Deconfinement and CP-breaking at $\theta=\pi$ in a softly-broken $\mathcal{N}=1$ SYM

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[Chen, Fukushima, Nishimura, Tanizaki, 2020]

- **Dashen's Phenomenon**

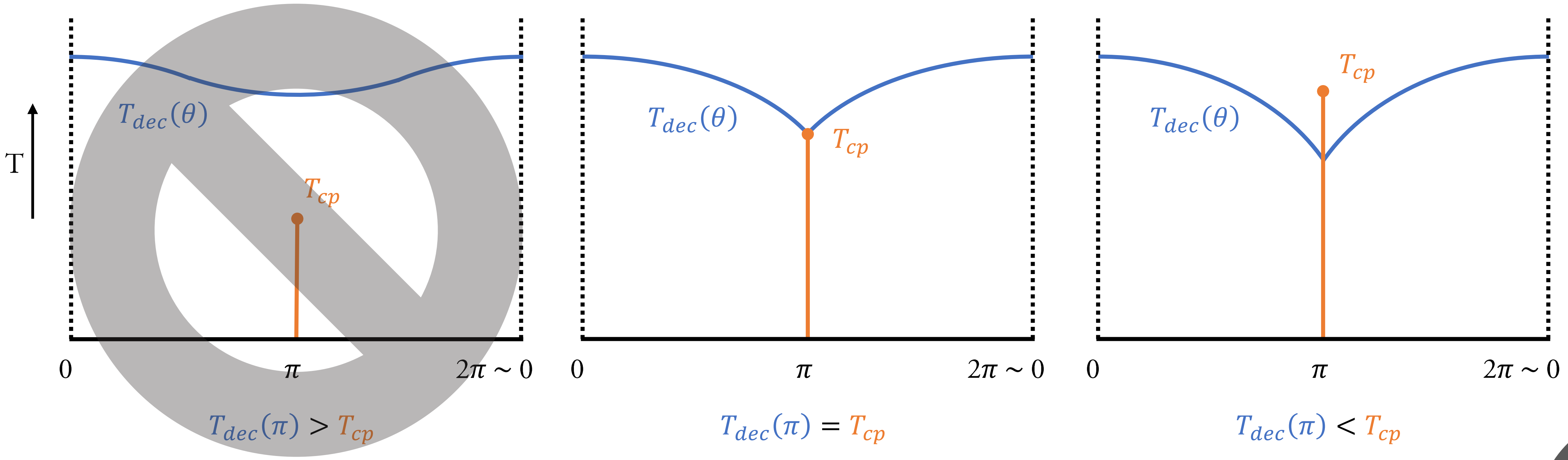
  CP symmetry is spontaneously broken @ $\theta=\pi$

- **Anomaly Matching** (pure YM)

  Mixed 't Hooft anomaly: center & CP symmetry @ $\theta=\pi$

  [Gaiotto, Kapustin, Komargodski, Seiberg, 2017]

- **Finite Temperature: Relation between Deconfinement and CP-restoration**



- **YM with an Adjoint Fermion**

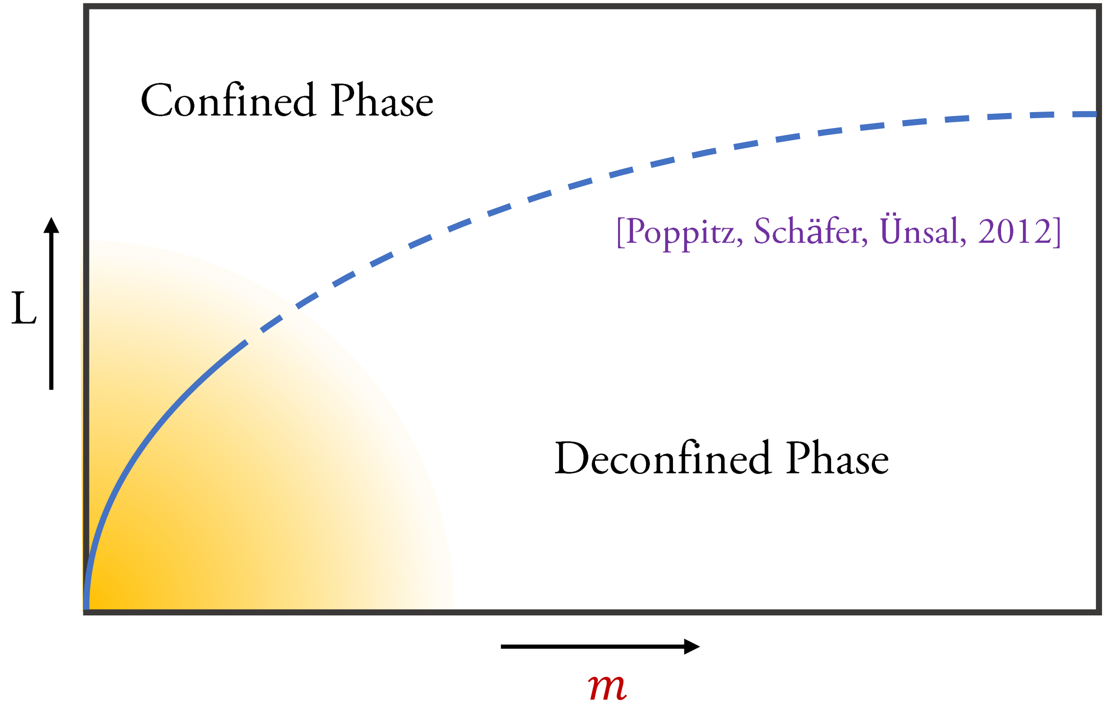
$$\mathcal{S} = \frac{1}{g^2}\int \mathrm{tr}\, F \wedge \star F - \frac{i\theta}{8\pi^2}\int \mathrm{tr}\, F \wedge F + \frac{2i}{g^2}\int d^4x\ \mathrm{tr}\,\bar{\lambda}\bar{\sigma}^{\mu} D_{\mu}\lambda - \frac{m}{g^2}\int d^4x\,(\mathrm{tr}\,\lambda\lambda + c.c.)$$

- **Periodic Boundary Condition on $\mathbb{R}^3 \times S^1$: A Non-thermal YM**

$$\begin{cases} m = 0 & \text{Non-thermal } \mathcal{N}=1 \text{ SYM} \\ m = \infty & \text{Thermal pure YM} \end{cases}$$

- **Calculable Region**

$$\begin{cases} \text{Weakly-coupled: } L \ll \Lambda^{-1} \\ \text{Softly-broken: } m \ll \Lambda \end{cases}$$



§ I : IR Effective D.o.F. (ϕ, σ)

- **Gauge Fixing: Unitary Gauge**

$$A_4(x^i, x^4) = \frac{1}{L}\phi(x^i)\tau^3$$

Polyakov loop
$$\begin{cases} \Omega \equiv \mathcal{P}\exp\left\{i\oint_0^L A_4\, dx^4\right\} = e^{i\phi\tau^3} \\ \mathrm{tr}\,\Omega = 2\cos\frac{\phi}{2} \end{cases}$$

$\# \ \tau^a \equiv \frac{\sigma^a}{2}$

- **Abelianization**

Almost confinement

$\Downarrow$

$\langle \mathrm{tr}\,\Omega \rangle \sim 0$

$\Downarrow$

$\langle \phi \rangle \sim \pi + 2\pi\mathbb{Z}$

Massless if $m = 0$

$\Uparrow$

$$\begin{cases} A_i = A_i^1\tau^1 + A_i^2\tau^2 + A_i\tau^3 \\ \lambda = \lambda^1\tau^1 + \lambda^2\tau^2 + \lambda\tau^3 \end{cases}$$

$\Downarrow$

Acquires a mass $\sim \frac{\langle\phi\rangle + 2\pi n}{L} \in \frac{(1+2\mathbb{Z})\pi}{L}$

$$S_{boson} = \int_{\mathbb{R}^3\times S^1} \left\{\frac{1}{g^2}\mathrm{tr}\, F\wedge\star F - \frac{i\theta}{8\pi^2}\mathrm{tr}\, F\wedge F\right\}$$

$\Downarrow$ Integrate out massive modes

$$S_{boson} = \int_{\mathbb{R}^3} \left\{\frac{1}{2Lg^2}|d\phi|^2 + \frac{L}{2g^2}|dA|^2 - \frac{i\theta}{8\pi^2} dA\wedge d\phi\right\}$$

- **Duality Transformation on $\mathbb{R}^3$**

U(1) gauge field $A \rightarrow$ U(1) compact scalar σ

$$S_{boson} = \int_{\mathbb{R}^3} \left\{\frac{1}{2Lg^2}|d\phi|^2 + \frac{g^2}{8L}\left|\frac{1}{2\pi}d\sigma + \frac{\theta}{4\pi^2}d\phi\right|^2\right\}$$

§ II : (ϕ, σ) As Order Parameters

- **Redundancy and Further Gauge Fixing**

$$\begin{cases} (\phi,\sigma) \simeq (\phi + 4\pi, \sigma) \\ (\phi,\sigma) \simeq (\phi, \sigma + 2\pi) \\ W_{\mathfrak{su}(2)}: (\phi,\sigma) \mapsto (-\phi, -\sigma) \end{cases} \implies (\phi,\sigma) \in \frac{\frac{\mathbb{R}}{4\pi} \times \frac{\mathbb{R}}{2\pi}}{W_{\mathfrak{su}(2)}} \xrightarrow{\text{Further gauge fixing}} \begin{cases} \phi \in [0,2\pi] \xrightarrow{\text{Almost confinement}} \phi \sim \pi \\ \sigma \simeq \sigma + 2\pi \end{cases}$$

$\# \ \sigma^1 \tau^3 \sigma^1 = -\tau^3$

- **Symmetry**

$$\begin{cases} \Omega \xrightarrow{\text{center}} -\Omega \\ \text{CP reverses the orientation} \end{cases} \implies \begin{array}{l} (\phi,\sigma) \xrightarrow{\text{center}} (\phi + 2\pi, \sigma) \\ \begin{cases} \theta = 0: \ (\phi,\sigma) \xrightarrow{\text{CP}} (\phi, -\sigma) \\ \theta = \pi: \ (\phi,\sigma) \xrightarrow{\text{CP}} (\phi, -\sigma - \phi) \end{cases} \end{array} \xrightarrow{\text{Further gauge fixing}} \begin{array}{l} (\phi,\sigma) \xrightarrow{\text{center}} (2\pi - \phi, -\sigma) \\ \begin{cases} \theta = 0: \ (\phi,\sigma) \xrightarrow{\text{CP}} (\phi, -\sigma) \\ \theta = \pi: \ (\phi,\sigma) \xrightarrow{\text{CP}} (\phi, -\sigma - \phi) \end{cases} \end{array}$$

- **Physical Interpretations**

The (ϕ, σ) generates the Wilson-'t Hooft loop along S^1. We have (very roughly) the Wilson loop $\sim e^{i\phi}$ and the 't Hooft loop $\sim e^{i\sigma}$.

§ III : IR Effective Potential of (ϕ, σ)

- **The Perturbative: Negligible**

When $m = 0$, SUSY **non-renormalizable theorems** guarantees no contribution to the effective potential. cf. [Intriligator, Seiberg, 1996]
When $m > 0$, explicit computations show they are of much higher order of the coupling g^2. cf. [Chen, Fukushima, Nishimura, Tanizaki, 2020]

- **The Nonperturbative: Monopole-instanton Counting**

2 fermionic zero modes [Nye, Singer, 2000] $\mathcal{V}_1 \sim \lambda\lambda \exp\left\{i\left(\sigma + \frac{\theta}{2\pi}\phi\right) - \frac{4\pi}{g^2}\phi\right\}$ $\mathcal{V}_0 \sim \lambda\lambda \exp\left\{i\theta - i\left(\sigma + \frac{\theta}{2\pi}\phi\right) + \frac{4\pi}{g^2}\phi\right\}$

When $m = 0$, fermion-boson vertex $\xrightarrow{\text{SUSY}}$ boson vertex = effective potential [SUSY version of **Polyakov mechanism**]. [Polyakov, 1977]
When $m > 0$, zero modes are lifted directly leading to effective potential. cf. [Poppitz, Schäfer, Ünsal, 2012]

$$\frac{\mathcal{V}(\phi,\sigma)}{V_0} = 4\cosh(2\phi') - 4\cos(2\sigma') - \gamma\left[\left(1 + \frac{g^2}{4\pi^2}\phi'\right)e^{-\phi'}\cos\left(\sigma' + \frac{\theta}{2}\right) + \left(1 - \frac{g^2}{4\pi^2}\phi'\right)e^{\phi'}\cos\left(\sigma' - \frac{\theta}{2}\right)\right]$$

$$\begin{cases} \phi' \equiv \frac{4\pi}{g^2}(\phi - \pi) \\ \sigma' \equiv \sigma + \frac{\theta}{2\pi}(\phi - \pi) \end{cases} \qquad V_0 \equiv \frac{9}{4\pi^2}\frac{L^3\Lambda^6}{g^2} \qquad \gamma \equiv \frac{8\pi^2}{3}\frac{m}{L^2\Lambda^3}$$

: A proxy of **temperature** in the calculable region

§ I : Gauge Groups SU(2)

Range $\begin{cases} \phi' \in \mathbb{R} \\ \sigma' \simeq \sigma' + 2\pi \end{cases}$

Symmetry $\begin{cases} (\phi',\sigma') \xrightarrow{\text{center}} (-\phi',-\sigma') \\ \begin{cases} \theta = 0: \ (\phi',\sigma') \xrightarrow{\text{CP}} (\phi',-\sigma') \\ \theta = \pi: \ (\phi',\sigma') \xrightarrow{\text{CP}} (\phi',-\sigma'-\pi) \end{cases} \end{cases}$

$$\gamma \equiv \frac{8\pi^2}{3}\frac{m}{L^2\Lambda^3}$$

$$\frac{\mathcal{V}(\phi',\sigma')}{V_0} = 4\cosh(2\phi') - 4\cos(2\sigma') - \gamma\left[\left(1+\frac{g^2}{4\pi^2}\phi'\right)e^{-\phi'}\cos\left(\sigma'+\frac{\theta}{2}\right) + \left(1-\frac{g^2}{4\pi^2}\phi'\right)e^{\phi'}\cos\left(\sigma'-\frac{\theta}{2}\right)\right]$$

- $\gamma = 0$

$$\begin{cases} \langle(\phi',\sigma')\rangle_1 = (0,0) \\ \langle(\phi',\sigma')\rangle_2 = (0,\pi) \end{cases}$$

$\Downarrow$

Center-symmetric

- $0 < \gamma < \gamma_{dec}$

$$\begin{cases} \dfrac{\mathcal{V}(0,0)}{V_0} = -4 - 2\gamma\cos\dfrac{\theta}{2} \\ \dfrac{\mathcal{V}(0,\pi)}{V_0} = -4 + 2\gamma\cos\dfrac{\theta}{2} \end{cases} \Longrightarrow \begin{cases} 0 \le \theta \le \pi: \ (0,0) \text{ is the true vacuum} \\ \pi \le \theta < 2\pi: \ (0,\pi) \text{ is the true vacuum} \end{cases}$$

$\Uparrow$

They are the only **center-symmetric** points.

$\Downarrow$

CP-breaking @ $\theta = \pi$

γ ~ temperature

§ I : Gauge Groups SU(2)

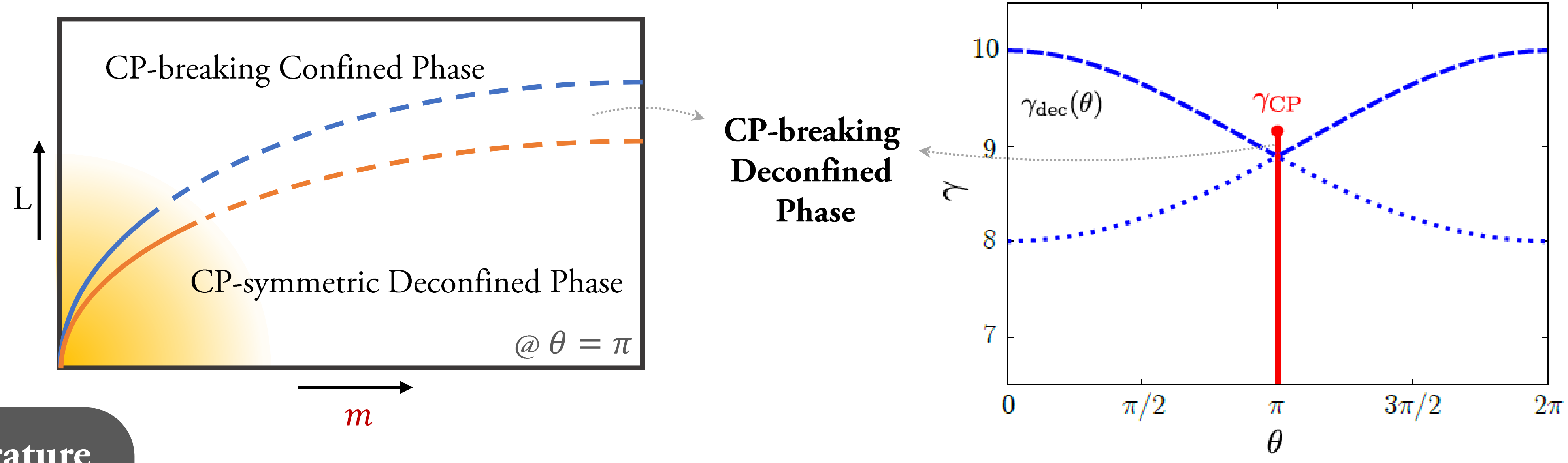
- 2nd-order Deconfinement phase transition [Poppitz, Schäfer, Ünsal, 2012]

$$\begin{cases} 0 \le \theta \le \pi: & \gamma_{dec}(\theta) = 8\left[1 + \frac{g^2}{4\pi^2}\left(1 + \cos\frac{\theta}{2}\right)\right] + \mathcal{O}(g^4) \\ \pi \le \theta < 2\pi: & \gamma_{dec}(\theta) = 8\left[1 + \frac{g^2}{4\pi^2}\left(1 - \cos\frac{\theta}{2}\right)\right] + \mathcal{O}(g^4) \end{cases}$$

$$\gamma_{dec}(\pi) = 8\left[1 + \frac{g^2}{4\pi^2}\right] + \mathcal{O}(g^4)$$

- CP-symmetry restores **strictly later than** Deconfinement phase transition [Chen, Fukushima, Nishimura, Tanizaki, 2020]

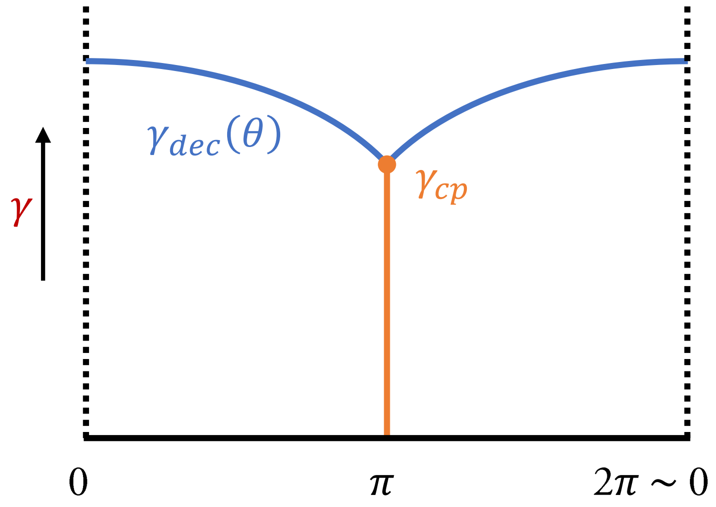
$$\gamma_{cp} = 8\left[1 + \left(\frac{1}{2} + \frac{3}{2\sqrt{2}}\operatorname{arcsinh} 1\right)\frac{g^2}{4\pi^2}\right] + \mathcal{O}(g^4) \approx 8\left[1 + 1.43\frac{g^2}{4\pi^2}\right] + \mathcal{O}(g^4) \quad \Longrightarrow \quad \gamma_{dec}(\pi) < \gamma_{cp}$$



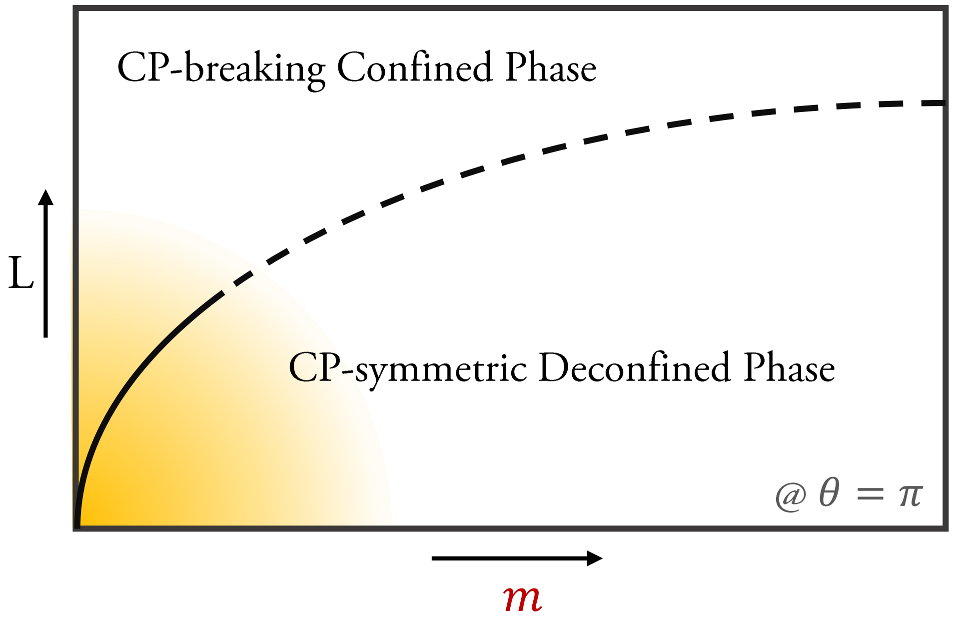
§ II : Gauge Groups Other Than SU(2)

- 1st-order Deconfinement phase transition [Anber, Poppitz, Teeple, 2014]
- CP-symmetry restores **at the same time** as Deconfinement phase transition [Chen, Fukushima, Nishimura, Tanizaki, 2020]

We evaluate the gauge groups with the rank ≤ 10 and the large N limit for SU(N).



$\gamma_{dec}(\pi) = \gamma_{cp}$



$\gamma \sim$ **temperature**

Conclusions

§ I : About Non-thermal Softly-broken $\mathcal{N}=1$ SYM @ $\theta = \pi$

- The prophecy of 't Hooft anomaly matching is verified.
- For most gauge groups, the CP-restoration and Deconfinement occur synchronously: $\gamma_{dec}(\pi) = \gamma_{cp}$.
- For SU(2), the CP-restoration is strictly posterior to Deconfinement: $\gamma_{dec}(\pi) < \gamma_{cp}$. A CP-breaking deconfined phase is observed.

§ II : About Thermal Pure YM @ $\theta = \pi$

- We optimistically anticipate that, for most gauge groups, the CP-restoration and Deconfinement occur synchronously: $T_{dec}(\pi) = T_{cp}$.
- We are more circumspect about SU(2). Other more peculiar scenarios might occur, such as no confinement but gapless, or the oblique confinement etc.

Thank You for Listening!

§ A : Mixed ‘t Hooft Anomaly between Center and CP @ θ=π

[Gaiotto, Kapustin, Komargodski, Seiberg, 2017]

- Center symmetry $\mathcal{Z}(G)$ is a 1-form symmetry. [Gaiotto, Kapustin, Seiberg, Willett, 2014]

- Couple TQFT to gauge theory. [Kapustin, Seiberg, 2014]

Turn on a background $\mathcal{Z}(G)^{[1]}$ gauge field $B^{[2]} \in H^4\big(\mathfrak{B}^2\mathcal{Z}(G), \mathrm{U}(1)\big)$.

$$\Downarrow$$

Couple $B^{[2]}$ to G gauge theory. # $H^4\big(\mathfrak{B}^2\mathcal{Z}(G), \mathrm{U}(1)\big) = H^4\big(\mathfrak{B}G/\mathcal{Z}(G), \mathrm{U}(1)\big)$.

$$\Downarrow$$

The instanton number $Q \equiv \frac{1}{8\pi^2}\int \mathrm{tr}\, F \wedge F$ is fractionalized to $Q[B]$.

G	$\mathcal{Z}(G)$	$Q[B]$ mod 1
$\mathrm{SU}(N)$	$\mathbb{Z}_N$	$1/N$
$\mathrm{Spin}(2k+1)$	$\mathbb{Z}_2$	0
$\mathrm{Spin}(4k)$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$1/2$
$\mathrm{Spin}(4k+2)$	$\mathbb{Z}_4$	$1/4$
$\mathrm{Sp}(2k)$	$\mathbb{Z}_2$	0
$\mathrm{Sp}(2k+1)$	$\mathbb{Z}_2$	$1/2$
E_6	$\mathbb{Z}_3$	$2/3$
E_7	$\mathbb{Z}_2$	$1/2$
$E_8,\ F_4,\ G_2$	1	0

[Aharony, Seiberg, Tachikawa, 2013; Witten, 2002]

- $\theta \xrightarrow{\mathrm{CP}} -\theta$, the CP symmetry @ θ=π requires the 2π-periodicity of θ.

G is supposed to be Connected, Compact, Simply-connected.

§ B : Generic Gauge Group G with Simple $\mathfrak{g}$

- **Gauge Fixing** $A_4(x^i, x^4) = \frac{1}{L}\phi(x^i)\cdot \boldsymbol{h}$ $\quad \boldsymbol{h} = h^{a=1,2,\cdots,r}$ is an orthonormal basis of a chosen Cartan subalgebra of $\mathfrak{g}$.

r is the rank of G.

- **Order Parameters**

$$(\phi, \sigma) \in \frac{\frac{\mathbb{R}^r}{2\pi\Lambda_r^\vee} \times \frac{\mathbb{R}^r}{2\pi\Lambda_w}}{W_\mathfrak{g}}$$

$$(\phi, \sigma) \xrightarrow{\text{center}} (\phi + 2\pi\boldsymbol{\mu}_c^\vee, \sigma) \qquad \boldsymbol{\mu}_c^\vee \in \Lambda_w^\vee/\Lambda_r^\vee \cong \mathcal{Z}(G)$$

$$\begin{cases} \theta = 0: \ (\phi, \sigma) \xrightarrow{\text{CP}} (\phi, -\sigma) \\ \theta = \pi: \ (\phi, \sigma) \xrightarrow{\text{CP}} (\phi, -\sigma - \phi) \end{cases} \qquad \begin{cases} \text{Center} \subseteq \text{Sym}\{M_{i=0,1,\cdots,r}\} \\ M_i \xrightarrow{\text{CP}} M_i^* \end{cases}$$

- **Effective Potential**

$$\boldsymbol{z} \equiv i\left(\sigma + \frac{\theta}{2\pi}\phi\right) - \frac{4\pi}{g^2}\phi$$

$$\begin{cases} M_i \equiv \exp\left\{\boldsymbol{\alpha}_i^\vee \cdot \boldsymbol{z} + \frac{8\pi^2}{c_2 g^2}\right\} \ \text{for } i = 1,2,\cdots,r \\ M_0 \equiv \exp\left\{\boldsymbol{\alpha}_0^\vee \cdot \boldsymbol{z} + \frac{8\pi^2(1-c_2)}{c_2 g^2} + i\theta\right\} \end{cases}$$

$\boldsymbol{\alpha}_i^\vee = \frac{2\boldsymbol{\alpha}_i}{|\boldsymbol{\alpha}_i|^2} \in \Lambda_r^\vee$, $\boldsymbol{\alpha}_i \in \Lambda_r$ are the simple roots.

$\boldsymbol{\alpha}_0^\vee = \boldsymbol{\alpha}_0$ is the affine root (the lowest root).

$|\boldsymbol{\alpha}_0|^2 = 2$.

$$\boxed{\frac{\mathcal{V}(\phi, \sigma)}{V_0} = \sum_{i=0}^{r}\sum_{j=0}^{r} M_i^*(\boldsymbol{\alpha}_i\cdot\boldsymbol{\alpha}_j)M_j - \gamma\sum_{i=0}^{r}\frac{M_i^* + M_i}{|\boldsymbol{\alpha}_i|^2}\left(1 - \frac{c_2 g^2}{8\pi^2}\ln|M_i|\right)}$$

$$V_0 \equiv \frac{9c_2^2}{16\pi^2}\frac{L^3\Lambda^6}{g^2} \qquad \boxed{\gamma \equiv \frac{32\pi^2}{3c_2^2}\frac{m}{L^2\Lambda^3}}$$

c_2 is the value of quadratic Casimir in the adjoint rep.

G is supposed to be Connected, Compact, Simply-connected.