

Baryo/Leptogenesis from Axion Inflation

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DESY, HAMBURG → CERN

Based on **1905.13318, 20xx.xxxxx, 1806.08769, 1812.08021, 1910.01205**

Collaboration with Y. Ema, V. Domcke, B. von Harling, K. Kamada, E. Morgante,
R. Sato, K. Schmitz, M. Yamada



1.

Introduction

Introduction

Cosmic Inflation v.s. Baryon Asymmetry

- ▶ **Inflation**: accelerated expansion of Universe
 - **Solve** horizon/flatness problems + **Provide** density perturbations.
 - Dilute unwanted relics, **but also SM particles such as baryons**.
 - ▶ **Baryogenesis**: production of baryon asymmetry after inflation.
 - Baryon to photon ratio (BBN) $\rightarrow \eta = \frac{n_B}{n_\gamma} \simeq 6 \times 10^{-10}$
- ➔ Baryogenesis from Inflation?
- Reheating requires couplings btw inflaton and the SM particles.



Introduction

Inflaton w/ Chern-Simons coupling

$$S = \int d^4x \left\{ \sqrt{-g} \left[\frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\phi}{4\Lambda} F_{\mu\nu} \tilde{F}^{\mu\nu} \right\}$$

- ▶ **Flat** potential v.s. **Coupling** to radiation
 - **Flat potential** protected by the shift sym. $\phi \mapsto \phi + c$
 - **Reheating** via the Chern-Simons coupling

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▶ **Flat** potential v.s. **Coupling** to radiation

- **Flat potential** protected by the shift sym. $\phi \mapsto \phi + c$
- **Reheating** via the Chern-Simons coupling

❖ **Helical-gauge field** production during inflation: $\dot{\phi} \neq 0$. BE

$$0 = \left[\partial_\eta^2 + k(k \pm 2\xi aH) \right] A_\pm(\eta, k)$$

where $\xi \equiv \frac{\dot{\phi}}{2\Lambda H}$

$$\Rightarrow 0 \neq \langle F_{\mu\nu} \tilde{F}^{\mu\nu} \rangle = -4 \langle \mathbf{E} \cdot \mathbf{B} \rangle$$



W.Garretson+ 9209238, M.Anber, L.Sorbo 0606534,...

➔ (Pre)Reheating, chiral GWs, **baryogenesis**, magnetogenesis,...

Helical gauge & Chiral fermion

Inflaton w/ Chern-Simons coupling to $U(1)_Y$

$$S = \int d^4x \left\{ \sqrt{-g} \left[\frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} + \frac{\phi}{4\Lambda} Y_{\mu\nu} \tilde{Y}^{\mu\nu} \right. \\ \left. + \sum_\alpha \psi_\alpha^\dagger \sigma \cdot (i\partial - g_Y Q_\alpha A_Y) \psi_\alpha + \dots \right\}$$

→ $U(1)_Y$

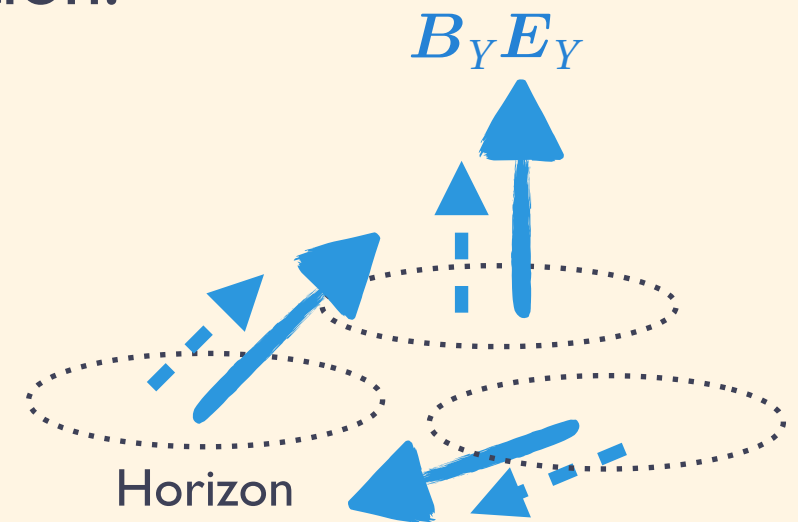
▼ **SM particles**
charged under $U(1)_Y$

❖ Helical-gauge field production during inflation.

$$0 = \left[\partial_\eta^2 + k(k \pm 2\xi aH) \right] A_{Y,\pm}(\eta, k)$$

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$$\Rightarrow \begin{aligned} 0 &\neq \langle Y_{\mu\nu} \tilde{Y}^{\mu\nu} \rangle \\ &= -4 \langle \mathbf{E}_Y \cdot \mathbf{B}_Y \rangle \end{aligned}$$



Helical gauge & Chiral fermion

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$U(1)_Y$

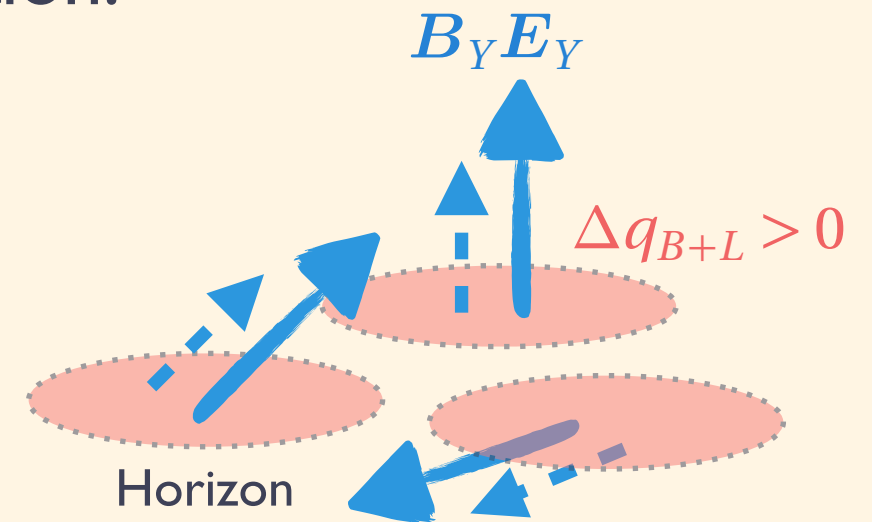
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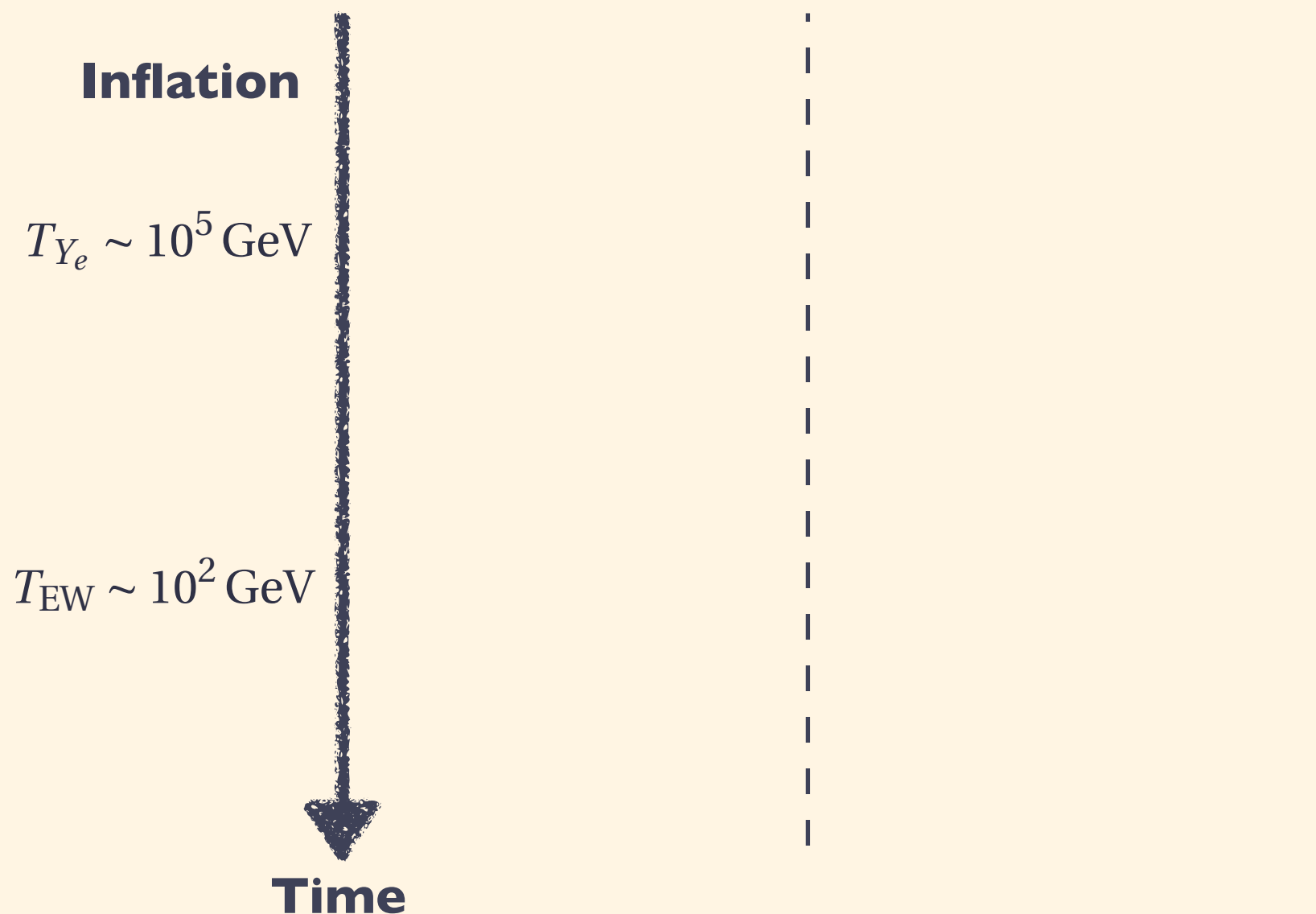
❖ B+L generation during inflation!

$$\partial_\mu J_{B+L}^\mu = \frac{3}{16\pi^2} \left(g_2^2 W_{\mu\nu}^a \tilde{W}^{a\mu\nu} - g_Y^2 Y_{\mu\nu} \tilde{Y}^{\mu\nu} \right) \neq 0$$

Our Idea

Baryogenesis from B+L asymmetry?

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = \frac{3}{32\pi^2} \left(g_2^2 W_{\mu\nu}^a \tilde{W}^{a\mu\nu} - g_Y^2 Y_{\mu\nu} \tilde{Y}^{\mu\nu} \right)$$



Our Idea

Baryogenesis from B+L asymmetry?

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu =$$

$$3\partial_\mu K_{CS}^\mu$$

$$-\frac{3\alpha_Y}{4\pi}\partial_\mu h_Y^\mu$$

Inflation

$$-\frac{3}{4}\frac{\alpha_Y}{\pi}\Delta h_Y$$

Inefficient

$$-\frac{3}{4}\frac{\alpha_Y}{\pi}\Delta h_Y$$

$$T_{Ye} \sim 10^5 \text{ GeV}$$

$$T_{EW} \sim 10^2 \text{ GeV}$$

Time

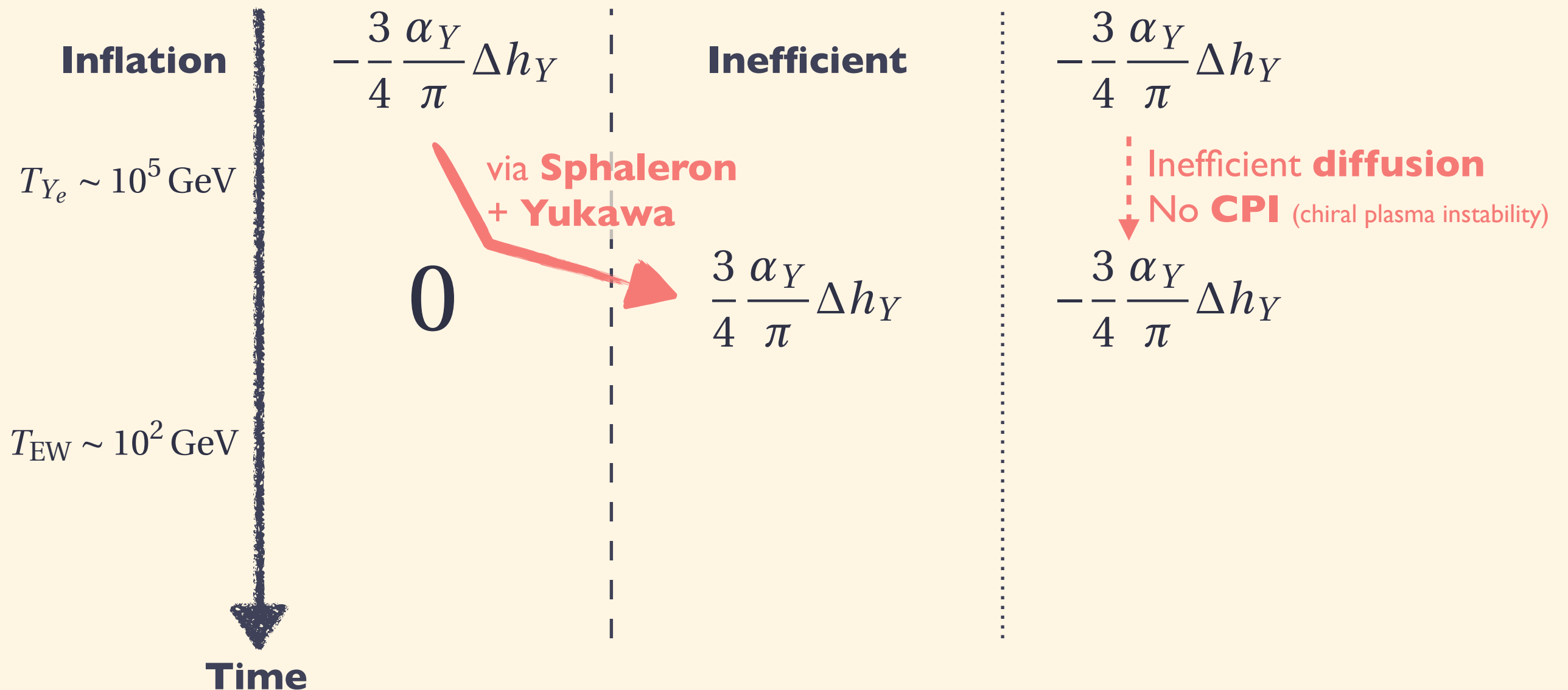
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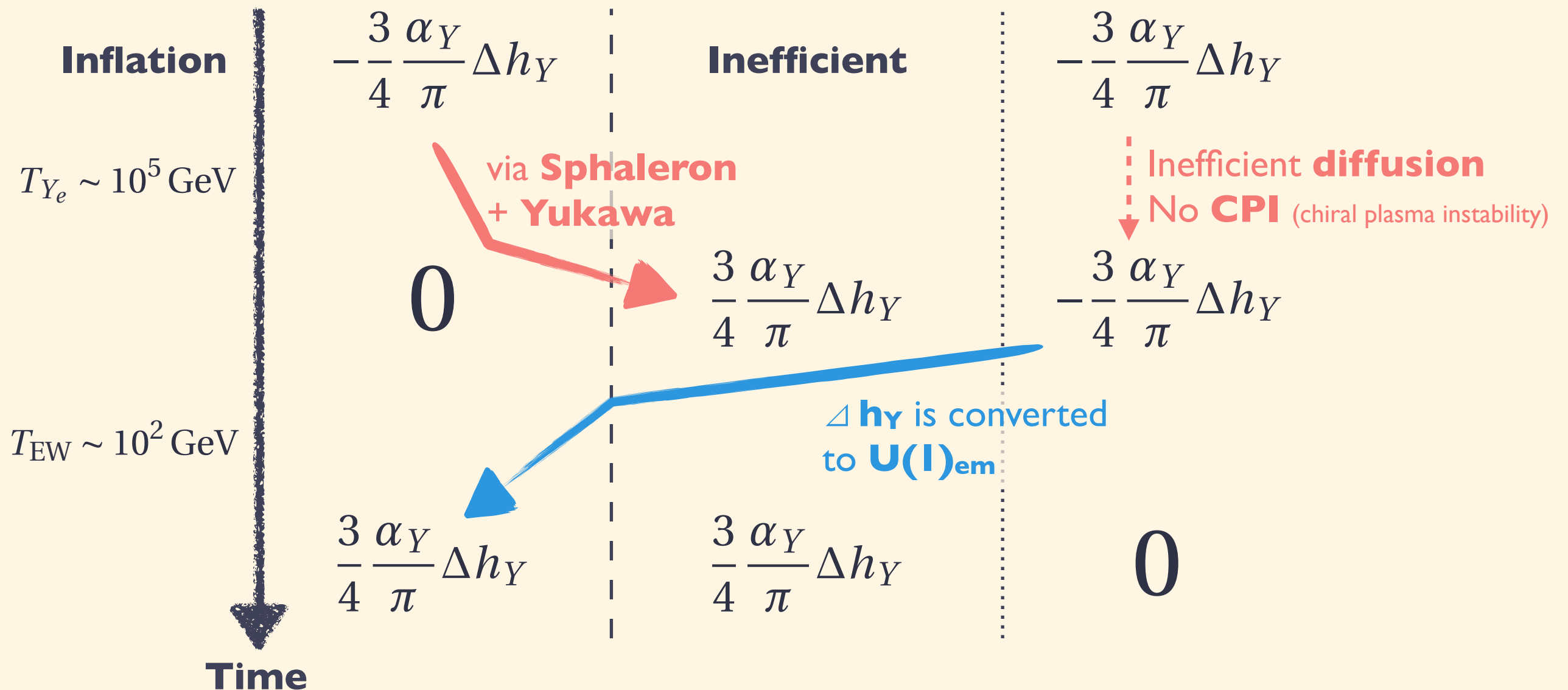
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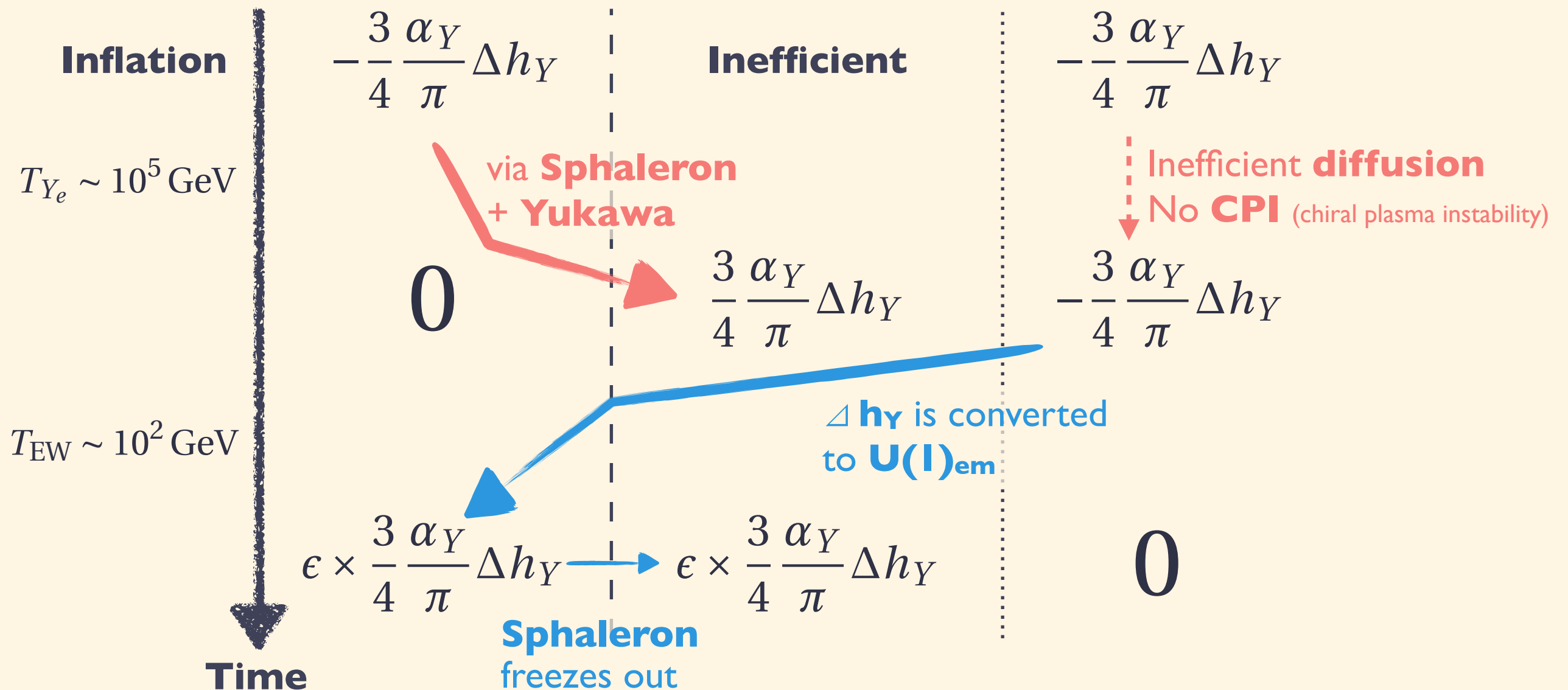
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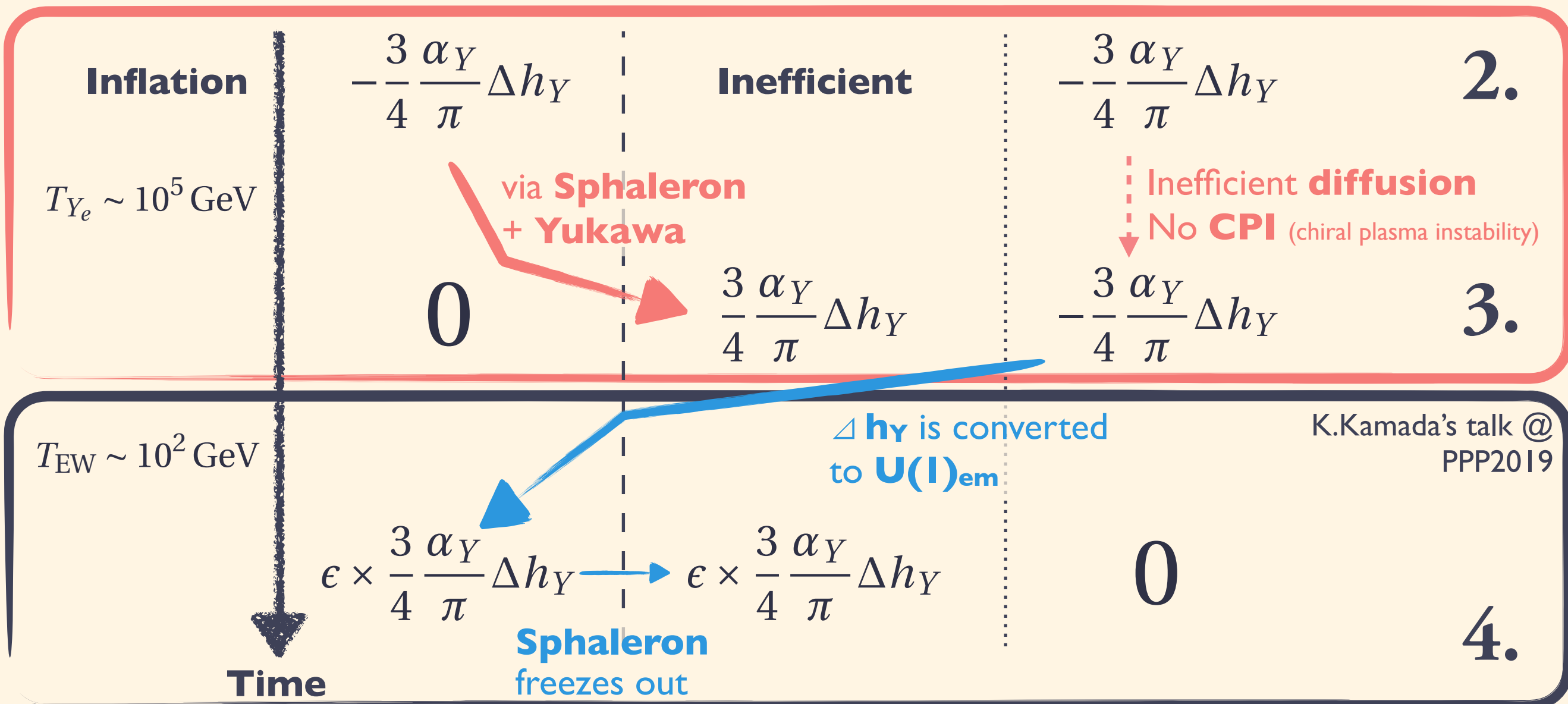
Outline of this Talk

Baryogenesis from B+L asymmetry?

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu =$$

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2.

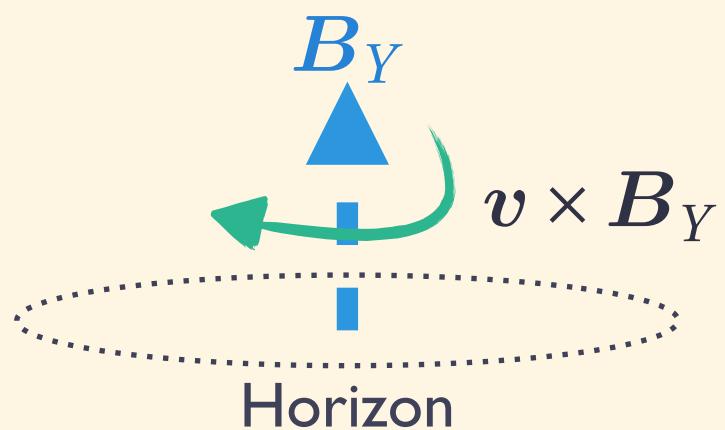
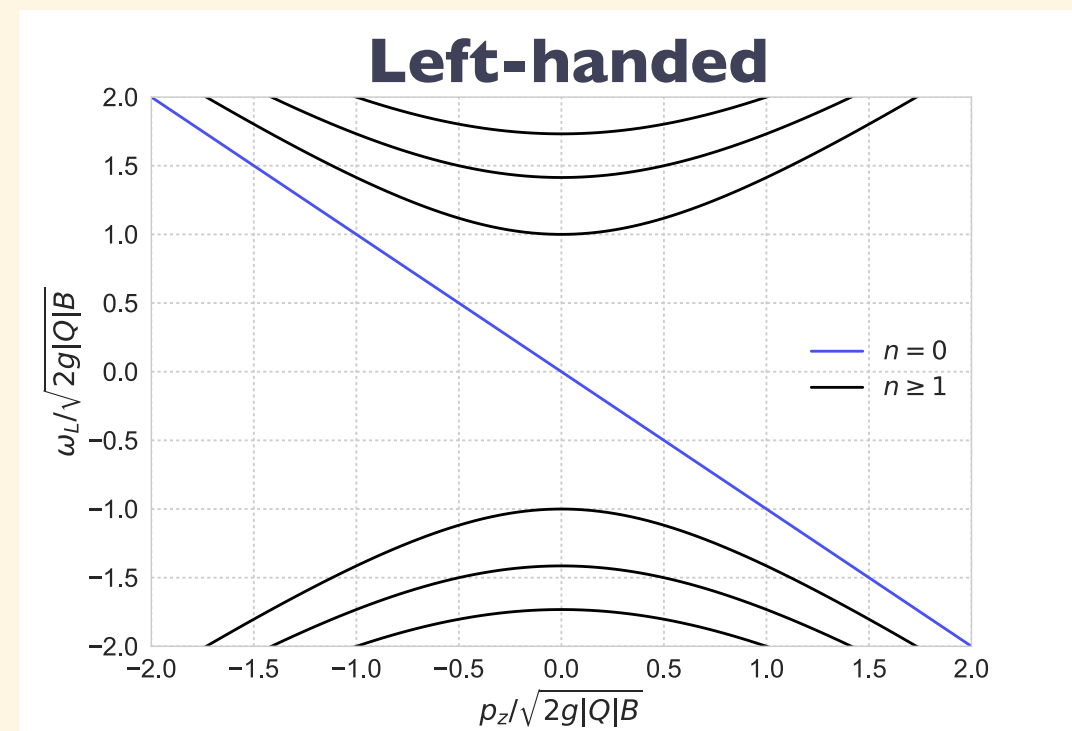
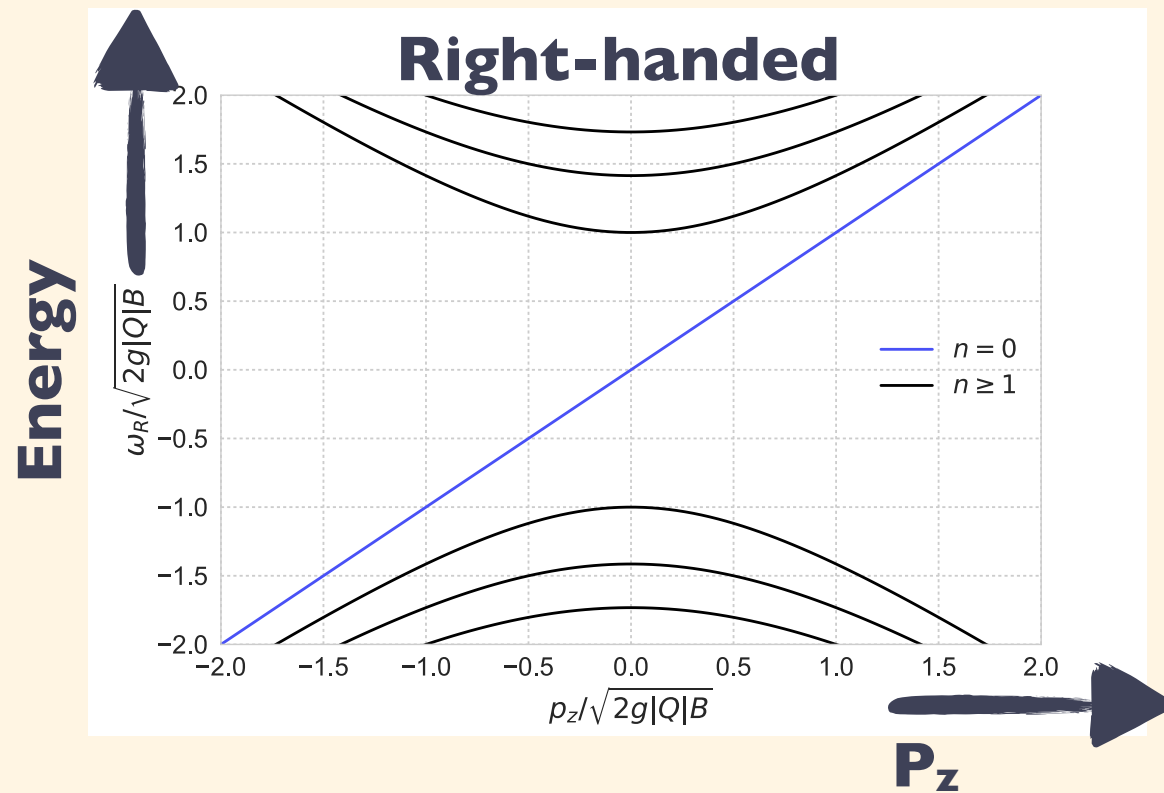
Production of

Helical gauge & Chiral fermion

Fermion Production

Landau Levels

- ▶ Turn off E_Y ; B_Y field modifies the **dispersion relation**.



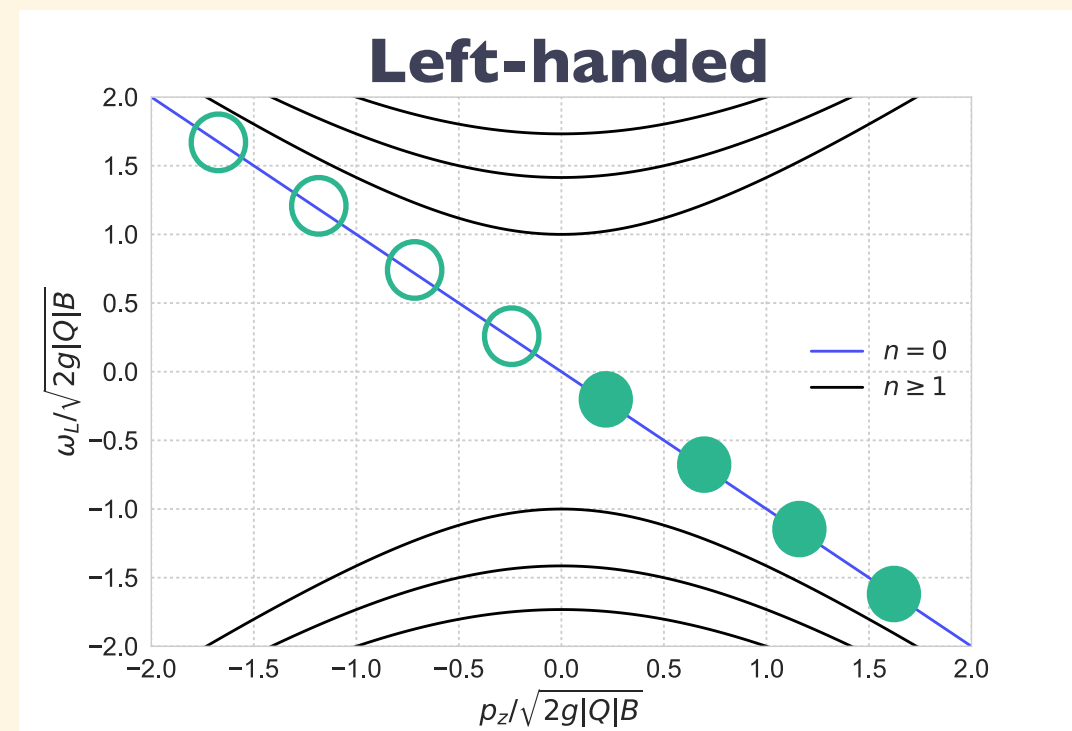
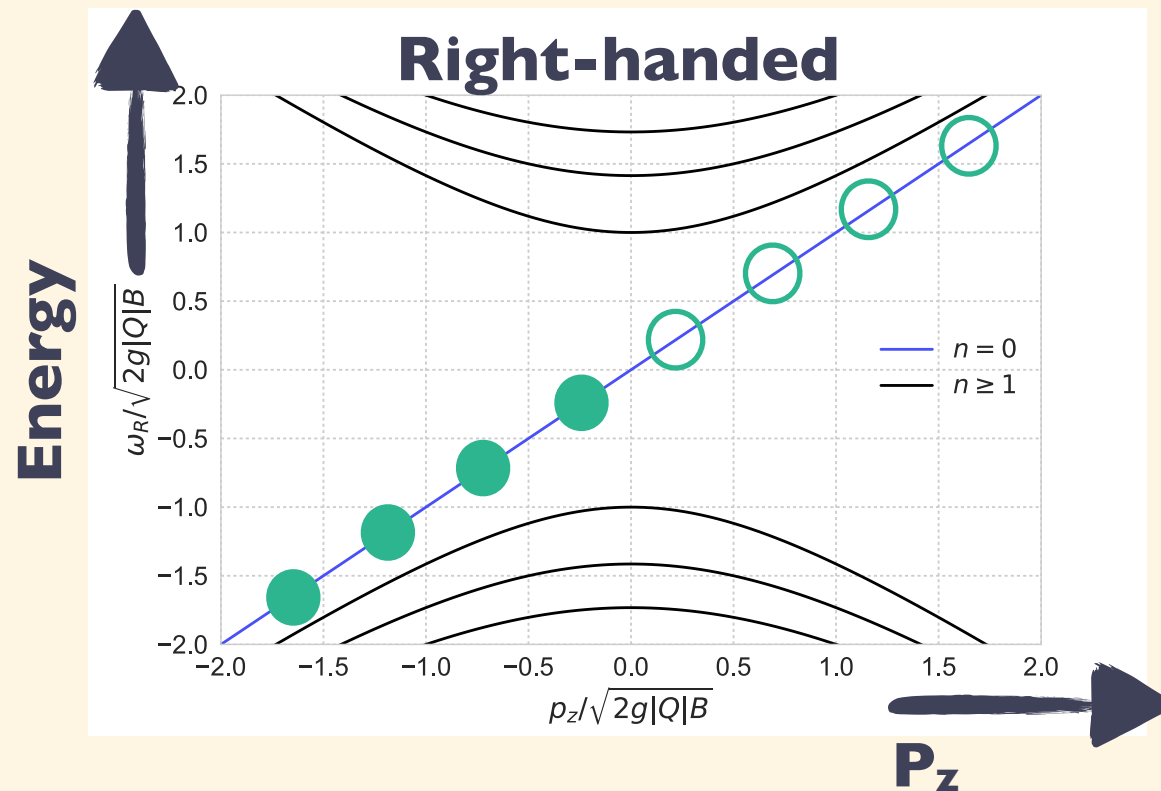
- **Landau level n** : transverse motion
- **P_z** : parallel motion

Fermion Production

Lowest Landau Level ($n=0$) & Chiral Anomaly

► Turn on E_Y and see what happens.

Nielsen, Ninomiya, Phys.Lett. **130B** (1983)

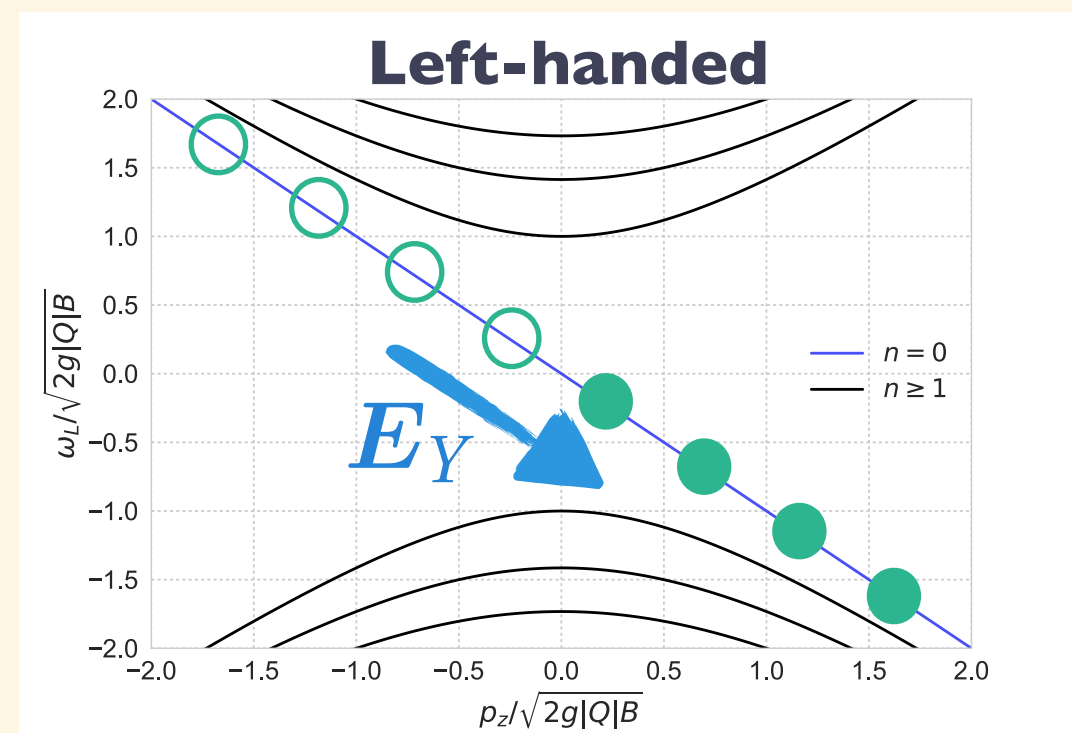
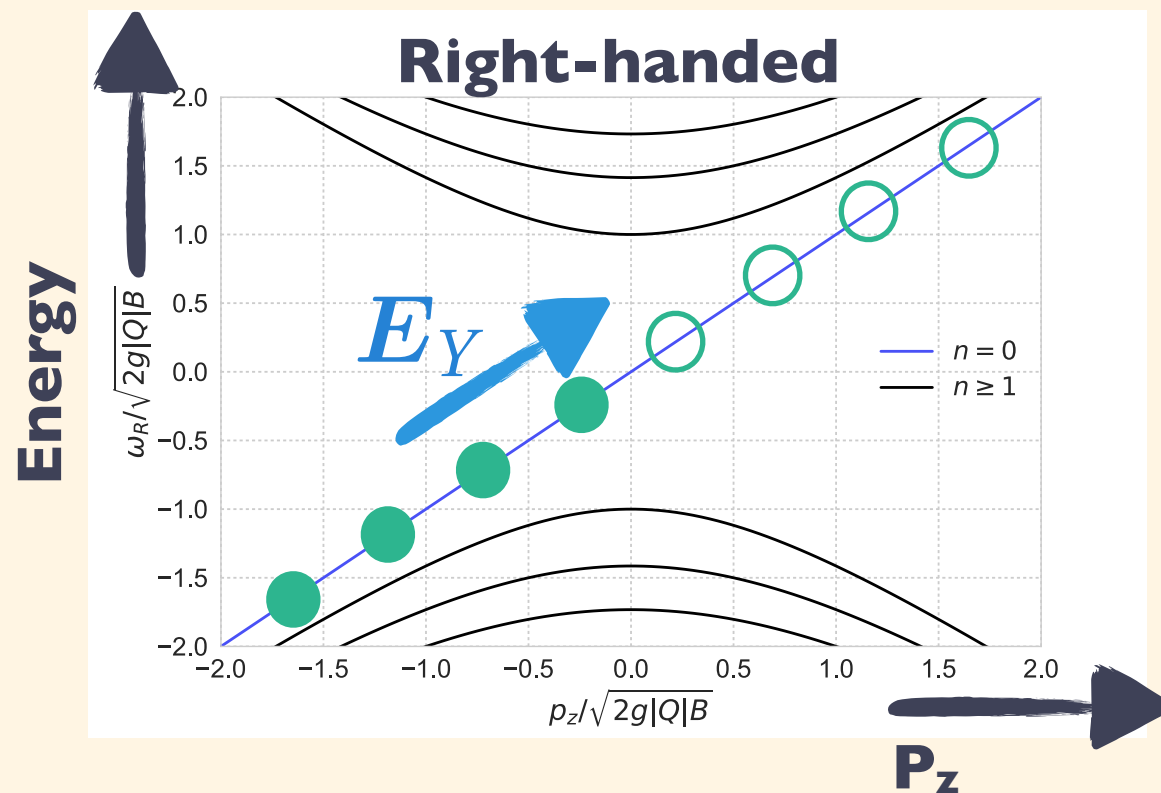


Fermion Production

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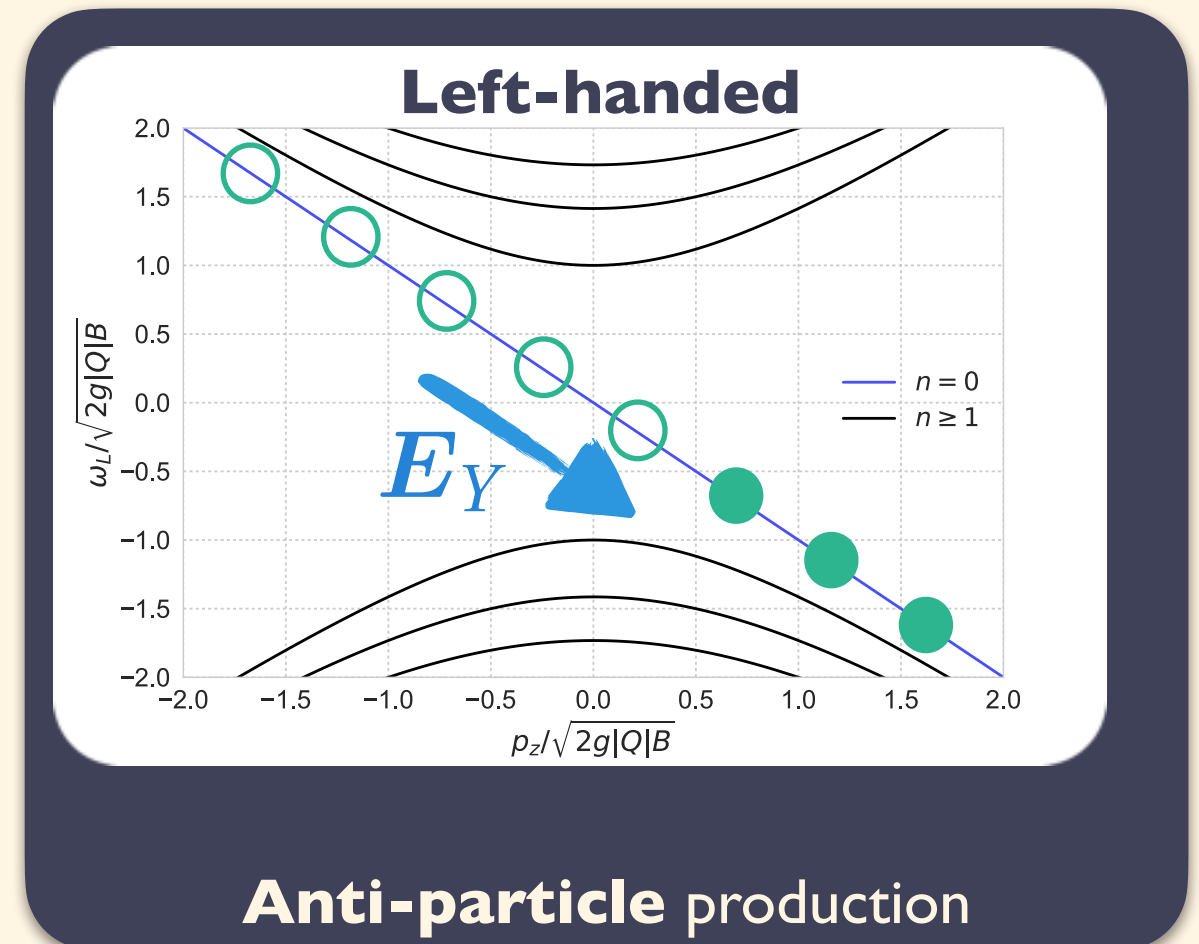
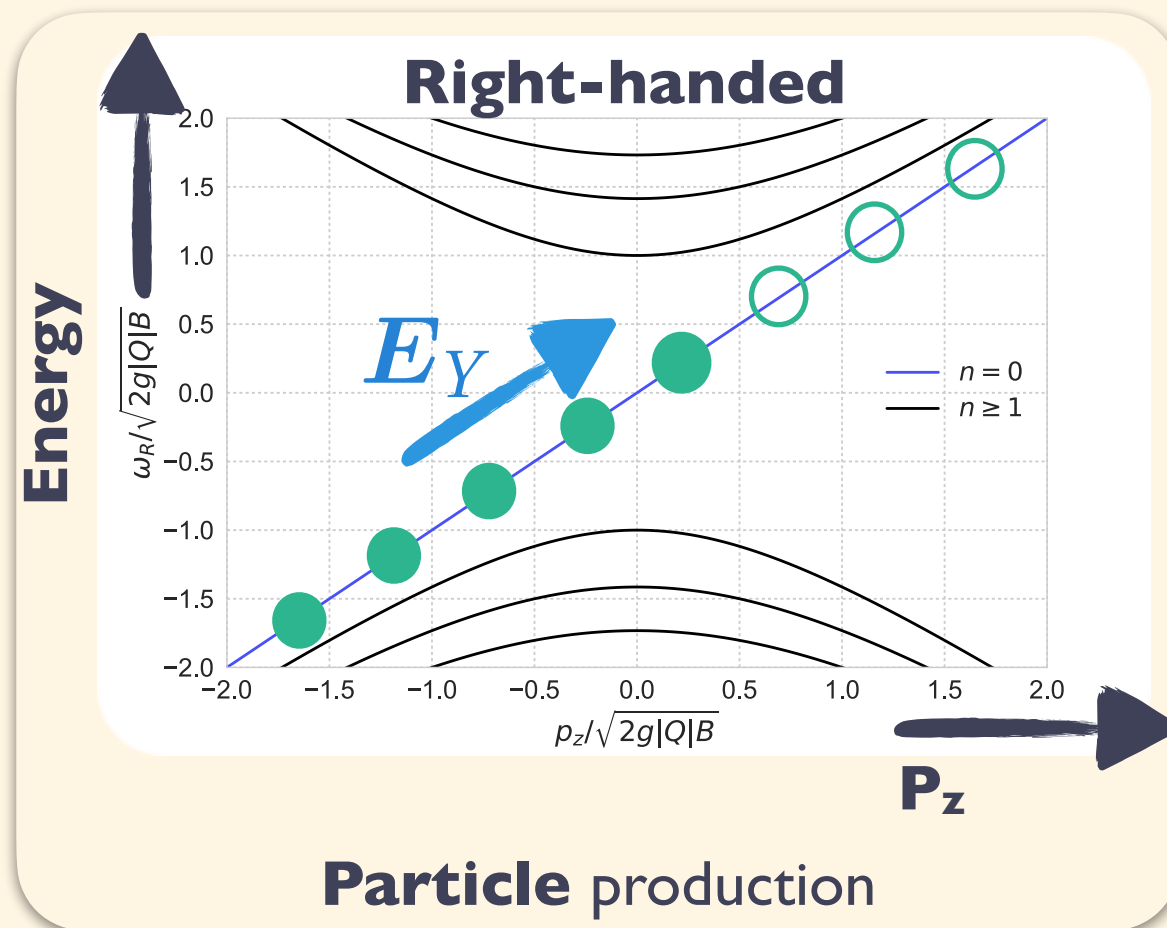


Fermion Production

Lowest Landau Level ($n=0$) & Chiral Anomaly

Nielsen, Ninomiya, Phys.Lett. **130B** (1983)

▶ Turn on E_Y and see what happens.



➔ Reproduce chiral anomaly!

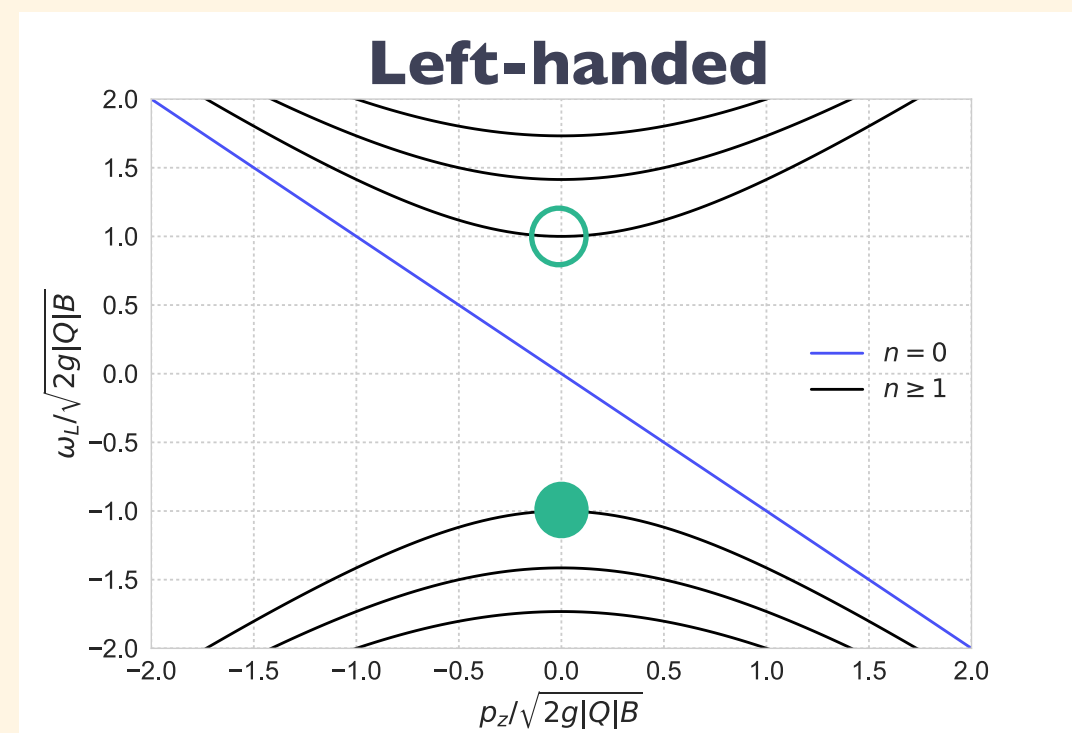
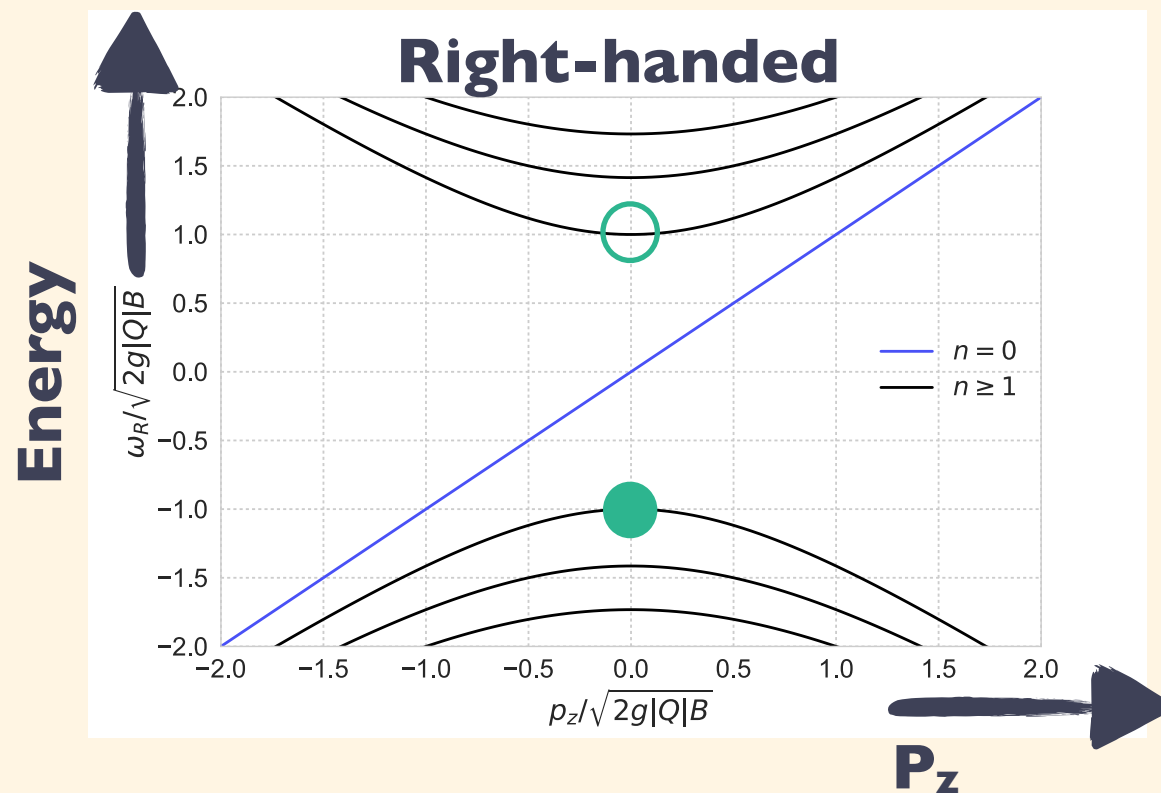
$$\dot{q}_\alpha = \dot{n}_\alpha - \dot{\bar{n}}_\alpha = \epsilon_\alpha N_\alpha \frac{g_Y^2 Q_\alpha^2}{4\pi^2} E_Y B_Y = -\epsilon_\alpha N_\alpha \frac{g_Y^2 Q_\alpha^2}{16\pi^2} Y^{\mu\nu} \tilde{Y}_{\mu\nu} \quad \text{w/ } \epsilon_\alpha = \pm \text{ for R/L, } N_\alpha = \text{dof}$$

Fermion Production

Higher Landau Levels ($n \geq 1$) & Pair Production

e.g., V.Domcke and **KM** 1806.08769

- ▶ Turn on E_y and see what happens.

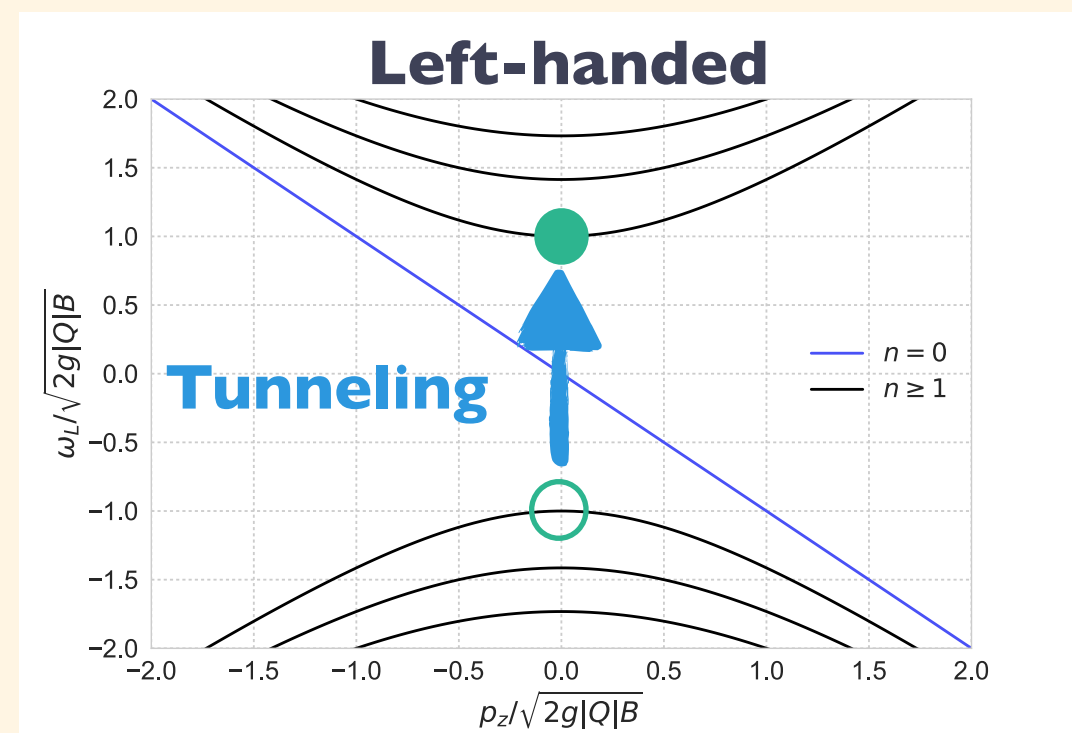
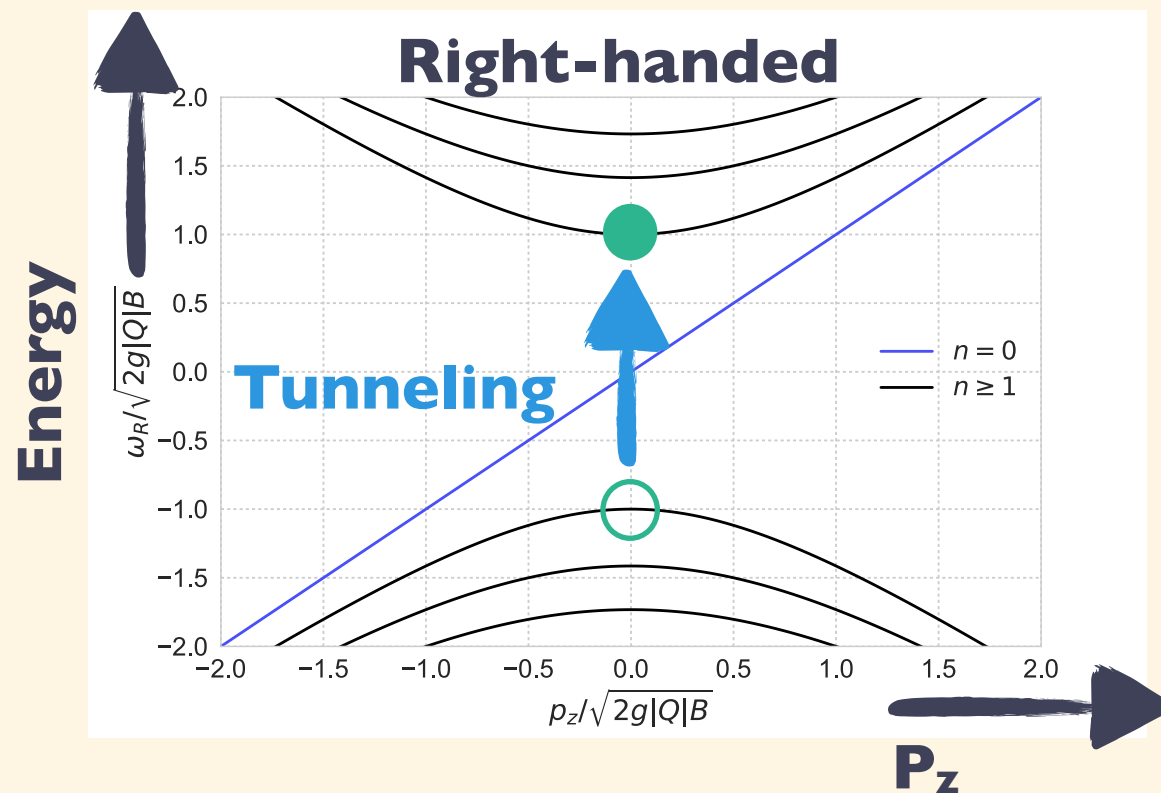


Fermion Production

Higher Landau Levels ($n \geq 1$) & Pair Production

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► Turn on E_Y and see what happens.



➔ Pair-production via Schwinger effect

$$\dot{n}_\alpha^{(n)} = \dot{\bar{n}}_\alpha^{(n)} = N_\alpha \frac{g_Y^2 Q_\alpha^2}{4\pi^2} E_Y B_Y e^{-\frac{2\pi n B_Y}{E_Y}}$$

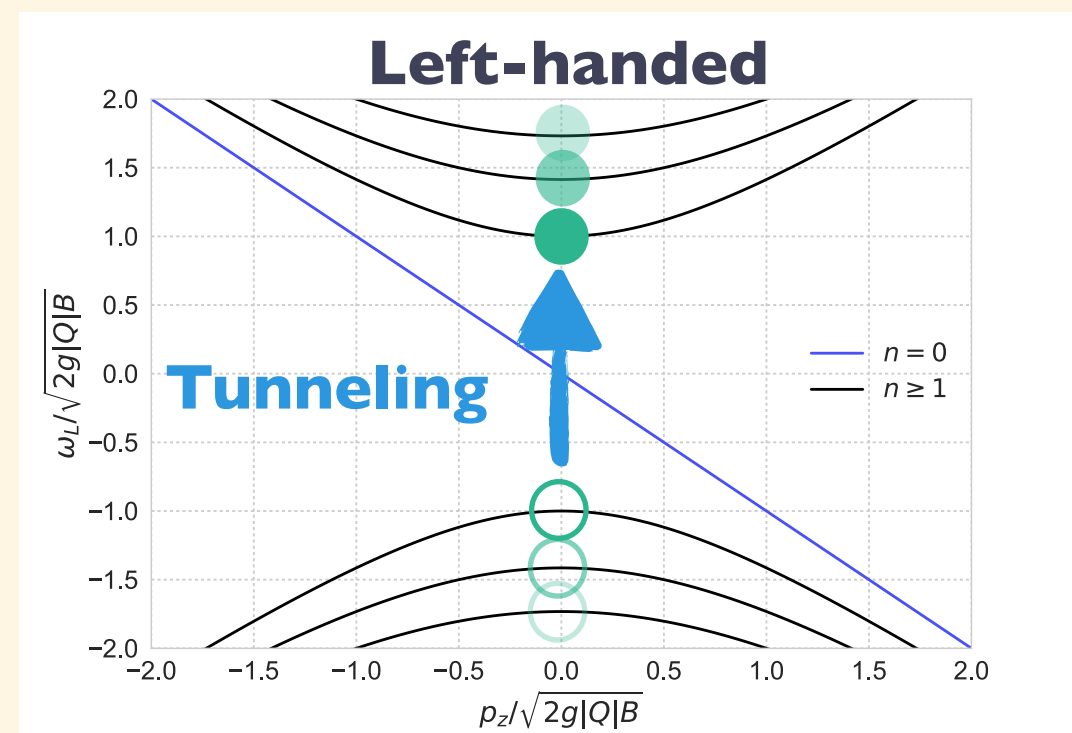
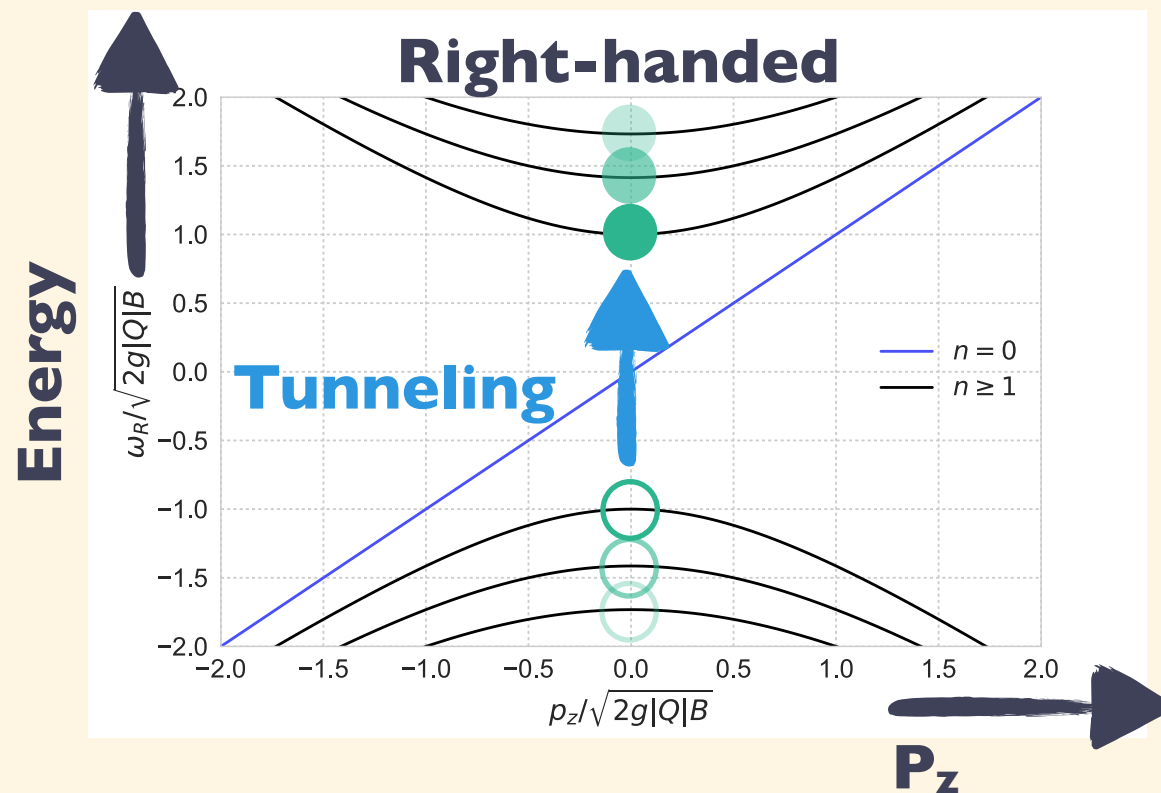
- Never contribute to asymmetries!

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$$\sum_{n=1} \dot{n}_\alpha^{(n)} = N_\alpha \frac{g_Y^2 Q_\alpha^2}{4\pi^2} E_Y B_Y \frac{1}{e^{\frac{2\pi B_Y}{E_Y}} - 1}$$

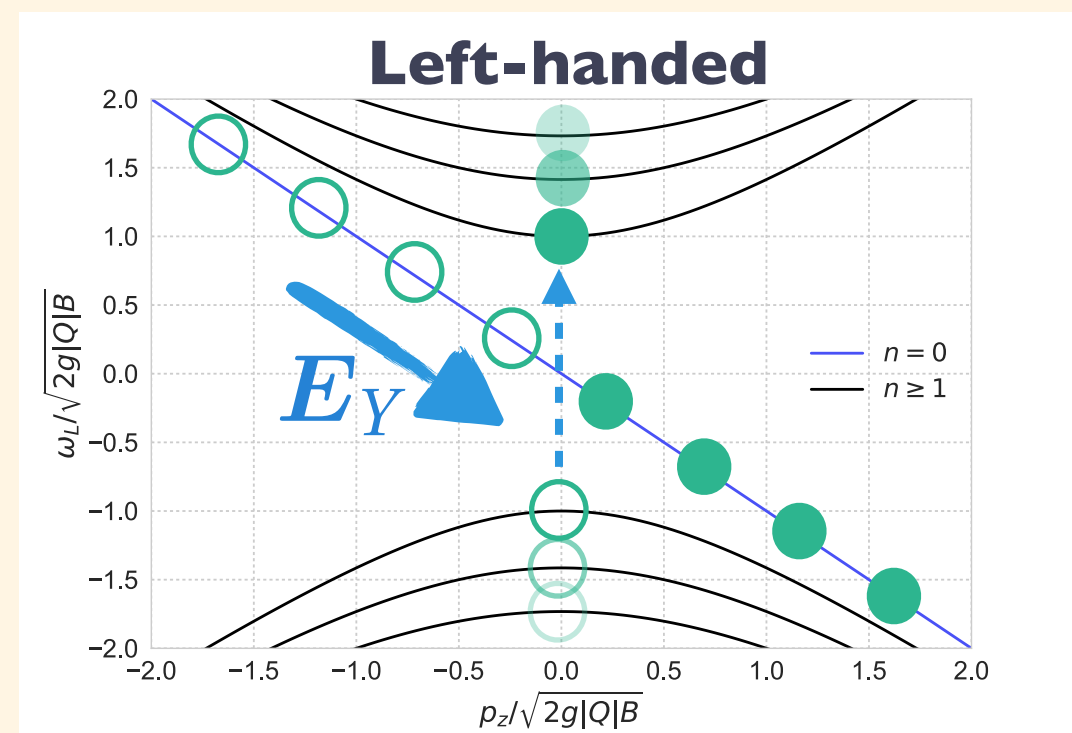
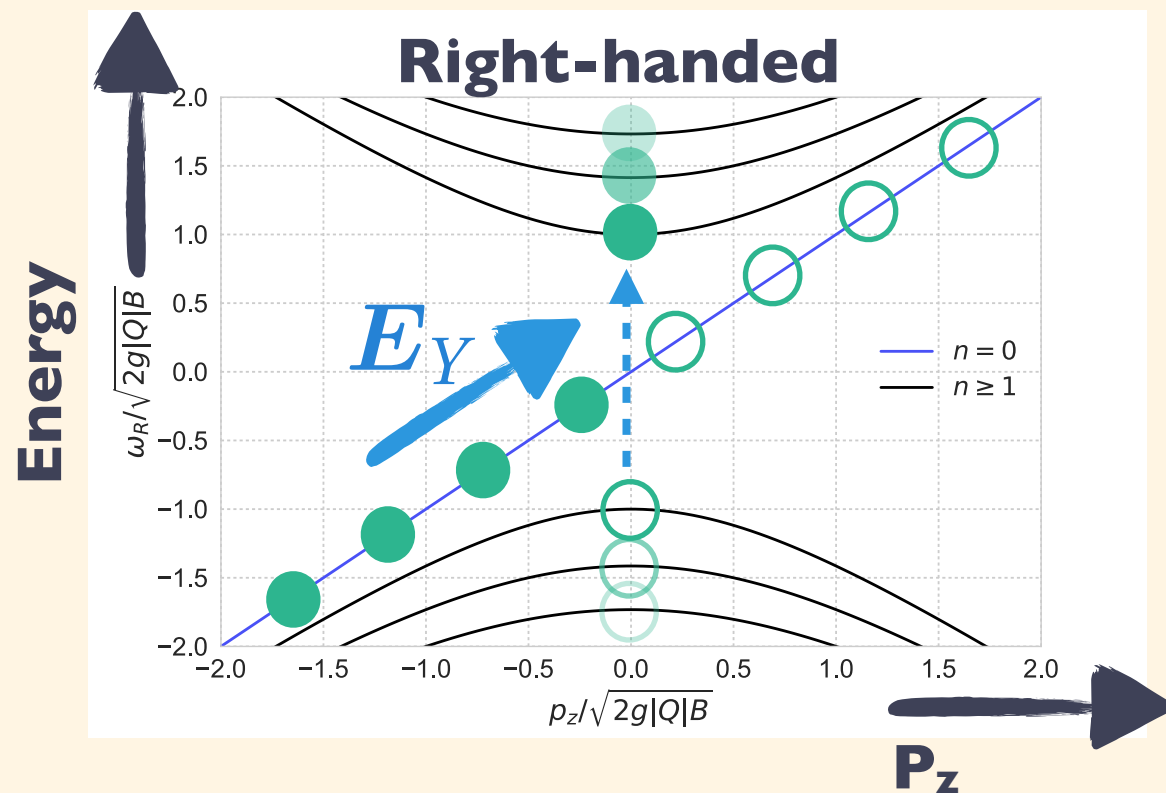
- Never contribute to asymmetries!

Fermion Production

Fermion Production in $\mathbf{B}_Y \parallel \mathbf{E}_Y$

e.g., V.Domcke and **KM** 1806.08769

- ▶ Turn on \mathbf{E}_Y and see what happens.



- **Chiral anomaly** from the lowest Landau level ($n=0$).
- **Pair production** from higher Landau levels ($n=1,2,\dots$).

Helical gauge & Chiral fermion

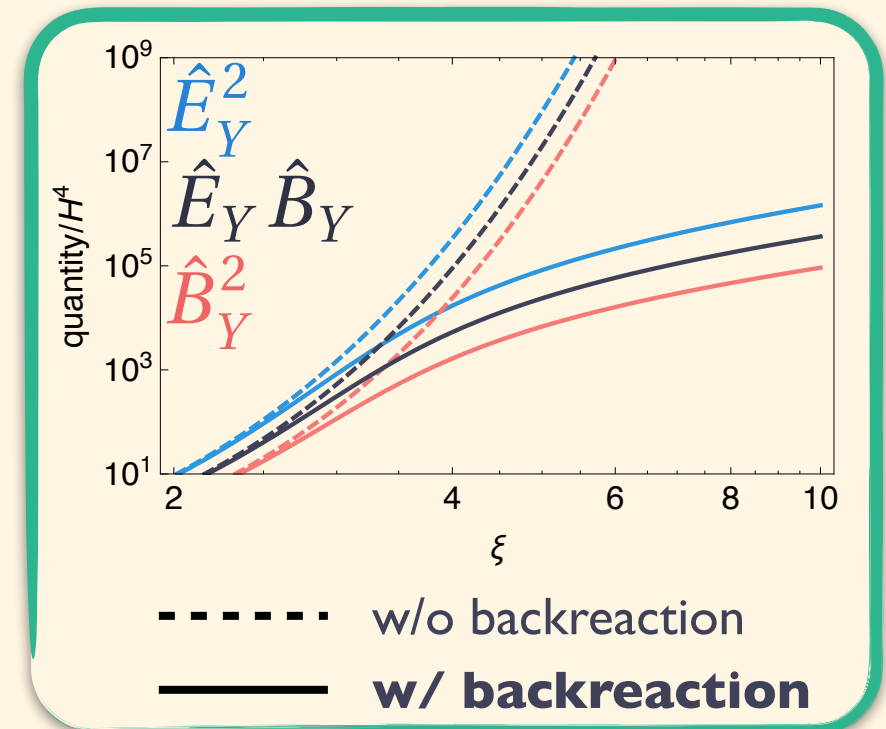
Implications on Axion Inflation

V.Domcke and **KM** 1806.08769

- ▶ **Backreaction** suppresses gauge field

$$0 = -\partial_t \mathbf{E}_Y + \nabla \times \mathbf{B}_Y + a \frac{g_Y^2}{4\pi^2} \frac{\dot{\phi}}{f_a} \mathbf{B}_Y - g_Y \mathbf{J}_Y$$

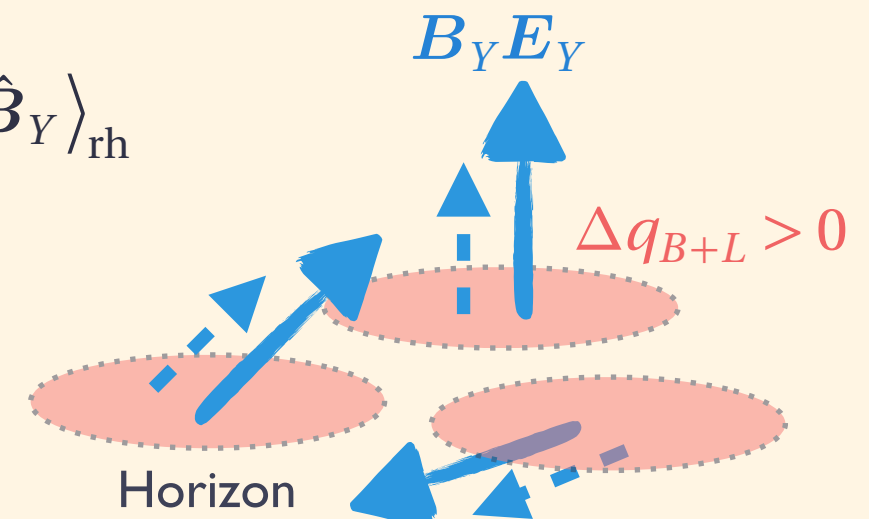
$$g_Y \mathbf{J}_Y = a \left[\sum_{\alpha} N_{\alpha} \frac{g_Y^3 |Q_{\alpha}|^3}{12\pi^2} \coth\left(\frac{\pi \hat{B}_Y}{\hat{E}_Y}\right) \frac{\hat{B}_Y}{H} \right] \mathbf{E}_Y$$



- ▶ **Primordial** generation of **B+L asym.**

$$\Delta q_{B+L}^{\text{rh}} = -\frac{3}{2} \frac{\alpha_Y}{\pi} \Delta h_Y^{\text{rh}} \quad \text{where} \quad \frac{\Delta h_Y^{\text{rh}}}{a_{\text{rh}}^3} = -\frac{2}{3H_{\text{rh}}} \langle \hat{\mathbf{E}}_Y \cdot \hat{\mathbf{B}}_Y \rangle_{\text{rh}}$$

$$\frac{\mu_{B+L}^{\text{rh}}}{T_{\text{rh}}} \sim 10^{-3} \left(\frac{H_{\text{rh}}}{10^{14} \text{ GeV}} \right)^{\frac{3}{2}} \left(\frac{\langle \hat{\mathbf{E}}_Y \cdot \hat{\mathbf{B}}_Y \rangle_{\text{rh}} / H_{\text{rh}}^4}{10^5} \right)$$



3.

Survival of
Helical Gauge Field

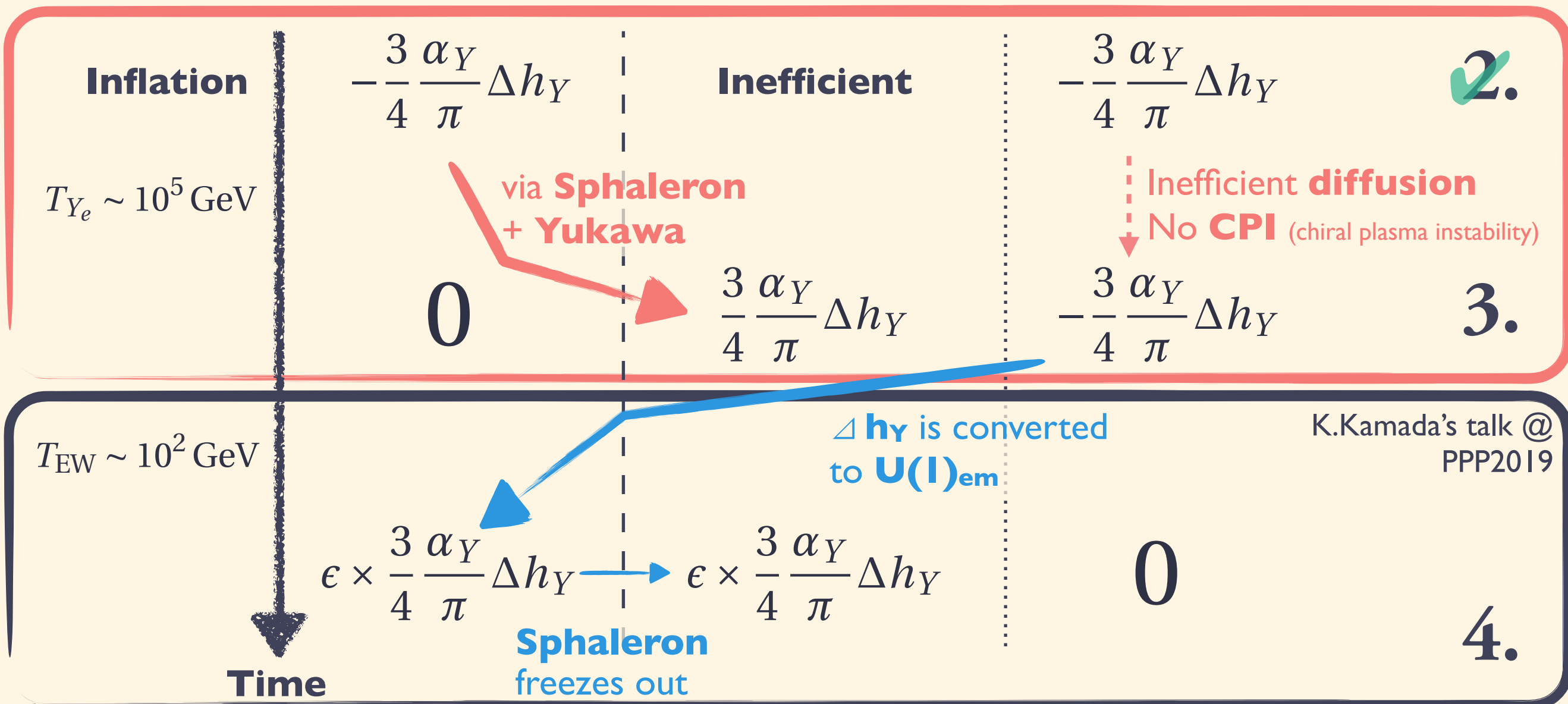
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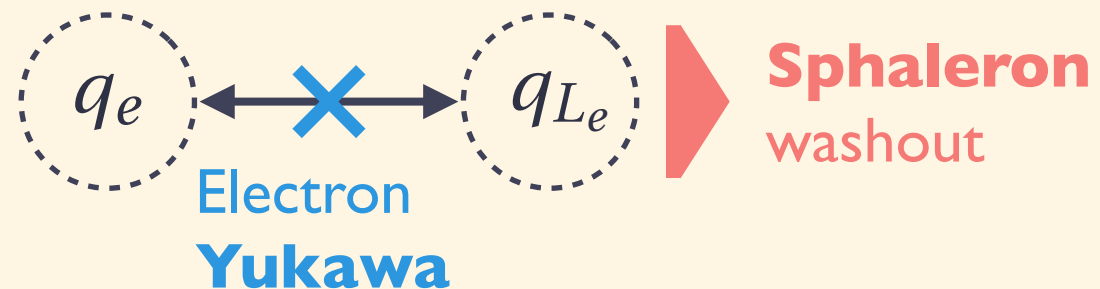


Survival of Helical Gauge Field

Avoid Chiral Plasma Instability

V.Domcke, B.Harling, E.Morgante, **KM**
1905.13318

- ▶ **Survival of q_e** from Sphaleron + Yukawa washout



Electron Yukawa is **NOT** equilibrated till

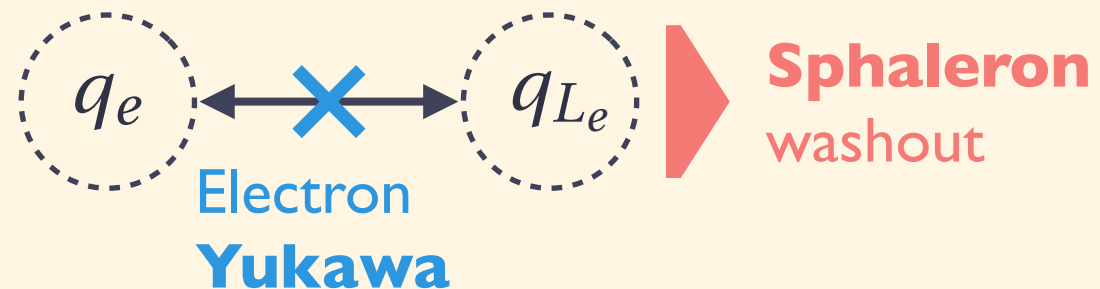
$$T < T_{Y_e} \sim 10^5 \text{ GeV}$$

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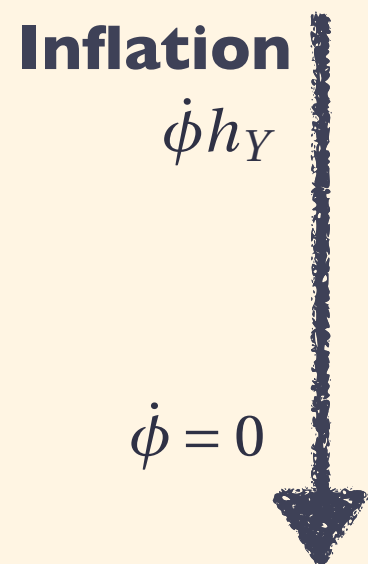
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- ▶ **Chiral Plasma Instability (CPI)** as an inverse process

$$\partial \cdot J_e = -\frac{g_Y^2}{16\pi^2} Y_{\mu\nu} \tilde{Y}^{\mu\nu} + (\text{Yukawa})$$

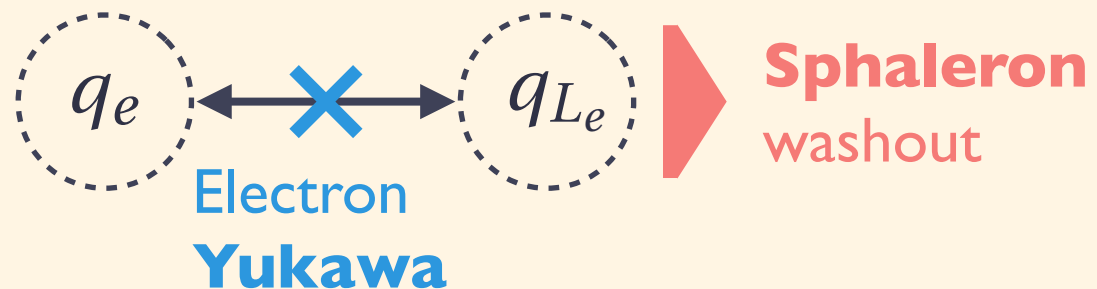


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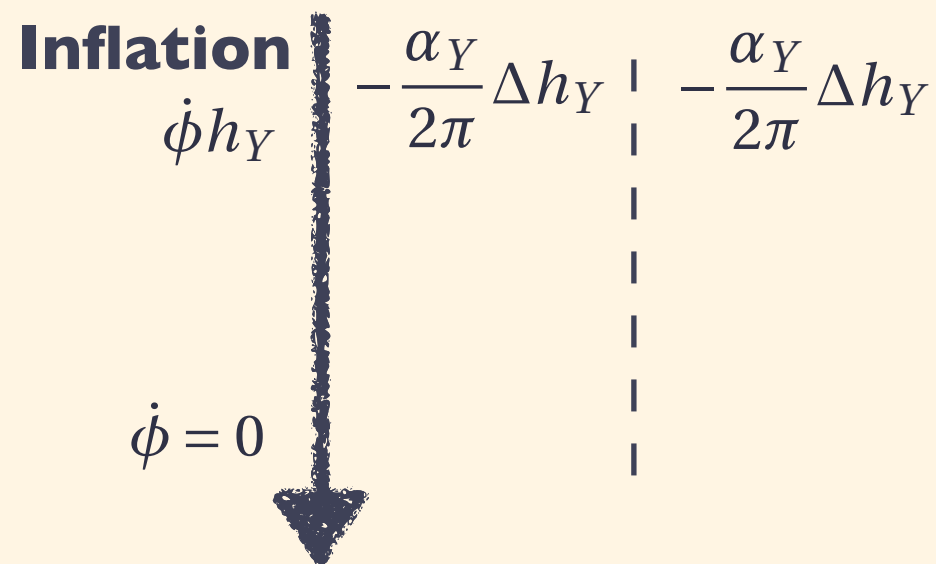
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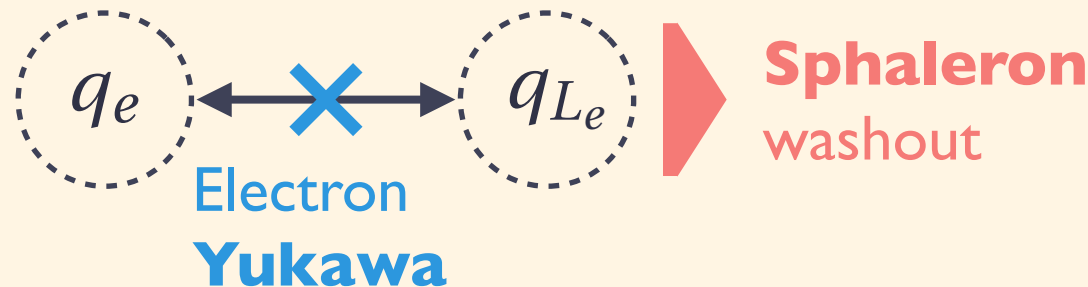


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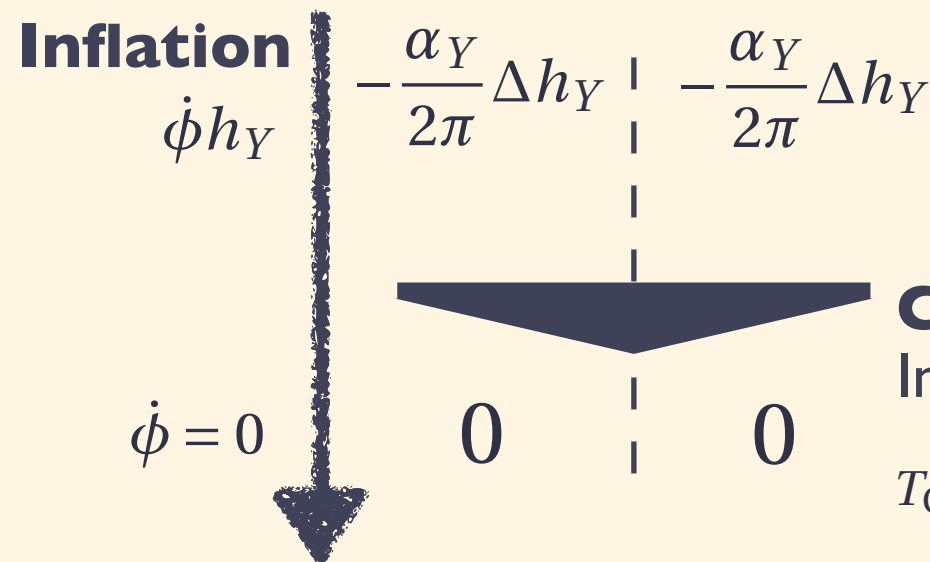
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$$\partial_\eta h_{Y,k}(\eta) = \frac{2k}{\sigma_Y} \left(\frac{k_{\text{CPI}}}{r_k(\eta)} - k \right) h_{Y,k}(\eta)$$

w/ $k_{\text{CPI}} = \frac{2\alpha_Y}{\pi} \frac{95}{18} \mu_e, \quad r_k(\eta) = \frac{k h_{Y,k}(\eta) / 2}{\rho_{B,k}(\eta)}$

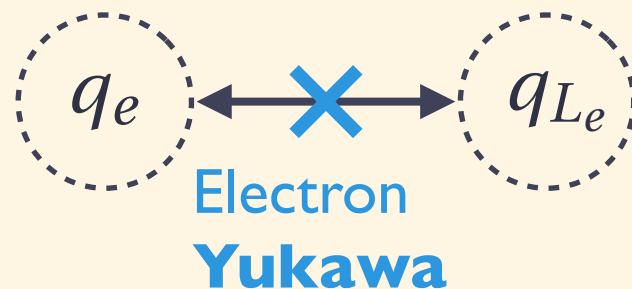
Joyce, Shaposhnikov 9703005, Akamatsu, Yamamoto 1302.2125,...

Survival of Helical Gauge Field

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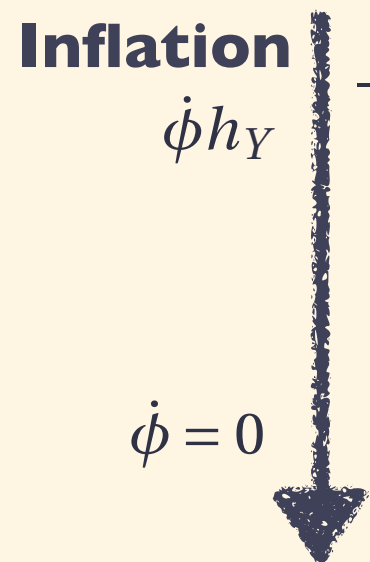


▶ **Sphaleron washout**

Electron Yukawa is **NOT** equilibrated till
 $T < T_{Y_e} \sim 10^5 \text{ GeV}$

- ▶ **Chiral Plasma Instability (CPI)** as an inverse process

$$\partial \cdot J_e = -\frac{\alpha_Y}{2\pi} \partial \cdot h_Y + (\text{Yukawa}) \longrightarrow \dot{q}_e = -\frac{\alpha_Y}{2\pi} \dot{h}_Y - \frac{711}{481} \Gamma_{Y_e} q_e$$



$$-\frac{\alpha_Y}{2\pi} \Delta h_Y \quad | \quad -\frac{\alpha_Y}{2\pi} \Delta h_Y$$

$$T_{\text{CPI}} < T_{Y_e} \sim 10^5 \text{ GeV}$$

Sphaleron + Yukawa

▶ 0

$$-\frac{\alpha_Y}{2\pi} \Delta h_Y$$

Chiral Plasma Instability

$$T_{\text{CPI}} \sim 10^5 \text{ GeV} \left(\frac{H_{\text{rh}}}{10^{14} \text{ GeV}} \right)^3 \left(\frac{\langle \hat{\mathbf{E}}_Y \cdot \hat{\mathbf{B}}_Y \rangle_{\text{rh}} / H_{\text{rh}}^4}{10^5} \right)^2$$

Survival of Helical Gauge Field

Avoid Magnetic Diffusion

V.Domcke, B.Harling, E.Morgante, **KM**
1905.13318

▶ Chiral Magneto Hydro Dynamics (ChMHD)

$$\frac{\partial}{\partial \eta} \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = \nu \nabla^2 \mathbf{v} + \frac{1}{\rho + P} \left(-\frac{1}{2} \nabla B_Y^2 + (\mathbf{B}_Y \cdot \nabla) \mathbf{B}_Y \right)$$

$$\frac{\partial \mathbf{B}_Y}{\partial \eta} = \frac{\nabla^2}{\sigma_Y} \mathbf{B}_Y + \nabla \times (\mathbf{v} \times \mathbf{B}_Y) + \frac{2\alpha_Y}{\pi} \frac{\mu_{Y,5}}{\sigma_Y} \nabla \times \mathbf{B}_Y \quad \text{w/ } \mu_{Y,5} = \sum_{\alpha} \epsilon_{\alpha} N_{\alpha} Q_{Y,\alpha}^2 \mu_{\alpha}$$

Survival of Helical Gauge Field

Avoid Magnetic Diffusion

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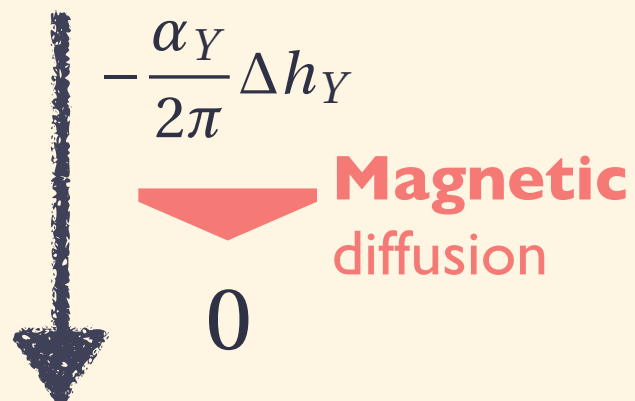
$$\frac{\partial}{\partial \eta} v + v \cdot \nabla v = v \nabla^2 v + \frac{1}{\rho + P} \left(-\frac{1}{2} \nabla B_Y^2 + (B_Y \cdot \nabla) B_Y \right)$$

Chiral Plasma
Instability

$$\frac{\partial B_Y}{\partial \eta} = \frac{\nabla^2}{\sigma_Y} B_Y + \nabla \times (v \times B_Y) + \frac{2\alpha_Y \mu_{Y,5}}{\pi \sigma_Y} \nabla \times B_Y \quad \text{w/ } \mu_{Y,5} = \sum_{\alpha} \epsilon_{\alpha} N_{\alpha} Q_{Y,\alpha}^2 \mu_{\alpha}$$

Magnetic
Diffusion

$$T_{\text{diff}} \sim \frac{\alpha_Y \ln \alpha_Y^{-1}}{5} H_{\text{rh}}$$



Survival of Helical Gauge Field

Avoid Magnetic Diffusion

V.Domcke, B.Harling, E.Morgante, **KM**
1905.13318

▶ Chiral Magneto Hydro Dynamics (ChMHD)

$$\frac{\partial}{\partial \eta} v + v \cdot \nabla v = \nu \nabla^2 v + \frac{1}{\rho + P} \left(-\frac{1}{2} \nabla B_Y^2 + (B_Y \cdot \nabla) B_Y \right)$$

Chiral Plasma
Instability

$$\frac{\partial B_Y}{\partial \eta} = \frac{\nabla^2}{\sigma_Y} B_Y + \nabla \times (v \times B_Y) + \frac{2\alpha_Y \mu_{Y,5}}{\pi \sigma_Y} \nabla \times B_Y \quad \text{w/ } \mu_{Y,5} = \sum_{\alpha} \epsilon_{\alpha} N_{\alpha} Q_{Y,\alpha}^2 \mu_{\alpha}$$

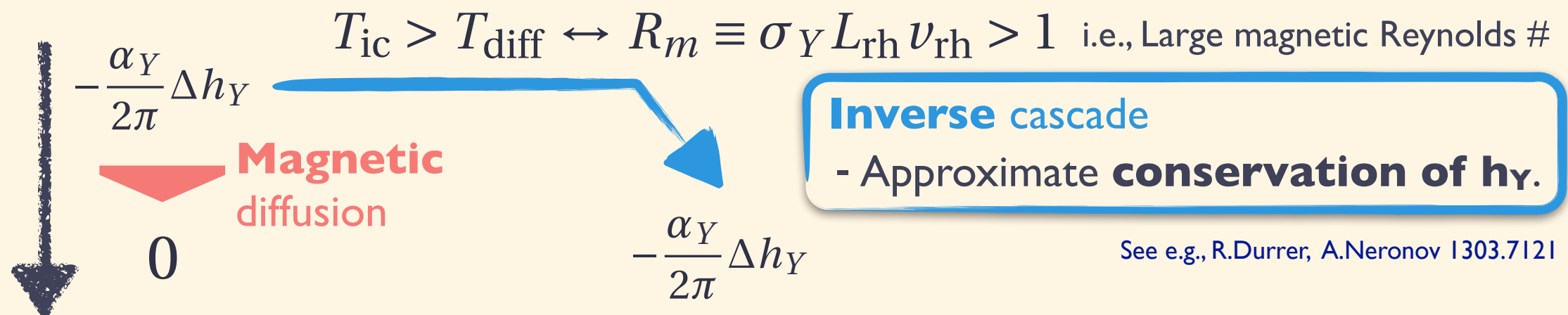
Magnetic
Diffusion

$$T_{\text{diff}} \sim \frac{\alpha_Y \ln \alpha_Y^{-1}}{5} H_{\text{rh}}$$

Inverse
cascade

$$T_{\text{ic}} \sim \nu_{\text{rh}} T_{\text{rh}}$$

▶ Inverse cascade: transfer from short to long wave length.



See e.g., R.Durrer, A.Neronov 1303.7121

4.

**Regeneration of
Baryon Asymmetry**

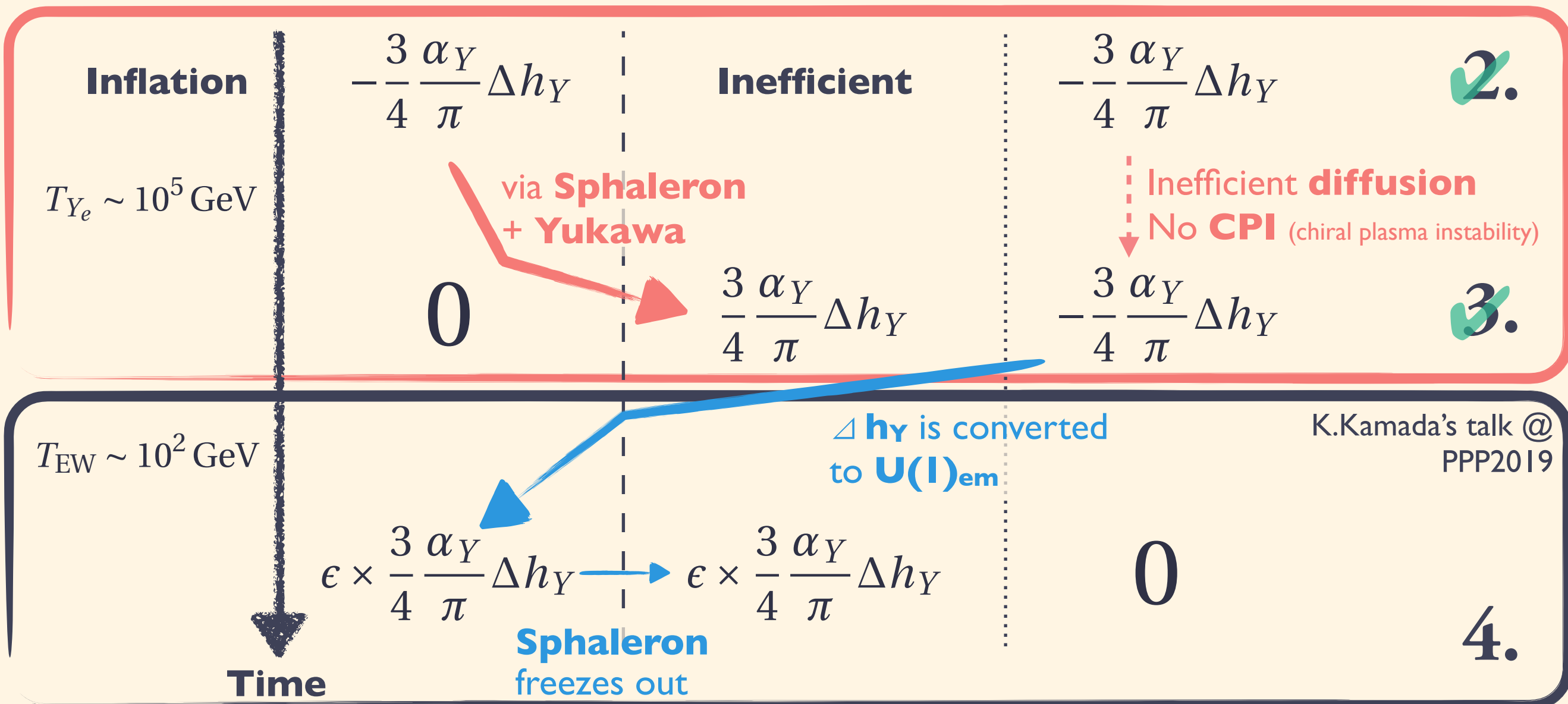
Outline of this Talk

Baryogenesis from B+L asymmetry?

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu =$$

$$3\partial_\mu K_{CS}^\mu$$

$$-\frac{3\alpha_Y}{4\pi}\partial_\mu h_Y^\mu$$



Regeneration of Baryon Asym.


Baryogenesis from Decaying Helicity

► Transport equation @ EW Crossover

T. Fujita, K.Kamada 1602.02109
K.Kamada, A.Long 1610.03074

- Slowest processes: EW sphaleron & Decaying helicity

$$\partial_\eta q_B = -\frac{111}{34} \Gamma_{W,\text{sph}} q_B + \frac{3}{2} (g_2^2 + g_Y^2) \sin(2\theta) (\partial_\eta \theta) \frac{\Delta h_Y^{\text{rh}}}{8\pi^2}$$


$$\begin{cases} \Gamma_{W,\text{sph}} \propto e^{-\frac{M_{\text{sph}}(T)}{T}} \\ M_{\text{sph}}(T) \propto v(T) \end{cases}$$

Regeneration of Baryon Asym.

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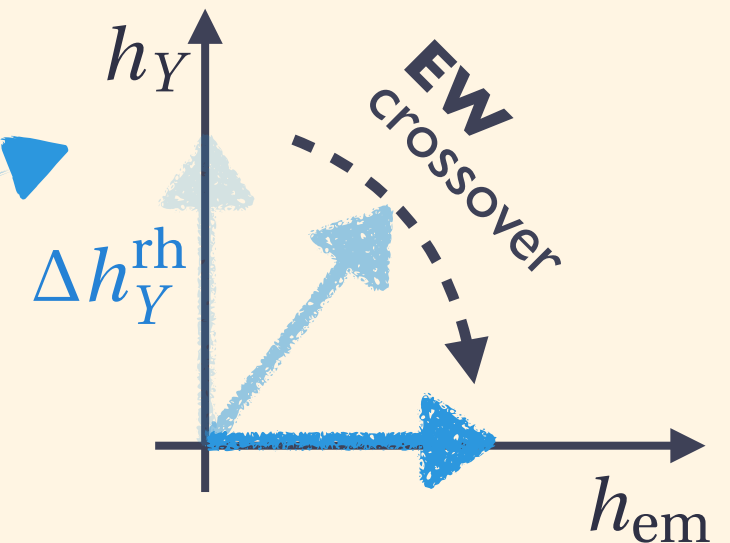
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Regeneration of Baryon Asym.

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► Transport equation @ EW Crossover

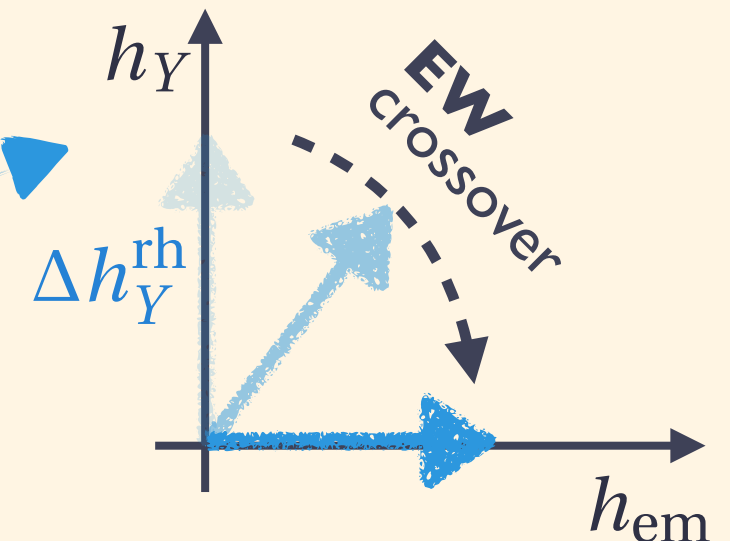
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$$\propto \partial_\eta h_Y$$



- EW sphaleron washout v.s. Decaying helicity

$$\eta_B = \frac{q_B}{s} \simeq \epsilon \times \frac{3\alpha_Y}{4\pi} \frac{\Delta h_Y}{s} \Big|_{\text{rh}}$$

$$\Delta h_Y^{\text{rh}} = -\frac{2}{3H_{\text{rh}}} \langle \hat{\mathbf{E}}_Y \cdot \hat{\mathbf{B}}_Y \rangle_{\text{rh}}$$

Sphaleron washout factor

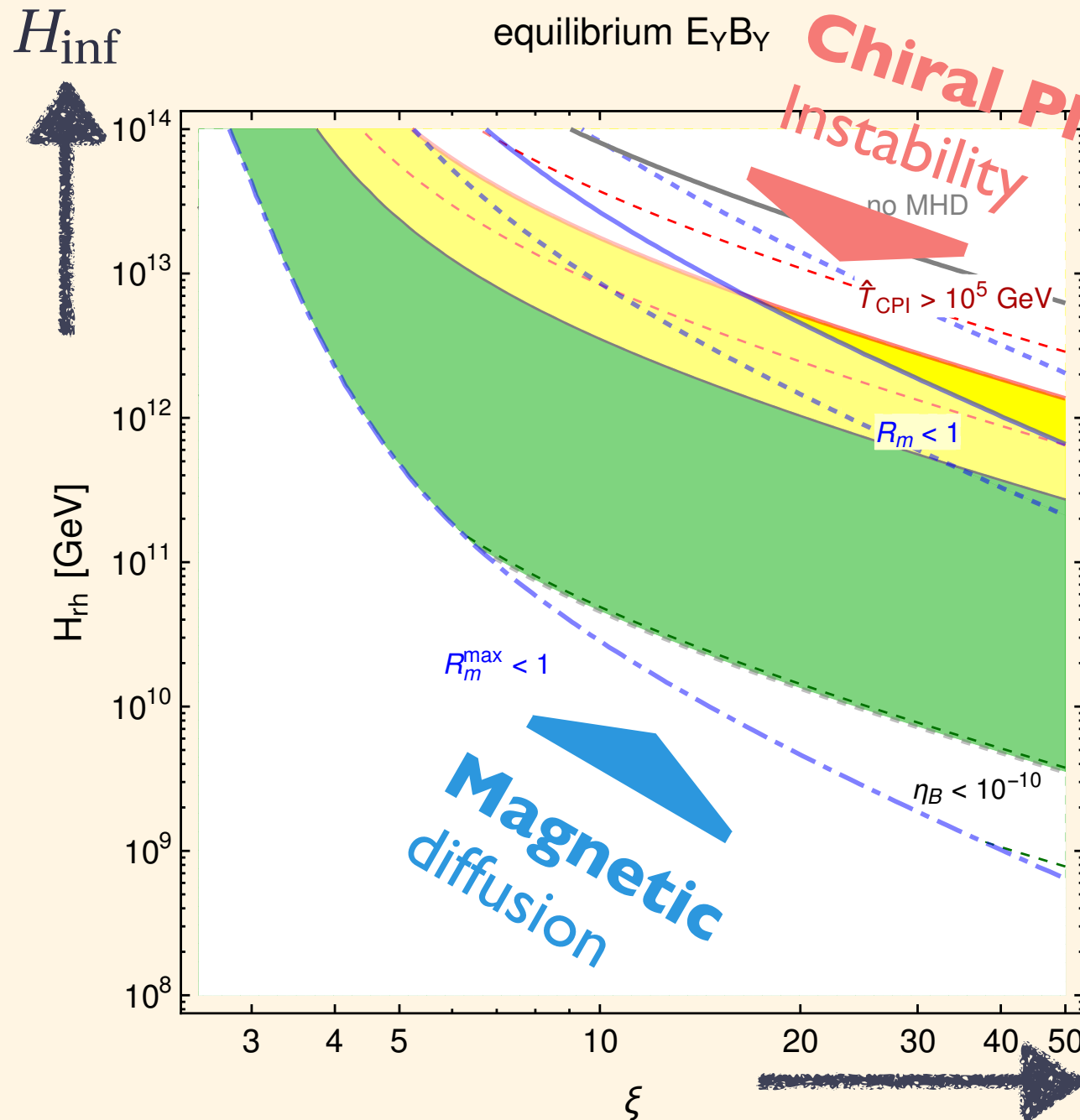
$$\epsilon \equiv \left[\frac{34}{111} \left(1 + \frac{\alpha_2}{\alpha_Y} \right) \frac{H}{\Gamma_{W,\text{sph}}} f(\theta, T) \right]_{T_{\text{EW}}}$$

- Huge uncertainties...

$$\text{w/ } f(\theta, T) = -T \frac{d\theta}{dT} \sin(2\theta)$$

Baryogenesis from axion inflation

Viability parameters for Baryogenesis



Observed
Baryon #

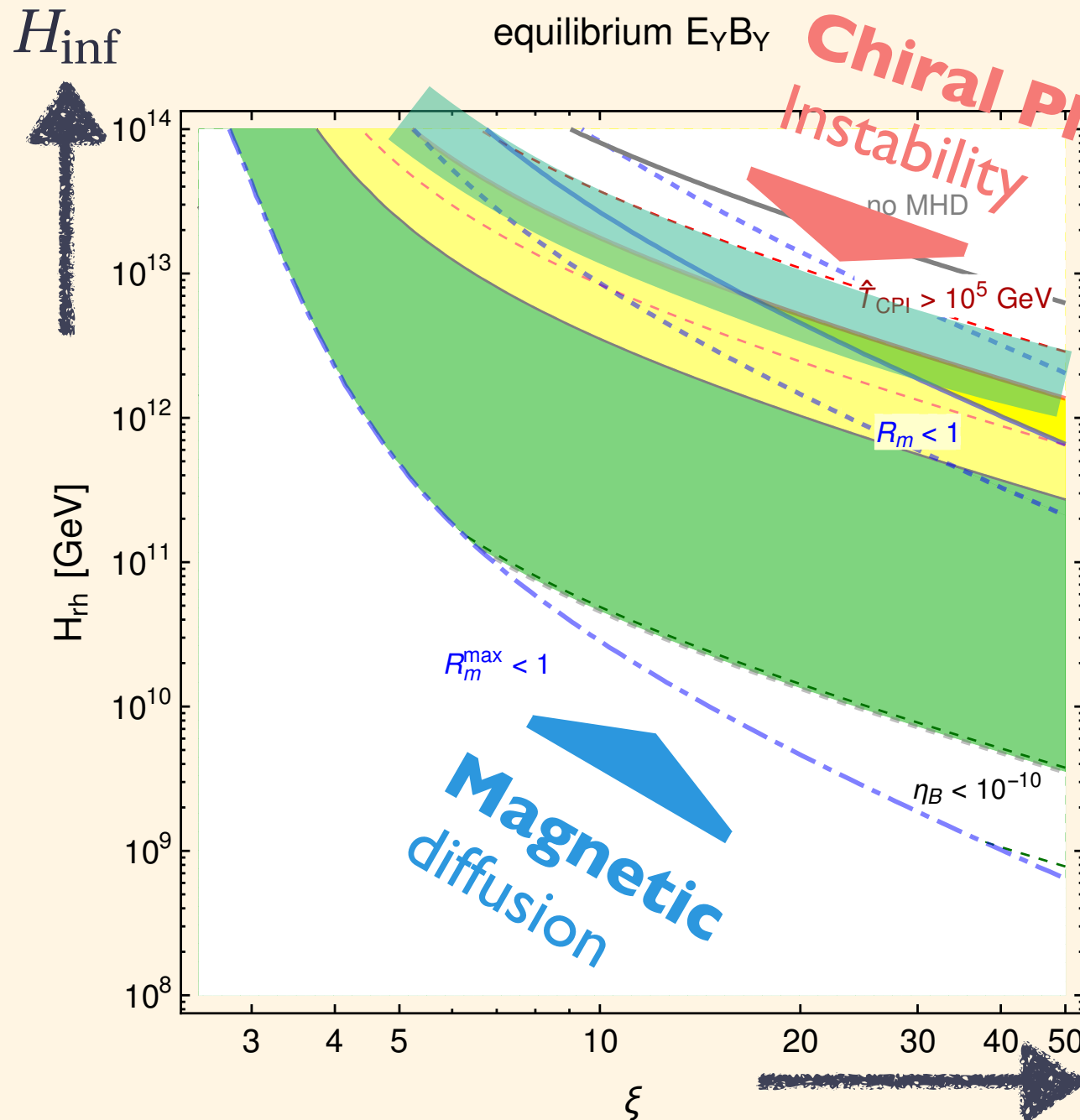
- Successful baryogenesis **only with SM + inflaton**
- ChMHD puts **lower/upper** bounds.
- **Mild dependence** on the Chern-Simons coupling

V.Domcke, B.Harling, E.Morgante, **KM**
1905.13318

$$\xi \equiv \frac{|\dot{\phi}|}{2\Lambda H} \sim \epsilon^{1/2} \frac{M_P}{\Lambda}$$

Baryogenesis from axion inflation

Viability parameters for Baryogenesis



Viable?

Competition btw overprod. and CPI.

Observed Baryon #

- Successful baryogenesis **only with SM + inflaton**

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5.

Leptogenesis from
axion inflation

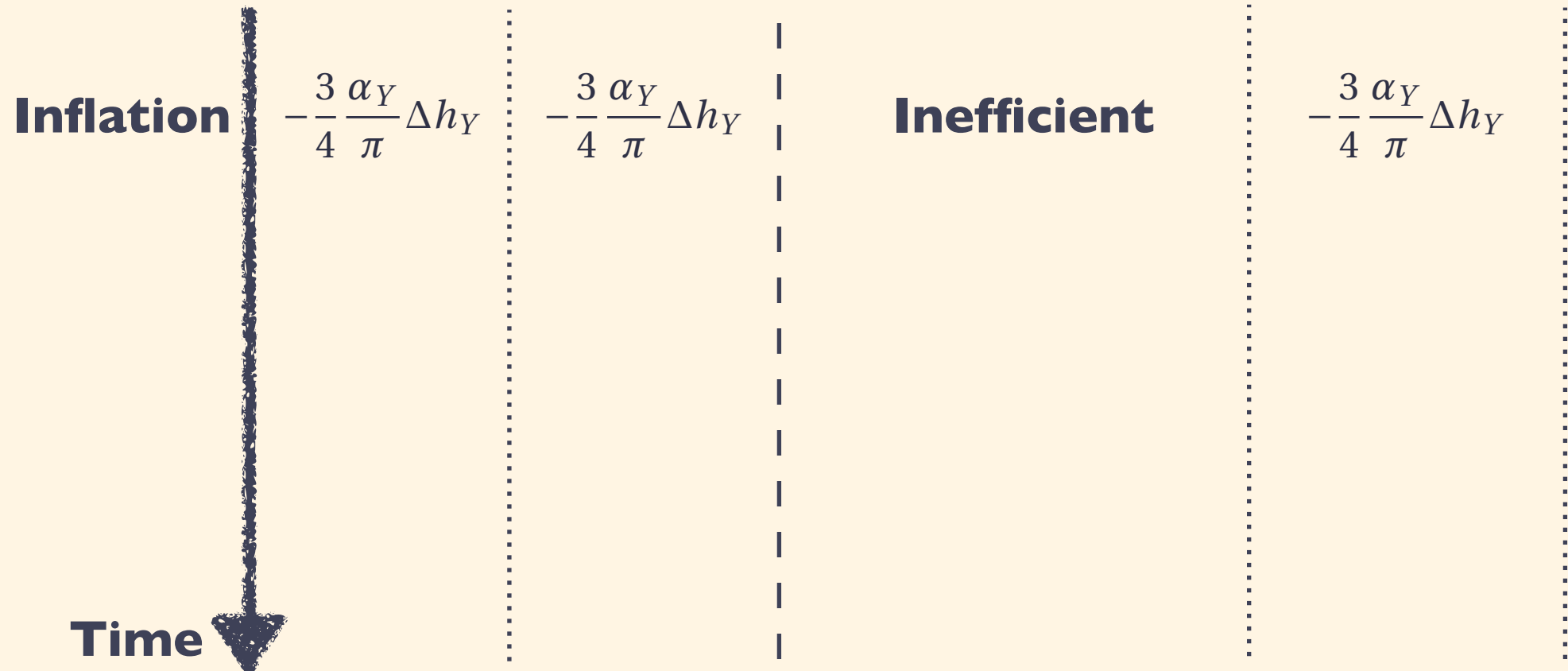
Leptogenesis?

What we have discussed so far...

- ▶ Setup: inflaton + SM

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1905.13318

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = \frac{3}{32\pi^2} \left(g_2^2 W_{\mu\nu}^a \tilde{W}^{a\mu\nu} - g_Y^2 Y_{\mu\nu} \tilde{Y}^{\mu\nu} \right)$$



Leptogenesis?

Leptogenesis from B+L asymmetry?

► Setup: inflaton + SM + **Majorana N_R**

V.Domcke, K.Kamada, **KM**, K.Schmitz, M.Yamada
20xx.xxxxx

$$\partial_\mu J_B^\mu$$

=

$$3\partial_\mu K_{CS}^\mu$$

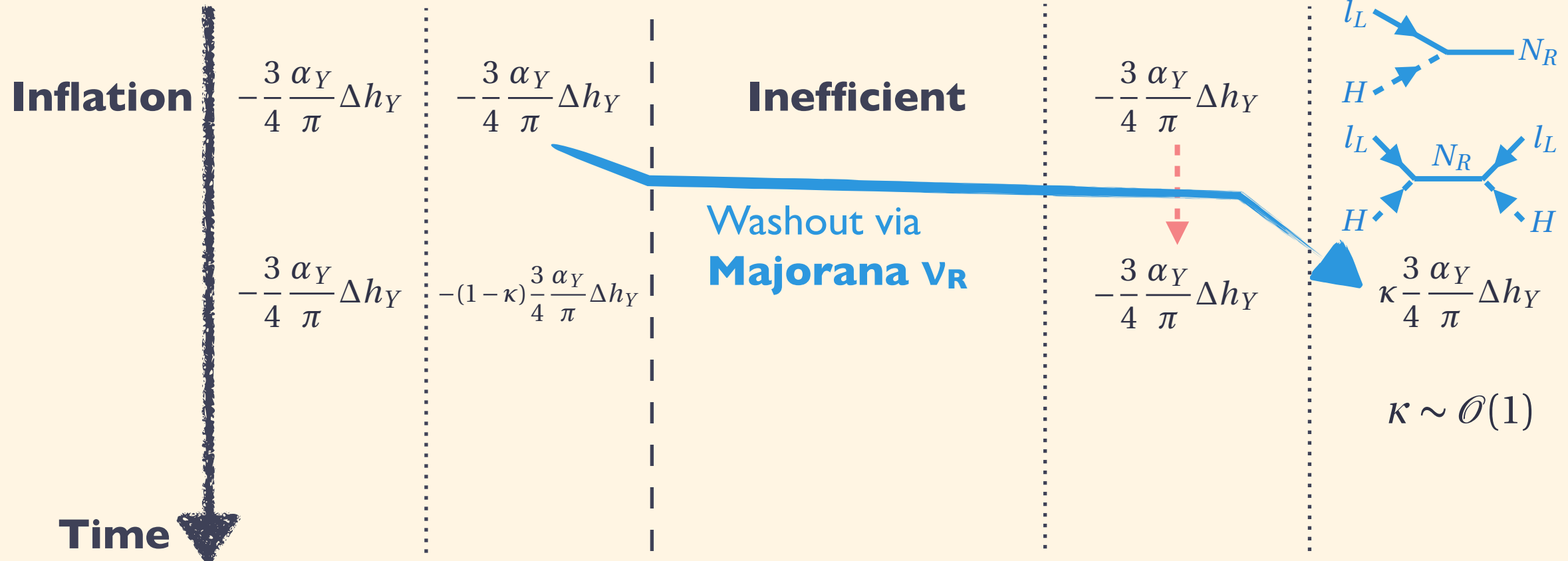
$$-\frac{3\alpha_Y}{4\pi}\partial_\mu h_Y^\mu$$

$$\partial_\mu J_L^\mu =$$

$$3\partial_\mu K_{CS}^\mu$$

$$-\frac{3\alpha_Y}{4\pi}\partial_\mu h_Y^\mu$$

$$+(ih\bar{N}L \cdot H + \text{H.c.})$$



Leptogenesis?

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V.Domcke, K.Kamada, **KM**, K.Schmitz, M.Yamada
20xx.xxxxx

$$\partial_\mu J_B^\mu = 3\partial_\mu K_{CS}^\mu - \frac{3\alpha_Y}{4\pi} \partial_\mu h_Y^\mu$$

$$\partial_\mu J_L^\mu = 3\partial_\mu K_{CS}^\mu - \frac{3\alpha_Y}{4\pi} \partial_\mu h_Y^\mu + (ih\bar{N}L \cdot H + \text{H.c.})$$

▶ **Resultant baryon asymmetry**

$$\frac{q_B}{s} \sim 10^{-9} \kappa \left(\frac{H_{\text{rh}}}{10^{10} \text{ GeV}} \right)^{\frac{3}{2}} \left(\frac{\langle \hat{E}_Y \cdot \hat{B}_Y \rangle_{\text{rh}} / H_{\text{rh}}^4}{10^5} \right)$$

Majorana N_R washout factor

- Depends on when N_R decouples.

$$\kappa \sim \mathcal{O}(0.1)$$

- N_R has to decouple before B+L is completely washed out.

$$T_{N_R} > T_{Y_e} \sim 10^5 \text{ GeV}$$

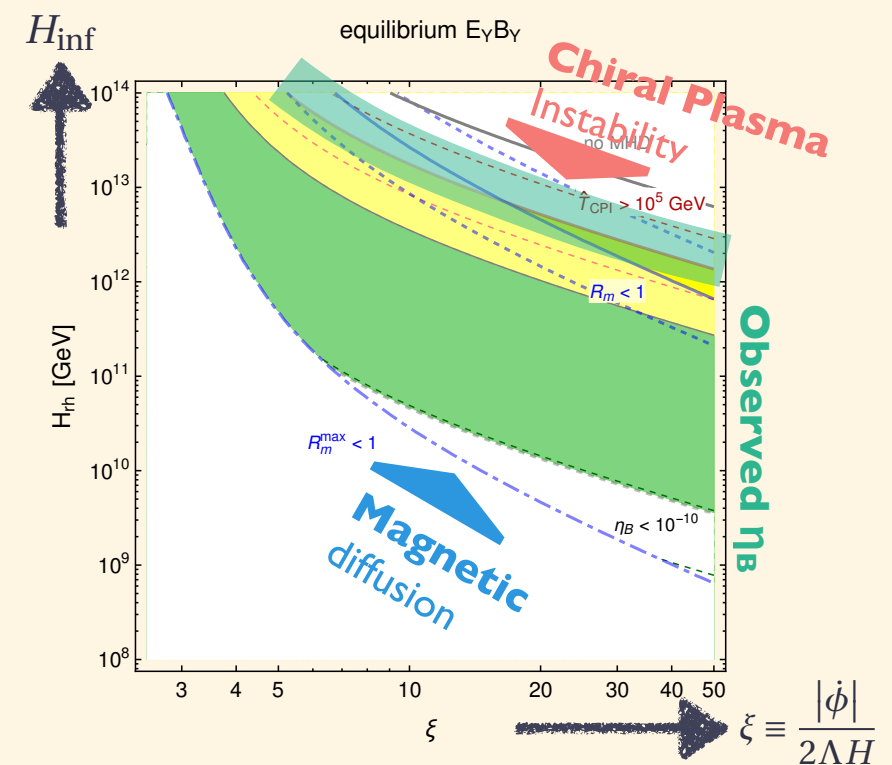
Summary

Inflaton w/ CS coupling to $U(1)_Y$

▶ Dual production of **B+L asymmetry & helical $U(1)_Y$**

▶ **Baryogenesis** only with SM + inflaton

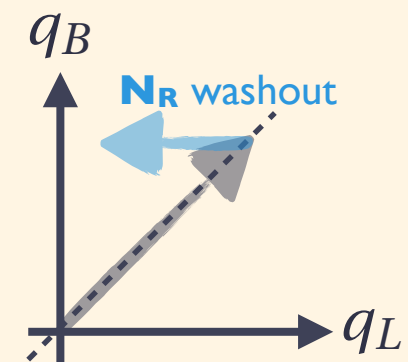
- BAU connected to H_{inf}
- Mild dependence on the CS coupling
- Upper/Lower bounds from ChMHD
- Uncertainties: SM physics & backreaction



▶ **Leptogenesis** with SM + inflaton + Majorana N_R

- Washout via N_R generates B-L asym. from B+L asym.

$$\frac{q_B}{s} \sim 10^{-9} K \left(\frac{H_{\text{rh}}}{10^{10} \text{ GeV}} \right)^{\frac{3}{2}} \left(\frac{\langle \hat{\mathbf{E}}_Y \cdot \hat{\mathbf{B}}_Y \rangle_{\text{rh}} / H_{\text{rh}}^4}{10^5} \right)$$



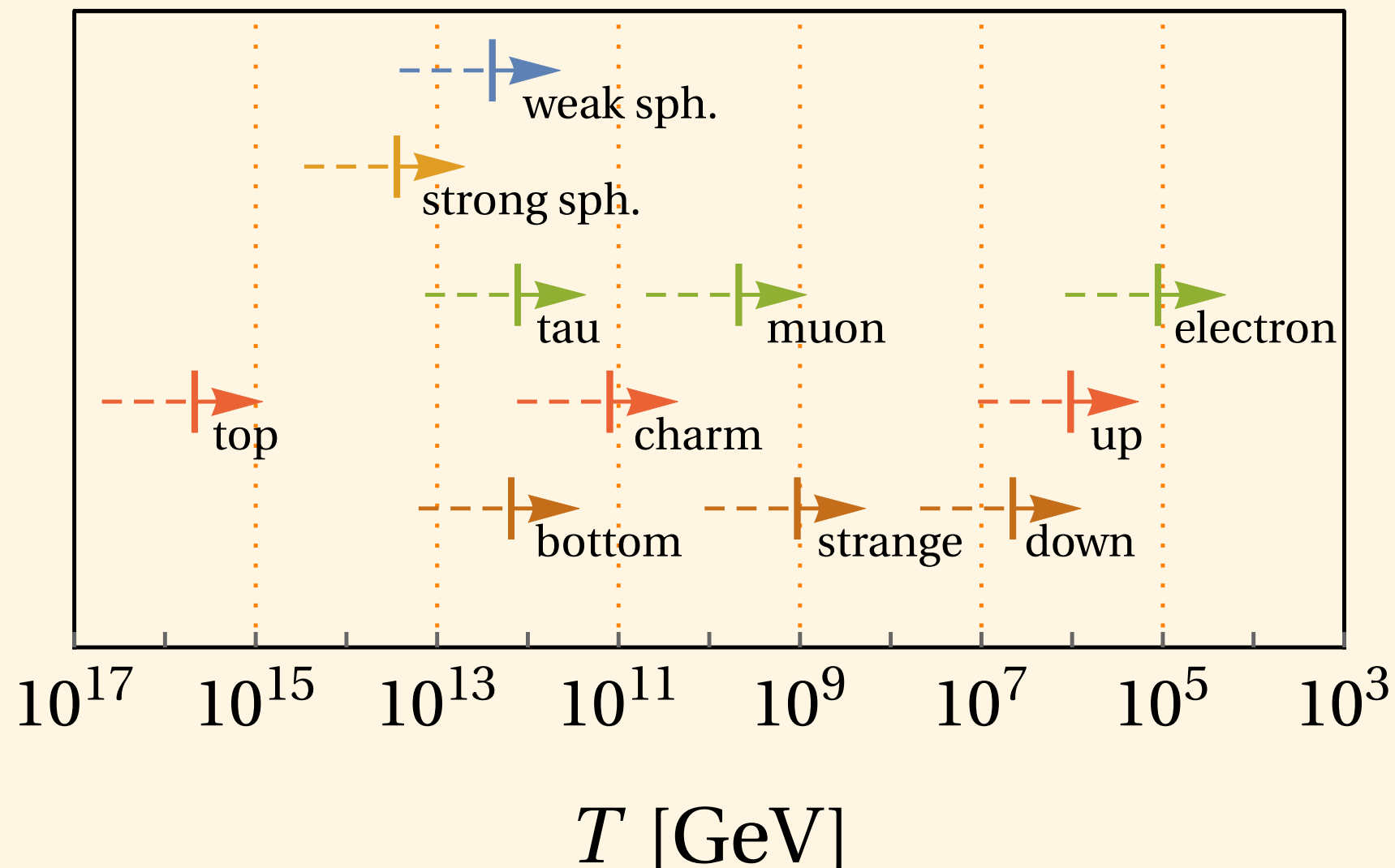
Back up

Decoupling Temperature

Summary plot of decoupling temperatures

▶ Assume radiation dominated Universe.

- During inflation, equilibration is more difficult because $H^2 > \frac{\rho_{\text{rad}}}{3M_P^2}$.



Field Rotation Invariance

Redundancies in inflaton-SM coupling

- ▶ **Warm up:** coupling to CS term in massless QED

$$\frac{g^2 \phi}{16\pi^2 f} F_{\mu\nu} \tilde{F}^{\mu\nu} \longrightarrow 0 = \partial_\mu F^{\mu\nu} - \frac{g^2 \partial_\mu \phi}{4\pi^2 f} \tilde{F}^{\mu\nu} - g J^\nu$$

Chiral rot. $\psi \mapsto e^{-i\gamma_5 \frac{\phi}{2f}} \psi$

$$-\frac{\phi}{2f} \partial_\mu J_5^\mu \longrightarrow 0 = \partial_\mu F^{\mu\nu} - g J^\nu \supset \text{loop} = \frac{g^2 \partial_\mu \phi}{4\pi^2 f} \tilde{F}^{\mu\nu}$$

Equivalent!

The diagram shows a fermion loop with a photon line (wavy) labeled 'A' and an inflaton line (dashed) labeled 'phi'. The loop is connected to the photon line at two vertices and to the inflaton line at two vertices. The diagram is used to illustrate the equivalence between the CS term and the divergence of the axial current.

- ▶ **Coupling to CS term of U(1)_Y**

$$\frac{g_Y^2 \phi}{16\pi^2 f} Y_{\mu\nu} \tilde{Y}^{\mu\nu} \longrightarrow 0 = D_\mu W^{a\mu\nu} - g_2 J^{a\nu}$$

B+L rot.

$$\frac{\phi}{f} \left(-\frac{1}{3} \partial_\mu J_{B+L}^\mu + \frac{g_2^2}{16\pi^2} W_{\mu\nu}^a \tilde{W}^{a\mu\nu} \right) \longrightarrow 0 = D_\mu W^{a\mu\nu} - \frac{g_2^2 \partial_\mu \phi}{4\pi^2 f} \tilde{W}^{a\mu\nu} - g_2 J^{a\nu}$$

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→ Field rotation never changes physics! (see also **KM+ 2006.03148**)

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➔ Field rotation never changes physics! (see also KM+ 2006.03 | 48)

Chiral Vortical Effect

Chiral vortical effect as NLO in μ/T

► Generalized Ohm' law w/ velocity & its **vortex**

- Assump: weak hyper gauge field & thermalized plasma.

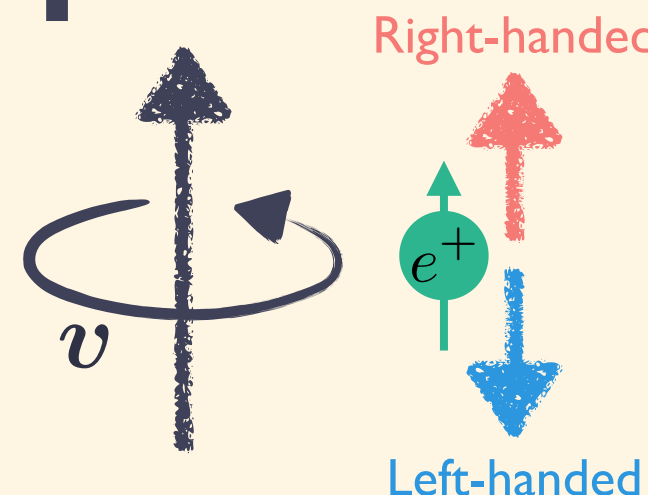
$$\hat{J}_Y = \sigma_Y (\hat{E}_Y + \mathbf{v} \times \hat{B}_Y) + \frac{2\alpha_Y}{\pi} \mu_{Y,5} \hat{B}_Y + c_{\text{CVE}} \frac{\nabla}{a} \times \mathbf{v}$$

$$\text{w/ } c_{\text{CVE}} = g_Y \left(\sum_{\alpha} \epsilon_{\alpha} N_{\alpha} Q_{Y,\alpha} \frac{\mu_{\alpha}^2}{4\pi^2} + \frac{T^2}{12} \sum_{\alpha} \epsilon_{\alpha} N_{\alpha} Q_{Y,\alpha} \right) = \frac{g_Y}{4\pi^2} \sum_{\alpha} \epsilon_{\alpha} N_{\alpha} Q_{Y,\alpha} \mu_{\alpha}^2$$

$$\sum_{\alpha} \epsilon_{\alpha} N_{\alpha} Q_{Y,\alpha} = 0 \quad \because \mathcal{A}(\text{U}(1)_Y - \text{grav.} - \text{grav.}) = 0$$

where $\epsilon_{\alpha} = \pm$ for R/L
 $N_{\alpha} = \text{d.o.f.}$

A.Vilenkin, Phys. Rev. D 21, 2260 (1980)



- **Small chemical potential** in the early Universe: $\mu/T \ll 1$.

$$\hat{J}_Y = \sigma_Y (\hat{E}_Y + \mathbf{v} \times \hat{B}_Y) + \frac{2\alpha_Y}{\pi} \mu_{Y,5} \hat{B}_Y + \frac{g_Y}{4\pi^2} \left(\sum_{\alpha} \epsilon_{\alpha} N_{\alpha} Q_{Y,\alpha} \mu_{\alpha}^2 \right) \frac{\nabla}{a} \times \mathbf{v}$$

$$\sim \frac{\mu}{T} \frac{\hat{B}_Y}{\alpha_Y^{-1} T^{-1}} \gg \sim g_Y \mu^2 \frac{v}{L} \sim \frac{\mu^2}{T^2} \frac{\hat{B}_Y}{g_Y^{-1} L} \quad @ \text{ equipartition: } \hat{B}_Y^2 \sim T^4 v^2$$

Induced Current

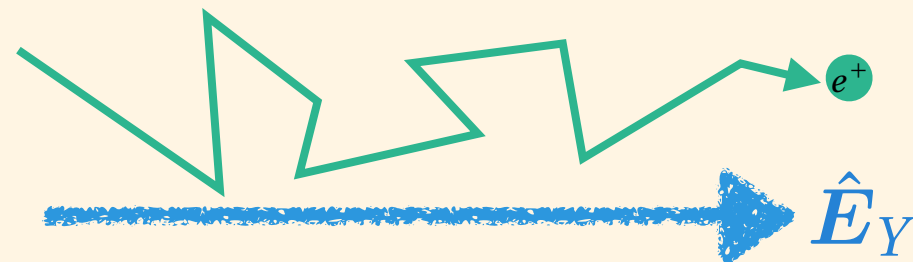
How does Induced Current look like?

- ▶ **Weak** electromagnetic field in thermalized plasma

$$\hat{J}_Y = \hat{\sigma}_Y \hat{E}_Y + \frac{2\alpha_Y}{\pi} \hat{\mu}_{Y,5} \hat{B}_Y \quad \text{i.e., Generalized Ohm's law (neglecting velocity field)}$$

- **No magnetic mass** for transverse mode of Abelian gauge field. E. Fradkin, Proc. of the Lebedev Institute 29, 6 (1965).
- This expression holds if **energy by acceleration** \ll **temperature**

$$g_Y |Q| \hat{E}_Y \hat{t}_{\text{int}} \ll \hat{T}$$



- ▶ **Strong** electromagnetic field ?

- Estimate the current operator for the **accelerated energy** \gg **temperature**

$$\hat{J}_Y^z = \sum_{\alpha} \frac{(g|Q_{\alpha}|)^3}{12\pi^2} \coth\left(\frac{\pi \hat{B}_Y}{\hat{E}_Y}\right) \hat{E}_Y \hat{B}_Y \frac{1}{H} \quad \rightarrow \quad \hat{J}_Y^z = \sum_{\alpha} \frac{(g|Q_{\alpha}|)^3}{6\pi^3} \frac{\hat{E}_Y^2}{H} \quad \text{for } B_Y \rightarrow 0$$

Reproduce e.g., Kobayashi, Afshordi 1408.4141

Massive Fermion Production

Schwinger effect for massive fermion

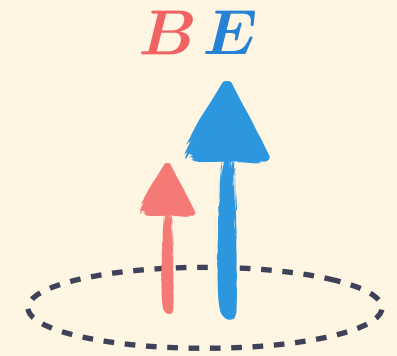
V.Domcke, Y.Ema, **KM**
1910.01205

- ▶ **Anomalous** current equation w/ mass term

$$\partial_\mu J_5^\mu = -\frac{g^2 Q^2}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} + 2im\bar{\psi}\gamma_5\psi$$

See e.g., K.Fukushima+ 1807.04416

$$\dot{q}_5 \simeq \frac{g^2 Q^2}{2\pi^2} EB \exp\left(-\frac{\pi m^2}{g|QE|}\right) \quad \frac{g^2 Q^2}{2\pi^2} EB \quad 2m \langle \bar{\psi} i\gamma_5 \psi \rangle \simeq \frac{g^2 Q^2}{2\pi^2} EB \left[\exp\left(-\frac{\pi m^2}{g|QE|}\right) - 1 \right]$$



- Production of W bosons is more violent. [Ambjorn, Olesen, Phys.Lett.B 218 \(1989\) 67](#); [Kajantie+, Nucl.Phys.B 544 \(1999\) 357-373](#)

➔ **Backreact to Higgs potential**

- ▶ **Induced current**

$$0 = -\partial_t \mathbf{E} + \nabla \times \mathbf{B} + a \frac{g^2}{4\pi^2} \frac{\dot{\phi}}{f_a} \mathbf{B} - \mathbf{gJ}$$

Vacuum cont. Involves gauge coupling running **finite terms**, i.e., Euler-Heisenberg

$$gQ \langle J \rangle_{\text{ind}} = a^3 \frac{(g|Q|)^3}{6\pi^2} \hat{E} |\hat{B}| \coth\left(\frac{\pi |\hat{B}|}{\hat{E}}\right) e^{-\frac{\pi m^2}{g|Q|\hat{E}}} \frac{1}{H}$$

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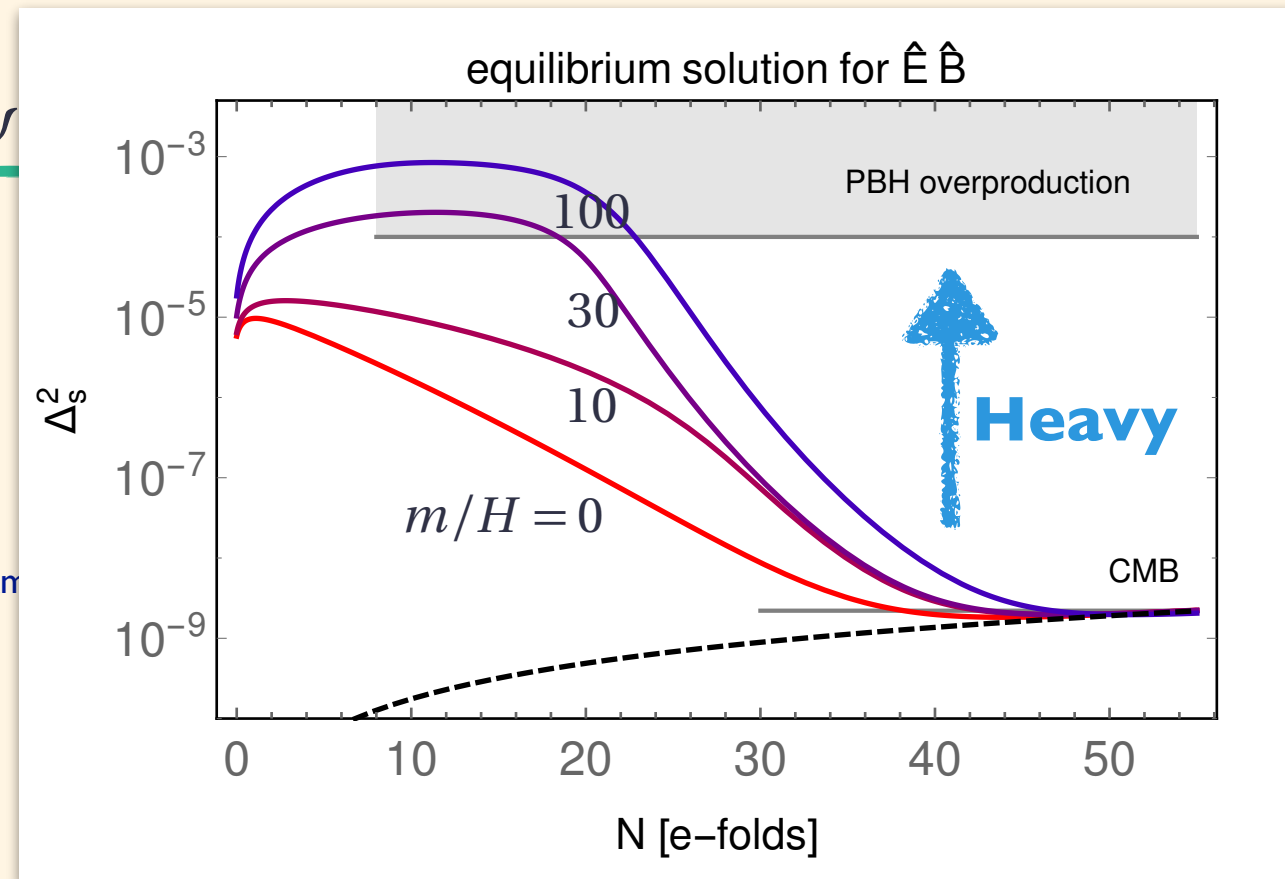
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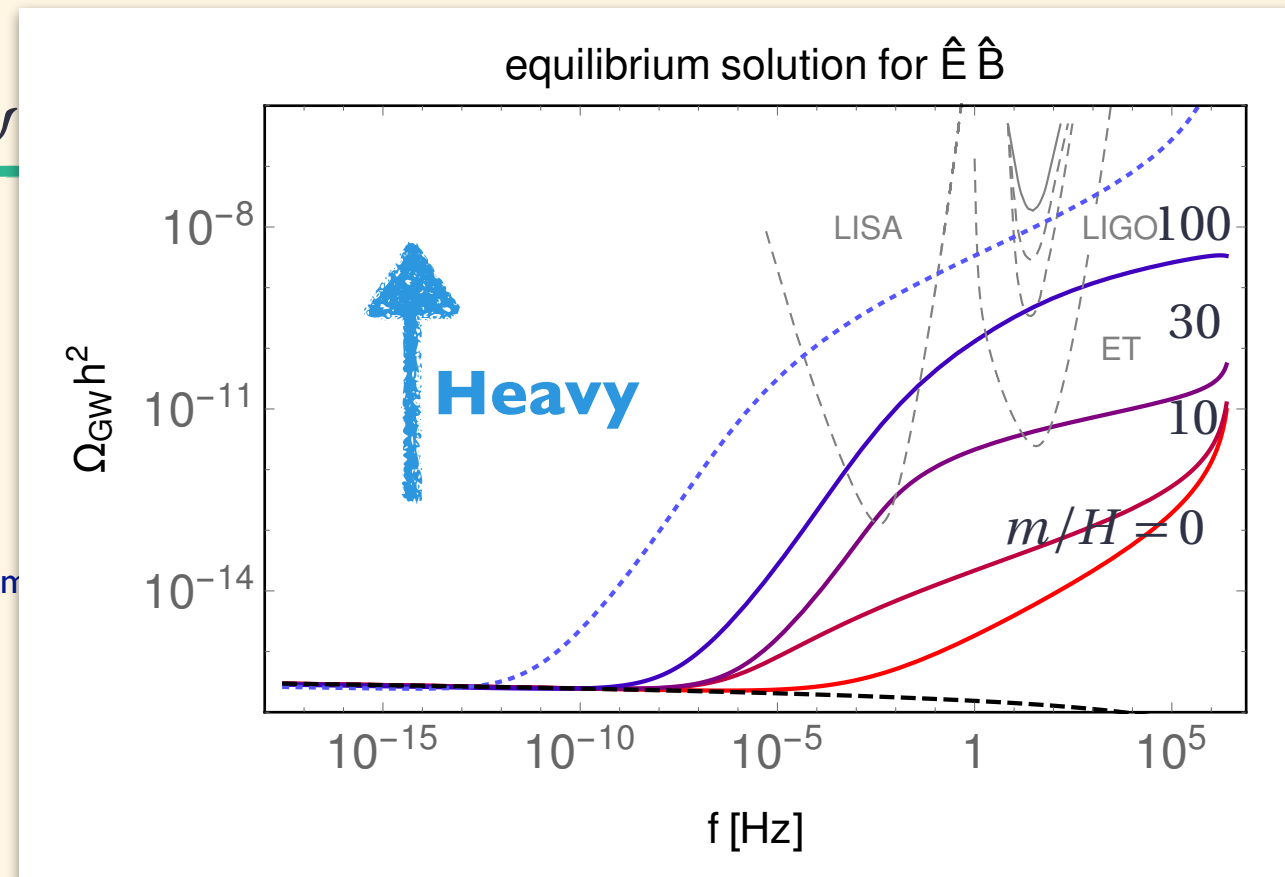
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Vacuum cont.

Involves gauge coupling running
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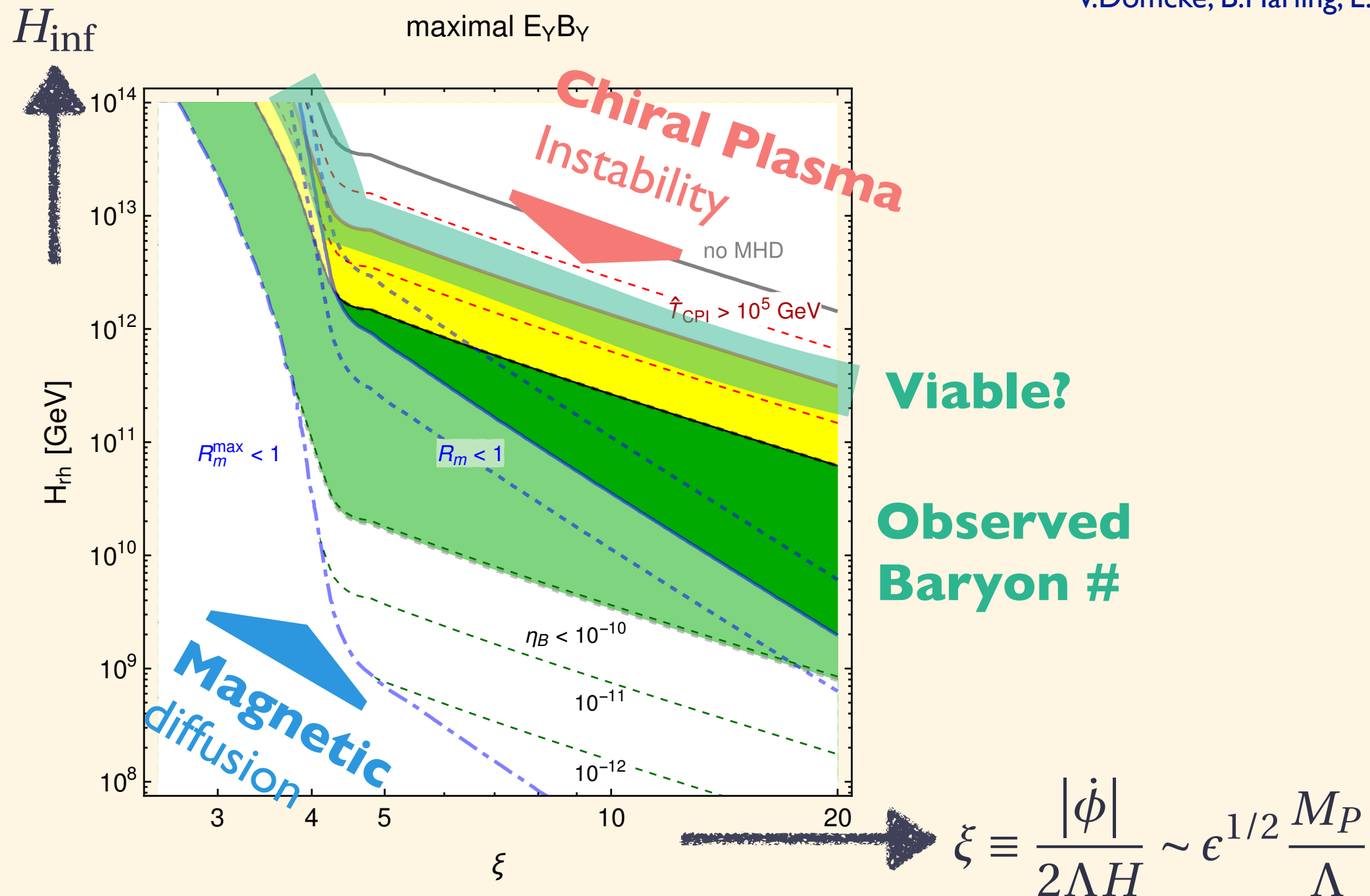
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Result

Viability parameters for Baryogenesis

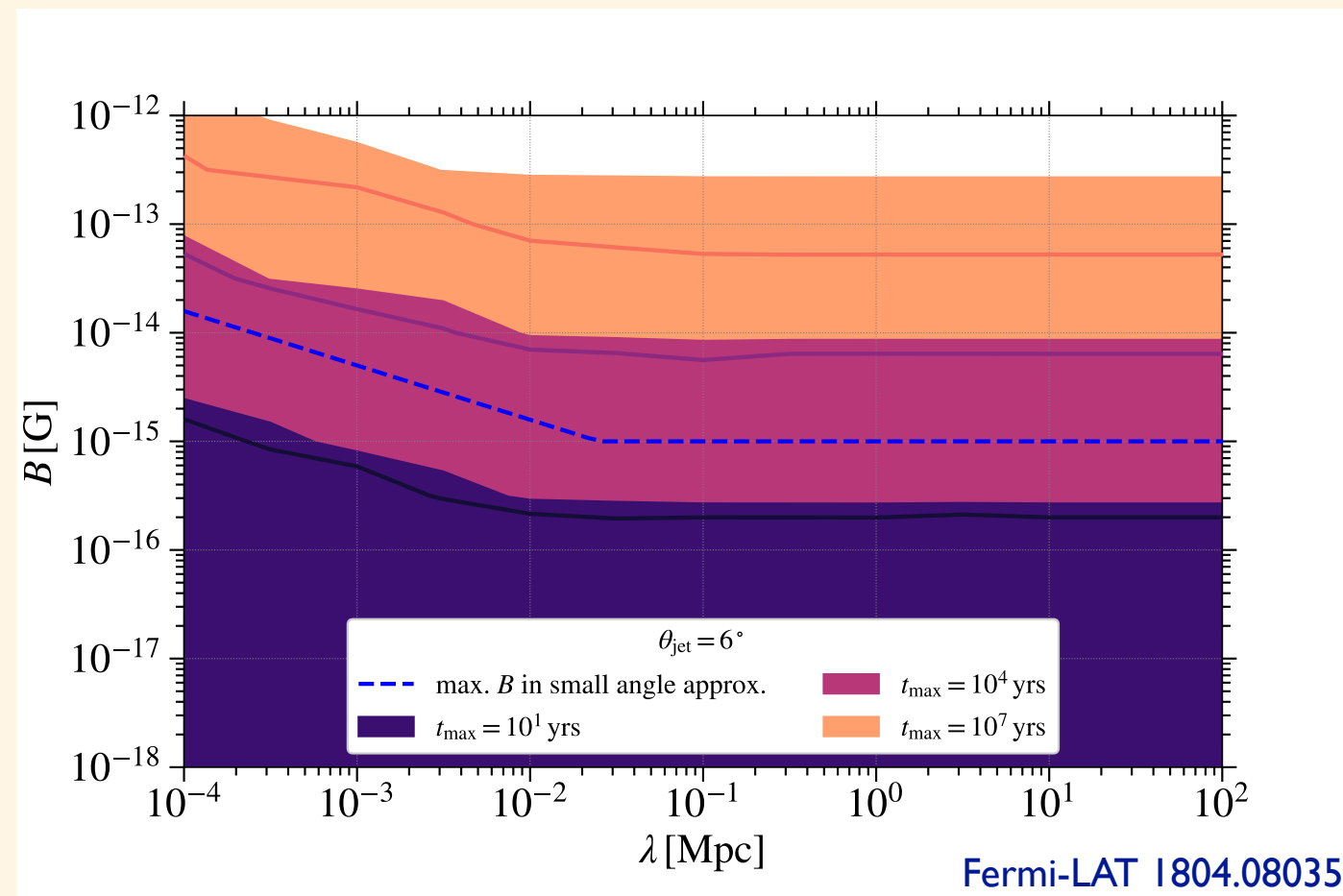
V.Domcke, B.Harling, E.Morgante, **KM**
1905.13318



Magnetic field

Implications to intergalactic B-field?

- ▶ Current lower limits to explain the intergalactic B-field.

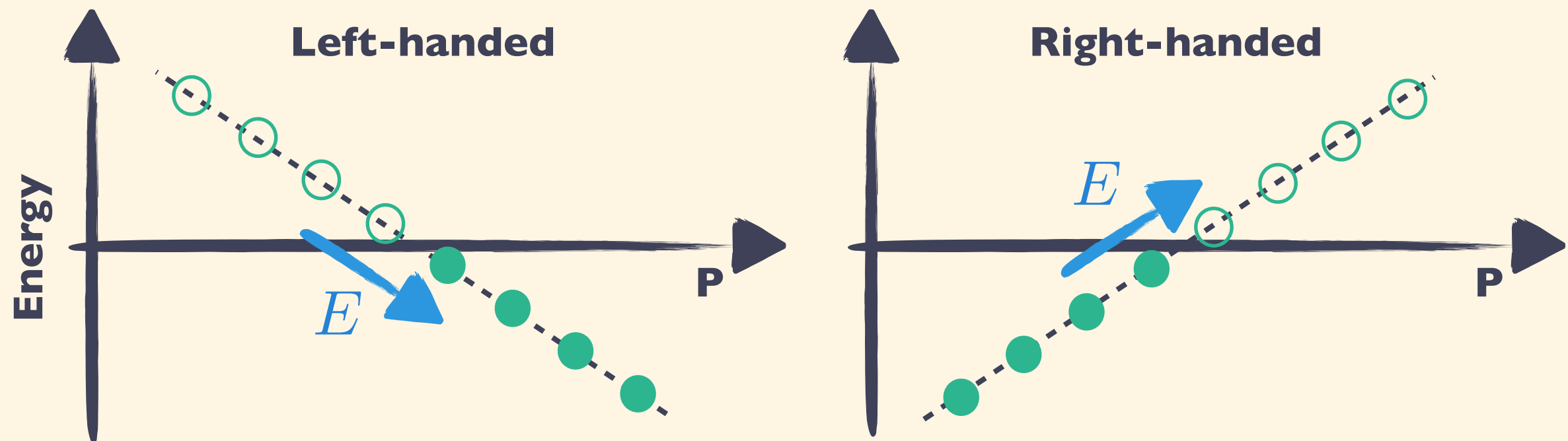


- Our baryogenesis model yields $B \lesssim 10^{-17} \text{G}$ for $\lambda \sim 0.1 \text{pc}$ **too small...**

Vacuum Contribution

Chiral anomaly in finite volume

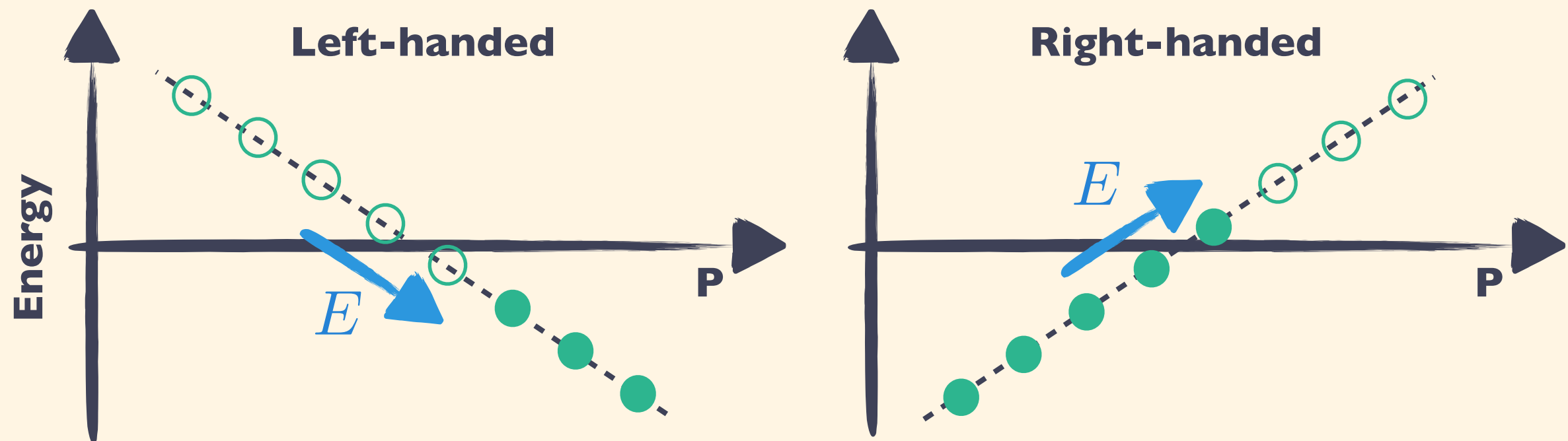
- ▶ Consider (1+1) QED on S^1 : $\partial \cdot J_5 = \frac{g}{2\pi} e^{\mu\nu} F_{\mu\nu} = \frac{gE}{\pi}$



Vacuum Contribution

Chiral anomaly in finite volume

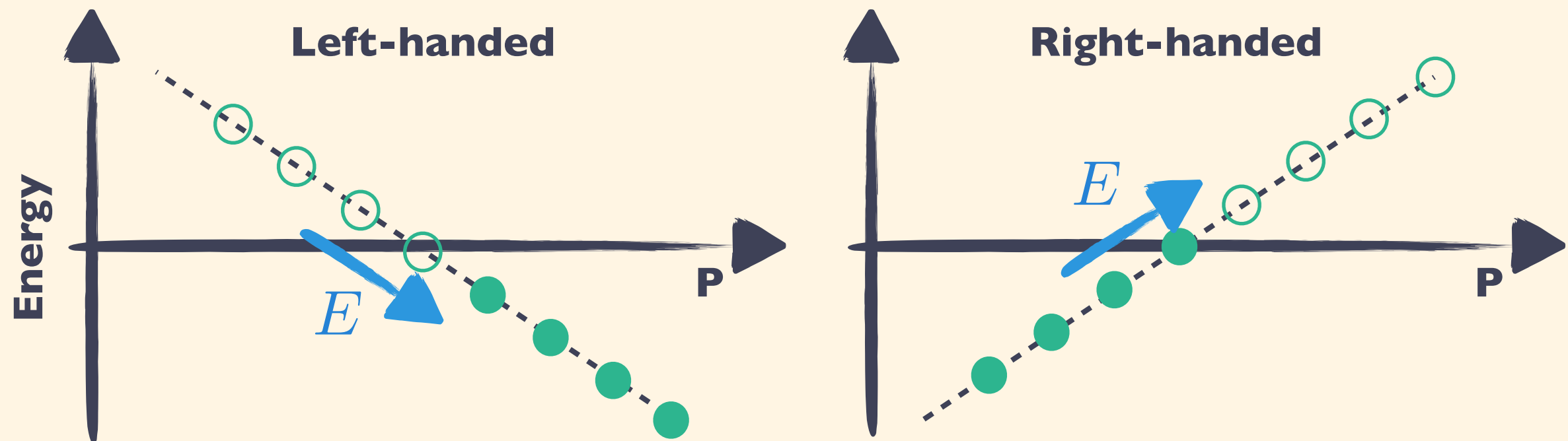
- ▶ Consider (1+1) QED on S^1 : $\partial \cdot J_5 = \frac{g}{2\pi} e^{\mu\nu} F_{\mu\nu} = \frac{gE}{\pi}$



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➔ Particles are not yet generated... How do we fulfill the anomaly eq?

$$Q_{R/L} = \sum_p \left[\left(\hat{b}^\dagger \hat{b} - \frac{1}{2} \right) - \left(\hat{d}^\dagger \hat{d} - \frac{1}{2} \right) \right] \longrightarrow Q_{R/L}^{(\text{vac})} = \lim_{\Lambda \rightarrow \infty} \sum_p \left[-\frac{1}{2} \text{sgn}(\omega_{R/L}) R \left(\frac{|\omega_{R/L}|}{\Lambda} \right) \right]$$

➔ Vacuum contribution accounts for the anomaly eq!

$$:Q_{R/L}: = 0 \quad Q_{R/L}^{(\text{vac})} = \frac{gE}{\pi} TL \quad \int dt dx \frac{gE}{\pi} = \frac{gE}{\pi} TL$$

Introduction

Inflaton w/ CS-coupling

$$S = \int d^4x \left\{ \sqrt{-g} \left[\frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] - \frac{1}{4} F_{a\mu\nu} F^{a\mu\nu} + \frac{\alpha\phi}{4\pi f_a} F_{a\mu\nu} \tilde{F}^{a\mu\nu} \right\} \text{non-Abelian}$$

▶ “Lightness” protected by the shift symmetry (classically): $\phi \mapsto \phi + c$

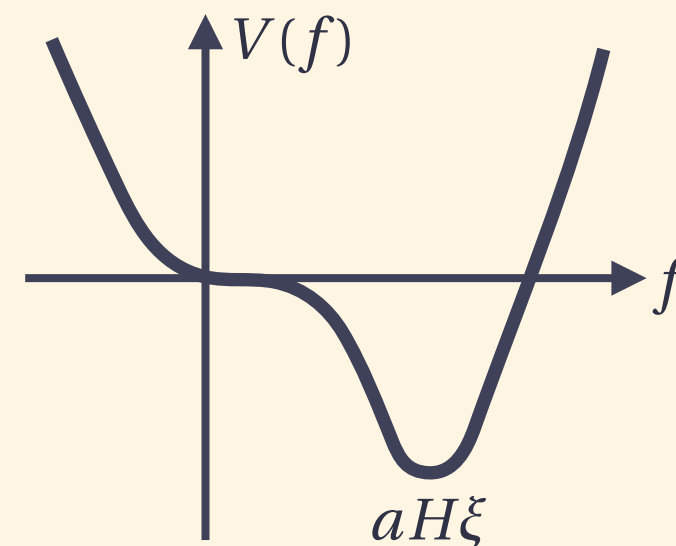
▶ Efficient **helical-gauge field** production by $\dot{\phi} \neq 0$.

+ **Homogeneous & isotropic** gauge field may develop.

$$f''(\eta) + 2f^3(\eta) - 2aH\xi f^2(\eta) = 0$$

$$\text{where } A_0^a = 0, \quad A_i^a = -g^{-1} f(\eta) \delta_i^a, \quad \xi \equiv \frac{\alpha\dot{\phi}}{2\pi f_a H}$$

$$0 \neq \langle F_{a\mu\nu} \tilde{F}^{a\mu\nu} \rangle = -4 \langle \mathbf{E}^a \cdot \mathbf{B}^a \rangle$$

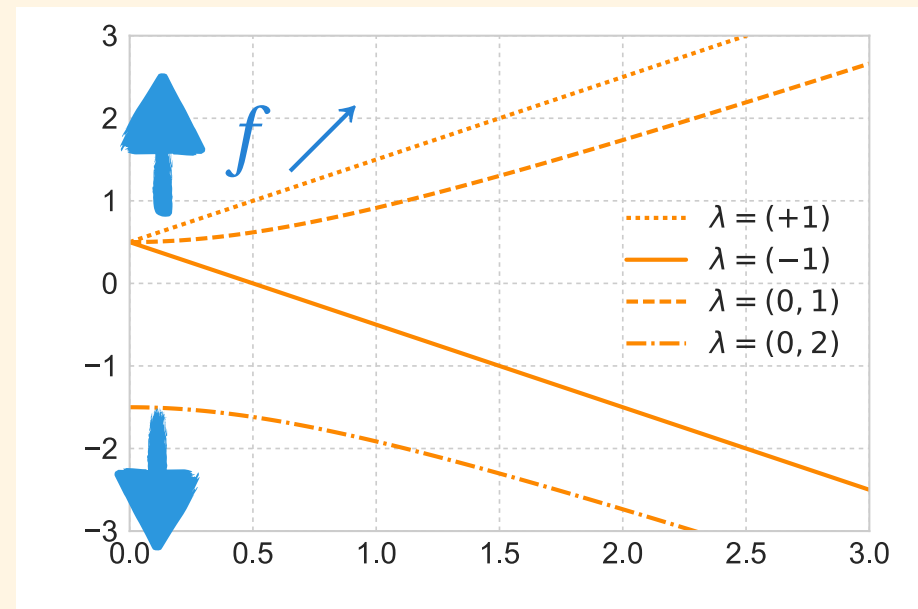
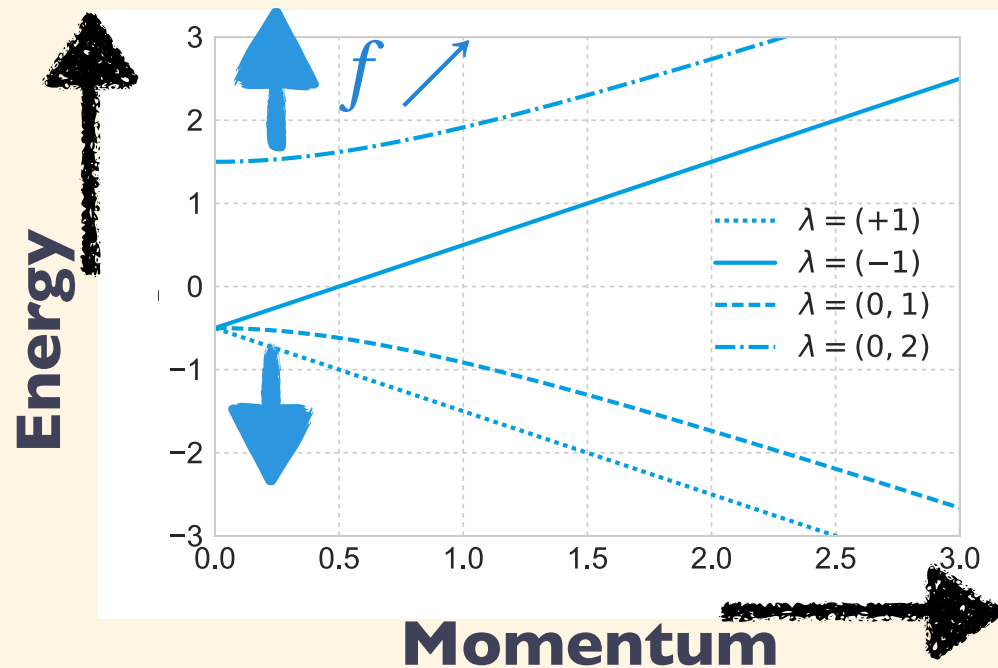


➔ axion inflation, relaxion, chiral GWs, baryogenesis, magnetogenesis,...

Fermion Production

Subtleties for $A_0^a = 0$, $A_i^a = -g^{-1} f(\eta) \delta_i^a$ V.Domcke, Y.Ema, **KM**, R.Sato
1812.08021

▶ **No Landau levels**, but we can still play the same game...



▶ The spectrum is different between the **initial** and **final** states.

➔ Need to include the contributions from **vacuum** not only **particles**.

$$Q_\alpha = \underbrace{:Q_\alpha:}_{\text{Fundamental repr: } \frac{1}{6} \text{ (Anomaly)}} + \underbrace{Q_\alpha^{(\text{vac})}}_{\frac{5}{6} \text{ (Anomaly)}} \quad \text{w/} \quad Q_\alpha^{(\text{vac})} \equiv \lim_{\hat{\Lambda} \rightarrow \infty} \text{vol}(\mathbb{R}^3) \int \frac{d^3 k}{(2\pi)^3} \left[-\frac{1}{2} \sum_\lambda \text{sgn}(\omega_\alpha^{(\lambda)}) R\left(\frac{|\omega_\alpha^{(\lambda)}|}{a\hat{\Lambda}}\right) \right]$$

The eta-invariant; see Atiyah and Singer

Chiral Plasma Instability

Chiral Plasma Instability

▶ Equation for Magnetic Helicity

$$\frac{\partial h_Y}{\partial \eta} = \int \frac{d^3 x}{\text{vol}(\mathbb{R}^3)} \frac{Y_{\mu\nu} \tilde{Y}^{\mu\nu}}{2} = -2 \int \frac{d^3 x}{\text{vol}(\mathbb{R}^3)} \mathbf{E}_Y \cdot \mathbf{B}_Y$$

$$0 = \nabla \times \mathbf{B}_Y - \mathbf{J}_Y, \quad \mathbf{J}_Y = \sigma_Y (\mathbf{E}_Y + \mathbf{v} \times \mathbf{B}_Y) + \frac{2\alpha_Y}{\pi} \mu_{Y,5} \mathbf{B}_Y \quad \text{w/ } \mu_{Y,5} = \sum_{\alpha} \epsilon_{\alpha} N_{\alpha} Q_{Y,\alpha}^2 \mu_{\alpha}$$

$$\frac{\partial}{\partial \eta} h_Y = \int \frac{d^3 x}{\text{vol}(\mathbb{R}^3)} \left(2\mathbf{B}_Y \cdot \frac{\nabla^2}{\sigma_Y} \mathbf{A}_Y + \frac{4\alpha_Y}{\pi} \frac{\mu_{Y,5}}{\sigma_Y} B_Y^2 \right)$$

▶ Mode equation

$$\frac{\partial}{\partial \eta} h_{Y,k} = -\frac{2k^2}{\sigma_Y} h_{Y,k} + \frac{8\alpha_Y}{\pi} \frac{\mu_{Y,5}}{\sigma_Y} \rho_{B,k} = -\frac{2k^2}{\sigma_Y} h_{Y,k} + \frac{4\alpha_Y}{\pi} \frac{\mu_{Y,5}}{\sigma_Y} k h_{Y,k} \quad \text{w/ } r_k \equiv \frac{k h_k(\eta)/2}{\rho_{B,k}(\eta)}$$

Kinetic Reynolds

Scaling in Turbulent & Viscous Regimes

▶ Velocity equation in ChMHD

$$\frac{\partial}{\partial \eta} v + \underbrace{v \cdot \nabla v}_{\sim \frac{v^2}{L}} = \underbrace{v \nabla^2 v}_{\sim \frac{v v}{L^2}} + \frac{1}{\rho + P} \left(-\frac{1}{2} \nabla B_Y^2 + (B_Y \cdot \nabla) B_Y \right) \sim \frac{B_Y^2}{L \rho}$$

Kinetic Reynolds # $Re = \frac{Lv}{\nu} \gg 1$: $\sim \frac{v^2}{L} \gg \sim \frac{v v}{L^2} \sim \frac{B_Y^2}{L \rho}$

- **Turbulent** regime: $R_m > Re > 1$

$$\rho v^2 \sim B_Y^2 \text{ \& const. = } h_Y \sim L B_Y^2 \blacktriangleright \partial_\eta B_Y \sim \frac{v B_Y}{L} \propto B_Y^4 \blacktriangleright B_Y \sim \left(\frac{\eta_t}{\eta} \right)^{\frac{1}{3}} B_{Y,t}, \quad L \sim \left(\frac{\eta}{\eta_t} \right)^{\frac{2}{3}} L_t$$

- **Viscous** regime: $R_m > 1 > Re$

$$\rho v^2 \sim Re B_Y^2 \text{ \& const. = } h_Y \sim L B_Y^2 \blacktriangleright \partial_\eta B_Y \sim \frac{v B_Y}{L} \propto B_Y^3 \blacktriangleright B_Y \sim \left(\frac{\eta_t}{\eta} \right)^{\frac{1}{2}} B_{Y,t}, \quad L \sim \left(\frac{\eta}{\eta_t} \right) L_t$$