Study of the phase diagram of dense two-color QCD within lattice simulation

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Probing the physics of high-density and low-temperature matter with ab initio calculations in 2-color QCD

Outline:

- Introduction
- QC_2D at small and moderate baryon densities
- QC_2D at large baryon density
- Deconfinement transition
- Conclusion

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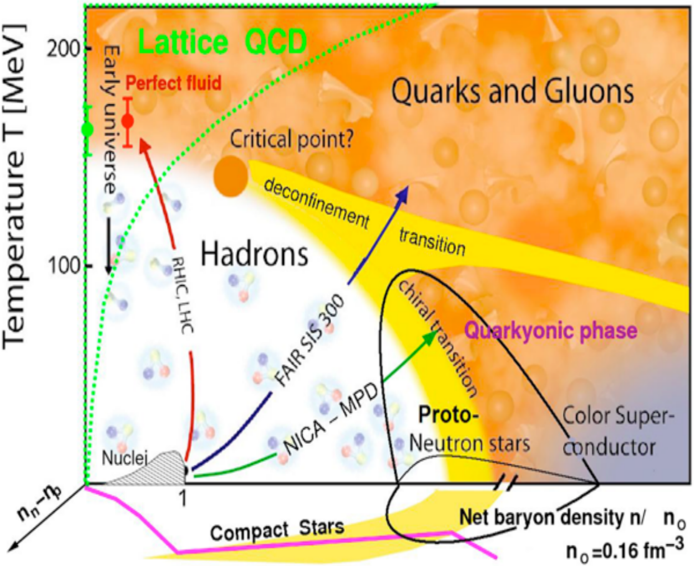
QCD phase diagram



QCD phase diagram

- Poor knowledge of dense QCD properties
- Phenomenological models (unknown systematic uncertainties)
- Lattice QCD (small densities $\frac{\mu}{T} < few$)

  Taylor expansion, Imaginary chemical potential,...
- **QCD-like theories**
  - SU(2) QCD with chemical potential
  - SU(3) QCD isospin chemical potential
  - ...

The sign problem problem

SU(3) QCD

- $Z = \int DU \exp\left(-S_G\right) \times \det\left(\hat{D} + m\right)$
- Eigenvalues go in pairs $\hat{D}$: $\pm i\lambda \Rightarrow$
  $\det\left(\hat{D} + m\right) = \prod_\lambda (\lambda^2 + m^2) > 0$
  i.e. one can use lattice simulation
- Introduce chemical potential: $\det\left(\hat{D} + m\right) \to \det\left(\hat{D} - \mu\gamma_4 + m\right) \Rightarrow$
  the determinant becomes complex (**sign problem**)

SU(2) QCD

- $(\gamma_5 C \tau_2) \cdot D^* = D \cdot (\gamma_5 C \tau_2)$
- Eigenvalues go in pairs $\hat{D} - \mu\gamma_4$: λ, λ^*
- For even N_f $\det\left(\hat{D} - \mu\gamma_4 + m\right) > 0 \Rightarrow$ **free from sign problem**

SU(2) QCD with baryon chemical potential

Differences between SU(2) and SU(3) at finite μ

- No phase of the fermion determinant
- The Lagrangian of the SU(2) QCD has the symmetry: $SU(2N_f)$ as compared to $SU_R(N_f) \times SU_L(N_f)$ for $SU(3)$ QCD
- Goldstone bosons $(N_f = 2)$ $\pi^+, \pi^-, \pi^0, d, \bar{d}$

QCD-like theories can be used to study dense QCD

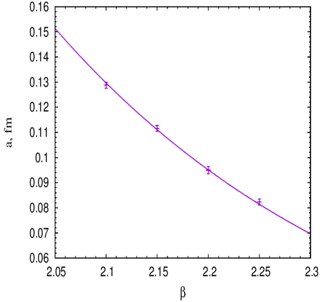
- Lattice study of QCD-like theories contains full dynamics of real system (contrary to phenomenological models)
- Adjust phenomenological models, check different approximations (analytic continuation)
- Study of different physical phenomena in dense medium

Lattice set up

- Staggered fermions
  $S_l = \sum_x (ma)\bar{\psi}_x\psi_x + \frac{1}{2}\sum_{x,\mu}\eta_\mu(x)\left(\bar{\psi}_{x+\mu}U_{x,\mu}\psi_x - \bar{\psi}_x U^+_{x,\mu}\psi_{x+\mu}\right)$
  $\lim_{a\to 0} S_l \to \int d^4x\bar{\psi}(\hat{D}+m)\psi$
- Diquark source in the action $\delta S \sim \lambda\psi^T(C\gamma_5)\times\sigma_2\times\tau_2\psi$
- Rooting $N_f = 2$
- $16^3\times 32$, $a = 0.11$ fm, $\mu = 900$ MeV ($a\mu \sim 0.5$),
  $m_\pi = 362(40)$ MeV, $\lambda/m = 0.2, 0.15, 0.1$
- $32^3\times 32$, $a = 0.044$ fm, $\mu = 2200$ MeV ($a\mu \sim 0.5$),
  $m_\pi = 740(40)$ MeV, $\lambda/m = 0.1$

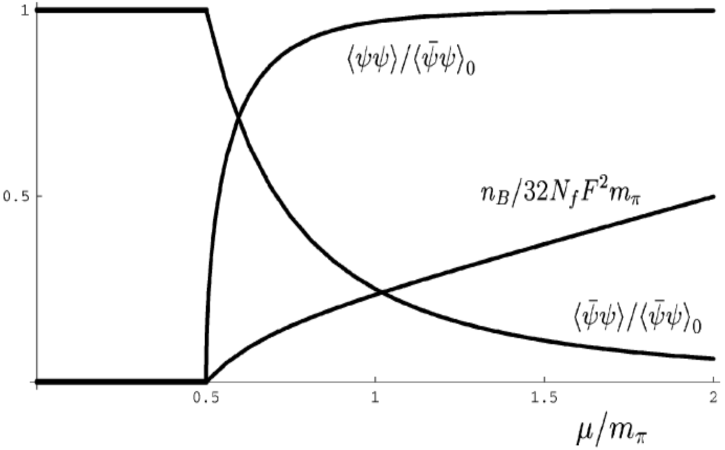
Continuum limit

- The symmetry breaking is different
  - Continuum: $SU(2N_f) \to Sp(2N_f)$
  - Staggered fermions: $SU(2N_f) \to O(2N_f)$
- Correct symmetry is restored in continuum limit
  - Naive limit $a \to 0$: two copies of $N_f = 2$ fundamental fermions
    $\frac{\lambda}{2} \sum_x \left(\psi_x^T \tau_2 \psi_x + \bar{\psi}_x \tau_2 \bar{\psi}_x^T \right) \Big|_{a \to 0} = \frac{\lambda}{2} \int d^4x \, (q_i^T C\gamma_5 \tau_2 q_j + \bar{q}_i C\gamma_5 \tau_2 \bar{q}_j^T) \times \begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix}_{ij}$
  - Correct spectrum of the Goldstone bosons
    (Phys.Rev.D 100 (2019) 11, 114507)
  - Correct β function for small a

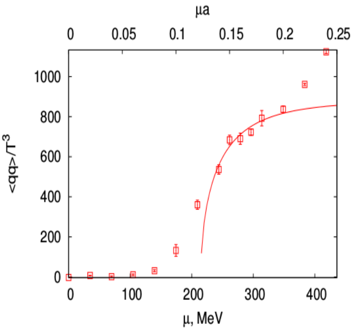


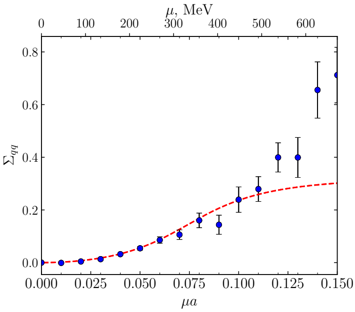
Small and moderate densities

Predictions of CHPT



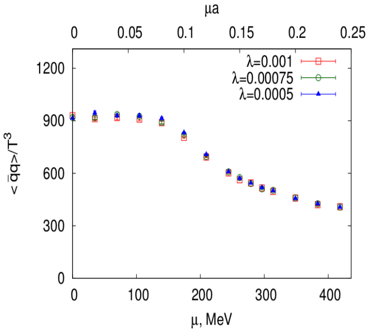
Diquark condensate

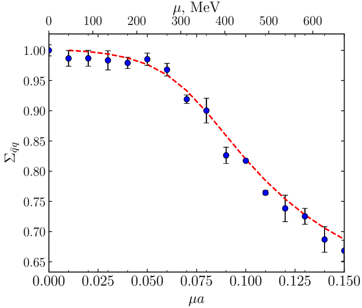




- Renormalized diquark condensate
  $\Sigma_{qq} = \frac{m}{4m_\pi^2 F^2}\left[\langle qq\rangle_\mu - \langle qq\rangle_0\right]$

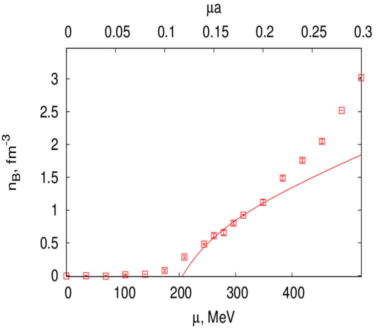
Chiral condensate

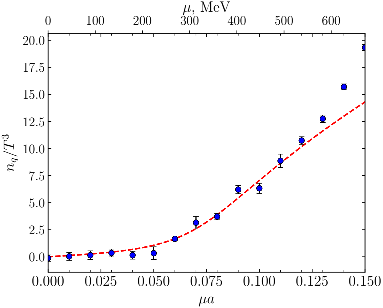




- Renormalized chiral condensate
  $\Sigma_{\bar{q}q} = \frac{m}{4m_\pi^2 F^2}\left[\langle\bar{q}q\rangle_\mu - \langle\bar{q}q\rangle_0\right] + 1$

Baryon density





Small and moderate densities

To summarise

- At small densities the data are well described by CHPT
- m_π from the fit and from the correlation function coincide
- Deviation from CHPT at moderate density
- Transition from dilute baryon gas to dense matter

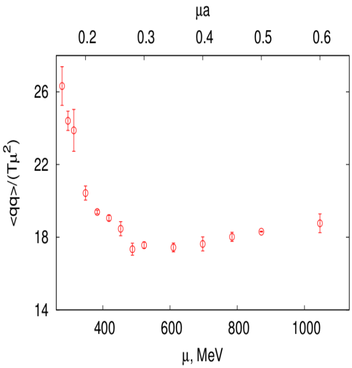
Large density

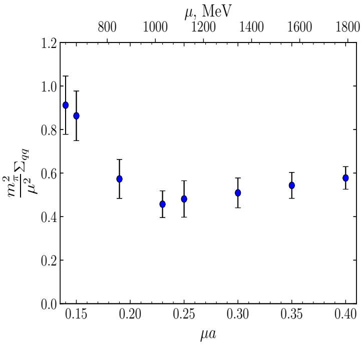
Phase diagram for $N_c \to \infty$

L. McLerran, R.D. Pisarski, Nucl.Phys. A796 (2007) 83-100

- Hadron phase $\mu < M_N/N_c$ ($p \sim O(1)$)
- Dilute baryon gas $\mu > M_N/N_c$ (width $\delta\mu \sim \frac{\Lambda_{QCD}}{N_c^2}$)
- Quarkyonic phase $\mu > \Lambda_{QCD}$ ($p \sim N_c$)
  - Degrees of freedom:
    - Baryons (on the surface)
    - Quarks (inside the Fermi sphere $|p| < \mu$)
  - No chiral symmetry breaking
  - The system is in confinement phase
- Deconfinement $\mu \gg \Lambda_{QCD}$ ($p \sim N_c^2$)

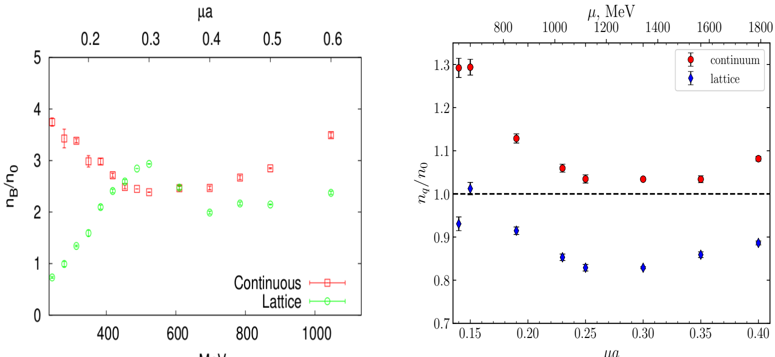
Diquark condensate





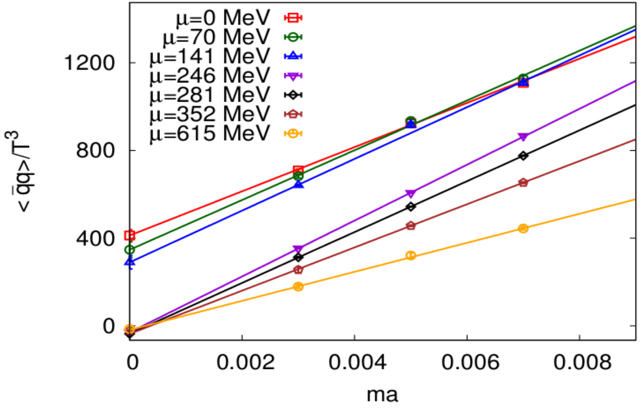
- Similar to Bardeen–Cooper–Schrieffer theory (BCS phase) $\mu > 800$ MeV, $\langle\psi\psi\rangle \sim \Delta(\mu)\mu^2$
- **Baryons (on the surface)**
- Degrees of freedom: Cooper pairs (diquarks qq or baryons)

Baryon density



- Free quarks $n_0 = N_f \times N_c \times (2s+1) \times \int \frac{d^3p}{(2\pi)^3}\theta(|p| - \mu) = \frac{4}{3\pi^2}\mu^3$
- **Quarks inside Fermi sphere**
- Quarks inside Fermi sphere dominate over the surface: $\frac{4}{3}\pi\mu^3 > 4\pi\mu^2\Lambda_{QCD} \Rightarrow \mu > 3\Lambda_{QCD}$
- We observe BEC-BCS transition
- Degrees of freedom: Quarks with the mass $\Delta(\mu)$

Chiral condensate (chiral limit $m \to 0$)



- No dynamical chiral symmetry breaking

Equation of state

- Pressure: $p(\mu) = \int_0^\mu d\xi n_q(\xi) + p(0)$
- Trace anomaly: $I(\mu) = \langle T_\mu^\mu \rangle = \epsilon(\mu) - 3p(\mu)$

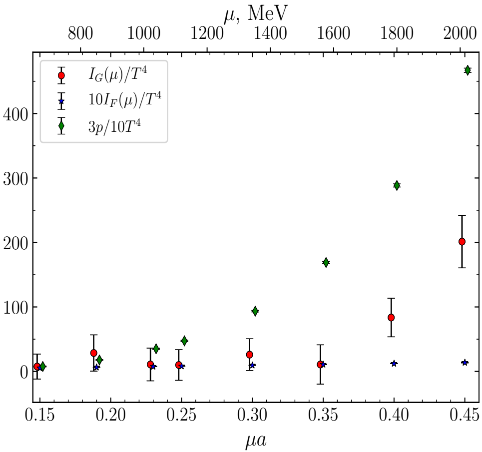
$$\frac{I(\mu)}{T^4} = \frac{I_G(\mu)}{T^4} + \frac{I_F(\mu)}{T^4}$$

$$\frac{I_G(\mu)}{T^4} = N_t^4 \beta(g) \left[\langle S_G \rangle_\mu - \langle S_G \rangle_0 \right]$$

$$\frac{I_F(\mu)}{T^4} = -N_t^4 \gamma(g) ma \left[\langle \bar{q}q \rangle_\mu - \langle \bar{q}q \rangle_0 \right]$$

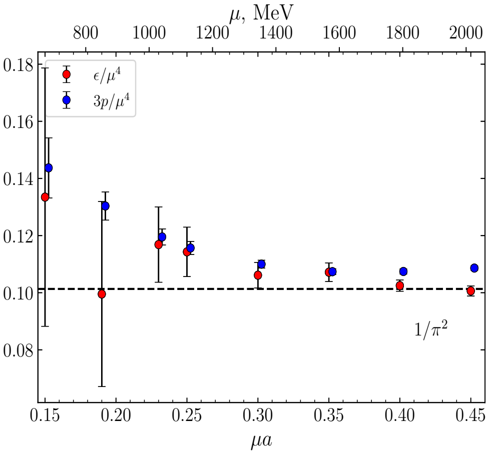
- Energy density: $\epsilon = I + 3p$
- Entropy: $s = \frac{\epsilon + p - \mu n_q}{T} \simeq 0$

Equation of state: trace anomaly



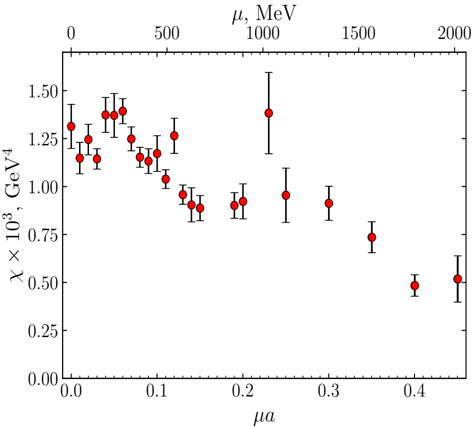
- Trace anomaly gives small contribution to EoS

Equation of state: energy density and pressure



- In BCS phase EoS of relativistic quarks $\epsilon \simeq 3p$
- $\Delta(\mu)$ is small in this region?

Topological susceptibility



- Topological susceptibility decreases with μ

Properties of Quarkyonic phase:

- Baryons (on the surface)

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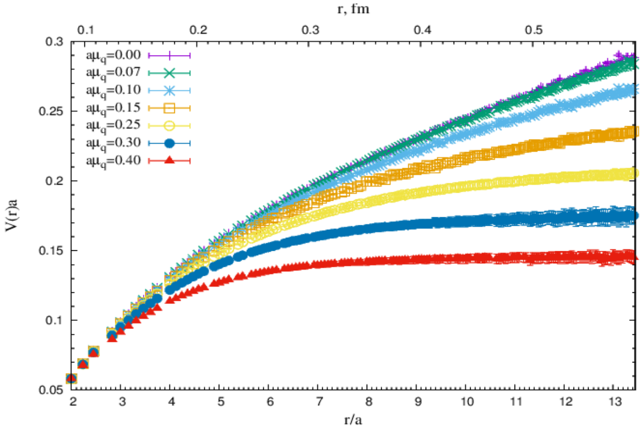
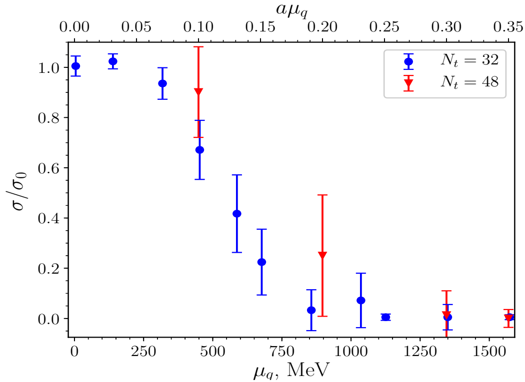
What about confinement in cold dense quark matter?

Potential between static quark-antiquark pair



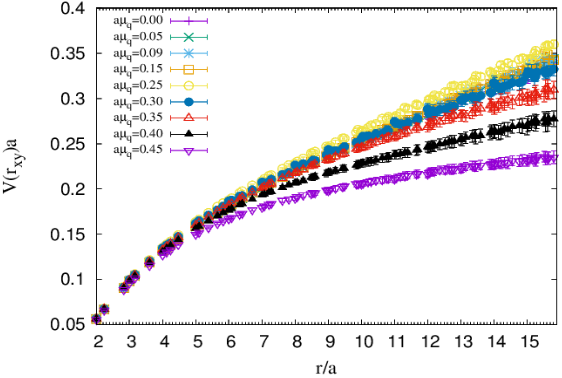
- We observe deconfinement in dense medium!

String tension

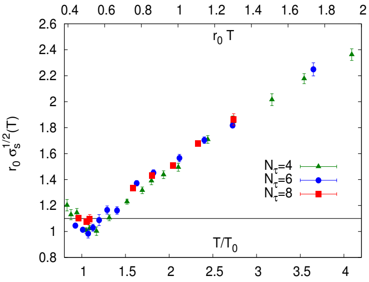


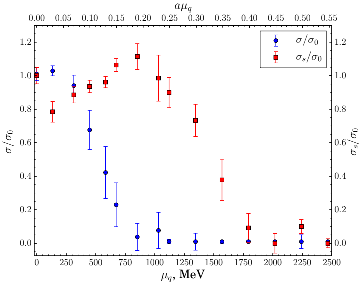
- Good fit by Cornell potential: $V(r) = A + \frac{B}{r} + \sigma r$
  $\mu \leq 1100$ MeV
- Good fit by Debye potential: $V(r) = A + \frac{B}{r} e^{-m_D r}$
  $\mu \geq 850$ MeV
- Confinement/deconfiniment transition in $\mu \in (850, 1100)$ MeV

Spatial potential $V(r)$



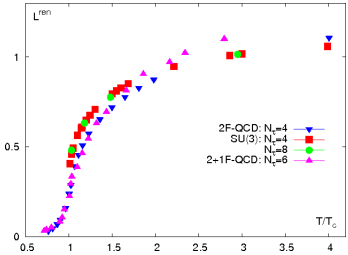
Spatial string tension

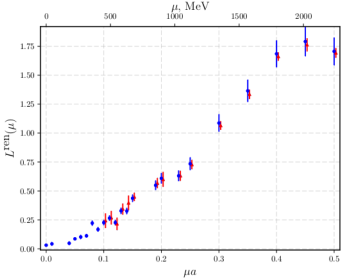




- Deconfinement at $\mu > 900 - 1100$ MeV?
- Spatial string tension disappears at $\mu \geq 1800$ MeV ($a\mu > 0.4$)
- Different behaviour of spatial string tension at finite temperature and finite density

Polyakov lines at finite temperature and density





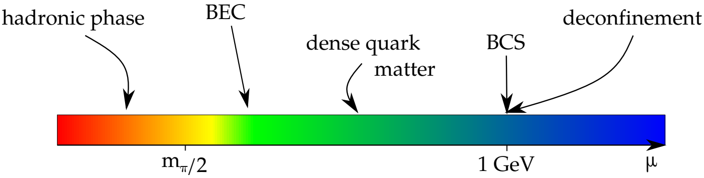
Polyakov lines and confinement/deconfinement transition

- Rapid transition at finite T
- Smooth transition at finite μ
- Unclear physics at $\mu > 2000$ MeV

Deconfinement in dense QC_2D matter

- The mechanism is not clear
- How color charges can be screened at finite density?
- Artifact of rooted staggered fermions?
- Finite temperature effect?
- We are carrying out additional study at few lattice spacings

Conclusion



- We observe handronic phase, BEC of diquarks and BCS phase
- EoS of dense QC_2D was calculated
- Topological susceptibility decreases with μ
- Deconfinement takes place at $\mu \sim 1$ GeV
- Vanishing of spatial string tension $\mu \sim 1.8$ GeV
- Deconfinement transition requires futher study