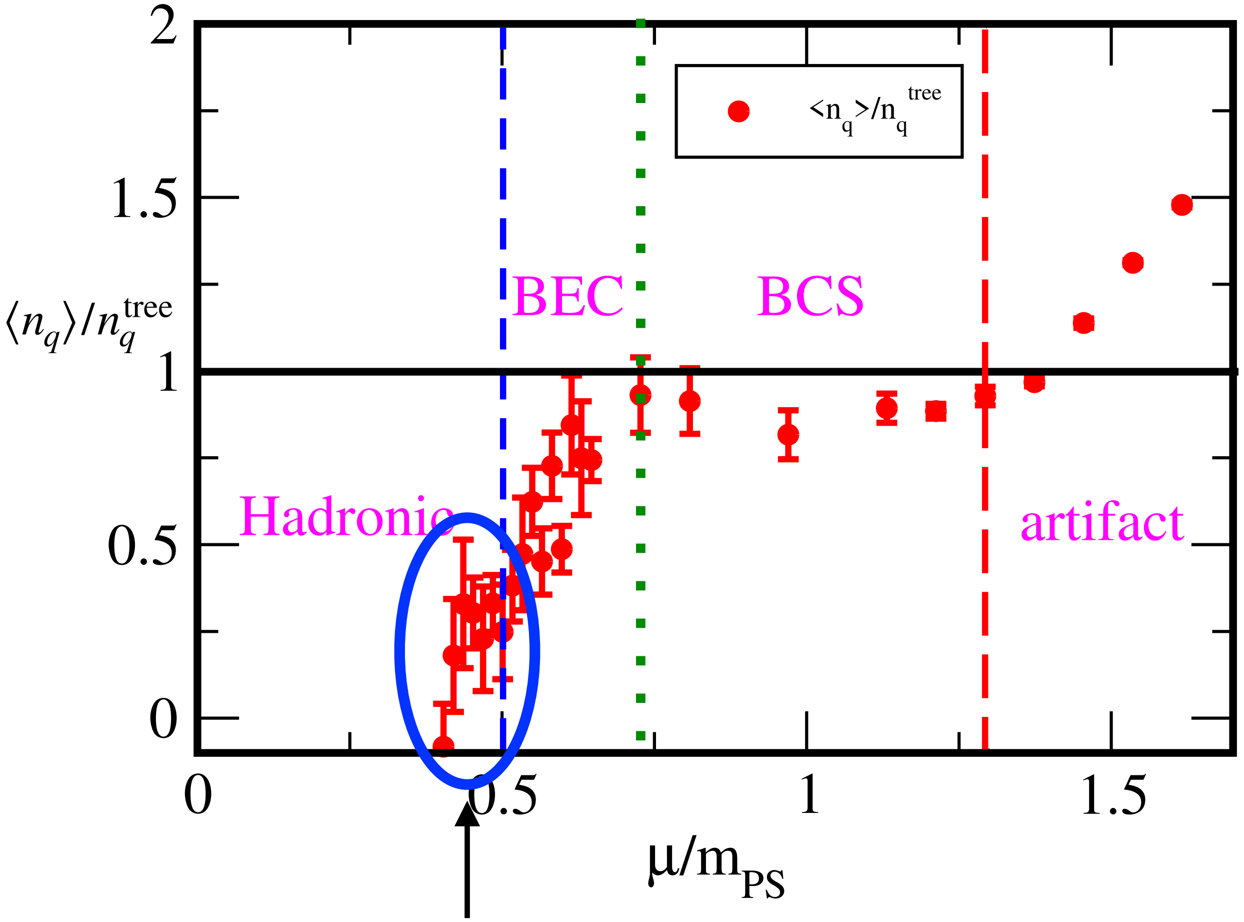
$$n_q = \sum_i \kappa \Big\langle \bar{\psi}_i(x)(\gamma_0 - 1)e^{\mu} U_t(x)\psi_i(x+\hat{t})$$
$$+ \bar{\psi}_i(x)(\gamma_0 + 1)e^{-\mu} U_t^{\dagger}(x-\hat{t})\psi_i(x-\hat{t})\Big\rangle$$

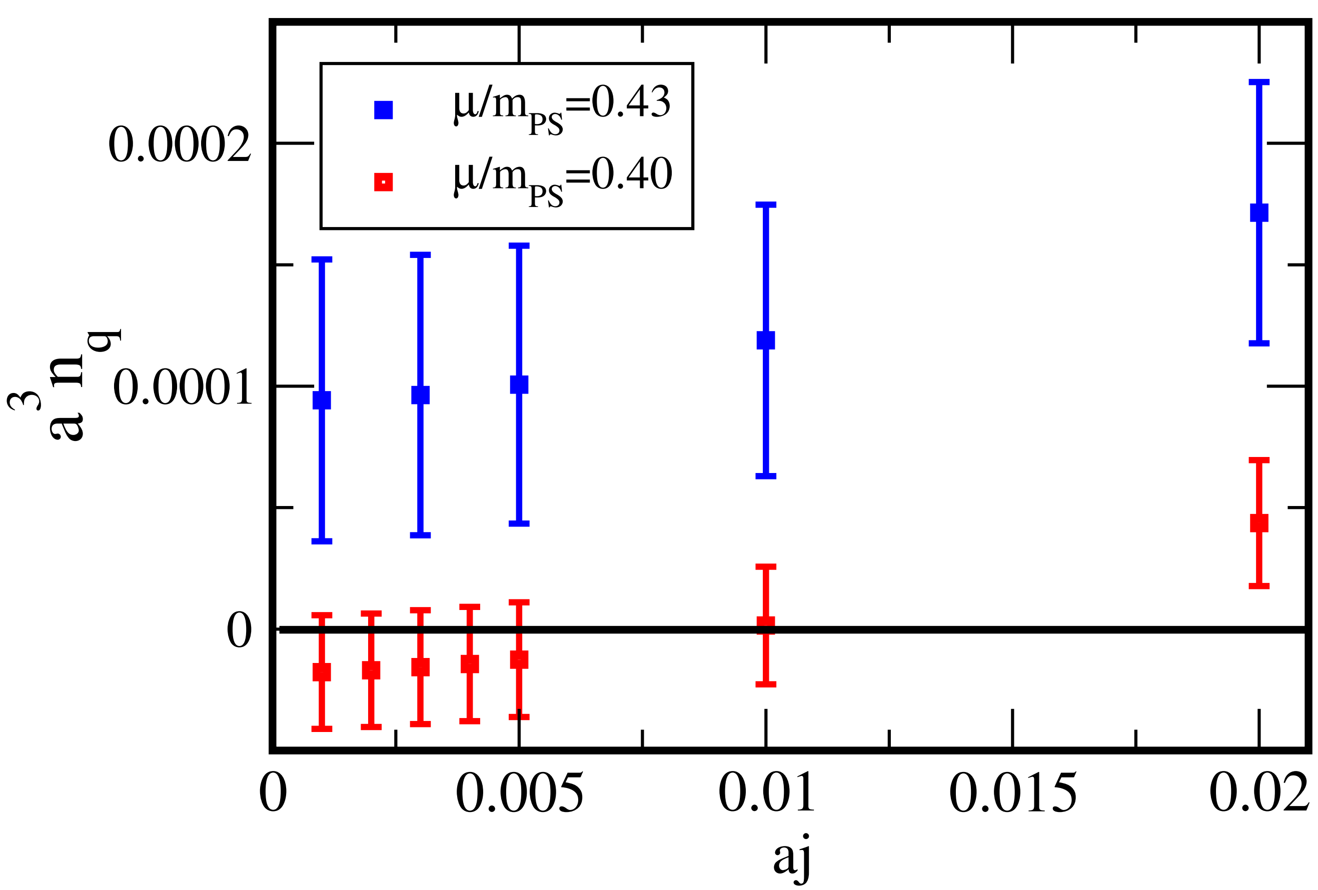
$\langle n_q \rangle / n_q^{\mathrm{tree}}$



BEC-BCS crossover occurs at $\mu \approx 0.72 m_{\mathrm{PS}}$

quark number density





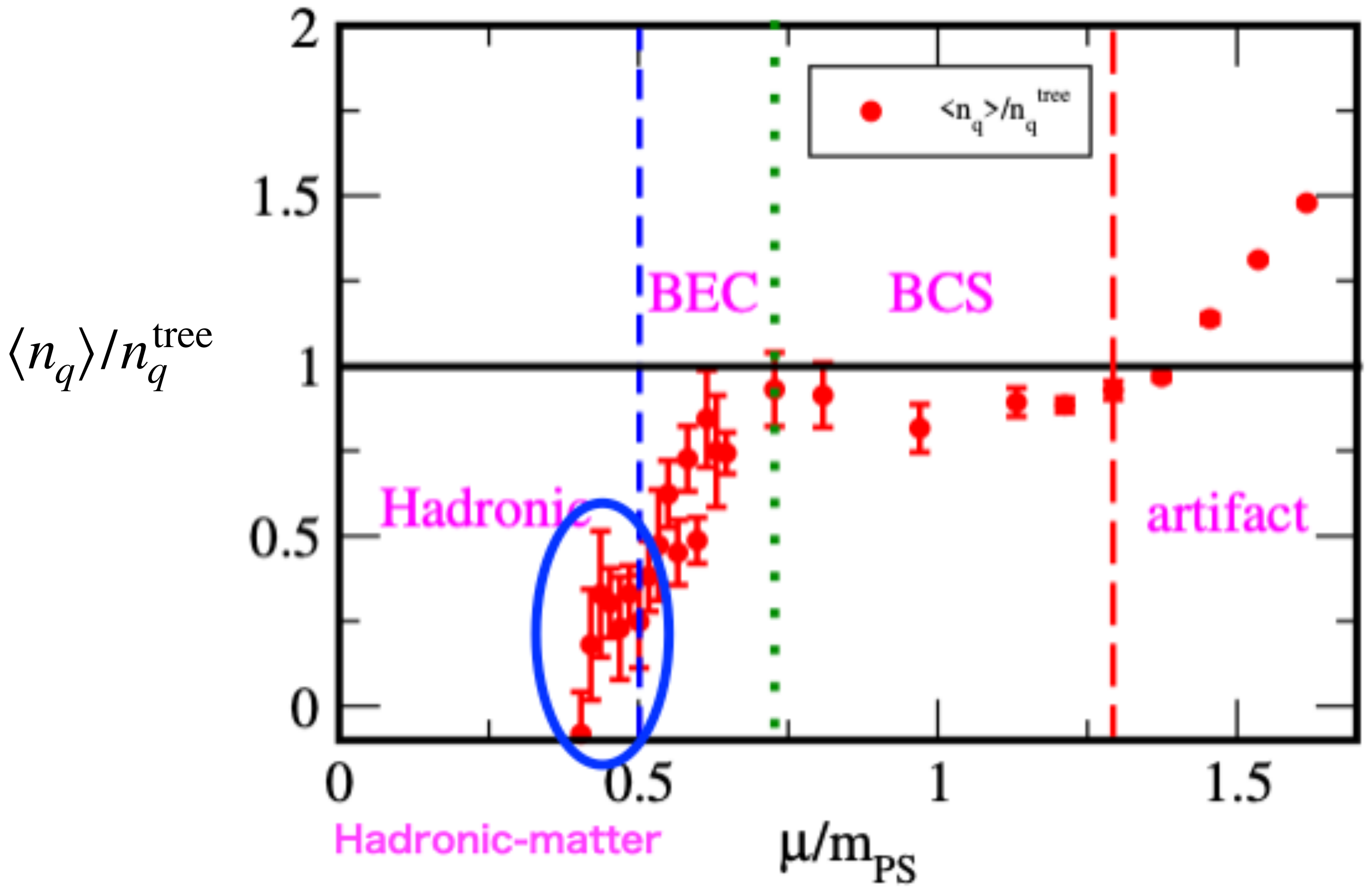
$\langle n_q \rangle \neq 0, \ \ \langle qq \rangle = 0$

Some quark d.o.f. exists

Superfluidity does not emerges (Hadronic phase)

Hadronic-matter phase (coexistence phase)

Summary of phase diagram at T=0.45Tc



@T=0.45Tc
(~90MeV)

$\mu \leq 0.42 m_{\rm PS}$: Hadronic phase

$0.42 m_{\rm PS} \lesssim \mu \lesssim 0.50 m_{\rm PS}$: Hadronic-matter phase

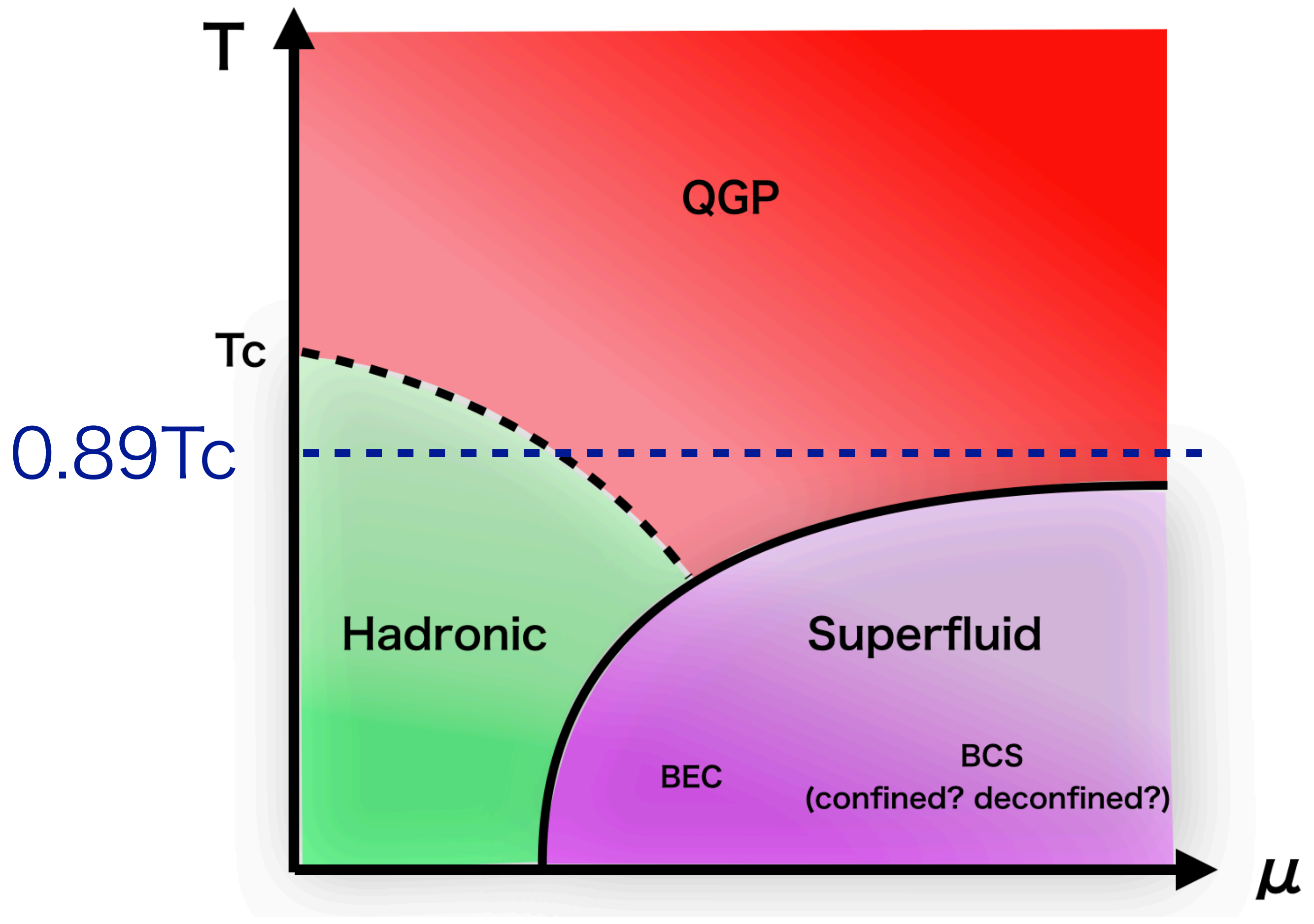
$0.50 m_{\rm PS} \lesssim \mu \lesssim 0.72 m_{\rm PS}$: BEC phase

$0.72 m_{\rm PS} \lesssim \mu \lesssim 1.28 m_{\rm PS}$: BCS phase

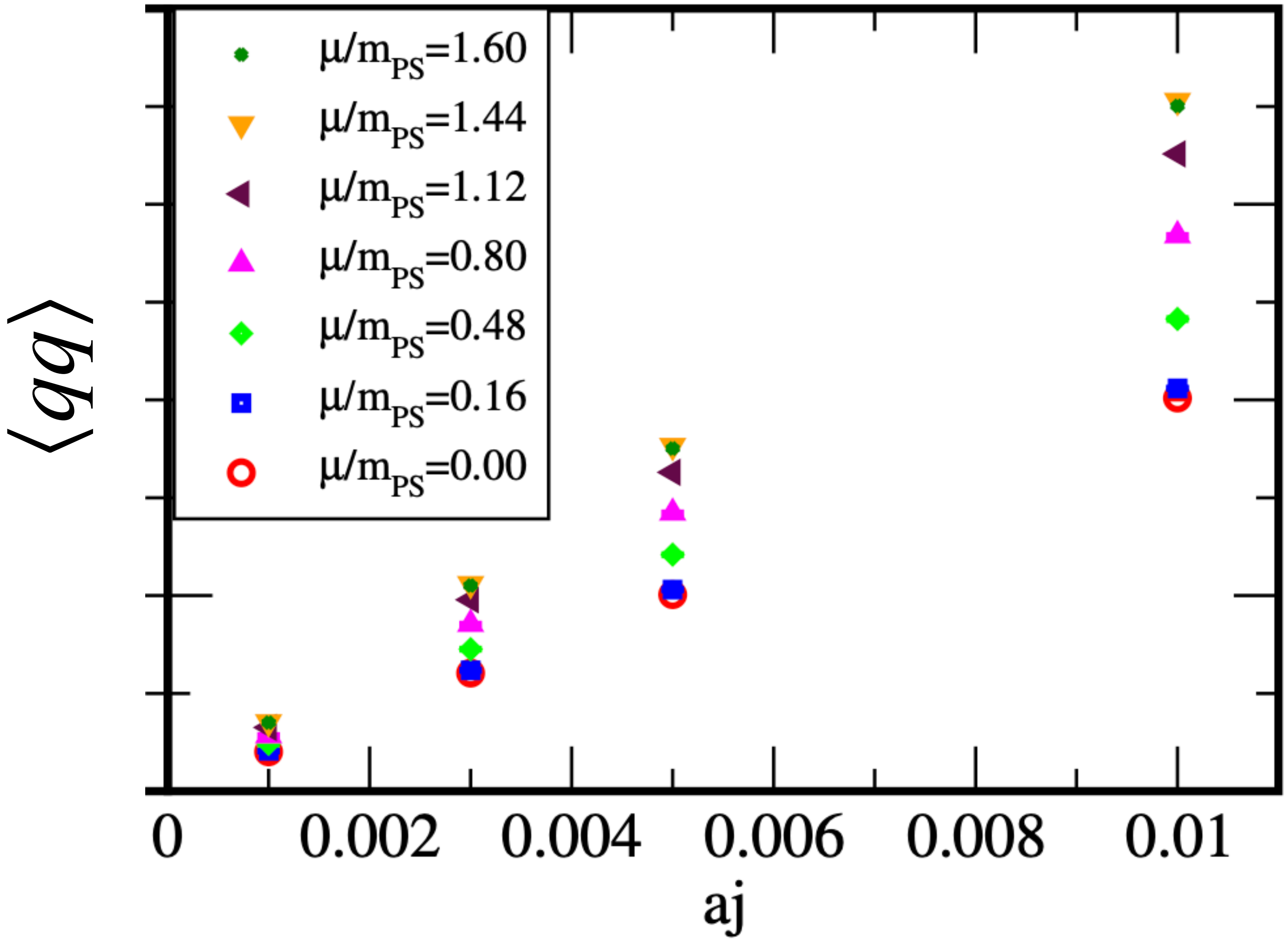
$1.28 m_{\rm PS} \lesssim \mu$: lattice artifact is strong

Results

Lattice size: 32^3x8 : T=0.89Tc (~ 180MeV)

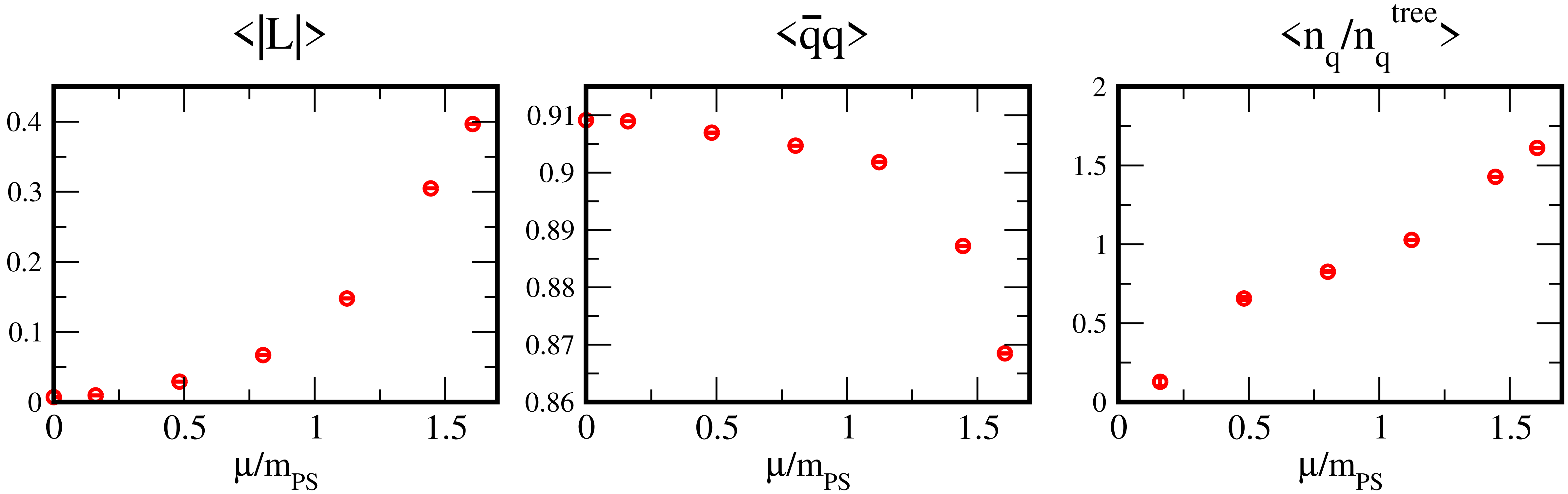


Diquark condensate



No superfluidity in whole μ regime

Polyakov loop, chiral condensate, number density



confined -> deconfined

chiral broken ->restored

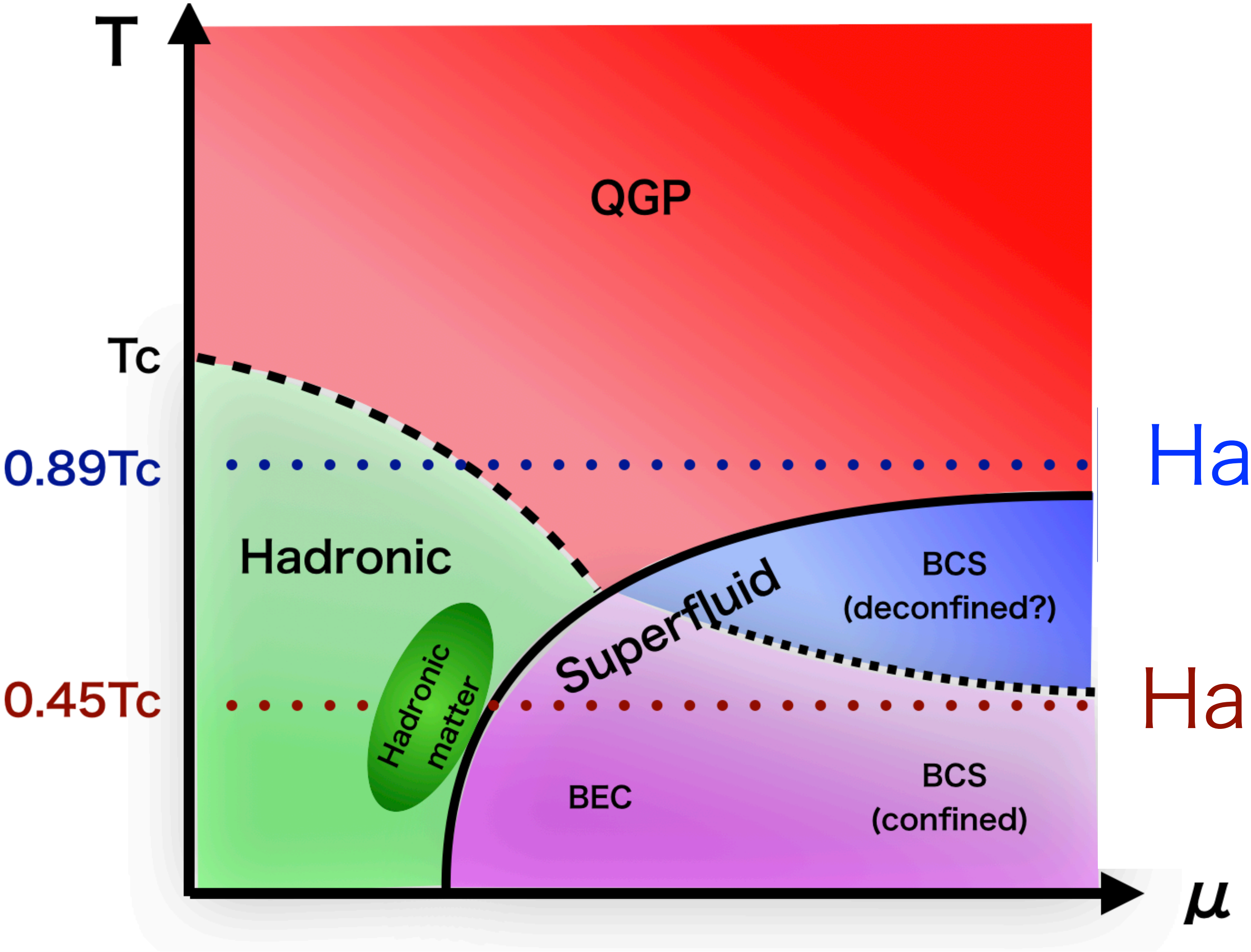
non-zero even in $\mu \ll m_{\mathrm{PS}}/2$

and no superfluidity

Hadronic -> QGP transition

Results

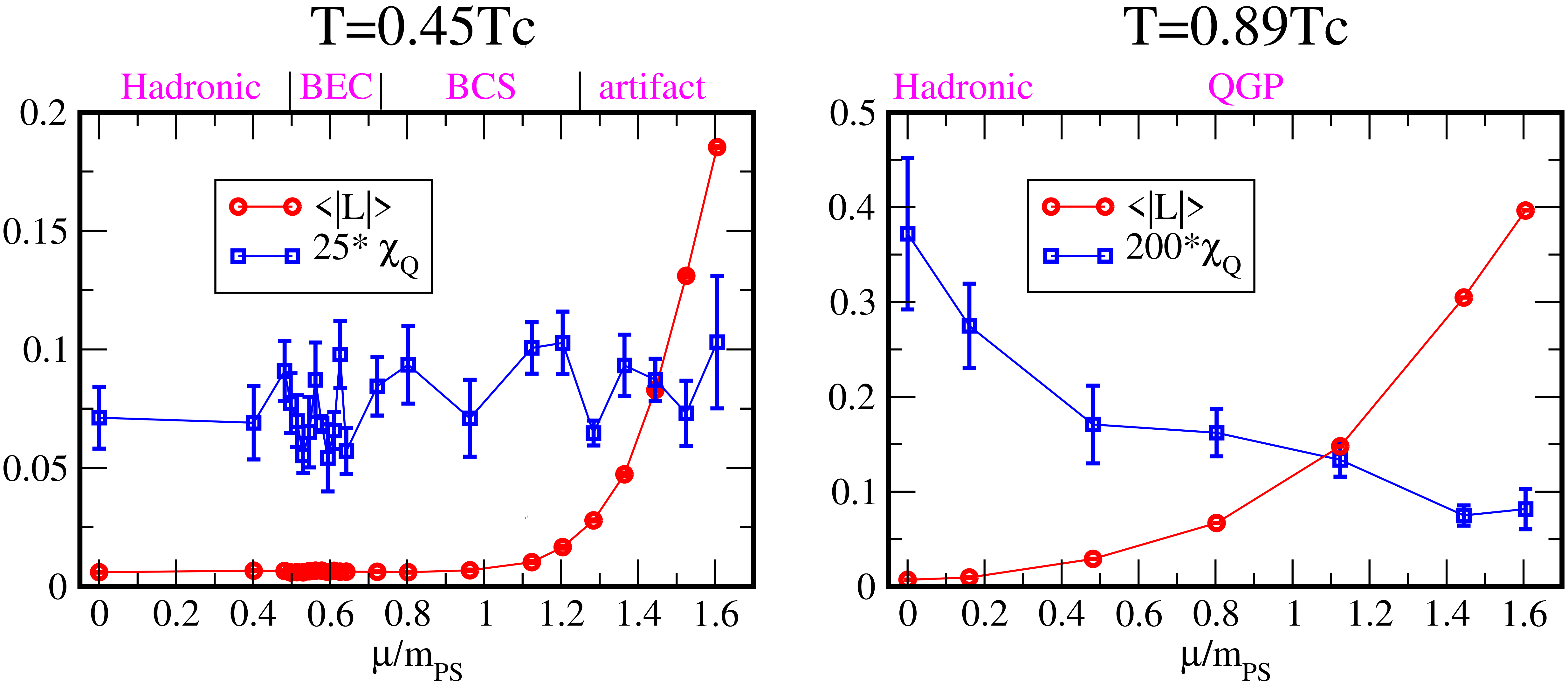
topological charge using gradient flow



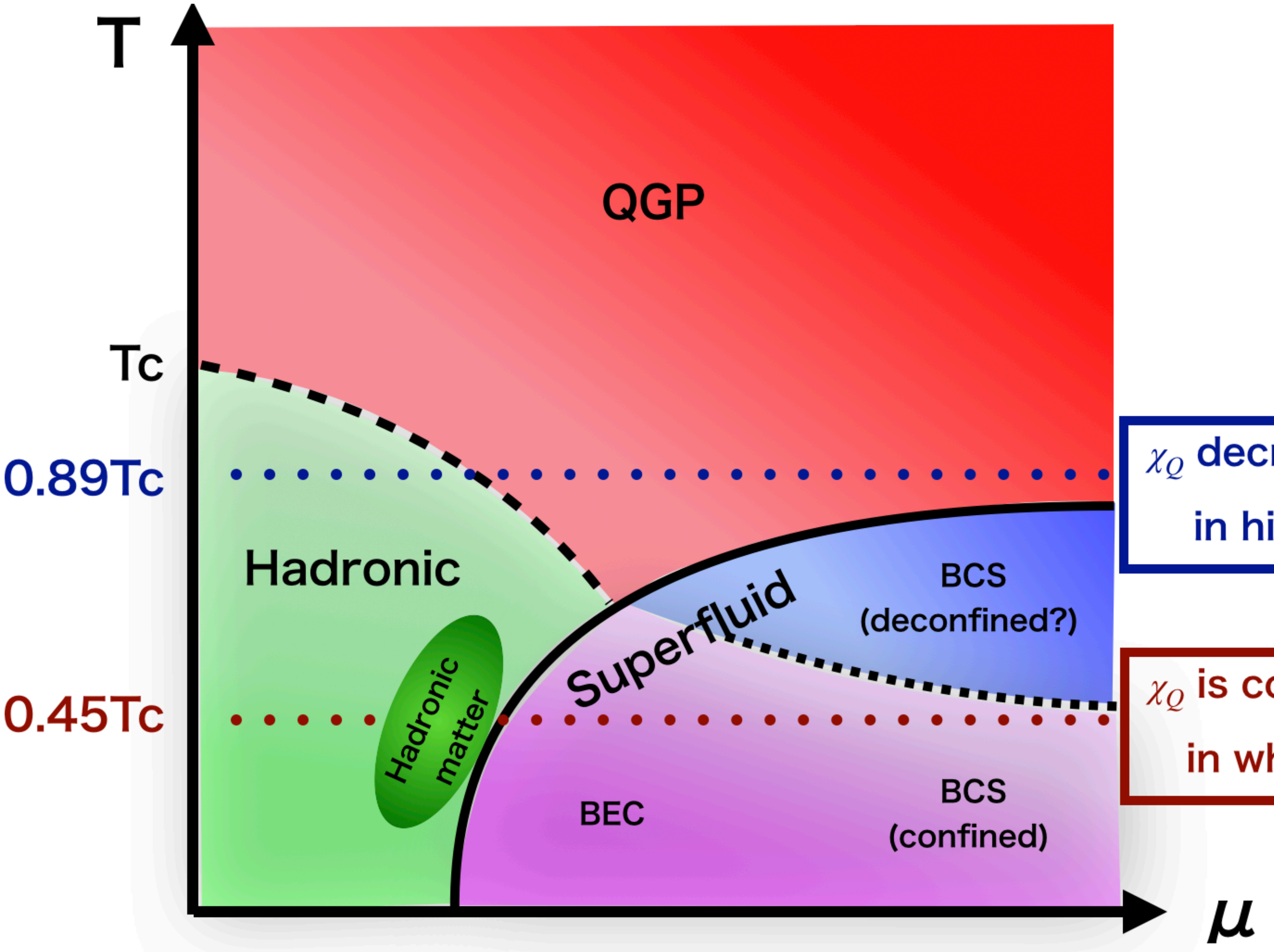
Hadronic -> QGP

Hadronic -> BEC-> BCS

Topological susceptibility and Polyakov loop



Summary

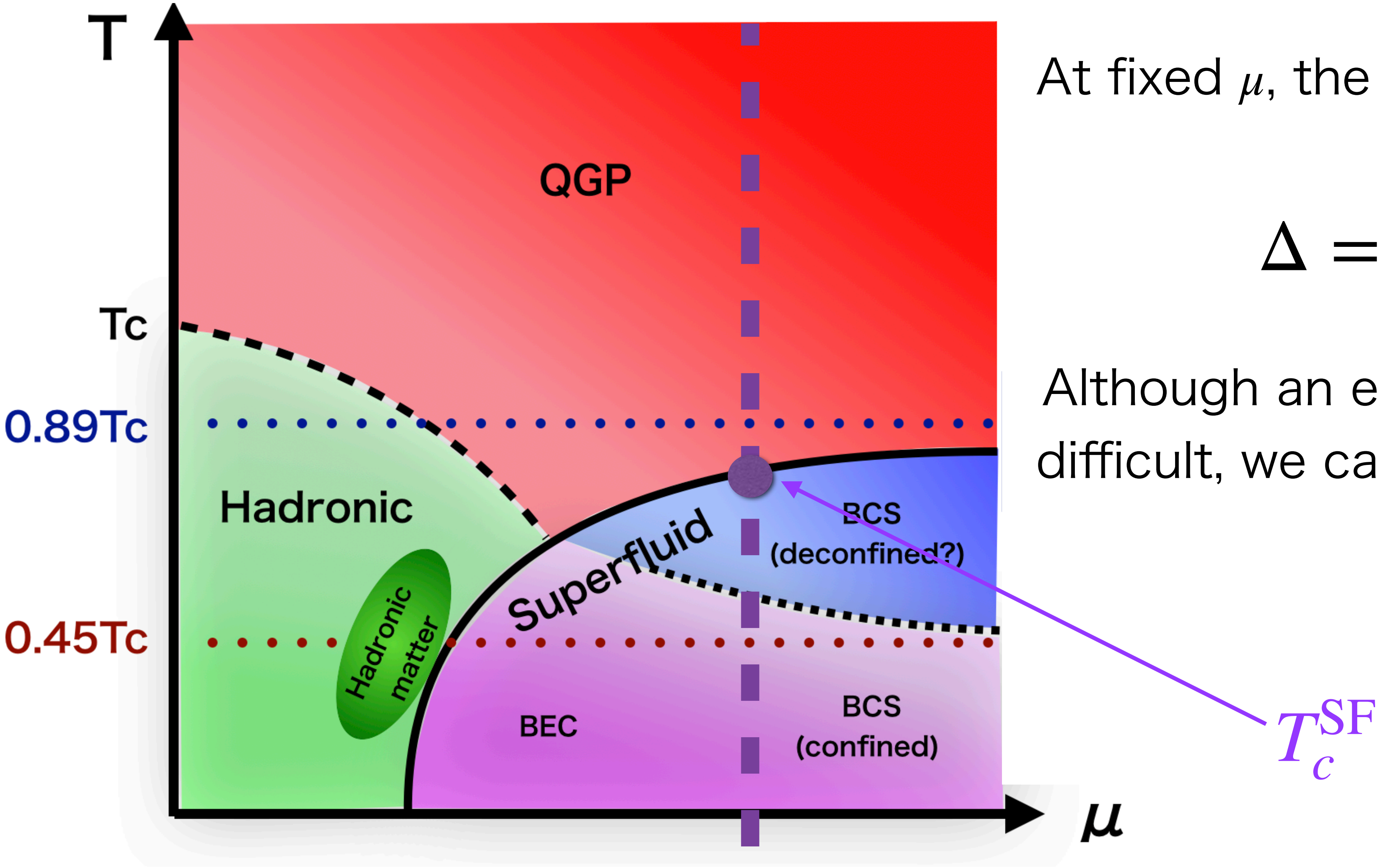


χ_Q **decreases in high** μ

χ_Q **is constant in whole** μ

低温高密度領域にはインスタントンがいる？

A role of instanton in high density



At fixed μ, the BCS relation is valid

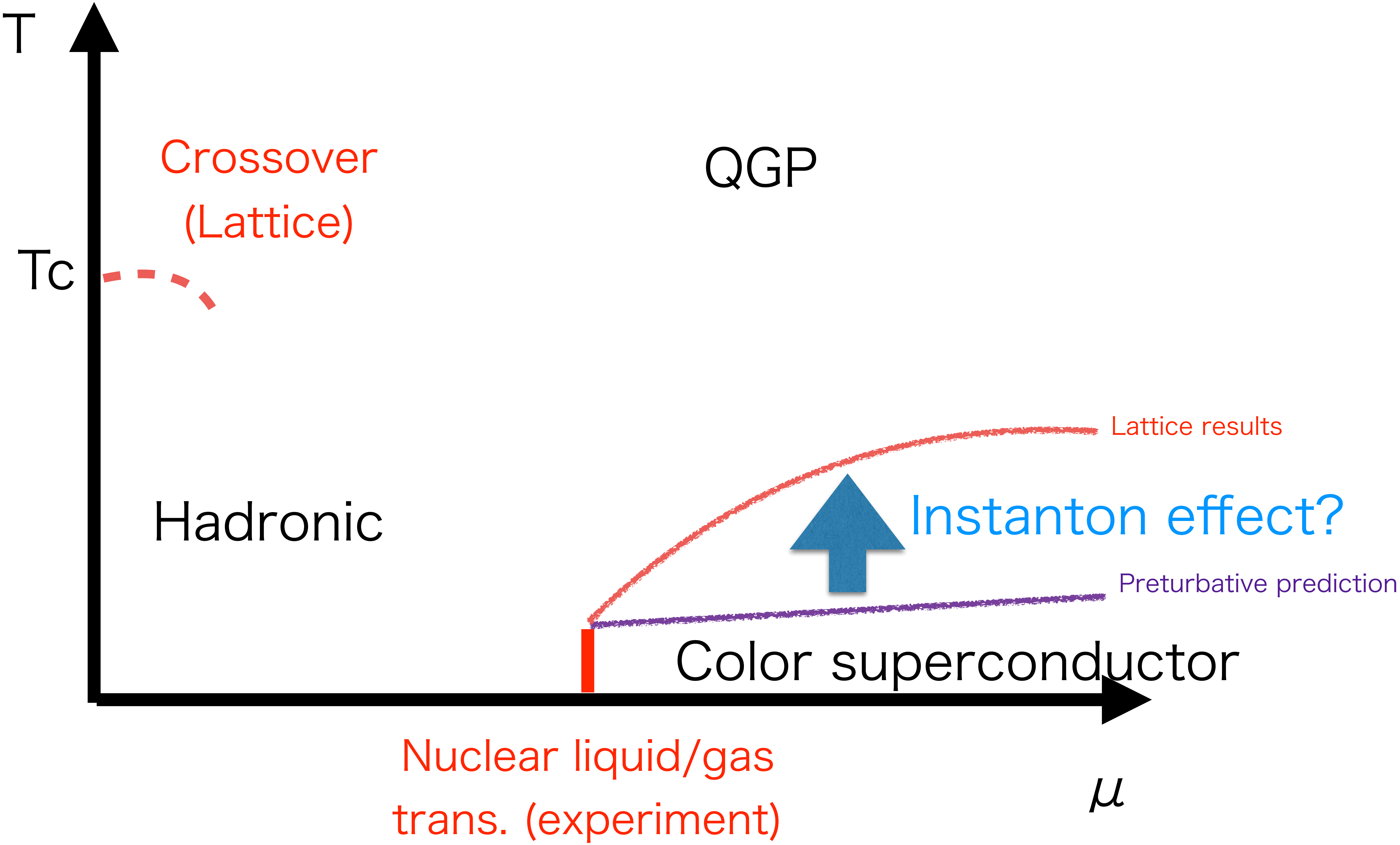
$$\Delta = \pi \frac{T_c^{\rm SF}}{e^{\gamma_E}}$$

Although an exact zero-T simulation is difficult, we can find a value of diquark gap.

speculation: diquark gap may get fat because of the interaction via nontrivial topological objects.

$T_c^{\rm SF}$ may be higher than analytical prediction

What is really known…



Crossover
(Lattice)

QGP

Hadronic

Lattice results

Instanton effect?

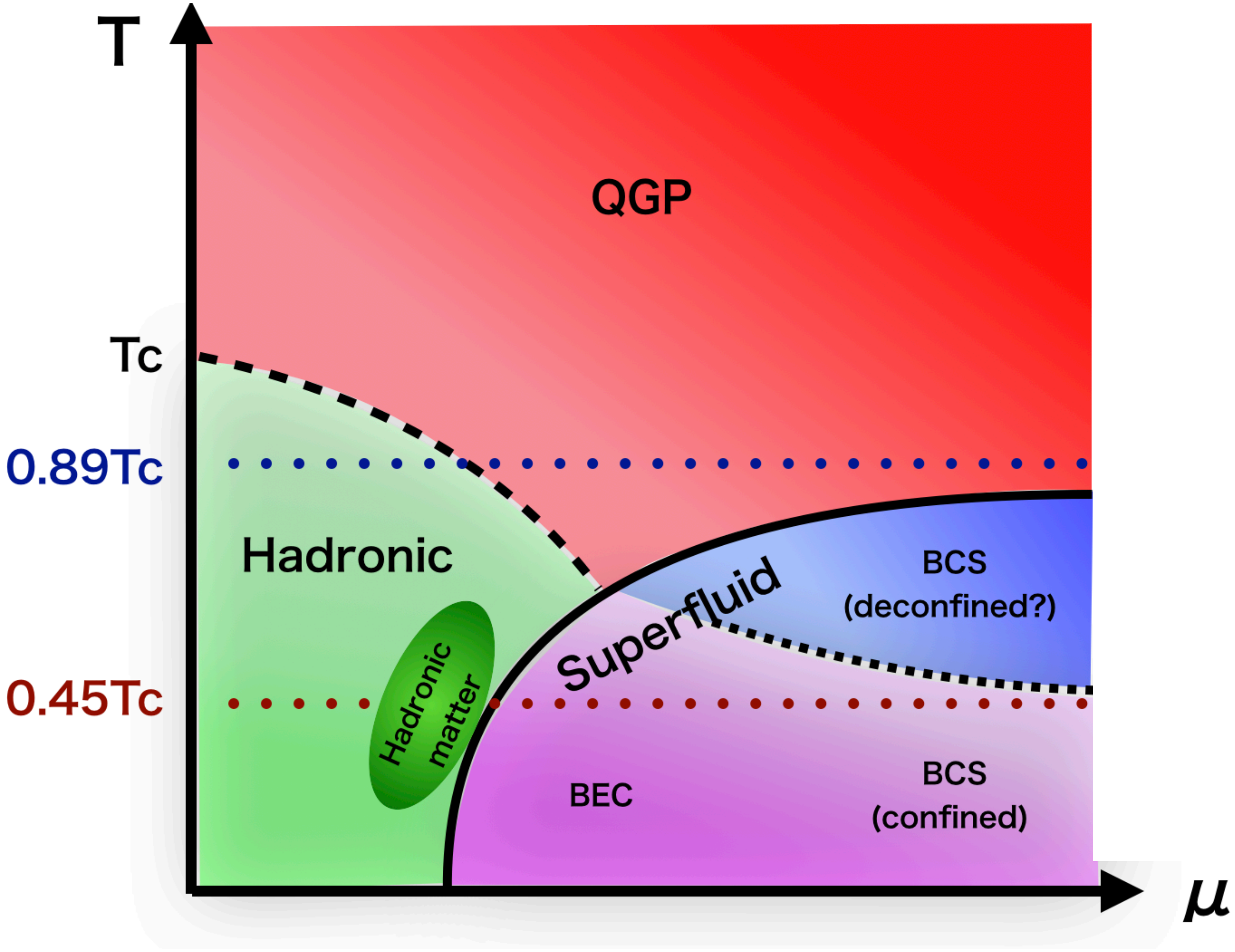
Preturbative prediction

Color superconductor

Nuclear liquid/gas
trans. (experiment)

Pochodzalla et al. PRL75 (1995) 1040

Phase diagram in Two-color QCD



BCS(deconfined) does not appear in our simulations, but it is widely believed.

A typical momentum of quarks is T.

If T is lower than the gap energy in SF phase, then quarks are quenched.

In three-color QCD, the transition would be 1st order, but in two-color QCD it must be 2nd order (or crossover).

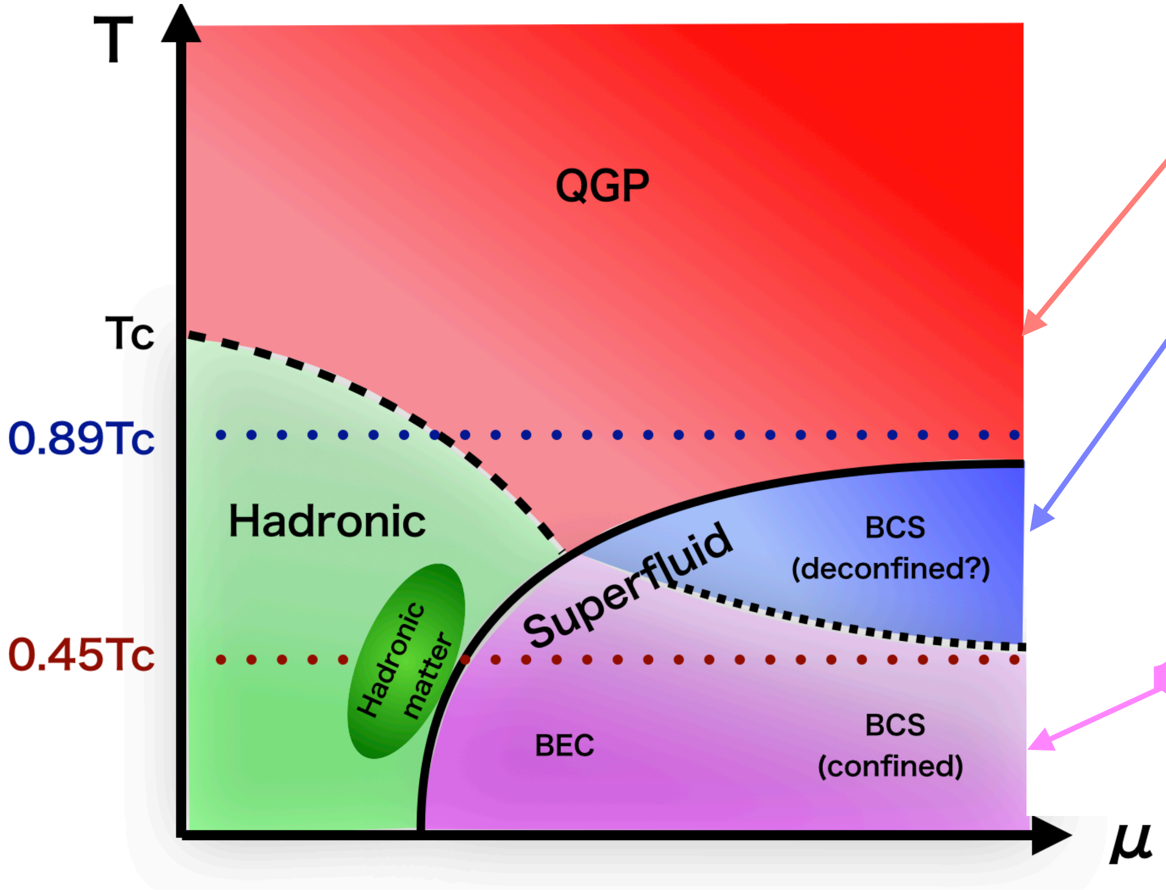
Confined or deconfined in high density

Three independent group' studies:

(1) Swansea (S. Hands et al) group : Wilson-Plaquette gauge + Wilson fermion

(2) Russia (Y.Kotov et al) group : tree level improved Symanzik gauge + rooted staggered fermion

(3) Our group : Iwasaki gauge + Wilson fermion, Tc=200 MeV to fix the scale



T=180 MeV (**deconfined**, hadron -> QGP phase transition occurs)

T=140 MeV (**deconfined** in high mu, <qq> is not zero, 2017, 2018, 2020)
T=130 MeV (**deconfined?** in 2019)
T= 93 MeV (**deconfined** in high mu ?, also <qq> is not zero?, 2017)

T=90 MeV (**confined** even in high mu)
T=87 MeV (**confined** in 2019)
T=55 MeV (**confined** in high mu, 2016)
T=47 MeV (**deconfined** coarse lattice in 2012, but **confined** in 2019)
T=45 MeV (**confined** in 2019)

All data seem to be in agreement with the phase diagram, though all data are not taken in the continuum limit and the scale setting may not be seriously estimated

まとめ

- 2カラーQCDの有限温度密度相図は第一原理計算で決まりつつある
- 低温高密度領域には非自明なトポロジカル配位が存在し、解析的な摂動真空とは異なる(diquark gapを大きくし、QGP/超流動相転移温度を大きくする？)
- 有限温度ではHadronic-matter相が現れる

何を知りたいか？

- 温度密度に依存した相図
- インスタントンの有無など非摂動的性質
- 核力、ハドロン質量の密度依存性
- 状態方程式(圧力、内部エネルギー、エントロピー)
- 輸送係数 (粘性、超流動密度)

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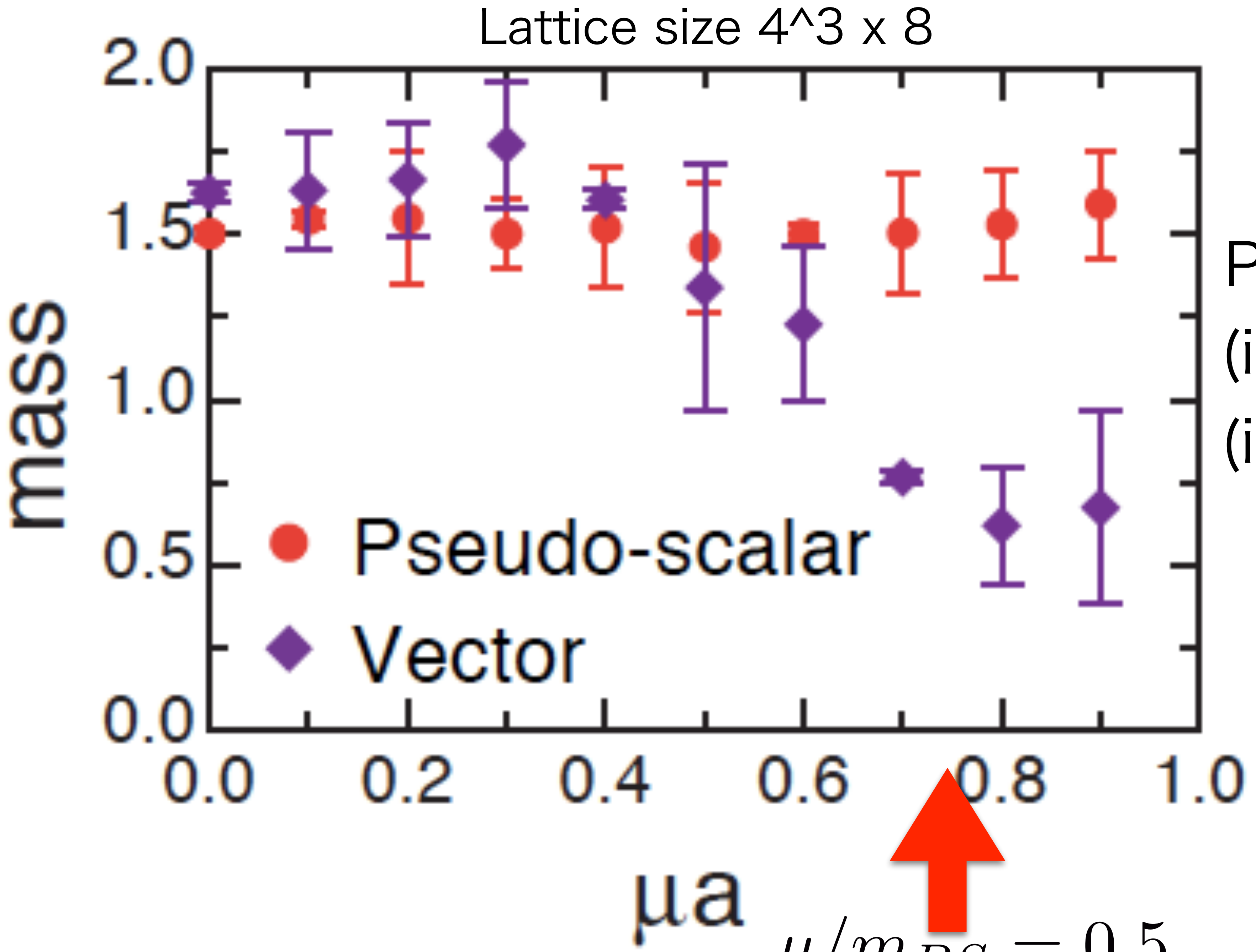
- 輸送係数 (粘性、超流動密度)

→ Gradient flow method
Sparse modeling method

Meson spectrum

Muroya et al. Phys.Lett. **B551** (2003) 305



$$m_{PS} < m_V$$

QCD inequality

PS has the lightest mass

(i) no disconn. diagram

(ii) Gamma5 Hermiticity

$$\Delta(-\mu)^\dagger = \gamma_5 \Delta(\mu) \gamma_5$$

$\mu / m_{PS} = 0.5$

超流動相では、rho mesonの方が軽い

(さらには、バリオン=diquarkが一番軽い)

Equation of State

N. Astrakhantsev, V. Braguta, E. Ilgenfritz, A. Kotov, A. Nikolaev[arXiv:2007.07640]

T~140MeV, dense-matter -> BCS (deconfinement)phase

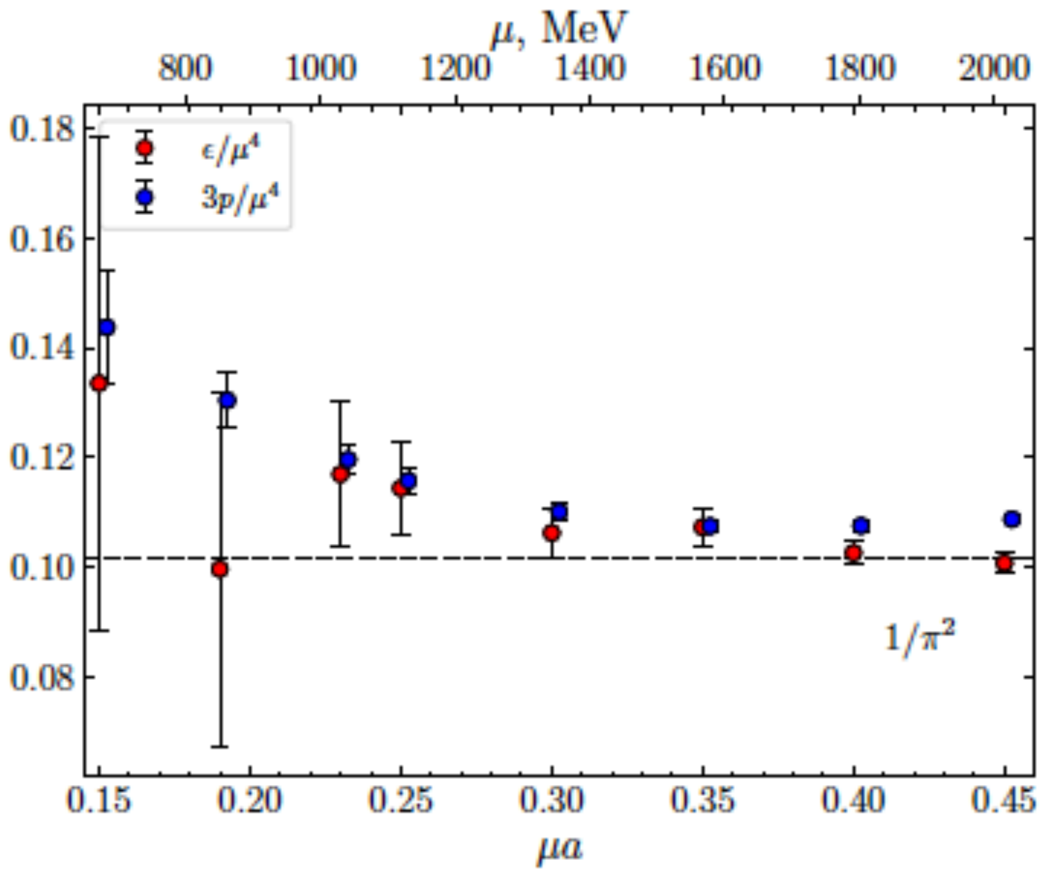


Figure 9. The energy density and the pressure divided by μ^4 as a function of chemical potential (blue circles are slightly shifted for the better visibility). The dashed line corresponds to the ϵ and $3p$ of a free relativistic quark gas $\epsilon = 3p = \mu^4/\pi^2$.

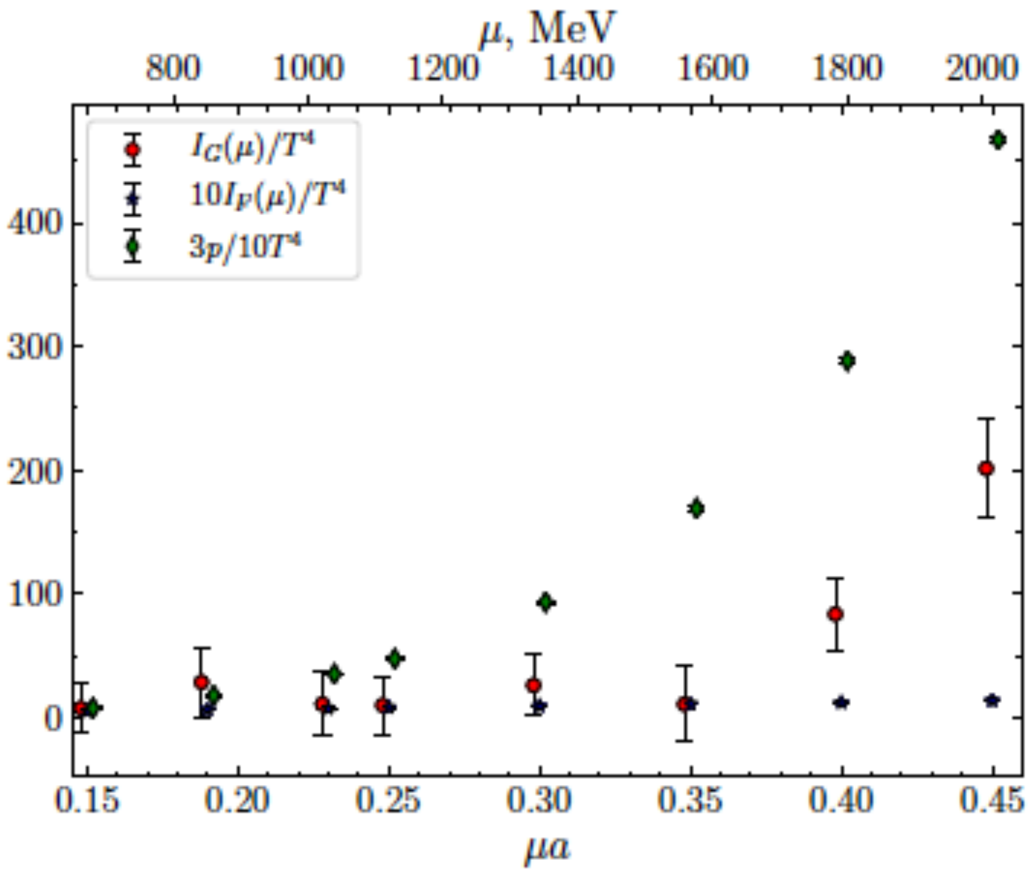


Figure 10. The gluon I_G and fermion I_F contributions to the anomaly, defined in (16) and (17) respectively, and pressure p as functions of the chemical potential. In order to plot these observables in one figure we rescaled them.

宣伝

online international workshop:

"Probing the physics of high-density and low-temperature matter with ab initio calculations in 2-color QCD"

3rd - 6th November 2020

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