Finite-Density Massless Two-Color QCD at Isospin Roberge-Weiss Point and 't Hooft Anomaly

Takuya Furusawa (TITech)

Furusawa, Tanizaki, Itou, PRResearch 2, 033253 (2020)

YITP workshop (online), Nov. 2020

Outline

1. Introduction
2. Symmetries & anomaly at isospin RW point NEW
3. Applications NEW
   Finite-T&-μ phase diagram, Similarity to AF
4. Summary & discussion

Massless 2-color QCD

SU(2) gauge theory w/ 2 massless Dirac fermions (fund. rep.)

$$\frac{1}{2g^2}\mathrm{tr}(F_{\mu\nu}[a])^2 + \bar{u}\gamma^\mu(\partial_\mu + ia_\mu)u + \bar{d}\gamma^\mu(\partial_\mu + ia_\mu)d$$

Good toy model for 3-color QCD at finite-T &-μ

No sign problem at finite density

⇒ Lattice simulation is available.

Nakamura (1984), Hands, Kogut, Lombardo, Morrison (1999),
Kogut, Stephanov, Toublan (1999), Kogut, Stephanov, Toublan, Verbaarschot, Zhitnitsky (2000)

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More on exact properties? ⇒ ’t Hooft anomaly

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’t Hooft Anomaly

G: global symmetry, A: background gauge field

$$Z[A+\delta_\theta A] = Z[A]$$

’t Hooft (1980), Frishman, Schwimmer, Banks, Yankielowicz (1981), Coleman, Grossman (1982)

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$$Z[A+\delta_\theta A] = Z[A]\underline{e^{i\mathscr{A}[\theta,A]}}$$

't Hooft anomaly

The 't Hooft anomaly is RG-invariant.

Anomaly at UV ⇒ The same anomaly at IR

Must show nontrivial IR behaviors!

○ SSB, CFT, Topological order, ✕ unique gapped

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Must show nontrivial IR behaviors!

Classic: Chiral symmetry (T=0)

Recent: Discrete & higher-form symmetries (T>=0)

’t Hooft (1980), Frishman, Schwimmer, Banks, Yankielowicz (1981), Coleman, Grossman (1982)

New anomalies at T>0

Center symmetry is key for anomaly at finite T.

e.g., Pure Yang-Mills gauge theory w/ theta term

Gaiotto, Kapustin, Komargodski, Seiberg (2017)

Anomaly btw. center and CP symmetries

→ Constraint on finite-T phase diagram

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→ Constraint on finite-T phase diagram

Fund. matters (including 2cQCD) break the center.

⇒ Twisted b.c. of matter fields

e.g., Zn QCD, QCD w/ imaginary μ @ RW pt., ⋯

Shimizu, Yonekura (2018), Tanizaki, Kikuchi, Misumi, Sakai (2018), Yonekura (2019), ⋯

Our model

2cQCD w/ imaginary isospin chemical potential

- ✔ Center symmetry at a special point
- ✔ No sign problem

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$$\left(\gamma^{\nu}\mathrm{D}_{\nu}(\mu) = \gamma^{\nu}(\partial_{\nu} + ia_{\nu}) + \mu\gamma^{0}\right)$$

Pseudo-reality of Dirac op. (usual 2cQCD):

$$\det\left[\gamma^{\nu}\mathrm{D}_{\nu}(\mu)\right] = \det\left[\gamma^{\nu}\mathrm{D}_{\nu}(\mu)\right]^{*}$$

$$\longrightarrow \quad \det\left[\gamma^{\nu}\mathrm{D}_{\nu}(\mu)\right] \times \det\left[\gamma^{\nu}\mathrm{D}_{\nu}(\mu)\right] \geq 0$$

up quark down quark

Hands, Montvay, Morrison, Oevers, Scorzato, Skullerud (200

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$$\left(\gamma^\nu \mathrm{D}_\nu(\mu) = \gamma^\nu(\partial_\nu + i a_\nu) + \mu\gamma^0\right)$$

Pseudo-reality of Dirac op. (our model):

$$\det\left[\gamma^\nu \mathrm{D}_\nu(\mu + i\mu_I)\right] = \det\left[\gamma^\nu \mathrm{D}_\nu(\mu - i\mu_I)\right]^*$$

$$\longrightarrow \quad \det\left[\gamma^\nu \mathrm{D}_\nu(\mu + i\mu_I)\right] \times \det\left[\gamma^\nu \mathrm{D}_\nu(\mu - i\mu_I)\right] \geq 0$$

up quark　　　　down quark

Furusawa, Tanizaki, Itou (2020)

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$$\left(\gamma^\nu \mathrm{D}_\nu(\mu) = \gamma^\nu(\partial_\nu + i a_\nu) + \mu\gamma^0\right)$$

In the following, we study

✔ symmetries and anomaly in this model

✔ anomaly constraint on finite (T, μ) phase diagram

✔ Similarity to quanrtum magnets

(2020)

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Imaginary isospin chemical potential

Fermion kinetic terms:

$$\mu_I = \theta_I / \beta$$

$$\bar{u}\gamma^\mu(\partial_\mu + ia_\mu)u + \bar{d}\gamma^\mu(\partial_\mu + ia_\mu)d + \underline{i\mu_I(\bar{u}\gamma^0 u - \bar{d}\gamma^0 d)}$$

Boundary condition:

$$u(\tau + \beta) = -u(\tau), \qquad d(\tau + \beta) = -d(\tau),$$

Furusawa, Tanizaki, Itou, PRResearch 2, 033253 (2020)

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Absorbed into B.C.

Boundary condition:

$$u(\tau+\beta) = -e^{i\theta_I}u(\tau), \qquad d(\tau+\beta) = -e^{-i\theta_I}d(\tau),$$

(Field redefinition:

$$u(\tau) \to e^{i\theta_I\tau/\beta}u(\tau) \quad d(\tau) \to e^{-i\theta_I\tau/\beta}d(\tau)$$

)

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$$u(\tau + \beta) = -\underline{e^{i\theta_I}}u(\tau), \qquad d(\tau + \beta) = -\underline{e^{-i\theta_I}}d(\tau),$$

$\Rightarrow \theta_I$ is periodic.

SU(2) gauge invariance

$$\theta_I \sim \theta_I \underline{+ \pi} \qquad u(x) \sim -u(x),\ d(x) \sim -d(x),$$

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Isospin Roberge-Weiss Point

$\theta_I = \pi/2$ is special (isospin RW point).

The boundary condition is invariant under $u(x) \leftrightarrow d(x)$ up to SU(2) gauge transformation.

$$\theta_I = \pi/2 \to -\pi/2 \sim \pi/2 \qquad \left(\theta_I \sim \theta_I + \pi\right)$$

Emergent symmetry [$(\mathbb{Z}_2)_{\text{center}}$] at the isospin RW point

$$u(x) \leftrightarrow d(x)$$

Gauging flavor symmetry

To find anomaly, let's gauge flavor subgroup at T, μ, & $\theta_I = \pi/2$:

[See our paper for complete discussion.]

$$\mathrm{U}(1)_{\mathrm{B}} \times \mathrm{U}(1)_{\mathrm{L},3} \subset G_{\mu,\theta_I}$$

$\mathrm{U}(1)_{\mathrm{B}}$: $\mathrm{U}(1)$ baryon symmetry

$\mathrm{U}(1)_{\mathrm{L},3}$: left-handed isospin $\mathrm{U}(1)$ symmetry

$$u_L \to e^{i\lambda_3/2} u_L,\ d_L \to e^{-i\lambda_3/2} d_L,$$

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- $\mathrm{U}(1)_\mathrm{B}$: $\mathrm{U}(1)$ baryon symmetry $\Leftarrow$ Gauged by A_B
- $\mathrm{U}(1)_{\mathrm{L},3}$: left-handed isospin $\mathrm{U}(1)$ symmetry $\Leftarrow$ Gauged by $A_{\mathrm{L},3}$

A_B & $A_{\mathrm{L},3}$ are τ-independent & have only spatial components.

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’t Hooft anomaly at isospin RW point

A_{B} & $A_{\mathrm{L,3}}$ violate $(\mathbb{Z}_2)_{\mathrm{center}}$ symmetry:

$$Z_{\mathrm{QC_2D}}[A_B, A_{L,3}] = Z^{\mathrm{Sym.}}_{\mathrm{QC_2D}}[A_B, A_{L,3}] \leftarrow [(\mathbb{Z}_2)_{\mathrm{center}} \text{ is explicit.}]$$

μ_I in action ↑ μ_I in boundary condition

$$\begin{cases} u(\tau) \to e^{i\theta_I \tau/\beta} u(\tau) \\ d(\tau) \to e^{-i\theta_I \tau/\beta} d(\tau) \end{cases}$$

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$$Z_{\rm QC_2D}[A_B, A_{L,3}] = Z^{\rm Sym.}_{\rm QC_2D}[A_B, A_{L,3}] \times \exp\left[-\frac{i}{4\pi}\int A_B \wedge dA_{\rm L,3}\right]$$

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Redefinition generates additional phase!

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't Hooft anomaly at isospin RW point

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$(\mathbb{Z}_2)_{\mathrm{center}}$ w/ background gauge fields:

$$Z_{\mathrm{QC_2D}}[A_B, A_{L,3}] \to Z_{\mathrm{QC_2D}}[A_B, A_{L,3}] \times \exp\left[\frac{i}{2\pi}\int A_B \wedge dA_{\mathrm{L},3}\right]$$

Mixed 't Hooft anomaly btw. $(\mathbb{Z}_2)_{\mathrm{center}} \ltimes \mathrm{U}(1)_{\mathrm{B}} \times \mathrm{U}(1)_{\mathrm{L},3}$

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’t Hooft anomaly matching

’t Hooft anomaly is invariant under RG flow.

⇒2cQCD must be nontrivial at IR:

○ SSB, CFT, Topological order, ✕ unique gapped

’t Hooft (1980), Frishman, Schwimmer, Banks, Yankielowicz (1981), Coleman, Grossman (1982)

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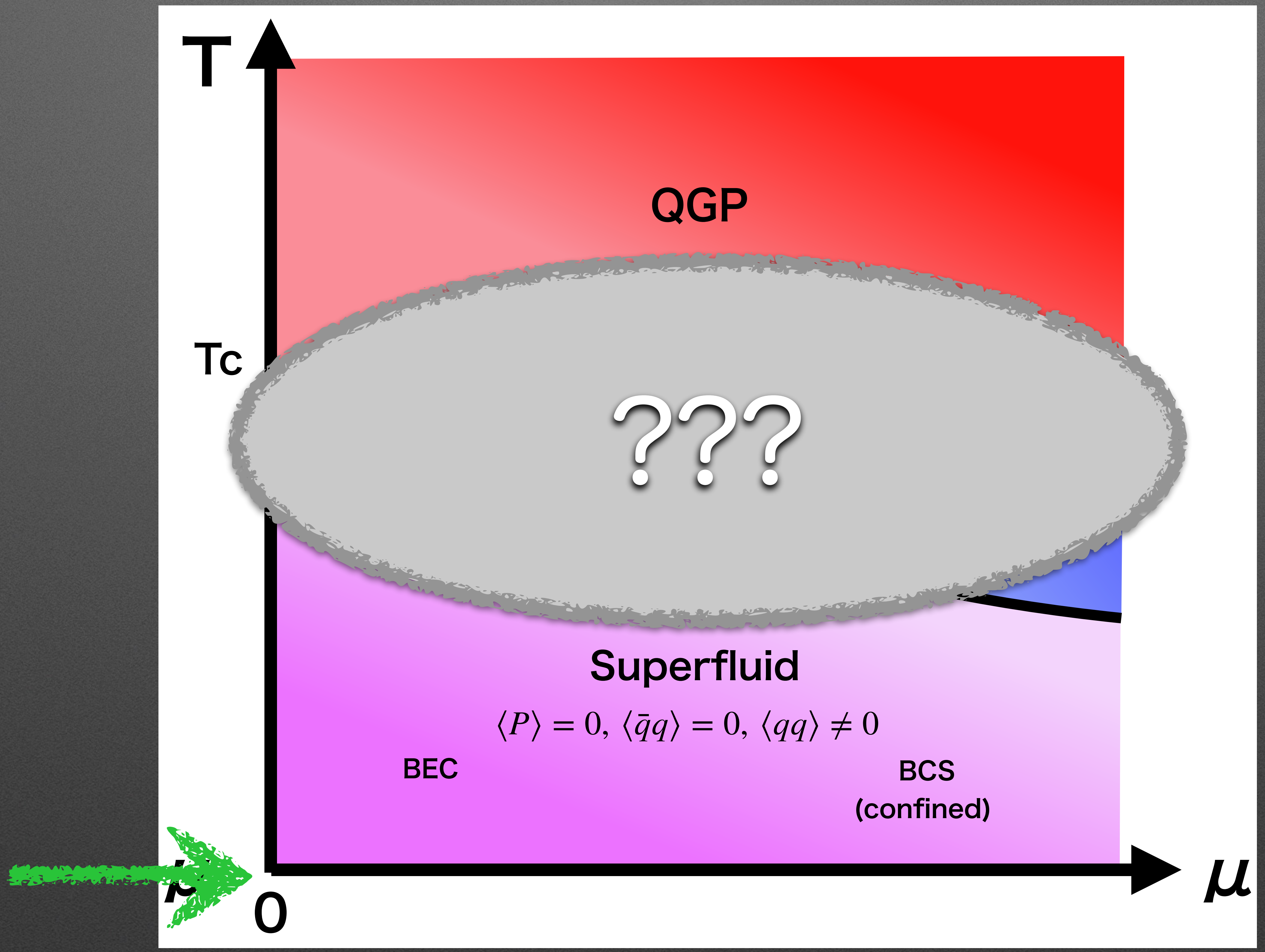
One of $(\mathbb{Z}_2)_{\mathrm{center}} \ltimes U(1)_B \times U(1)_{L,3}$ must be broken:

$(\mathbb{Z}_2)_{\mathrm{center}}$: Quark gluon plasma

$U(1)_B$: Baryon superfluidity

$U(1)_{L,3}$: Chiral symmetry breaking

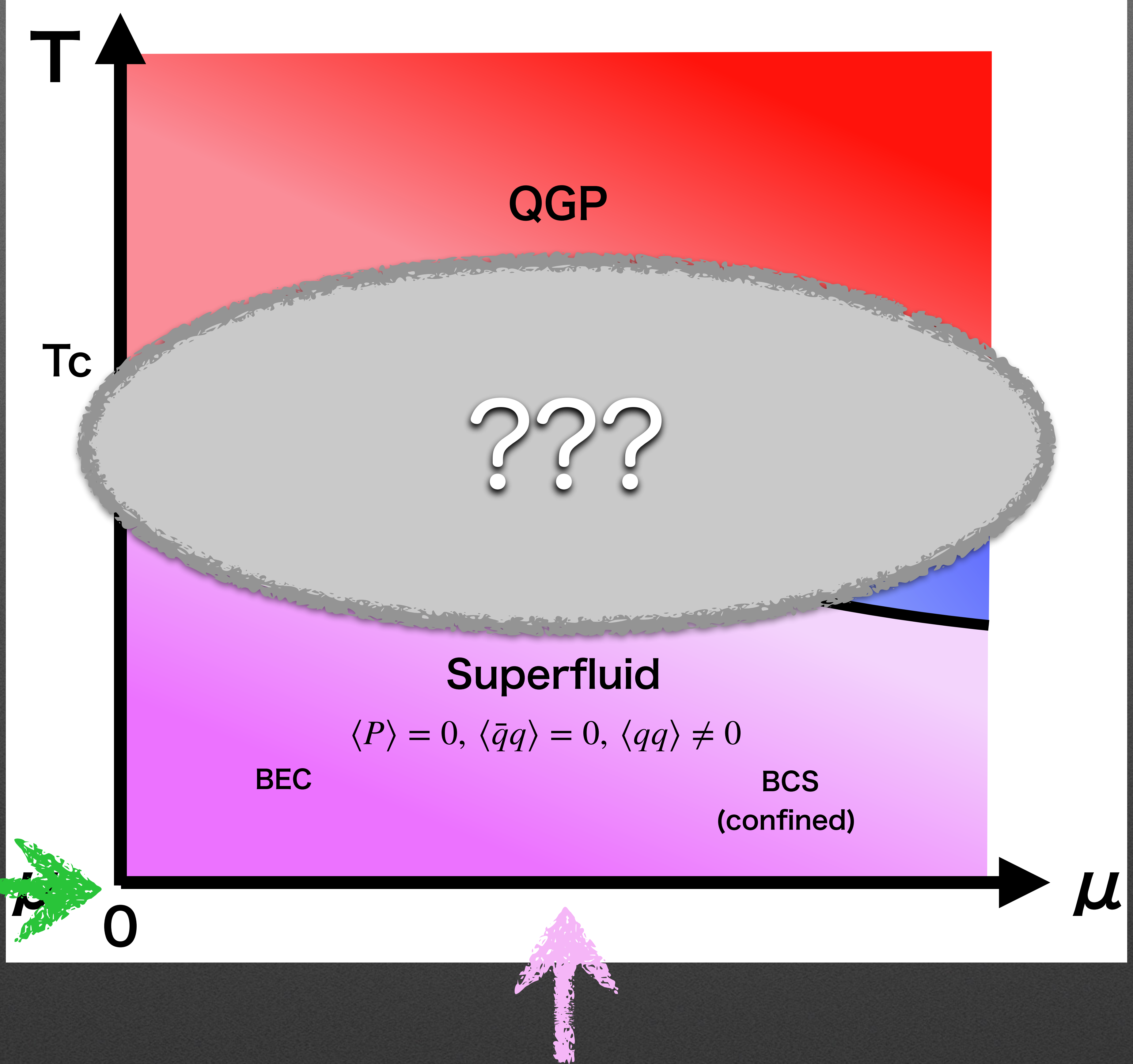
Possible phase diagram



$SO(6) \to SO(5)$
(pert. anom.)

Kogut, Stephanov, Toublan (1999),
Kogut, Stephanov, Toublan, Verbaarschot, Zhitnitsky (2000).

Possible phase diagram



$SO(6) \to SO(5)$
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BSF phase for $\mu > 0$ (ChPT)

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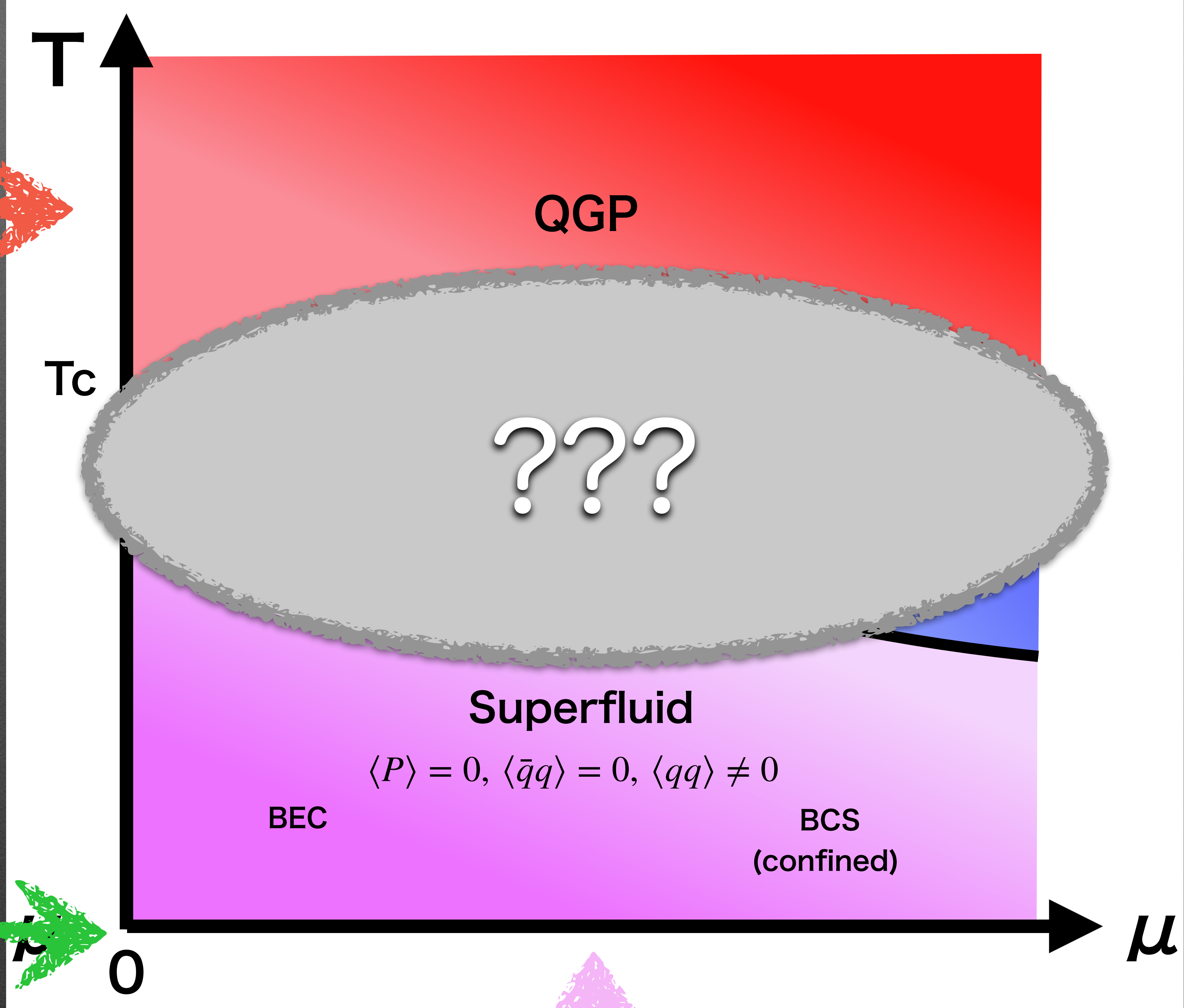
Possible phase diagram

QGP phase
for high T
(pert. theory)

Gross, Pisarski, Yaffe (1981).



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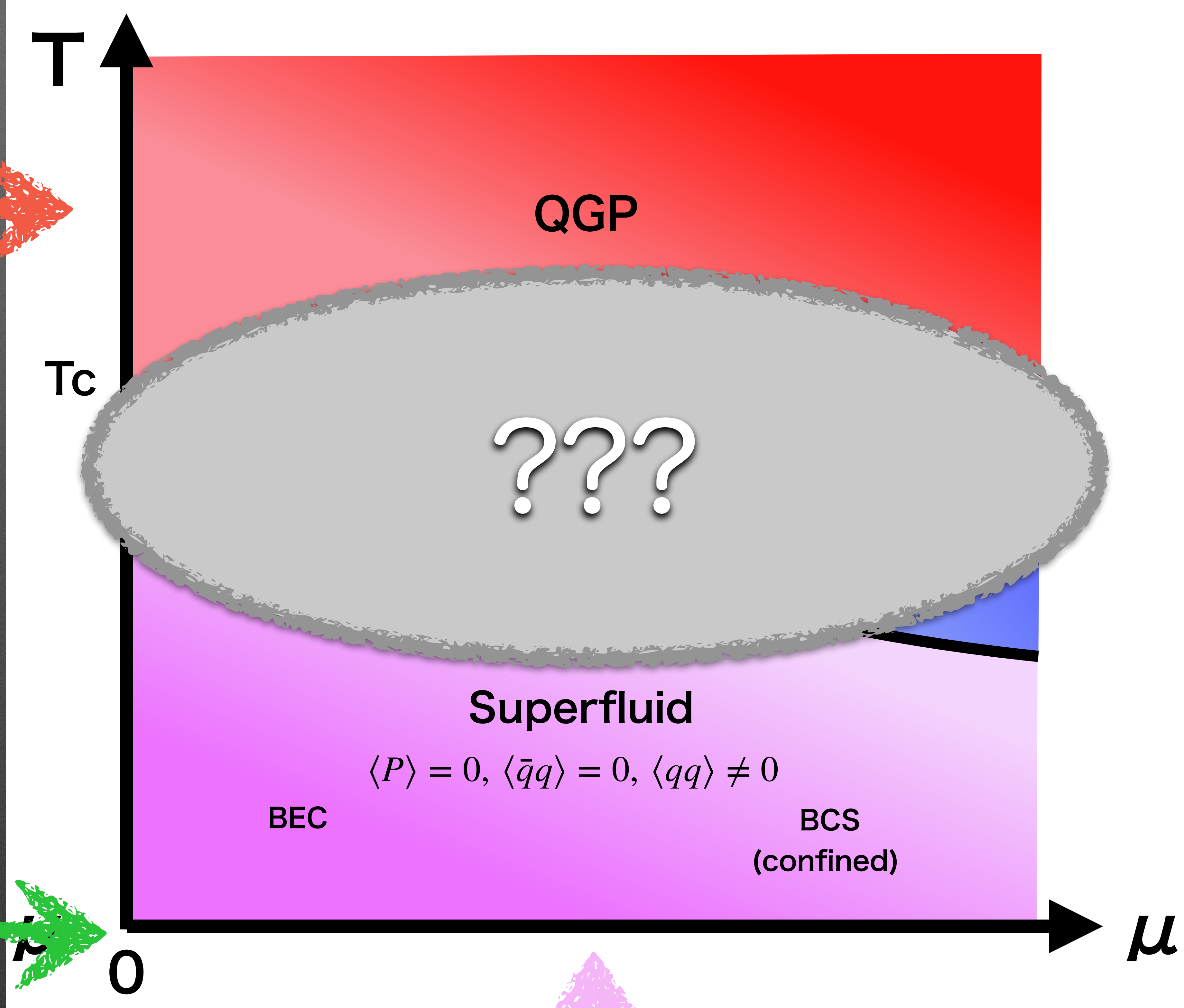

$SO(6) \to SO(5)$
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QGP starts at T_{QGP}
BSF breaks at T_{BSF}
Anomaly constraint
$T_{\mathrm{QGP}} \leq T_{\mathrm{BSF}}$

Furusawa, Tanizaki, Itou (2020)

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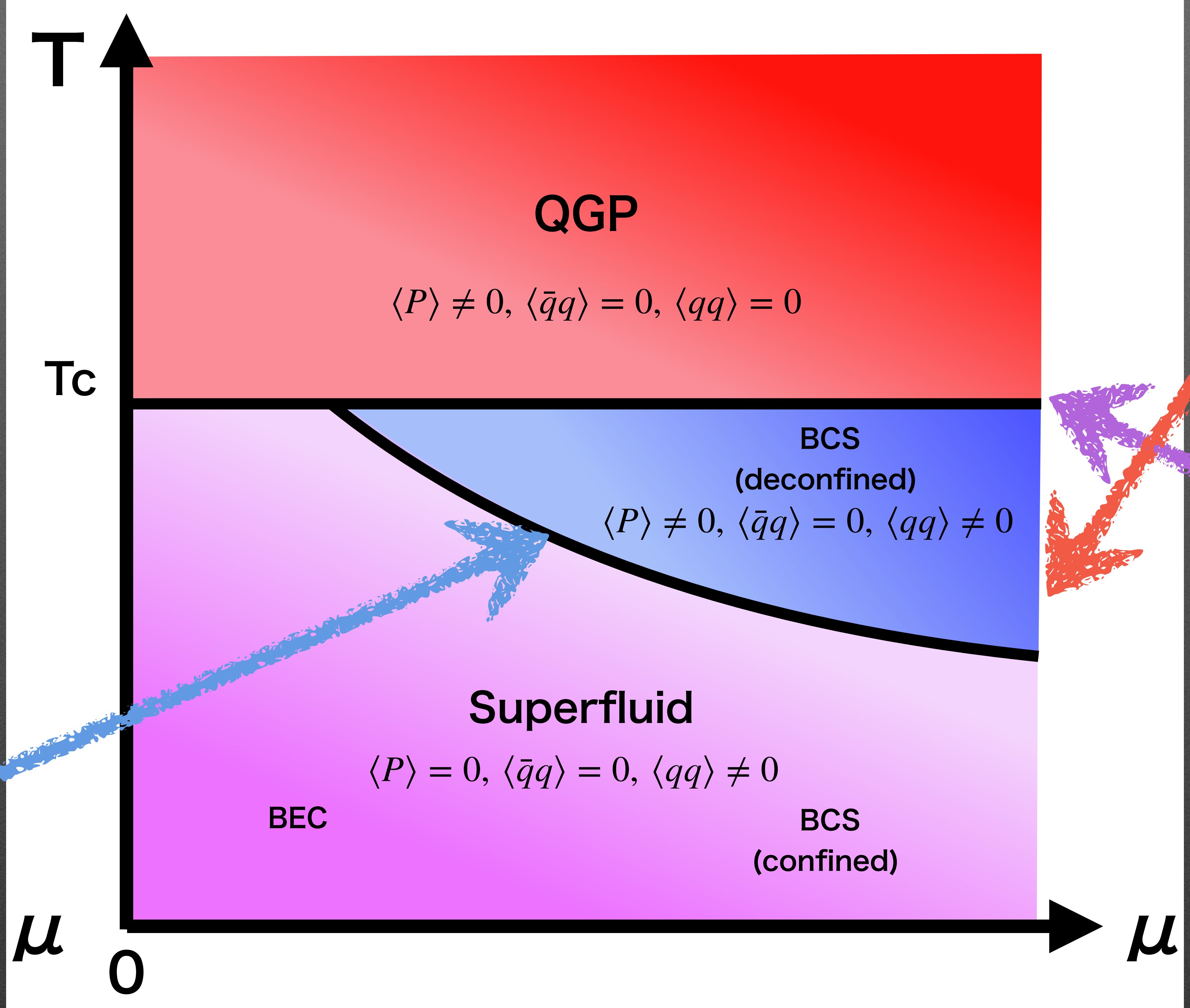
Possible phase diagram

$T_{\rm QGP} = T_{\rm BSF}$ →



QGP starts at $T_{\rm QGP}$

BSF breaks at $T_{\rm BSF}$

Anomaly constraint

$T_{\rm QGP} \leq T_{\rm BSF}$

Furusawa, Tanizaki, Itou (2020)

Coexisting phase is allowed.

cf. Iida, Itou, Lee, (2020)

(One of the simplest phase diagrams)

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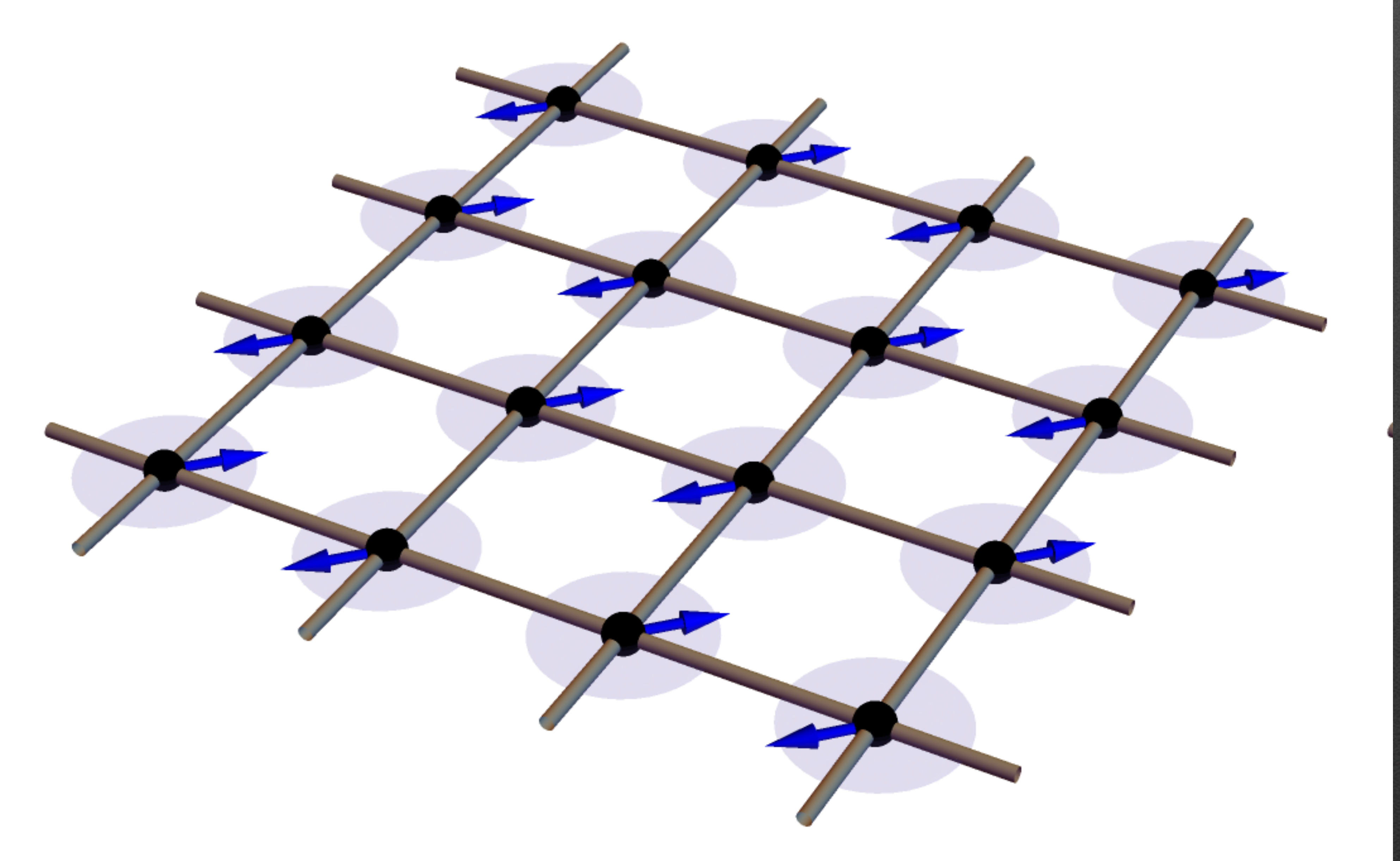
QFT of antiferromagnets

2+1D CP1 model:

Spin vector: $S_\alpha \sim \phi^\dagger \sigma^\alpha \phi$

$$|(\partial_\mu - ib_\mu)\phi|^2 + r|\phi|^2 + \lambda|\phi|^4 + \lambda_{EP}|\phi^\dagger\sigma^z\phi|^2 + \cdots$$

ϕ: 2-comp. complex scalar, b_μ: U(1) dynamical gauge field



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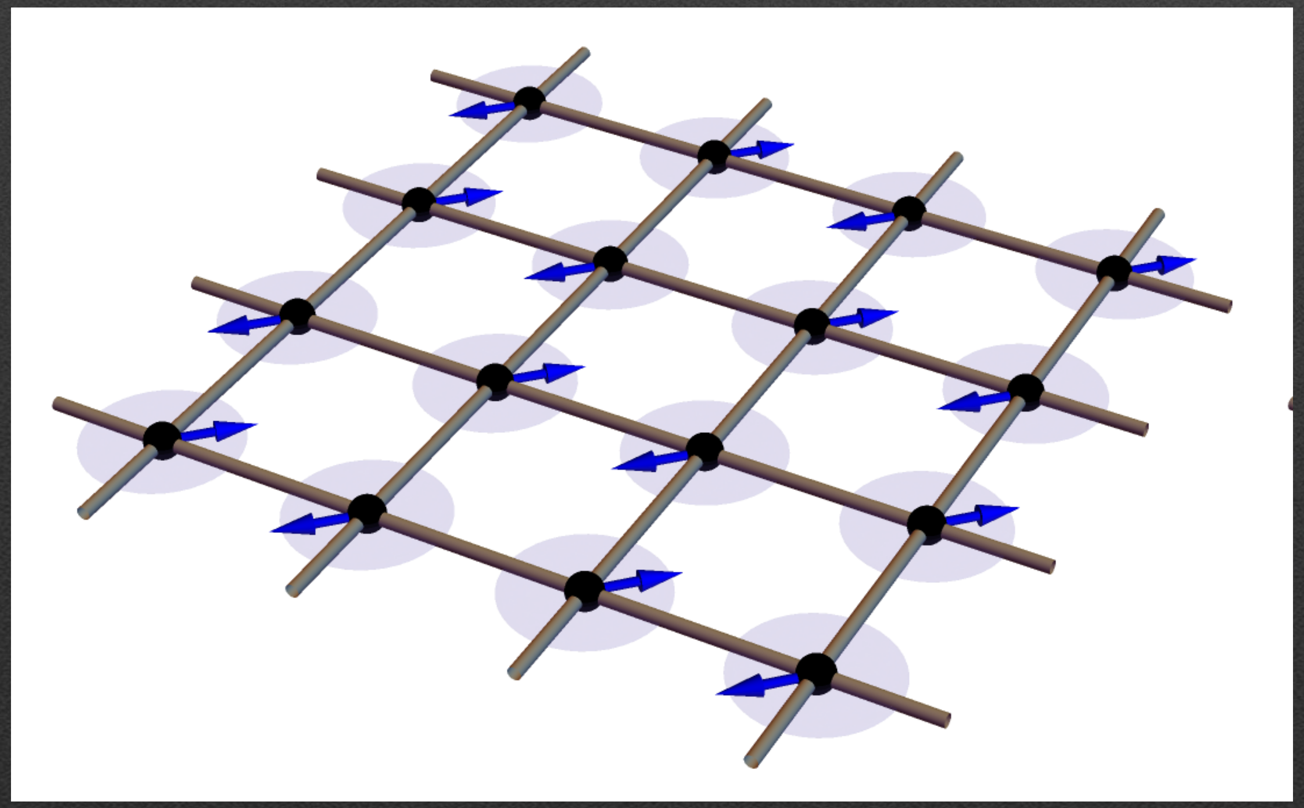
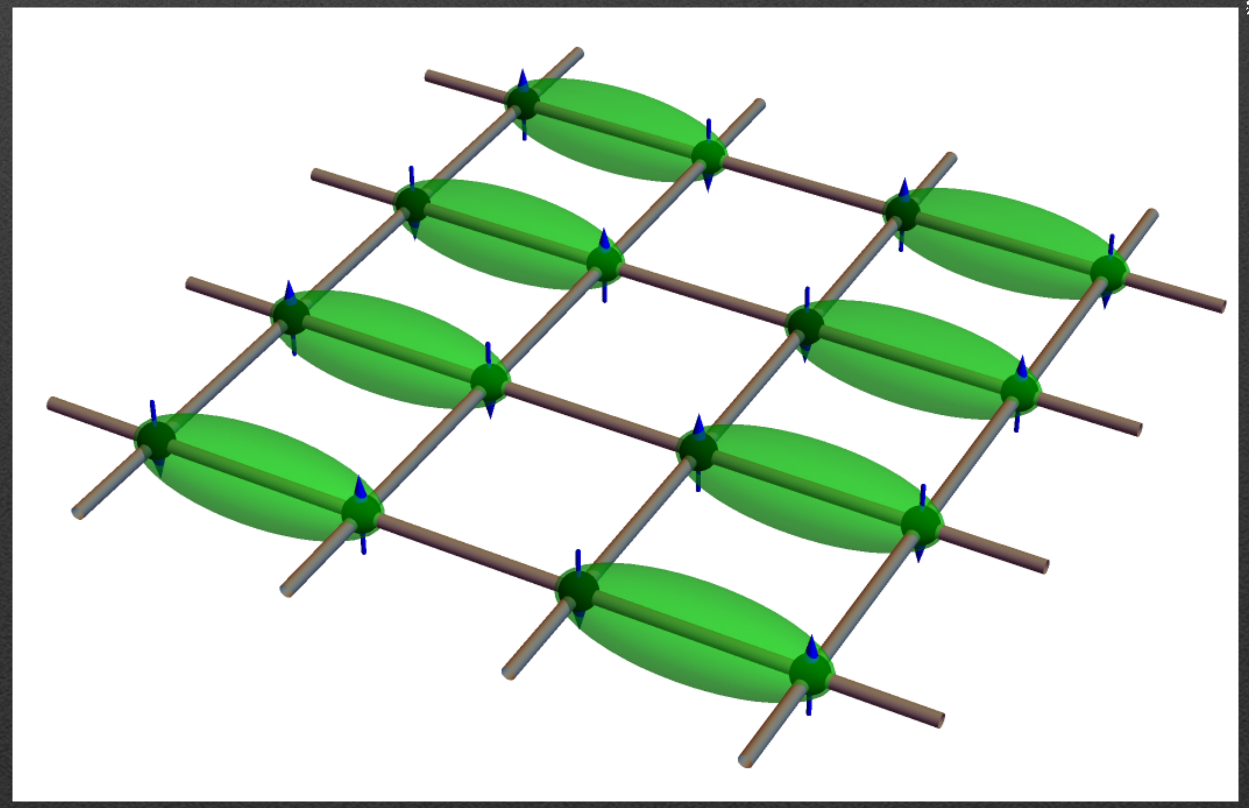
Higgs phase (Neel) Coulomb phase (VBS)

r



$r = r_c$



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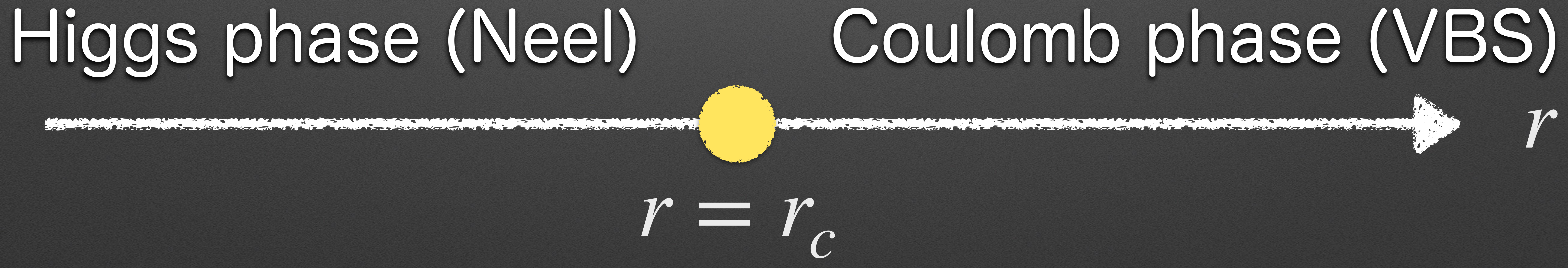

Example of unconventional QCP beyond LG theory.

Wang, Nahum, Metlitski, Xu, Senthil (2017)

Symmetries in CP1 model

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Easy-plane potential breaks $SO(3)_{\rm spin} \to (\mathbb{Z}_2)_{\rm spin} \ltimes SO(2)_{\rm spin}$.

Flip S_z

Rotate S_x&S_y

Symmetries in CP1 model

2+1D CP1 model:

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"Hidden" global symmetry: $U(1)_{\text{magnetic}}$ ← Current: $J_\mu = \epsilon^{\mu\nu\rho}\partial_\nu b_\rho/(2\pi)$

Metlitski,Thorngren (2018), Komargodski, Sulejmanpasic, Unsal (2018), Komargodski, Sharon, Thorngren, Zhou, (2019).

Symmetries in CP1 model

2+1D CP1 model:

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“Hidden” global symmetry: $\mathrm{U}(1)_{\mathrm{magnetic}}$

✔ $(\mathbb{Z}_2)_{\mathrm{spin}} \ltimes \mathrm{SO}(2)_{\mathrm{spin}} \times \mathrm{U}(1)_{\mathrm{magnetic}}$ has a mixed anomaly.

The same anomaly structure with $(\mathbb{Z}_2)_{\mathrm{center}} \ltimes \mathrm{U}(1)_{\mathrm{B}} \times \mathrm{U}(1)_{\mathrm{L},3}$

Metlitski,Thorngren (2018), Komargodski, Sulejmanpasic, Unsal (2018), Komargodski, Sharon, Thorngren, Zhou, (2019).

Symmetries in CP1 model

2+1D CP1 model:

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Novel QCP in 2cQCD at isospin RW point?

Symmetry enhancement at $\lambda_{\mathrm{EP}} = 0$ & $r = r_{\mathrm{c}}$:

$$\mathrm{SO(3)_{spin}} \times \mathrm{U(1)_{magnetic}} \rightarrow \mathrm{SO(5)}$$

Wang, Nahum, Metlitski, Xu, Senthil (2017)

$\Rightarrow$ SO(5) symmetry in 2cQCD?

Th

$)_{\mathrm{L,3}}$

Metlitsk

., (2019).

Summary & Discussion

2 color QCD with imaginary isospin chemical potential.

- $(\mathbb{Z}_2)_{\rm center}$ symmetry at isospin RW point
- $(\mathbb{Z}_2)_{\rm center} \ltimes \mathrm{U}(1)_{\rm B} \times \mathrm{U}(1)_{\rm L,3}$ anomaly at finite T & μ

Nonperturbative constraint from anomaly matching

- Finite-T &-μ phase diagram $T_{\rm QGP} \leq T_{\rm ChSB}$ or $T_{\rm BSF}$
- The same anomaly as 2+1D antiferromagnets
  ⇒ Unconventional quantum critical point at finite T?
  [See our paper for details. PRResearch 2, 033253 (2020)]

Sign problem free ⇒ testable by lattice simulation!