フレーバー対称性と素粒子標準模型有効場の理論

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Based on

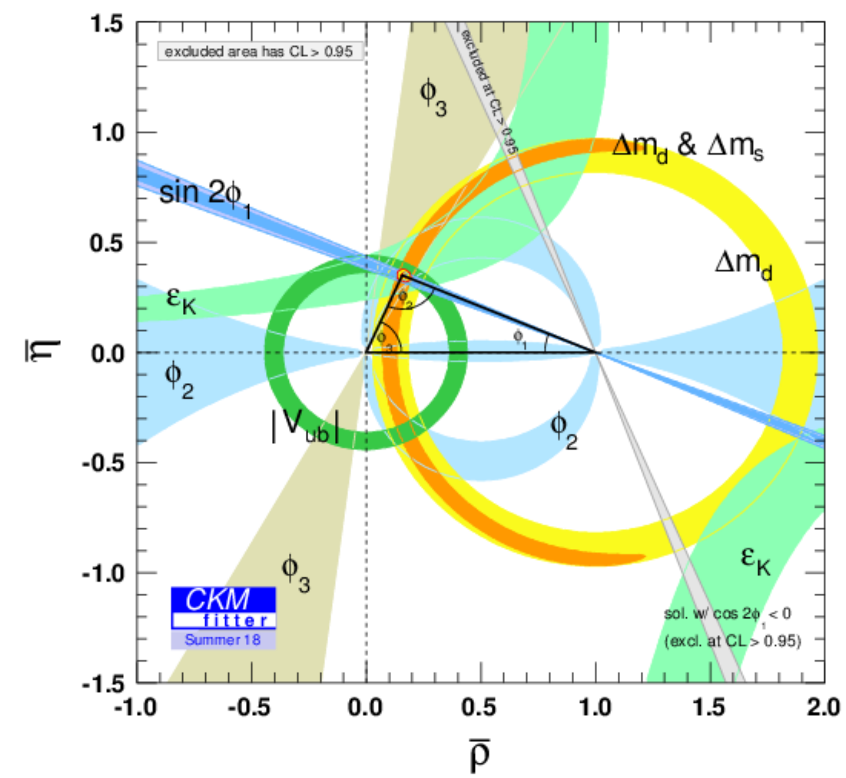
Darius A. Faroughy, Gino Isidori, Felix Wilsch and KY
(University of Zurich) [1909.02519]

Universität Zürich[UZH]

The Flavor Problem

- Theoretical arguments based on the hierarchy problem $\rightarrow$ TeV scale NP

- The measurements of quark flavor-violating observables show a remarkable overall success of the SM



New flavor-breaking sources of O(1) at the TeV scale are definitely excluded

$$\mathscr{L}_{eff} = \mathscr{L}_{SM} + \sum_i \frac{C_i}{\Lambda^2} \mathcal{O}_i^{d=6} \quad \text{(NP)}$$

$$|C_{NP}| \sim 1 \longrightarrow \Lambda_{NP} \sim \begin{cases} 500 \text{ TeV} & : \quad B_s \\ 2000 \text{ TeV} & : \quad B_d \\ 10^4 - 10^5 \text{ TeV} & : \quad K^0 \end{cases}$$

The Flavor Problem

Operator	Bounds on Λ (TeV)		Bounds on c_{ij} ($\Lambda = 1$ TeV)		Observables
	Re	Im	Re	Im	
$(\bar{s}_L \gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	Δm_K; ε_K
$(\bar{s}_R d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	Δm_K; ε_K
$(\bar{c}_L \gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	Δm_D; $\lvert q/p \rvert$, ϕ_D
$(\bar{c}_R u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	Δm_D; $\lvert q/p \rvert$, ϕ_D
$(\bar{b}_L \gamma^\mu d_L)^2$	5.1×10^2	9.3×10^2	3.3×10^{-6}	1.0×10^{-6}	Δm_{B_d}; $S_{B_d \to \psi K}$
$(\bar{b}_R d_L)(\bar{b}_L d_R)$	1.9×10^3	3.6×10^3	5.6×10^{-7}	1.7×10^{-7}	Δm_{B_d}; $S_{B_d \to \psi K}$
$(\bar{b}_L \gamma^\mu s_L)^2$	1.1×10^2	1.1×10^2	7.6×10^{-5}	7.6×10^{-5}	Δm_{B_s}
$(\bar{b}_R s_L)(\bar{b}_L s_R)$	3.7×10^2	3.7×10^2	1.3×10^{-5}	1.3×10^{-5}	Δm_{B_s}

- if we insist with the theoretical prejudice that NP has to emerge in the TeV region, we have to conclude that NP have a highly non-generic flavor structure

→ Flavor symmetry

Flavor symmetry in SM

$$\mathscr{L}_{SM}^{\text{fermion}} = \mathscr{L}_{\text{gauge}} + \mathscr{L}_{\text{Yukawa}}$$

fermion sector $\sum_{i=1}^{3} \sum_{\psi_i} \bar{\psi}_i i \not{D} \psi_i$

- in gauge sector $\mathscr{L}_{\text{gauge}}$, there is 3 identical replica of the basic fermion family $[\psi = Q_L, u_R, d_R, L_L, e_R]$

⇒ big flavor symmetry is found in gauge sector

$$U(3)^5 = U(3)_{Q_L} \times U(3)_{u_R} \times U(3)_{d_R} \times U(3)_{L_L} \times U(3)_{e_R}$$
$$= SU(3)^5 \times U(1)^5$$

controll flavor dynamics ($SU(3)^5$)

can be identified with B, L and hypercharge ($U(1)^5$)

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fermion sector $\sum_{i=1}^{3} \sum_{\psi_i} \bar{\psi}_i i \not{D} \psi_i$ $\mathscr{L}_Y = \bar{Q}_L^i Y_D^{ij} d_R^j H + \bar{Q}_L^i Y_U^{ij} u_R^j \tilde{H} + \bar{L}_L^i Y_E^{ij} e_R^j H + (h.c.)$

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controll flavor dynamics (→ $SU(3)^5$); can be identified with B, L and hypercharge (→ $U(1)^5$)

- $U(3)^5$ flavor symmetry is broken only by the Yukawa couplings $Y_{D,U,E}$

Flavor symmetry in SM + NP

$$\mathscr{L}^{\text{fermion}}_{SM+NP} = \mathscr{L}_{\text{gauge}} + \mathscr{L}_{\text{Yukawa}} + \mathscr{L}_{NP}$$

fermion sector $\sum_{i=1}^{3} \sum_{\psi_i} \bar{\psi}_i i \not{D} \psi_i$ $\quad \mathscr{L}_Y = \bar{Q}^i_L Y^{ij}_D d^j_R H + \bar{Q}^i_L Y^{ij}_U u^j_R \tilde{H} + \bar{L}^i_L Y^{ij}_E e^j_R H + (h.c.)$

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controll flavor dynamics (→ $SU(3)^5$) — ($U(1)^5$ ←) can be identified with B, L and hypercharge

- $U(3)^5$ flavor symmetry is broken only by the Yukawa couplings $Y_{D,U,E}$

- Assumption that flavor structure in NP is also controlled by Yukawa is the most reasonable solution to the flavor problem

⇒ Minimal Flavor Violation paradigm

Minimal Flavor Violation (MFV)

D'Ambrosio, Giudice, Isidori, Strumia [hep-ph/0207036]

$$\mathcal{L}_Y = \bar{Q}^i_L Y^{ij}_D d^j_R H + \bar{Q}^i_L Y^{ij}_U u^j_R \tilde{H} + \bar{L}^i_L Y^{ij}_E e^j_R H + (h.c.)$$

- assume that $G_F \equiv SU(3)^5$ is a good symmetry, promoting the $Y_{U,D,E}$ to be dynamical fields with non-trivial transformation properties under G_F :

  under $G_F = SU(3)_{Q_L} \times SU(3)_{u_R} \times SU(3)_{d_R} \times SU(3)_{L_L} \times SU(3)_{e_R}$

  $$Y_U \sim (3,\bar{3},1,1,1),\ \ Y_D \sim (3,1,\bar{3},1,1),\ \ Y_E \sim (1,1,1,3,\bar{3})$$

  $$Q_L \sim (3,1,1,1,1), u_R \sim (1,3,1,1,1), d_R \sim (1,1,3,1,1),$$
  $$L_L \sim (1,1,1,3,1),\ \ e_R \sim (1,1,1,1,3)$$

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$\bar{3}_{Q_L}$ $\quad 3_{Q_L} \times \bar{3}_{u_R}$ $\quad 3_{u_R}$

→ G_F invariant

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$$L_L \sim (1,1,1,3,1),\ e_R \sim (1,1,1,1,3)$$

We then define that an effective theory satisfies the criterion of MFV if all higher-dimensional operators, constructed from SM and $Y_{U,D,E}$ fields (spurion)

$$\mathscr{L}_{NPinMFV} = \sum_i \frac{C_i}{\Lambda^2} \mathscr{O}_i^{d=6}(\text{SM fields} + Y_{U,D,E})$$

Minimal Flavor Violation (MFV)

- By introducing $Y_{U,D,E}$ fields, we can write higher-dimensional operators in G_F invariant way

$$G_F = SU(3)_{Q_L} \times SU(3)_{u_R} \times SU(3)_{d_R}$$

$$Y_U \sim (3,\bar{3},1)$$

$$(\bar{Q}_L^i \qquad \gamma_\mu Q_L^j)$$

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G_F invariant

$Y_U Y_U^\dagger$ is transforming as $(8,1,1)$

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G_F invariant

$Y_U \sim (3,\bar{3},1)$

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e.g.) $b_i \to b_j$ FCNC transition

int basis $(\bar{b}_L^i Y_U Y_U^\dagger \gamma_\mu b_L^j)$

$$Y_D = \lambda_d \qquad \lambda_d = \mathrm{diag}(m_d, m_s, m_b)/v$$

$$Y_U = V_{CKM}^\dagger \lambda_u \quad \text{where} \quad \lambda_u = \mathrm{diag}(m_u, m_c, m_t)/v \sim \mathrm{diag}(0,0,1)$$

$$Y_E = \lambda_e \qquad \lambda_e = \mathrm{diag}(m_e, m_\mu, m_\tau)/v$$

$$(Y_U Y_U^\dagger)^{ij} = (V^\dagger \lambda_u^2 V)^{ij} \simeq \lambda_t^2 V_{ti}^* V_{tj}$$

mass basis $\lambda_t^2 V_{ti}^* V_{tj} (\bar{b}_L^i \gamma_\mu b_L^j)$ $\propto \left(\dfrac{m_t}{v}\right)^2$ most big effect

Minimal Flavor Violation (MFV)

$$A(d_i \to d_j) = A_{SM} + A_{NP}$$

$$\frac{C_{SM}}{16\pi^2 v^2}\lambda_t^2 V_{ti}^* V_{tj} \qquad \frac{C_{NP}}{\Lambda^2}\lambda_t^2 V_{ti}^* V_{tj}$$

$$\propto \text{(CKM factor)} \left[\frac{C_{SM}}{16\pi^2 v^2} + \frac{C_{NP}}{\Lambda^2}\right]$$

In MFV, flavor violation is completely determined by Yukawa couplings and all CP violation originates from the CKM phase

- Different flavor transitions are correlated, differences are only CKM

$$A(b \to s) = (V_{tb} V_{ts}^*)\left[\frac{C_{SM}}{16\pi^2 v^2} + \frac{C_{NP}}{\Lambda^2}\right]$$

$$A(s \to d) = (V_{ts} V_{td}^*)\left[\quad '' \quad\right]$$

exactly same structure

very predictive

Minimal Flavor Violation (MFV)

- $b_i \to b_j$ FCNC transitions in MFV

$(\bar{L}L)$ type $\quad (\bar{b}^i_L \, Y_U Y_U^\dagger \, b^j_L)$

$(\bar{L}R)$ type $\quad (\bar{b}^i_L \, Y_U Y_U^\dagger Y_D \, b^j_R)$

$(\bar{R}R)$ type $\quad (\bar{b}^i_R \, Y_D^\dagger Y_U Y_U^\dagger Y_D \, b^j_R)$

From MFV to $U(2)^5$

$U(3)^5 = U(3)_{Q_L} \times U(3)_{u_R} \times U(3)_{d_R} \times U(3)_{L_L} \times U(3)_{e_R}$ flavor symmetry

- Largest flavor symmetry group compatible with the SM gauge symmetry
- MFV = minimal breaking of $U(3)^5$ by SM Yukawa couplings

MFV virtue

Naturally small effects in FCNC observables assuming TeV-scale NP

MFV main problem

No explanation for Yukawa hierarchies (masses and mixing angles)

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$U(2)^5$ flavor symmetry

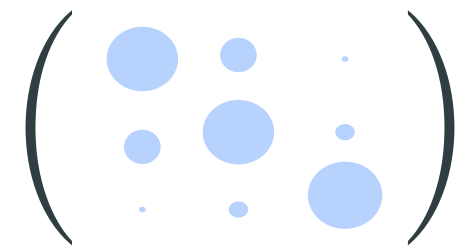
Barbieri, Isidori, Jones-Perez, Lodone, Straub [1105.2296]

SM flavor puzzle

SM flavor sector contains a large number of free parameters

[3 lepton masses + 6 quark masses + 3+1 CKM parameters] ← fixed by data

Striking hierarchy   Mass : 3rd > 2nd > 1st   Almost diagonal CKM matrix

$M_{u,d} \sim$  $V_{CKM} \sim$ 

- $U(2)^5$ symmetry gives "natural" explanation of why 3rd Yukawa couplings are large (being allowed by the symmetry)

  distinguish the first two generations of fermions from the 3rd $\psi = (\psi_1, \psi_2, \psi_3)$

- The symmetry is a good approximation in the SM Yukawa

  exact symmetry for $m_u, m_d, m_c, m_s = 0$ & $V_{CKM} = 1$

  ⇒ we only need small breakings terms

$U(2)^5$ flavor symmetry

Barbieri, Isidori, Jones-Perez, Lodone, Straub [1105.2296]

- The set of breaking terms necessary to reproduce the quark spectrum, while keeping small FCNCs beyond the SM

Under $U(2)^3 = U(2)^q \times U(2)^u \times U(2)^d$ symmetry

quark

$$Q^{(2)} = (Q^1, Q^2) \sim (2,1,1) \qquad Q^3 \sim (1,1,1)$$
$$u^{(2)} = (u^1, u^2) \sim (1,2,1) \qquad t \sim (1,1,1)$$
$$d^{(2)} = (d^1, d^2) \sim (1,1,2) \qquad b \sim (1,1,1)$$

Spurion
(U(2) breaking term)

$$V_q \sim (2,1,1), \ \Delta_u \sim (2,\bar{2},1), \ \Delta_d \sim (2,1,\bar{2})$$

Unbroken symmetry

$$Y_u = y_t \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$U(2)_q$ $U(2)_u$

After breaking

$$\left(\begin{array}{cc|c} \multicolumn{2}{c|}{\Delta_u} & V_q \\ \hline 0 & 0 & 1 \end{array}\right)$$

U(2) breaking term

$$|V| \sim |V_{ts}|$$
$$|\Delta_u| \sim y_c$$

$U(2)$ flavour symmetry provides natural link to the Yukawa couplings

From MFV to $U(2)^5$

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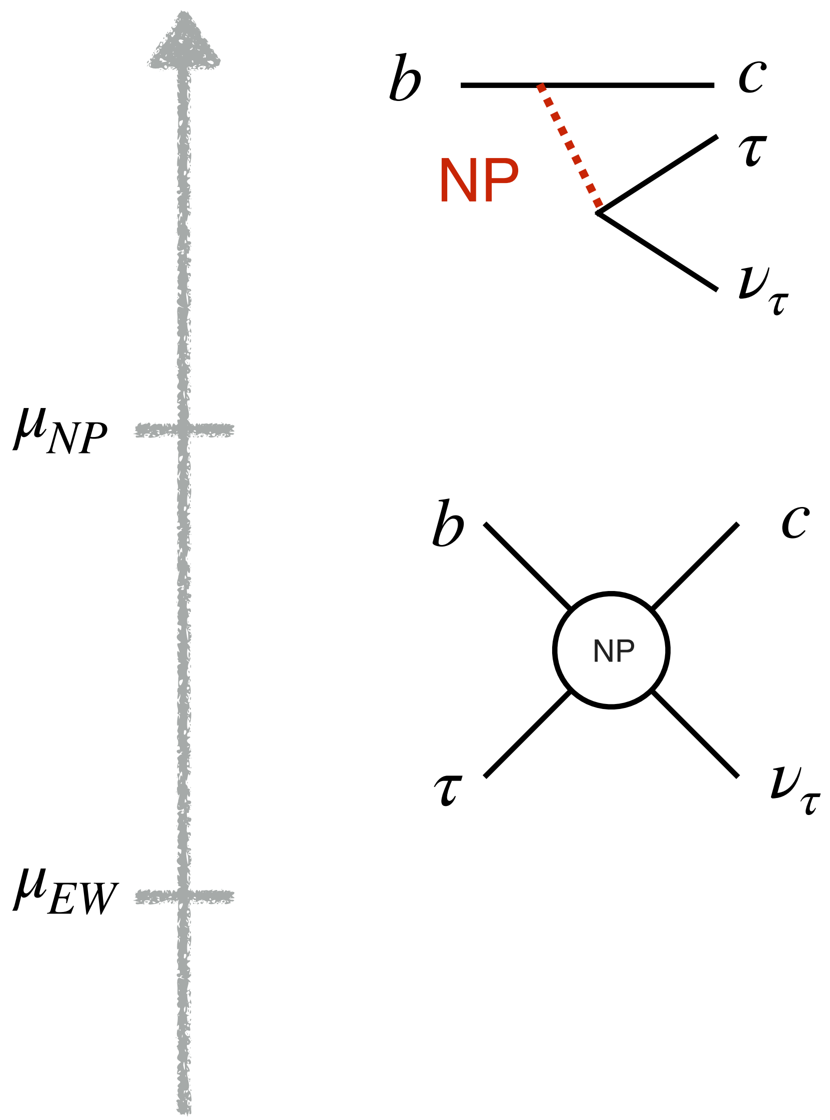
$U(2)^5 = U(2)_{Q_L} \times U(2)_{u_R} \times U(2)_{d_R} \times U(2)_{L_L} \times U(2)_{e_R}$ flavor symmetry

- acting on 1st & 2nd generations only
- The exact symmetry limit is good starting point for the SM quark spectrum $(m_u, m_d, m_c, m_s = 0 \;\&\; V_{CKM} = 1)$ $\Rightarrow$ we only need small breaking terms
- B-anomalies are compatible with U(2) flavor symmetry cf [1909.02519]

SM Effective Field Theory (SMEFT)

B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek [1008.4884].

- SMEFT is a effective theory based on $SU(3)_c \times SU(2)_L \times U(1)_Y$ at scale $\mu_{\rm EW} < \mu < \mu_{\rm NP}$



full theory

Integrate heavy particle

SMEFT

$$\mathscr{L}_{\rm eff} \sim \sum_i \frac{C_i}{\Lambda^2} \mathscr{O}_i^{d=6}$$

SM Effective Field Theory (SMEFT)

B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek [1008.4884].

- Complete non-redundant classification of baryon- and lepton-number conserving dimension-six operators in the SMEFT has been presented (Warsaw basis)

w/o flavor index 59 dim six operators in SMEFT

$1: X^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$Q_{\widetilde G}$	$f^{ABC}\widetilde G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$
$Q_{\widetilde W}$	$\epsilon^{IJK}\widetilde W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$

$2: H^6$	
Q_H	$(H^\dagger H)^3$

$3: H^4D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$(H^\dagger D_\mu H)^* (H^\dagger D_\mu H)$

$5: \psi^2H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar l_p e_r H$
Q_{uH}	$(H^\dagger H)(\bar q_p u_r \widetilde H$
Q_{dH}	$(H^\dagger H)(\bar q_p d_r H$

$4: X^2H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde G}$	$H^\dagger H\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde W}$	$H^\dagger H\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu}B^{\mu\nu}$
$Q_{H\widetilde B}$	$H^\dagger H\, \widetilde B_{\mu\nu}B^{\mu\nu}$
Q_{HWB}	$H^\dagger\tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde WB}$	$H^\dagger\tau^I H\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$

$6: \psi^2XH + \text{h.c.}$	
Q_{eW}	$(\bar l_p\sigma^{\mu\nu}e_r)\tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar l_p\sigma^{\mu\nu}e_r)H B_{\mu\nu}$
Q_{uG}	$(\bar q_p\sigma^{\mu\nu}T^A u_r)\widetilde H\, G_{\mu\nu}^A$
Q_{uW}	$(\bar q_p\sigma^{\mu\nu}u_r)\tau^I\widetilde H\, W_{\mu\nu}^I$
Q_{uB}	$(\bar q_p\sigma^{\mu\nu}u_r)\widetilde H\, B_{\mu\nu}$
Q_{dG}	$(\bar q_p\sigma^{\mu\nu}T^A d_r)H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar q_p\sigma^{\mu\nu}d_r)\tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar q_p\sigma^{\mu\nu}d_r)H\, B_{\mu\nu}$

$7: \psi^2H^2D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar l_p\gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar l_p\tau^I\gamma^\mu l_r)$
Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar e_p\gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar q_p\gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar q_p\tau^I\gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar u_p\gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar d_p\gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde H^\dagger D_\mu H)(\bar u_p\gamma^\mu d_r)$

$8: (\bar LL)(\bar LL)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu\tau^I q_r)(\bar q_s\gamma^\mu\tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu\tau^I l_r)(\bar q_s\gamma^\mu\tau^I q_t)$

$8: (\bar RR)(\bar RR)$	
Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$
Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$
Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$
Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$
Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$

$8: (\bar LL)(\bar RR)$	
Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$

$8: (\bar LR)(\bar RL) + \text{h.c.}$	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_{tj})$

$8: (\bar LR)(\bar LR) + \text{h.c.}$	
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\epsilon_{jk}(\bar q_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\epsilon_{jk}(\bar q_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\epsilon_{jk}(\bar q_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar l_p^j\sigma_{\mu\nu}e_r)\epsilon_{jk}(\bar q_s^k\sigma^{\mu\nu}u_t)$

SM Effective Field Theory (SMEFT)

B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek [1008.4884].

- Complete non-redundant classification of baryon- and lepton-number conserving dimension-six operators in the SMEFT has been presented (Warsaw basis)

w/o flavor index 59 dim six operators in SMEFT

$1: X^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde W}$	$\epsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

$2: H^6$	
Q_H	$(H^\dagger H)^3$

$3: H^4 D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

$5: \psi^2 H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar l_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar q_p u_r \widetilde H)$
Q_{dH}	$(H^\dagger H)(\bar q_p d_r H)$

$4: X^2 H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde G}$	$H^\dagger H\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde W}$	$H^\dagger H\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde B}$	$H^\dagger H\, \widetilde B_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde W B}$	$H^\dagger \tau^I H\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$

$6: \psi^2 X H + \text{h.c.}$	
Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde H\, G_{\mu\nu}^A$
Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde H\, W_{\mu\nu}^I$
Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde H\, B_{\mu\nu}$
Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7: \psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar l_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar l_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar e_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar q_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar q_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar u_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar d_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde H^\dagger D_\mu H)(\bar u_p \gamma^\mu d_r)$

$8: (\bar L L)(\bar L L)$	
Q_{ll}	$(\bar l_p \gamma_\mu l_r)(\bar l_s \gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar q_s \gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar q_p \gamma_\mu \tau^I q_r)(\bar q_s \gamma^\mu \tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar l_p \gamma_\mu l_r)(\bar q_s \gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar l_p \gamma_\mu \tau^I l_r)(\bar q_s \gamma^\mu \tau^I q_t)$

$8: (\bar R R)(\bar R R)$	
Q_{ee}	$(\bar e_p \gamma_\mu e_r)(\bar e_s \gamma^\mu e_t)$
Q_{uu}	$(\bar u_p \gamma_\mu u_r)(\bar u_s \gamma^\mu u_t)$
Q_{dd}	$(\bar d_p \gamma_\mu d_r)(\bar d_s \gamma^\mu d_t)$
Q_{eu}	$(\bar e_p \gamma_\mu e_r)(\bar u_s \gamma^\mu u_t)$
Q_{ed}	$(\bar e_p \gamma_\mu e_r)(\bar d_s \gamma^\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar u_p \gamma_\mu u_r)(\bar d_s \gamma^\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar u_p \gamma_\mu T^A u_r)(\bar d_s \gamma^\mu T^A d_t)$

$8: (\bar L L)(\bar R R)$	
Q_{le}	$(\bar l_p \gamma_\mu l_r)(\bar e_s \gamma^\mu e_t)$
Q_{lu}	$(\bar l_p \gamma_\mu l_r)(\bar u_s \gamma^\mu u_t)$
Q_{ld}	$(\bar l_p \gamma_\mu l_r)(\bar d_s \gamma^\mu d_t)$
Q_{qe}	$(\bar q_p \gamma_\mu q_r)(\bar e_s \gamma^\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar u_s \gamma^\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar u_s \gamma^\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar d_s \gamma^\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar d_s \gamma^\mu T^A d_t)$

$8: (\bar L R)(\bar R L) + \text{h.c.}$	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_{tj})$

$8: (\bar L R)(\bar L R) + \text{h.c.}$	
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\epsilon_{jk}(\bar q_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\epsilon_{jk}(\bar q_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\epsilon_{jk}(\bar q_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$

w/ flavor index $(n_g = 3)$ 2499 dim six operators in SMEFT

1350 CP-even and 1149 CP-odd

huge number of free parameters —flavor symmetry→ reduce number of independent parameters

Our work

- We analyse how $U(3)^5$ and $U(2)^5$ flavor symmetries act on SMEFT, providing an organising principle to classify the large number of dim6 operators involving fermion fields

Class	Operators	No symmetry 3 Gen.		No symmetry 1 Gen.	
1–4	X^3, H^6, H^4D^2, X^2H^2	9	6	9	6
5	ψ^2H^3	27	27	3	3
6	ψ^2XH	72	72	8	8
7	ψ^2H^2D	51	30	8	1
8	$(\bar{L}L)(\bar{L}L)$	171	126	5	–
	$(\bar{R}R)(\bar{R}R)$	255	195	7	–
	$(\bar{L}L)(\bar{R}R)$	360	288	8	–
	$(\bar{L}R)(\bar{R}L)$	81	81	1	1
	$(\bar{L}R)(\bar{L}R)$	324	324	4	4
	total:	1350	1149	53	23

$U(3)^5$: ?

$U(2)^5$: ?

CP-even CP-odd

1) Case for $U(3)^5$ and MFV

2) Case for $U(2)^5$

[3) Case for beyond $U(3)^5$ and $U(2)^5$]

Operator classification

59 dim six operators in SMEFT

class 1-4 : w/o fermion ope.

class 5-7 : w/ 2-fermion ope.

$1 : X^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

$2 : H^6$	
Q_H	$(H^\dagger H)^3$

$3 : H^4 D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

$5 : \psi^2 H^3 + \mathrm{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H}$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H$

$4 : X^2 H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

$6 : \psi^2 X H + \mathrm{h.c.}$	
Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7 : \psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

Operator classification

class 8: w/ 4-fermion ope.

59 dim six operators in SMEFT

$8:(\bar{L}L)(\bar{L}L)$		$8:(\bar{R}R)(\bar{R}R)$		$8:(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$8:(\bar{L}R)(\bar{R}L)+\mathrm{h.c.}$		$8:(\bar{L}R)(\bar{L}R)+\mathrm{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$	$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
		$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
		$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
		$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu} u_t)$

1) $U(3)^5$ and MFV

e.g. class 5 : $(\bar{L}R)$ bilinear

No symmetry → (# parameters) = (flavor index)^2

non-hermitian ope. → Re + Im

$(\bar{L}R)$ type ope. → ✗ $(\bar{q}\, u)$, $(\bar{q}\, d)$: not allowed in exact $U(3)^5$

→ $(\bar{q} Y_u u)$, $(\bar{q} Y_d d)$: allowed w/ Y_u

→ $(\bar{q}^i (Y_u Y_u^\dagger) Y_d d^j)$: allowed w/ more $Y_{u,e,d}$:

$5:\ \psi^2 H^3 + \text{h.c.}$		No sym. CP-ev	No sym. CP-odd	exact $U(3)^5$	$\sim \mathcal{O}(Y_{u,d,e})$ CP-ev	$\sim \mathcal{O}(Y_{u,d,e})$ CP-odd	$\sim \mathcal{O}(Y_d Y_u^2)$ CP-ev	$\sim \mathcal{O}(Y_d Y_u^2)$ CP-odd
Q_{eH}	$(H^\dagger H)(\bar{\ell}_p e_r H)$	9	9	0	1	1	1	1
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \tilde{H})$	9	9	0	1	1	1	1
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$	9	9	0	1	1	2	2
		27	27	0	3	3	4	4

1) $U(3)^5$ and MFV

		No symmetry				$U(3)^5$					
Class	Operators	3 Gen.		1 Gen.		Exact		$\mathcal{O}(Y^1_{e,d,u})$		$\mathcal{O}(Y^1_e, Y^1_d Y^2_u)$	
1–4	X^3, H^6, H^4D^2, X^2H^2	9	6	9	6	9	6	9	6	9	6
5	$\psi^2 H^3$	27	27	3	3	–	–	3	3	4	4
6	$\psi^2 XH$	72	72	8	8	–	–	8	8	11	11
7	$\psi^2 H^2 D$	51	30	8	1	7	–	7	–	11	1
	$(\bar{L}L)(\bar{L}L)$	171	126	5	–	8	–	8	–	14	–
	$(\bar{R}R)(\bar{R}R)$	255	195	7	–	9	–	9	–	14	–
8	$(\bar{L}L)(\bar{R}R)$	360	288	8	–	8	–	8	–	18	–
	$(\bar{L}R)(\bar{R}L)$	81	81	1	1	–	–	–	–	–	–
	$(\bar{L}R)(\bar{L}R)$	324	324	4	4	–	–	–	–	4	4
total:		1350	1149	53	23	41	6	52	17	85	26

1) $U(3)^5$ and MFV

		No symmetry				$U(3)^5$					
Class	Operators	3 Gen.		1 Gen.		Exact		$\mathcal{O}(Y^1_{e,d,u})$		$\mathcal{O}(Y^1_e, Y^1_d Y^2_u)$	
1–4	X^3, H^6, H^4D^2, X^2H^2	9	6	9	6	9	6	9	6	9	6
5	ψ^2H^3	27	27	3	3	–	–	3	3	4	4
6	ψ^2XH	72	72	8	8	–	–	8	8	11	11
7	ψ^2H^2D	51	30	8	1	7	–	7	–	11	1
	$(\bar{L}L)(\bar{L}L)$	171	126	5	–	8	–	8	–	14	–
	$(\bar{R}R)(\bar{R}R)$	255	195	7	–	9	–	9	–	14	–
8	$(\bar{L}L)(\bar{R}R)$	360	288	8	–	8	–	8	–	18	–
	$(\bar{L}R)(\bar{R}L)$	81	81	1	1	–	–	–	–	–	–
	$(\bar{L}R)(\bar{L}R)$	324	324	4	4	–	–	–	–	4	4
total:		1350	1149	53	23	41	6	52	17	85	26

~2500 → ~100

MFV

II) Case for $U(2)^5$

Yukawa in U(2)

$$Y_e = y_\tau \begin{pmatrix} \Delta_e & x_\tau V_\ell \\ 0 & 1 \end{pmatrix}, \qquad Y_u = y_t \begin{pmatrix} \Delta_u & x_t V_q \\ 0 & 1 \end{pmatrix}, \qquad Y_d = y_b \begin{pmatrix} \Delta_d & x_b V_q \\ 0 & 1 \end{pmatrix}$$

$V_q \sim (2,1,1)$, $\Delta_u \sim (2,\bar{2},1)$, $\Delta_d \sim (2,1,\bar{2})$ $\quad$ $y_{\tau,t,b}$ and $x_{\tau,t,b}$: $\mathcal{O}(1)$ free complex parameters

Transformation for spurions

$$V_{q(\ell)} = e^{i\bar{\phi}_{q(\ell)}} \begin{pmatrix} 0 \\ \epsilon_{q(\ell)} \end{pmatrix}, \quad \Delta_e = O_e^{\intercal} \begin{pmatrix} \delta'_e & 0 \\ 0 & \delta_e \end{pmatrix}, \quad \Delta_u = U_u^{\dagger} \begin{pmatrix} \delta'_u & 0 \\ 0 & \delta_u \end{pmatrix}, \quad \Delta_d = U_d^{\dagger} \begin{pmatrix} \delta'_d & 0 \\ 0 & \delta_d \end{pmatrix}$$

$$\epsilon_i = \mathcal{O}(y_t|V_{ts}|) = \mathcal{O}(10^{-1})$$

$$\delta_i = \mathcal{O}\left(\frac{y_c}{y_t}, \frac{y_s}{y_b}, \frac{y_\mu}{y_\tau}\right) = \mathcal{O}(10^{-2})$$

$$\delta'_i = \mathcal{O}\left(\frac{y_u}{y_t}, \frac{y_d}{y_b}, \frac{y_e}{y_\tau}\right) = \mathcal{O}(10^{-3})$$

$$1 \gg \epsilon_i \gg \delta_i \gg \delta'_i > 0$$

$$O_e = \begin{pmatrix} c_e & s_e \\ -s_e & c_e \end{pmatrix}, \qquad U_q = \begin{pmatrix} c_q & s_q\, e^{i\alpha_q} \\ -s_q\, e^{-i\alpha_q} & c_q \end{pmatrix}$$

II) Case for $U(2)^5$

$$\psi = (\psi_1, \psi_2, \psi_3)$$

L ℓ_3

e.g.) leptonic ($\bar{L}L$) bilinear

$$\bar{\ell}_p \Gamma \Lambda_{LL}^{pr} \ell_r \,, \qquad \Lambda_{LL} = \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_1 + c_1 \epsilon_\ell^2 & \beta_1 \epsilon_\ell \\ 0 & \beta_1^* \epsilon_\ell & a_2 \end{pmatrix} + \mathcal{O}(\delta_e^2)$$

a : $\mathcal{O}(V^0)$

β : $\mathcal{O}(V)$

c : $\mathcal{O}(V^2)$

※ laten $(a, b, c, , ,)$: real, greek$(\alpha, \beta, \gamma, , ,)$: complex

Spurions	Operator	Explicit expression in flavour components
V^0	$a_1 \bar{L}L + a_2 \bar{\ell}_3 \ell_3$	$a_1 \left(\bar{\ell}_1 \ell_1 + \bar{\ell}_2 \ell_2\right) + a_2 \left(\bar{\ell}_3 \ell_3\right)$
V^1	$\beta_1 \bar{L} V_\ell \ell_3 + \text{h.c.}$	$\beta_1 \epsilon_\ell \left(\bar{\ell}_2 \ell_3\right) + \text{h.c.}$
V^2	$c_1 \bar{L} V_\ell V_\ell^\dagger L$	$c_1 \epsilon_\ell^2 \left(\bar{\ell}_2 \ell_2\right)$
Δ^1, $\Delta^1 V^1$	–	–
Δ^2	$h_1 \bar{L} \Delta_e \Delta_e^\dagger L$	$\approx h_1 \left[\delta_e^2 (\bar{\ell}_2 \ell_2) - s_e \delta_e^2 (\bar{\ell}_1 \ell_2 + \bar{\ell}_2 \ell_1) + (s_e^2 \delta_e^2 + \delta_e'^2)(\bar{\ell}_1 \ell_1)\right]$
$\Delta^2 V^1$	$\lambda_1 \bar{L} \Delta_e \Delta_e^\dagger V_\ell \ell_3 + \text{h.c.}$	$\approx \lambda_1 \epsilon_\ell \delta_e^2 (\bar{\ell}_2 \ell_3 - s_e \bar{\ell}_1 \ell_3) + \text{h.c.}$
$\Delta^2 V^2$	$\mu_1 \bar{L} \Delta_e \Delta_e^\dagger V_\ell V_\ell^\dagger L + \text{h.c.}$	$\approx \mu_1 \epsilon_\ell^2 \delta_e^2 (\bar{\ell}_2 \ell_2 - s_e \bar{\ell}_1 \ell_2) + \text{h.c.}$

II) Case for $U(2)^5$

e.g.) leptonic ($\bar{R}R$) bilinear

$$\bar{e}_p \Gamma \Lambda^{pr}_{RR} e_r \ , \qquad \Lambda_{RR} = \begin{pmatrix} a_1 & 0 & \sigma_1^* \epsilon_\ell s_e \delta'_e \\ 0 & a_1 & \sigma_1^* \epsilon_\ell \delta_e \\ \sigma_1 \epsilon_\ell s_e \delta'_e & \sigma_1 \epsilon_\ell \delta_e & a_2 \end{pmatrix} + \mathcal{O}(\delta_e^2)$$

a : $\mathcal{O}(V^0)$
β : $\mathcal{O}(V)$
c : $\mathcal{O}(V^2)$

Spurions	Operator ($\bar{e}e$ type)	Explicit expression in flavour components
V^0	$a_1 \bar{E}E + a_2 \bar{e}_3 e_3$	$a_1 (\bar{e}_1 e_1 + \bar{e}_2 e_2) + a_2 (\bar{e}_3 e_3)$
V^1, V^2, Δ^1	–	
$\Delta^1 V^1$	$\sigma_1 \bar{e}_3 V_\ell^\dagger \Delta_e E$ + h.c.	$\approx \sigma_1 \epsilon_\ell [\delta_e(\bar{e}_3 e_2) + s_e \delta'_e(\bar{e}_3 e_1)]$ + h.c.
Δ^2	$h_1 \bar{E} \Delta_e^\dagger \Delta_e E$	$h_1 \left[\delta_e^2(\bar{e}_2 e_2) + \delta_e'^2(\bar{e}_1 e_1)\right]$
$\Delta^2 V^1$	–	
$\Delta^2 V^2$	$m_1 \bar{E} \Delta_e^\dagger V_\ell V_\ell^\dagger \Delta_e E$	$\approx m_1 \epsilon_\ell^2 \left[\delta_e^2(\bar{e}_2 e_2) + s_e \delta'_e \delta_e(\bar{e}_1 e_2 + \bar{e}_2 e_1) + s_e^2 \delta_e'^2(\bar{e}_1 e_1)\right]$

II) Case for $U(2)^5$

Results for bilinear structure

Class	N. indep. structures	$U(2)^5$ breaking terms									
		V^0		V^1		V^2		Δ^1		$\Delta^1 V^1$	
5 & 6: $(\bar{L}R)$	11	11	11	11	11	–	–	11	11	11	11
7: $(\bar{L}L)$	4	8	–	4	4	4	–	–	–	–	–
7: $(\bar{R}R)$	3	6	–	–	–	–	–	–	–	3	3
7: Q_{Hud}	1	1	1	–	–	–	–	–	–	2	2
total:	19	26	12	15	15	4	–	11	11	16	16

II) Case for $U(2)^5$

$$\psi = (\underbrace{\psi_1, \psi_2}_{L}, \underbrace{\psi_3}_{\ell_3})$$

4 fermion operator $(\bar{L}L)(\bar{L}L)$

$Q_{\ell\ell},\ Q_{qq}^{(1)}$ and $Q_{qq}^{(3)}$ case

$$\begin{aligned}
V^0: \quad & \left[a_1(\bar{L}^p L^p)(\bar{L}^r L^r) + a_2(\bar{L}^p L^r)(\bar{L}^r L^p) + a_3(\bar{L}L)(\bar{\ell}_3\ell_3)\right. \\
& \left. + a_4(\bar{L}\ell_3)(\bar{\ell}_3 L) + a_5(\bar{\ell}_3\ell_3)(\bar{\ell}_3\ell_3)\right] , \\
V^1: \quad & \left[\beta_1(\bar{L}^p V_\ell^p \ell_3)(\bar{L}^r L^r) + \beta_2(\bar{L}V_\ell\ell_3)(\bar{\ell}_3\ell_3) + \beta_3(\bar{L}^p V_\ell^p L^r)(\bar{L}^r\ell_3) \ + \ \text{h.c.}\right] , \\
V^2: \quad & \left[c_1(\bar{L}^p V_\ell^p V_\ell^{\dagger\, r} L^r)(\bar{L}^s L^s) + c_2(\bar{L}^p V_\ell^p V_\ell^{\dagger\, r} L^r)(\bar{\ell}_3\ell_3) + c_3(\bar{L}^p V_\ell^p \ell_3)(\bar{\ell}_3 V_\ell^{\dagger\, r} L^r)\right. \\
& \left. + c_4(\bar{L}^p V_\ell^p L^r)(\bar{L}^r V_\ell^{\dagger\, s} L^s) + (\gamma_1(\bar{L}^p V_\ell^p \ell_3)(\bar{L}^r V_\ell^r \ell_3) \ + \ \text{h.c.})\right] , \\
V^3: \quad & \left[\xi_1(\bar{L}^p V_\ell^p V_\ell^{\dagger\, r} L^r)(\bar{L}^s V_\ell^s \ell_3) \ + \ \text{h.c.}\right] .
\end{aligned}$$

II) Case for $U(2)^5$

4 fermion operator $(\bar{L}L)(\bar{L}L)$

$Q_{\ell\ell}$, $Q_{qq}^{(1)}$ and $Q_{qq}^{(3)}$ case

a : $\mathcal{O}(V^0)$

β : $\mathcal{O}(V)$

c : $\mathcal{O}(V^2)$

	(11)	(12)	(13)	(21)	(22)	(23)	(31)	(32)	(33)
(11)	a_1 a_2				$2a_1$ $c_1\epsilon_\ell^2$	$\beta_1\epsilon_\ell$		$\beta_1^*\epsilon_\ell$	a_3
(12)				$2a_2$ $c_4\epsilon_\ell^2$			$\beta_3^*\epsilon_\ell$		
(13)				$\beta_3\epsilon_\ell$			a_4		
(21)		$2a_2$ $c_4\epsilon_\ell^2$	$\beta_3\epsilon_\ell$						
(22)	$2a_1$ $c_1\epsilon_\ell^2$				a_1 a_2 $c_1\epsilon_\ell^2$ $c_4\epsilon_\ell^2$	$\beta_1\epsilon_\ell$ $\beta_3\epsilon_\ell$ $\xi_1\epsilon_\ell^3$		$\beta_1^*\epsilon_\ell$ $\beta_3^*\epsilon_\ell$ $\xi_1^*\epsilon_\ell^3$	a_3 $c_2\epsilon_\ell^2$
(23)	$\beta_1\epsilon_\ell$				$\beta_1\epsilon_\ell$ $\beta_3\epsilon_\ell$ $\xi_1\epsilon_\ell^3$	$\gamma_1\epsilon_\ell^2$		a_4 $c_3\epsilon_\ell^2$	$\beta_2\epsilon_\ell$
(31)		$\beta_3^*\epsilon_\ell$	a_4						
(32)	$\beta_1^*\epsilon_\ell$				$\beta_1^*\epsilon_\ell$ $\beta_3^*\epsilon_\ell$ $\xi_1^*\epsilon_\ell^3$	a_4 $c_3\epsilon_\ell^2$		$\gamma_1^*\epsilon_\ell^2$	$\beta_2^*\epsilon_\ell$
(33)	a_3				a_3 $c_2\epsilon_\ell^2$	$\beta_2\epsilon_\ell$		$\beta_2^*\epsilon_\ell$	a_5

Table 12: The $\Sigma_{\ell\ell}^{ij,nm}$ tensor in the interaction basis as defined in Eq. (27): the entries are as indicated in rows (ij) and columns (nm), respectively. All terms in each cell should be added.

II) Case for $U(2)^5$

Operators	$U(2)^5$ [terms summed up to different orders]													
	Exact		$\mathcal{O}(V^1)$		$\mathcal{O}(V^2)$		$\mathcal{O}(V^1, \Delta^1)$		$\mathcal{O}(V^2, \Delta^1)$		$\mathcal{O}(V^2, \Delta^1 V^1)$		$\mathcal{O}(V^3, \Delta^1 V^1)$	
Class 1–4	9	6	9	6	9	6	9	6	9	6	9	6	9	6
$\psi^2 H^3$	3	3	6	6	6	6	9	9	9	9	12	12	12	12
$\psi^2 X H$	8	8	16	16	16	16	24	24	24	24	32	32	32	32
$\psi^2 H^2 D$	15	1	19	5	23	5	19	5	23	5	28	10	28	10
$(\bar{L}L)(\bar{L}L)$	23	–	40	17	67	24	40	17	67	24	67	24	74	31
$(\bar{R}R)(\bar{R}R)$	29	–	29	–	29	–	29	–	29	–	53	24	53	24
$(\bar{L}L)(\bar{R}R)$	32	–	48	16	64	16	53	21	69	21	90	42	90	42
$(\bar{L}R)(\bar{R}L)$	1	1	3	3	4	4	5	5	6	6	10	10	10	10
$(\bar{L}R)(\bar{L}R)$	4	4	12	12	16	16	24	24	28	28	48	48	48	48
total:	124	23	182	81	234	93	212	111	264	123	349	208	356	215

~300

~600

Normal

~2500

U(2)^5

II) Case for $U(2)^5$

e.g. relevant operators for semileptonic B decays

$$\mathcal{O}_{\ell q}^{(1)} = (\bar{\ell}_L^{\alpha}\gamma^{\mu}\ell_L^{\beta})(\bar{q}_L^{i}\gamma_{\mu}q_L^{j})\,,$$

$$\mathcal{O}_{\ell q}^{(3)} = (\bar{\ell}_L^{\alpha}\gamma^{\mu}\tau^{I}\ell_L^{\beta})(\bar{q}_L^{i}\gamma_{\mu}\tau^{I}q_L^{j})\,,$$

$$\mathcal{O}_{\ell d} = (\bar{\ell}_L^{\alpha}\gamma^{\mu}\ell_L^{\beta})(\bar{d}_R^{i}\gamma_{\mu}d_R^{j})\,,$$

$$\mathcal{O}_{qe} = (\bar{q}_L^{i}\gamma^{\mu}q_L^{j})(\bar{e}_R^{\alpha}\gamma_{\mu}e_R^{\beta})\,,$$

$$\mathcal{O}_{ed} = (\bar{e}_R^{\alpha}\gamma^{\mu}e_R^{\beta})(\bar{d}_R^{i}\gamma_{\mu}d_R^{j})\,,$$

$$\mathcal{O}_{\ell edq} = (\bar{\ell}_L^{\alpha}e_R^{\beta})(\bar{d}_R^{i}q_L^{j})\,,$$

$$\mathcal{O}_{\ell equ}^{(1)} = (\bar{\ell}_L^{a,\alpha}e_R^{\beta})\epsilon_{ab}(\bar{q}_L^{a,i}u_R^{j})\,,$$

$$\mathcal{O}_{\ell equ}^{(3)} = (\bar{\ell}_L^{a,\alpha}\sigma_{\mu\nu}e_R^{\beta})\epsilon_{ab}(\bar{q}_L^{b,i}\sigma^{\mu\nu}u_R^{j})$$

II) Case for $U(2)^5$

e.g. relevant operators for semileptonic B decays

only few yield sizable effects if we impose a minimally broken $U(2)^5$ symmetry $\sim \mathcal{O}(V^2)$

$$\mathcal{O}_{\ell q}^{(1)} = (\bar{\ell}_L^\alpha \gamma^\mu \ell_L^\beta)(\bar{q}_L^i \gamma_\mu q_L^j)\,,$$

$$\mathcal{O}_{\ell q}^{(3)} = (\bar{\ell}_L^\alpha \gamma^\mu \tau^I \ell_L^\beta)(\bar{q}_L^i \gamma_\mu \tau^I q_L^j)\,,$$

$$\mathcal{O}_{\ell d} = (\bar{\ell}_L^\alpha \gamma^\mu \ell_L^\beta)(\bar{d}_R^i \gamma_\mu d_R^j)\,,$$

$$\mathcal{O}_{qe} = (\bar{q}_L^i \gamma^\mu q_L^j)(\bar{e}_R^\alpha \gamma_\mu e_R^\beta)\,,$$

$$\mathcal{O}_{ed} = (\bar{e}_R^\alpha \gamma^\mu e_R^\beta)(\bar{d}_R^i \gamma_\mu d_R^j)\,,$$

$$\mathcal{O}_{\ell edq} = (\bar{\ell}_L^\alpha e_R^\beta)(\bar{d}_R^i q_L^j)\,,$$

$$\mathcal{O}_{\ell equ}^{(1)} = (\bar{\ell}_L^{a,\alpha} e_R^\beta)\epsilon_{ab}(\bar{q}_L^{a,i} u_R^j)\,,$$

$$\mathcal{O}_{\ell equ}^{(3)} = (\bar{\ell}_L^{a,\alpha} \sigma_{\mu\nu} e_R^\beta)\epsilon_{ab}(\bar{q}_L^{b,i} \sigma^{\mu\nu} u_R^j)$$

Summary

- NP may have a highly non-generic flavor structure

→ Flavor symmetry   MFV and $U(2)$ flavor symmetry

- We analyze how $U(3)^5$ and $U(2)^5$ flavor symmetries act on SMEFT

2499 in SMEFT
huge number of free parameters —flavor symmetry→ reduce number of independent parameters

$U(3)^5$ **and MFV**   drastic reduction : ~ 25 times smaller

$U(2)^5$ drastic reduction : ~ one order smaller

- This classification can be a useful first step toward a systematic analysis in motivated flavor versions of the SMEFT

Backup

$U(2)^5$ flavor symmetry

Yukawa after removing unphysical parameters

$$Y_u = |y_t| \begin{pmatrix} U_q^\dagger O_u^\intercal \hat{\Delta}_u & |V_q||x_t| e^{i\phi_q} \vec{n} \\ 0 & 1 \end{pmatrix}$$

$$Y_d = |y_b| \begin{pmatrix} U_q^\dagger \hat{\Delta}_d & |V_q||x_b| e^{i\phi_q} \vec{n} \\ 0 & 1 \end{pmatrix}$$

$$Y_e = |y_\tau| \begin{pmatrix} O_e^\intercal \hat{\Delta}_e & |V_e||x_\tau| \vec{n} \\ 0 & 1 \end{pmatrix}$$

$\hat{\Delta}_{u,d,e}$: 2×2 diagonal positive matrix

$O_{u,e}$: 2×2 orthogonal matrix

$$U_q = \begin{pmatrix} c_d & s_d e^{i\alpha_d} \\ -s_d e^{-i\alpha_d} & c_d \end{pmatrix}, \quad \vec{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Structure of Yukawa is fixed under $U(2)$ symmetry

→ elements in diagonal matrixes are described by CKM elements & fermions masses

$$Y_f \xrightarrow{Q_L \to L_d^\dagger Q_L \quad d_R \to R_d^\dagger d_R} \mathrm{diag}(Y_f) = L_f^\dagger Y_f R_f \quad (f = u, d)$$

where

$$L_d \approx \begin{pmatrix} c_d & -s_d e^{i\alpha_d} & 0 \\ s_d e^{-i\alpha_d} & c_d & s_b \\ -s_d s_b e^{-i(\alpha_d+\phi_q)} & -c_d s_b e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix} \qquad R_d \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{m_s}{m_b} s_b \\ 0 & -\frac{m_s}{m_b} s_b e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$s_d/c_d = |V_{td}/V_{ts}|$, $\alpha_d = -\mathrm{Arg}(V_{td}/V_{ts})$, $s_t = s_b - V_{cb}$, s_u

$U(2)^5$ flavor symmetry

Yukawa after removing unphysical parameters

$$Y_u = |y_t| \begin{pmatrix} U_q^\dagger O_u^\intercal \hat{\Delta}_u & |V_q||x_t| e^{i\phi_q} \vec{n} \\ 0 & 1 \end{pmatrix}$$

$$Y_d = |y_b| \begin{pmatrix} U_q^\dagger \hat{\Delta}_d & |V_q||x_b| e^{i\phi_q} \vec{n} \\ 0 & 1 \end{pmatrix}$$

$$Y_e = |y_\tau| \begin{pmatrix} O_e^\intercal \hat{\Delta}_e & |V_e||x_\tau| \vec{n} \\ 0 & 1 \end{pmatrix}$$

$\hat{\Delta}_{u,d,e}$: 2×2 diagonal positive matrix

$O_{u,e}$: 2×2 orthogonal matrix

$$U_q = \begin{pmatrix} c_d & s_d e^{i\alpha_d} \\ -s_d e^{-i\alpha_d} & c_d \end{pmatrix}, \ \vec{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

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$$Y_f \xrightarrow{Q_L \to L_d^\dagger Q_L \quad d_R \to R_d^\dagger d_R} \mathrm{diag}(Y_f) = L_f^\dagger Y_f R_f \quad (f = u, d)$$

Parameters — constrained

quark: $s_d/c_d = |V_{td}/V_{ts}|$, $\alpha_d = -\mathrm{Arg}(V_{td}/V_{ts})$, $s_t = s_b - V_{cb}$, s_u — $s_b/c_b = |x_b||V_q|$, ϕ_q

lepton: $s_\tau/c_\tau = |x_\tau||V_\ell|$, s_e