# COLOR FLUX TUBES IN TWO-COLOR QCD AT LOW TEMPERATURE AND HIGH DENSITY

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with

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Probing the physics of high-density and low-temperature matter with ab initio calculations in 2-color QCD, YITP workshop, 3rd - 6th November, 2020, Online

# Outline

- Introduction
- Two-color QCD with Nf=2
- Dual superconductor picture of the QCD vacuum
- Color flux tubes
- Summary

# Introduction

- Purpose
  - Understanding the physics of QCD in extreme conditions at finite temperature and density.
  - In the real world, we encountered the above situations.
    - For example, in neutron stars and relativistic heavy ion collisions.
  - What we want to investigate
    - Phase diagram on T-μ plane
    - Thermodynamic quantities
    - Nonperturbative properties (color confinement, χSB, …)

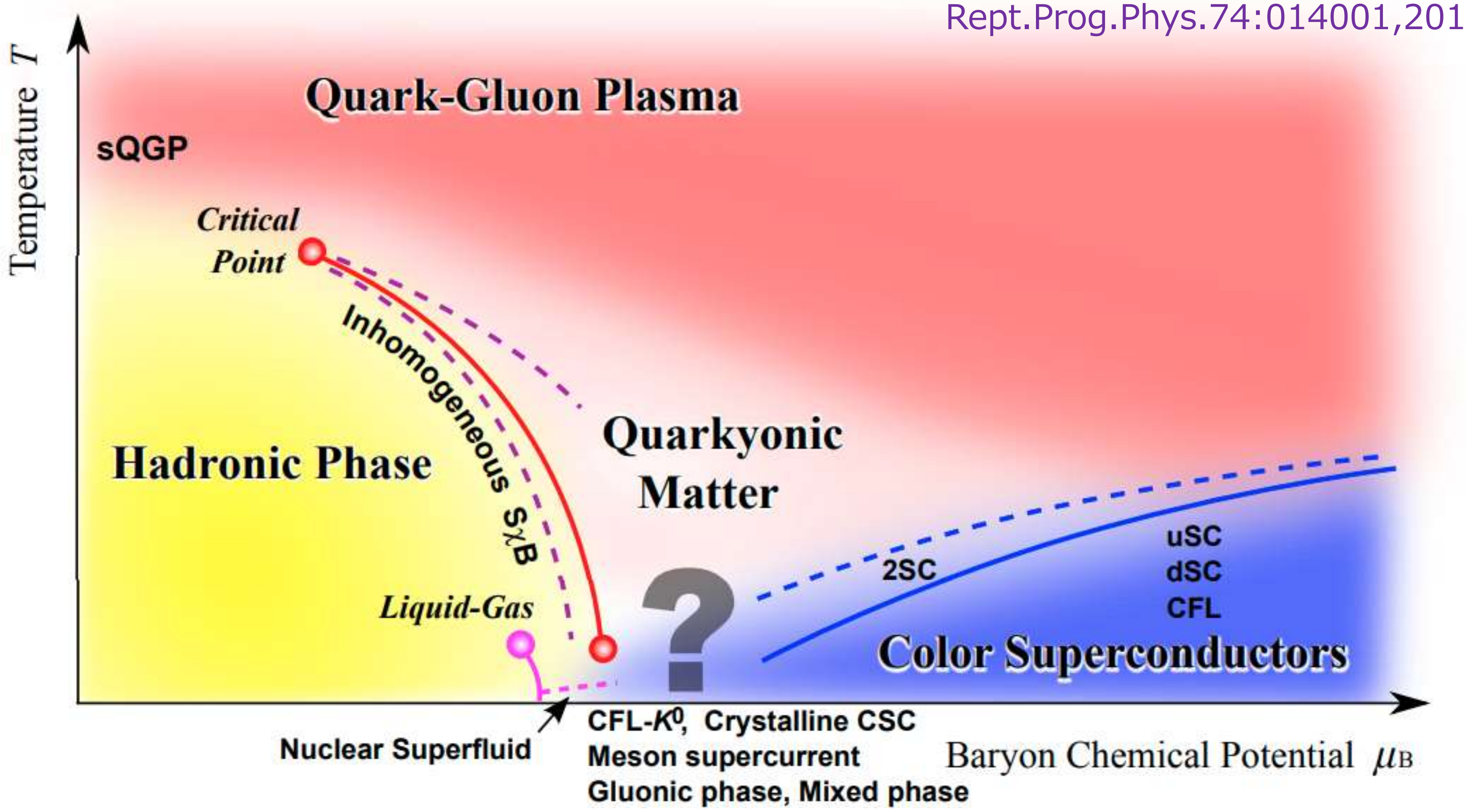
  and others

- Many analytical calculations at finite temperature and density in QCD have been done in areas where perturbative calculations are useful, and various physics are known. However, perturbative calculations are limited to a part of the phase diagram in the T-μ plane.
- At zero baryon chemical potential, various non-perturbative quantities like the Polyakov loop and the chiral condensate can be calculated using lattice QCD simulations.
- Many lattice calculations have been carried out in the finite density regime, which is difficult to compute analytically. However, the applicability of this method is still limited due to the sign problem.

# QCD phase diagram

- Schematic picture

K. Fukushima and T. Hatsuda
Rept.Prog.Phys.74:014001,2011



# Two-color QCD with Nf=2

- In this work, we examine the temperature and density dependence of the phase and color flux tube structure of dense two-color QCD with two-flavor Wilson fermions by using a lattice simulation to avoid the sign problem.
- 2-color QCD has the same properties as 3-color QCD, e.g. color confinement and spontaneous chiral symmetry breaking.
- It is expected to provide insights for 3-color QCD.

# Lattice setup

K. Iida, E. Itou and T.-G. Lee, JHEP01 (2020) 181

- Lattice action
  - Gauge part : Iwasaki gauge action

$$S_g = \beta \sum_x \left( c_0 \sum_{\substack{\mu<\nu \\ \mu,\nu=1}}^{4} W_{\mu\nu}^{1\times 1}(x) + c_1 \sum_{\substack{\mu\neq\nu \\ \mu,\nu=1}}^{4} W_{\mu\nu}^{1\times 2}(x) \right)$$

  - Fermion part : Two-flavor Wilson fermion action including the quark number operator and the diquark source term

$$S_F = \bar{\psi}_1 \Delta(\mu) \psi_1 + \bar{\psi}_2 \Delta(\mu) \psi_2 - J \bar{\psi}_1 (C\gamma_5) \tau_2 \bar{\psi}_2^T + \bar{J} \psi_2^T (C\gamma_5) \tau_2 \psi_1$$

$$\Delta(\mu)_{x,y} = \delta_{x,y} - \kappa \sum_{i=1}^{3} \left[ (1-\gamma_i) U_{x,i} \delta_{x+\hat{i},y} + (1+\gamma_i) U_{y,i}^{\dagger} \delta_{x-\hat{i},y} \right]$$
$$- \kappa \left[ e^{+\mu} (1-\gamma_4) U_{x,4} \delta_{x+\hat{4},y} + e^{-\mu} (1+\gamma_4) U_{y,4}^{\dagger} \delta_{x-\hat{4},y} \right]$$

# Definition of the phases

K. Iida, E. Itou and T.-G. Lee
JHEP01 (2020) 181

| | Hadronic | | QGP | Superfluid | |
|---|---|---|---|---|---|
| | | Hadronic matter | | BEC | BCS |
| $\langle \lvert L \rvert \rangle$ | zero | zero | non-zero | | |
| $\langle qq \rangle$ | zero | zero | zero | non-zero | $\propto \mu^2$ |
| $\langle n_q \rangle$ | $\langle n_q \rangle = 0$ | $\langle n_q \rangle > 0$ | $\langle n_q \rangle \geq 0$ | non-zero | $\langle n_q \rangle / n_q^{\text{tree}} \approx 1$ |

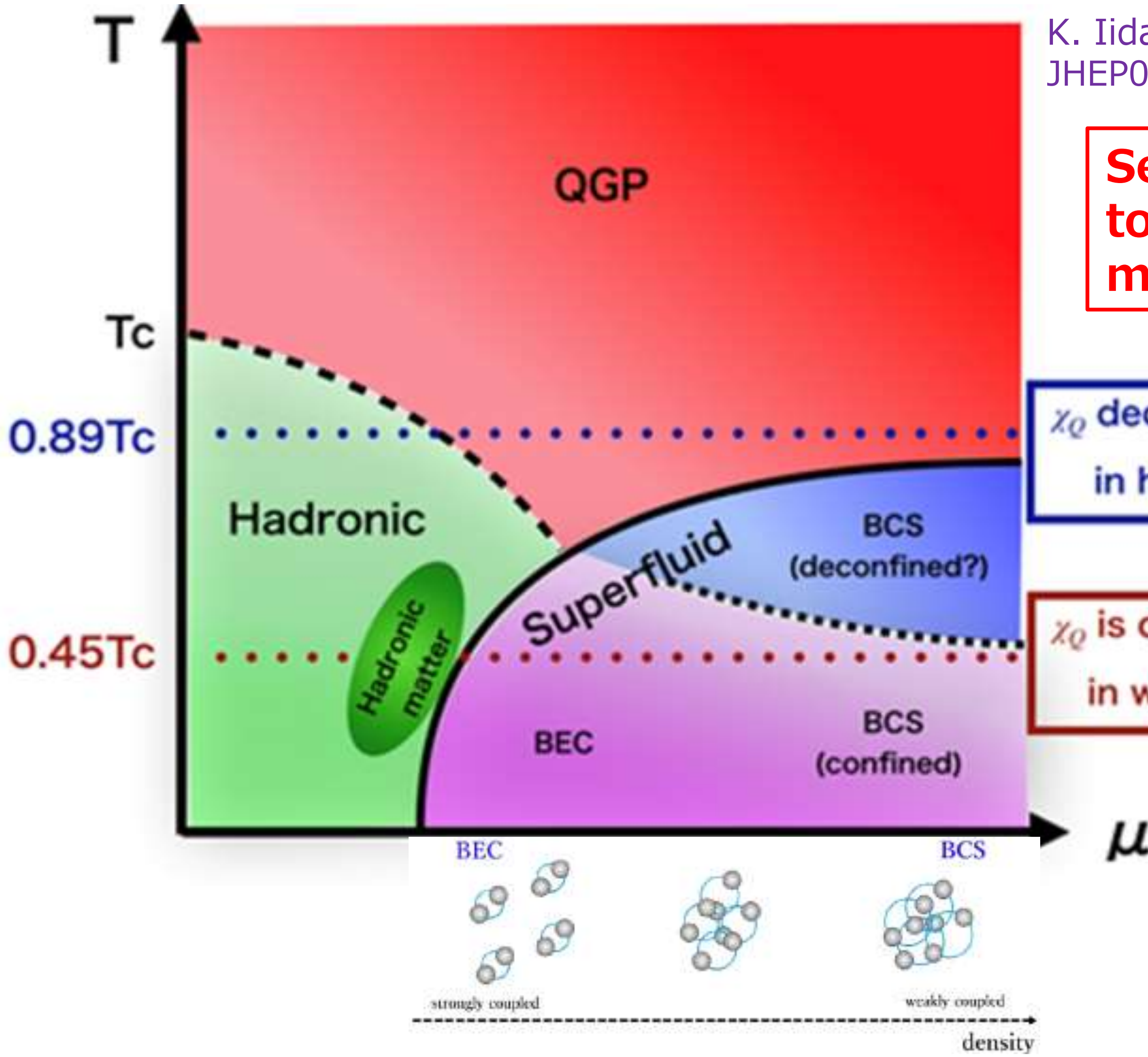
- $\langle \lvert L \rvert \rangle$ : Polyakov loop
- $\langle qq \rangle$ : diquark condensate
- $\langle n_q \rangle$ : quark number density

$$L = \frac{1}{N_s^3} \sum_{\vec{x}} \prod_{\tau} U_4(\vec{x}, \tau)$$

$$\langle qq \rangle \equiv \frac{\kappa}{2} \langle \bar{\psi}_1 K \bar{\psi}_2^T - \psi_1 K \psi_2^T \rangle$$

$$a^3 n_q = \sum \kappa \langle \bar{\psi}_i(x)(\gamma_0 - \mathbb{I}_4) e^{\mu} U_4(x) \psi_i(x + \hat{4}) + \bar{\psi}_i(x)(\gamma_0 + \mathbb{I}_4) e^{-\mu} U_4^{\dagger}(x - \hat{4}) \psi_i(x - \hat{4}) \rangle.$$

# Schematic picture of two-color QCD phase diagram




K. Iida, E. Itou and T.-G. Lee
JHEP01 (2020) 181

**See Ito's talk tomorrow for more details!**

$\chi_Q$ decreases in high $\mu$

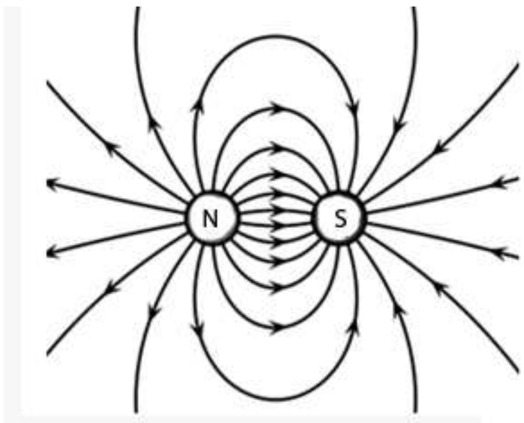
$\chi_Q$ is constant in whole $\mu$

# Dual superconductor picture of the QCD vacuum

- In QCD vacuum, color magnetic monopoles condense instead of the formation of Cooper pairs in the BCS theory of normal superconductivity. (dual superconductor)
  G. ‘tHooft (1976), S.Mandelstam (1976)
- Color confinement is due to the dual Meissner effect by condensation of color magnetic monopoles.
- There are many numerical evidence for the dual superconductor picture of the QCD vacuum. Shiba-Suzuki (1994), Y.Matsubara et al.(1994), Cea-Cosmai(1995), G.S.Bali et al.(1998), A.Di Giacomo et al.(1999), …
- Color electric field between quark and antiquark is squeezed into tube-like structure. The formation of color flux provides a linear potential between quark and antiquark.
- In this talk, we focus on the nature of color electric flux tube at low temperature and high density of two-color QCD.

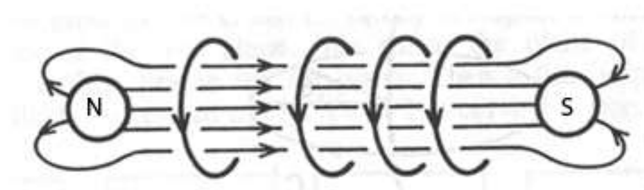
# Flux tubes

- Magnetic field of a magnetic dipole




In empty space

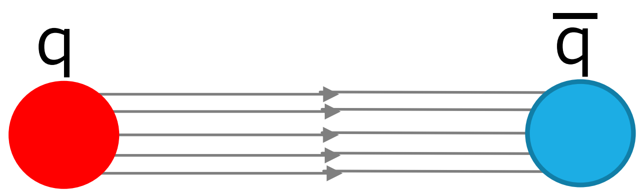
K. Huang, Int. J. Mod, Phys. A30,1530056 (2015)




In a normal superconductor

dual

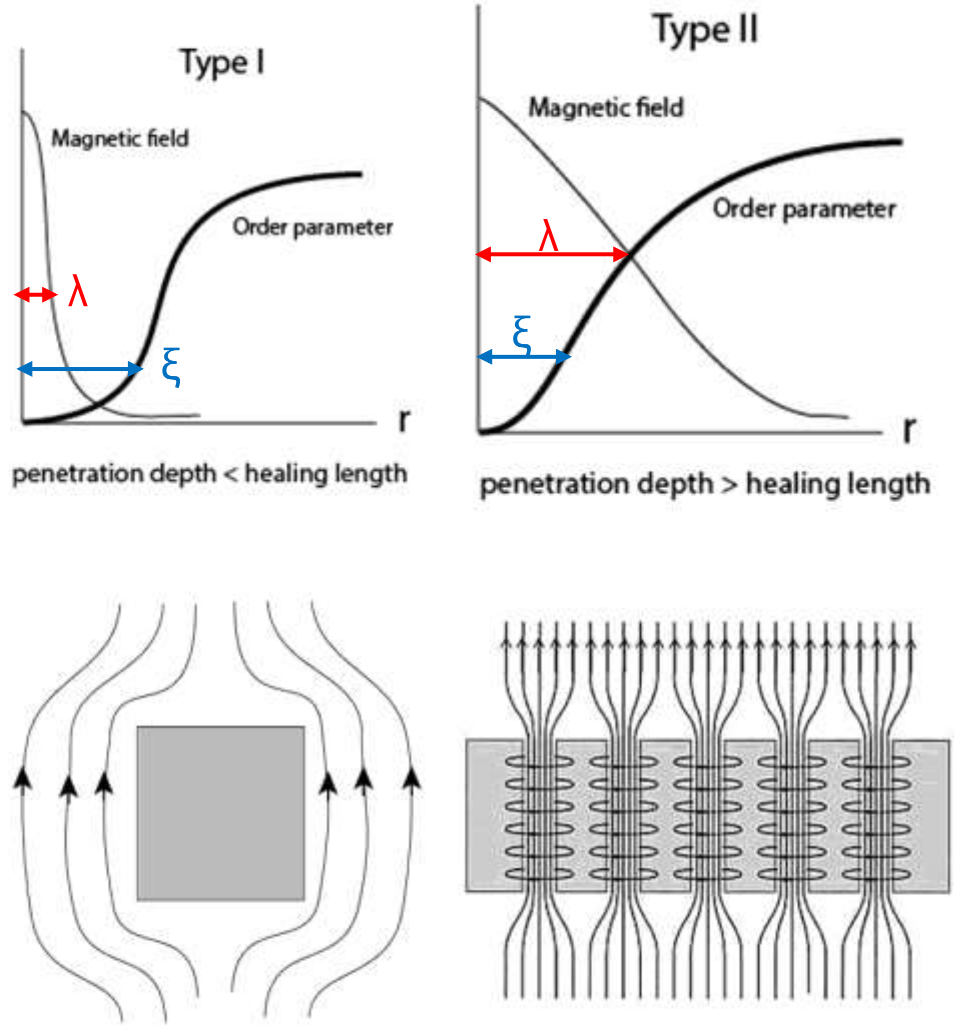
- Color electric field of quark and antiquark pair




In the QCD vacuum

# Magnetic flux in superconductor

- Two characteristic lengths
  - London penetration length ($\lambda$) :Lengths of magnetic field penetration into a superconductor
  - Coherence length (healing length) ($\xi$) :Length of a variation of superconducting order parameter
- Ginzburg-Landau parameter $\kappa = \lambda/\xi$
  - Type I : $\kappa < 1/\sqrt{2}$
  - Type II : $\kappa > 1/\sqrt{2}$



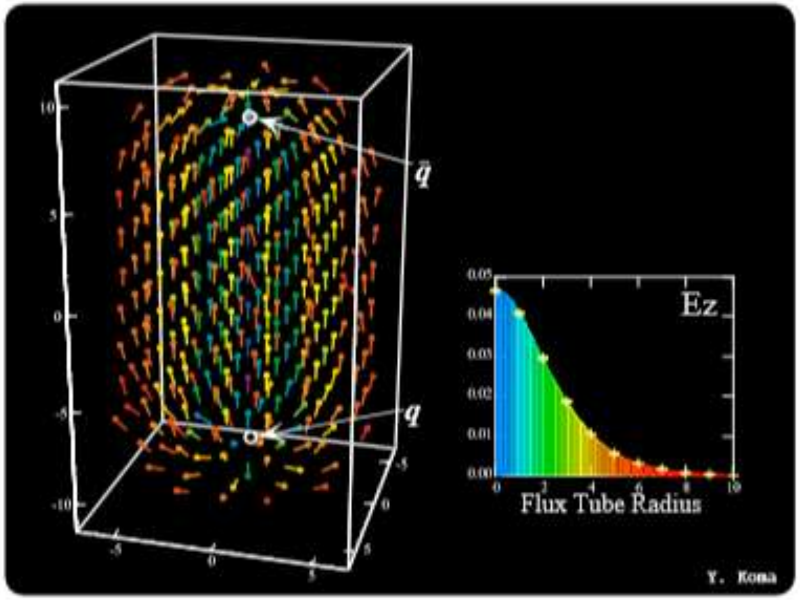

K. Huang, Int. J. Mod, Phys. A30,1530056 (2015)

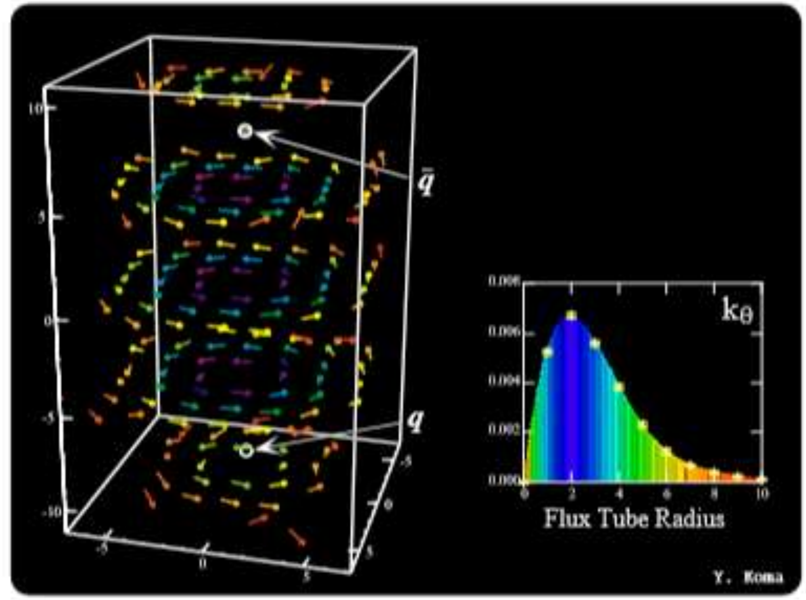
# Previous works of flux tubes in QCD

- quenched SU(2)
  - border between type 1 and type 2 (Y.Matsubara et al. '94, T.Suzuki et al. '07, P.Cea et al. '12)
  - Type 1 (G.S.Bali et al. '98)
  - Type 2 (P.Cea et al. '95)
  - weak Type 1 (S.Kato et al., '14)
- quenched SU(3)
  - border between type 1 and type 2 (Y.Matsubara '94)
  - Type 1 (P.Cea et al. '12)
- Finite temperature quenched SU(3) (P.Cea et al. '16)
- 2+1 flavor QCD
  - Type 1 (P.Cea et al. '17)
- 2+1 flavor QCD in external magnetic field (C.Bonati et al. '18)

The results for the type of vacuum are different depending on the physical quantity used and the analysis method.

# Flux tube profiles in SU(2) gauge theory




- Quenched SU(2) QCD
- Maximally Abelian gauge
- Extract U(1) gauge fields and U(1) monopole
- Ez : U(1) electric field (longitudinal), k_θ : U(1) monopole

Y. Koma, M. Koma, E.-M. Ilgenfritz, T. Suzuki and M. I. Polikarpov Phys. Rev. D68, 094018 (2003).

# Measuring the color fields on the lattice

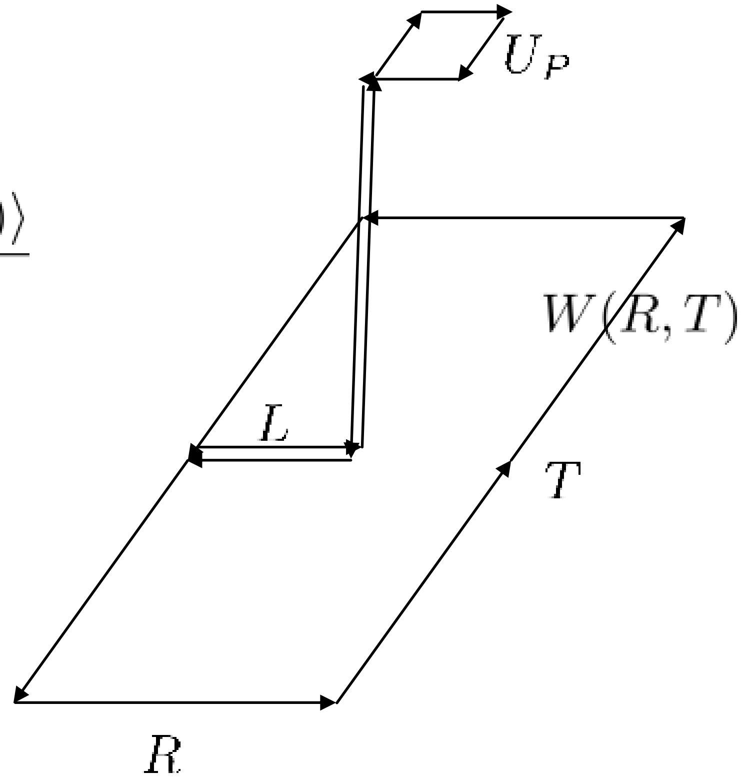
- Connected correlator

$$\rho_W = \frac{\langle \mathrm{Tr}(W L U_P L^\dagger) \rangle}{\langle \mathrm{Tr}(W) \rangle} - \frac{1}{N} \frac{\langle \mathrm{Tr}(U_P) \mathrm{Tr}(W) \rangle}{\langle \mathrm{Tr}(W) \rangle}$$

$W$ : Wilson loop

$L$ : Schwinger line

$U_P$ : Plaquette

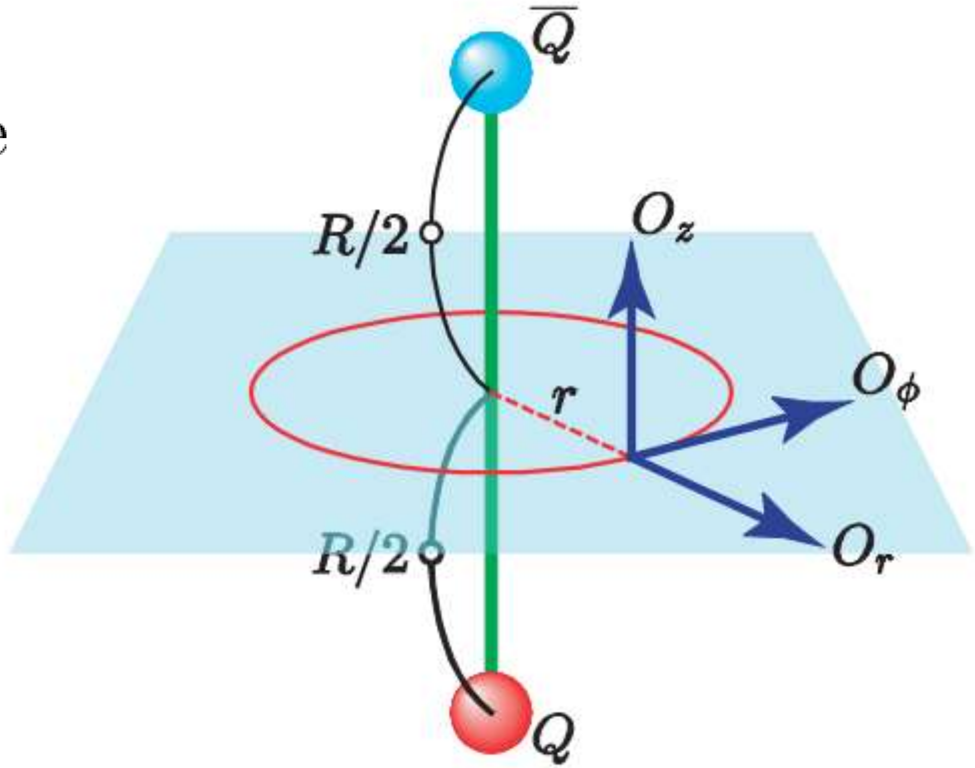



A. Di Giacomo, M. Maggiore, S. Olejnik, Nucl.Phys. B347 (1990) 441,
P. Cea, L. Cosmai, Phys.Rev. D52 (1995) 5152

# Color flux tube

- Cylindrical coordinate system $(r, \phi, z)$

  - $r$ : radial distance
  - $\phi$ : azimuthal angle
  - $z$ : axial coodinate




T. Suzuki et al., Phys.Rev.D80 (2009) 054504

# Profile of Ez

- Exponential form (London theory)

$$E_z(r) = E_z(0)\exp\left(-\frac{r}{\lambda}\right)$$

  - λ : London penetration length characteristic length of the exponential decrease of (electric) field in (dual) superconductor

- $K_0$ form (London theory for vortex as a line singularity : Abrikosov vortex)

$$E_z(r) = \frac{\phi}{2\pi\lambda^2}K_0\left(\frac{r}{\lambda}\right), \quad \lambda \gg \xi$$

Kn (r) : n-th order modified
Bessel function of the second kind

$$r \to \infty \quad E(r) \propto \frac{1}{\sqrt{r}}\exp\left(-\frac{r}{\lambda}\right)$$

$$r \to 0 \quad E(r) \propto \log\frac{\lambda}{r}$$

P. Cea and L. Cosmai, Phys. Rev. D52 (1995) 5152
G. S. Bali, K. Schilling, C.Schlichter, Phys. Rev. D51 (1995) 5165

- Clem form (cylindrical vortex with core)

$$E_z(r) = \frac{\phi}{2\pi\lambda^2\alpha} \frac{K_0[(r^2/\lambda^2 + \alpha^2)^{\frac{1}{2}}]}{K_1[\alpha]}$$

$$\frac{1}{\alpha} = \frac{\lambda}{\xi_v} \qquad \kappa = \frac{\lambda}{\xi} = \frac{\sqrt{2}}{\alpha}\sqrt{1 - \frac{K_0^2(\alpha)}{K_1^2(\alpha)}}$$

$\phi$ : external flux

$\lambda$ : London penetration length

$\xi_v$ : variational core-radius

$\kappa = \lambda/\xi$ : Ginzburg-Landau parameter

J. R. Clem, J. Low Temp. Phys 18, 427 (1975)
P. Cea, L. Cosmai and A. Papa, Phys. Rev. D86(2012)

# Lattice setup

- Lattice volume

$$16^4 \text{ lattice}$$

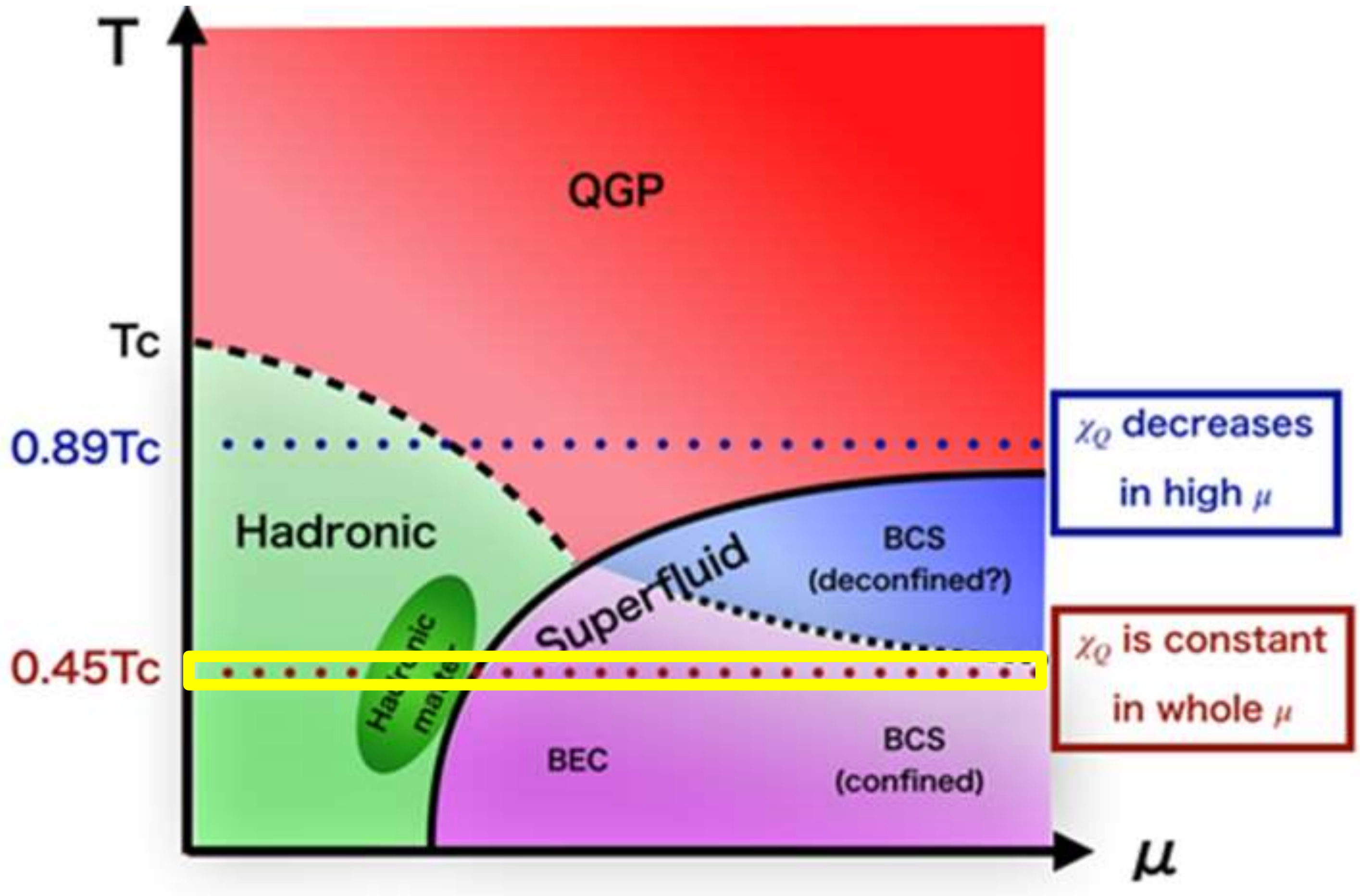
- Parameters

$$\beta = 0.800,\ \kappa = 0.159, \quad T = 0.45T_c$$
$$m_{PS}/m_V = 0.823(9),\ am_{PS} = 0.623(3),$$

| $(\mu, j)$ | $\mu$ / $m_{PS}$ | phase |
|---|---|---|
| (0.00, 0.00) | 0.00 | Hadronic |
| (0.25, 0.00) | 0.40 | near border |
| (0.35, 0.01) | 0.56 | BEC |
| (0.50, 0.01) | 0.80 | near border |
| (0.70, 0.01) | 1.12 | BCS |

- Smoothing of the gauge configurations : APE smearings for spatial links , one hypercubic smearing for temporal links

# The area used in this work



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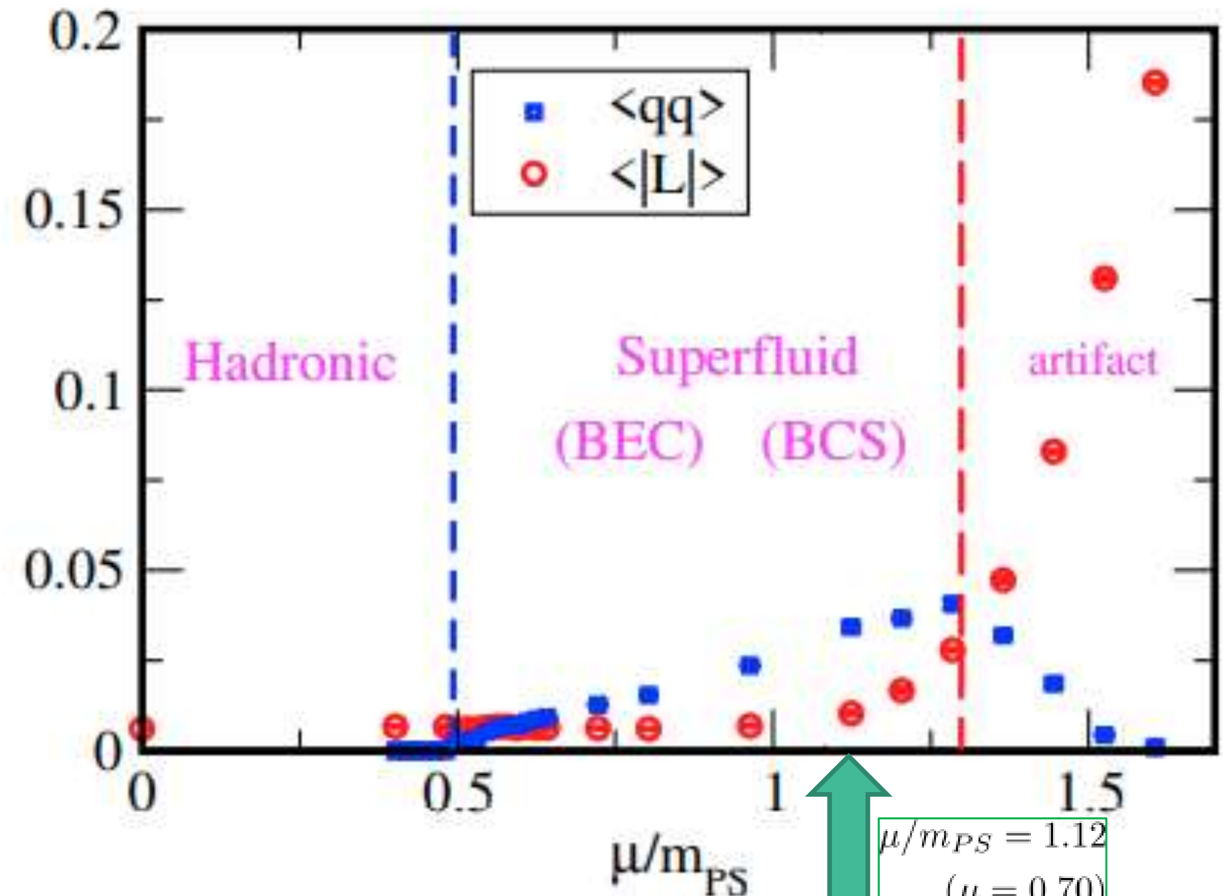
# Polyakov loop and diquark condensate

- Confinement phase in all μ except for artifact region




$\mu/m_{PS} = 1.12$
$(\mu = 0.70)$

**T=0.45Tc**

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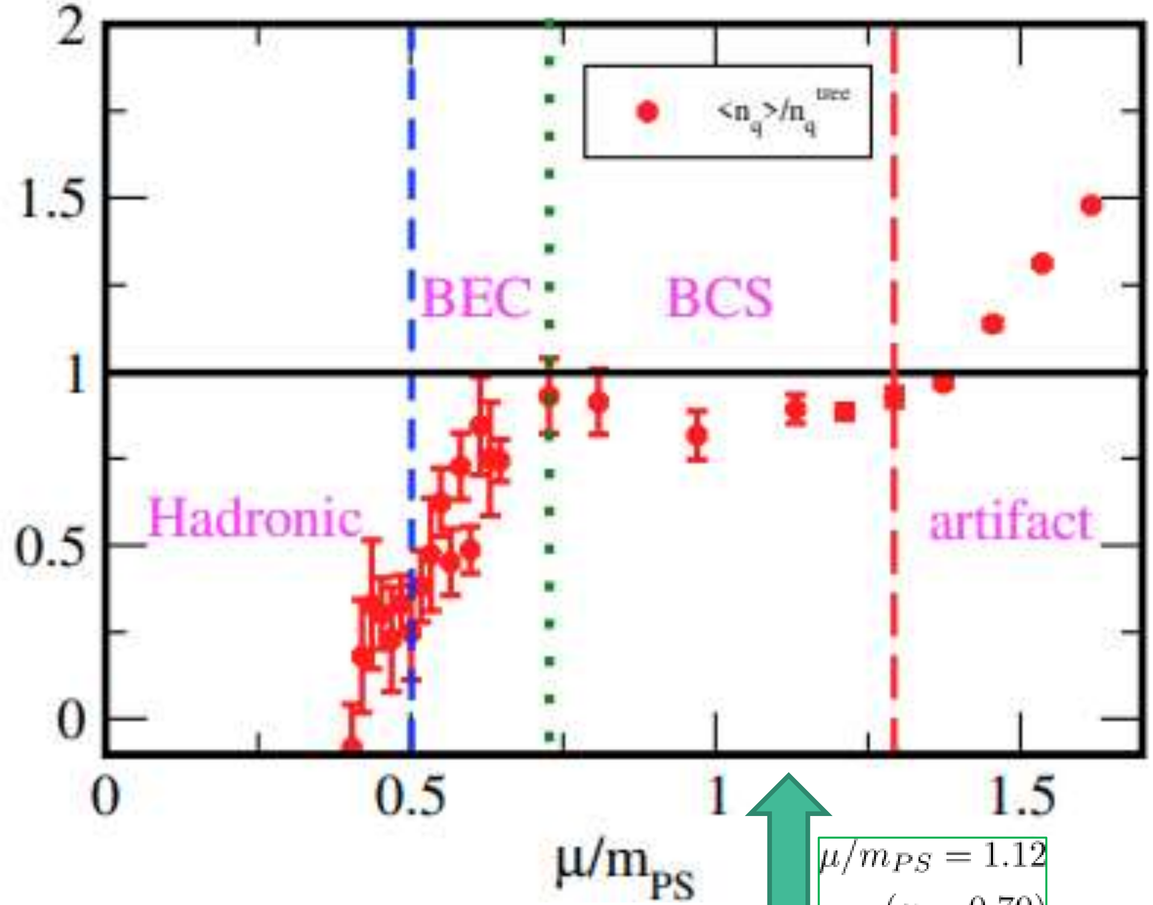
# Quark number density

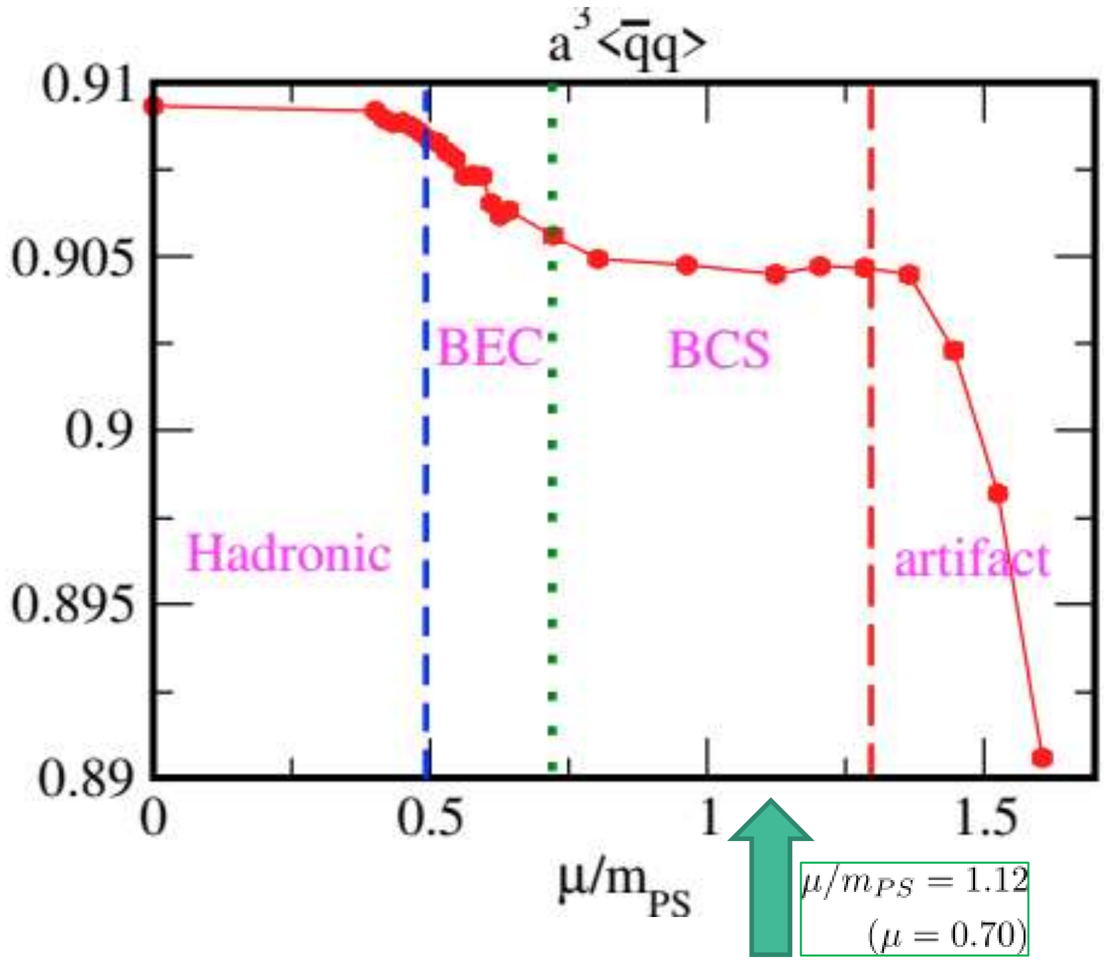



$\mu/m_{PS} = 1.12$
$(\mu = 0.70)$

**T=0.45Tc**

K. Iida, E. Itou and T.-G. Lee, JHEP01 (2020) 181

# Chiral condensate

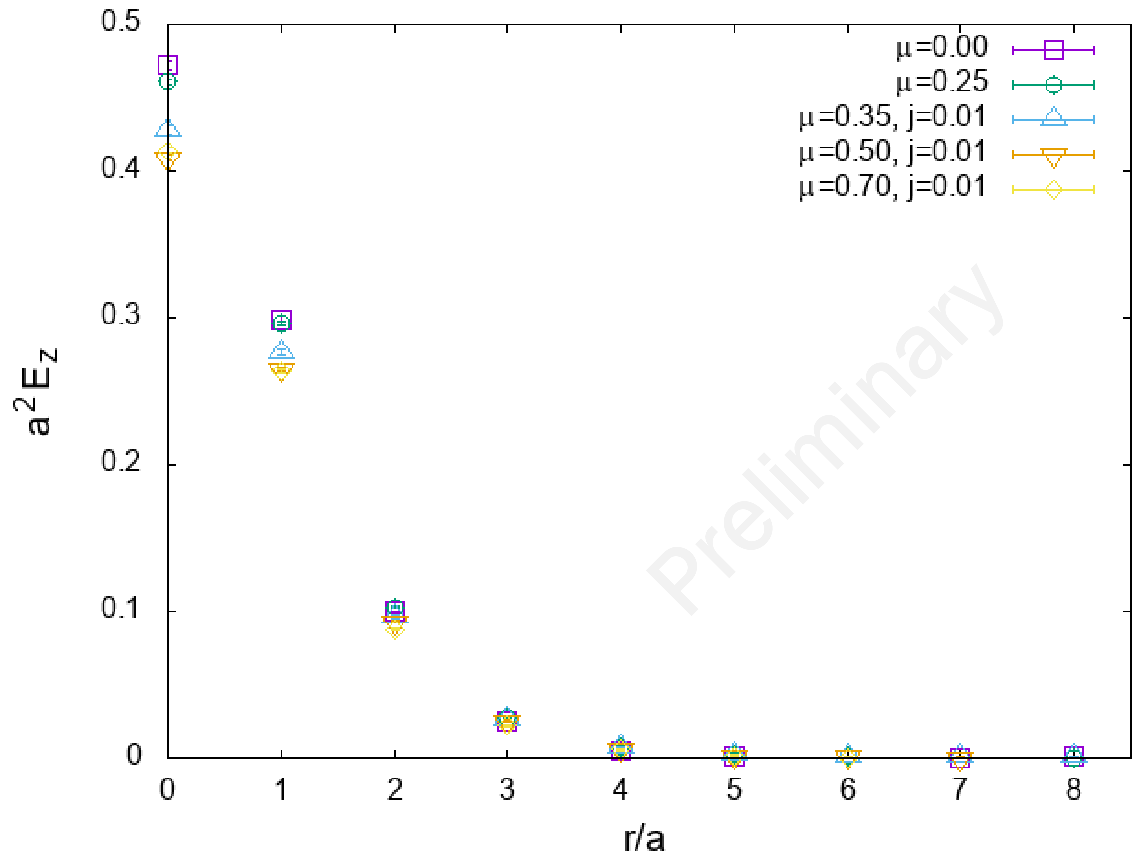



T=0.45Tc

# Results (preliminary)

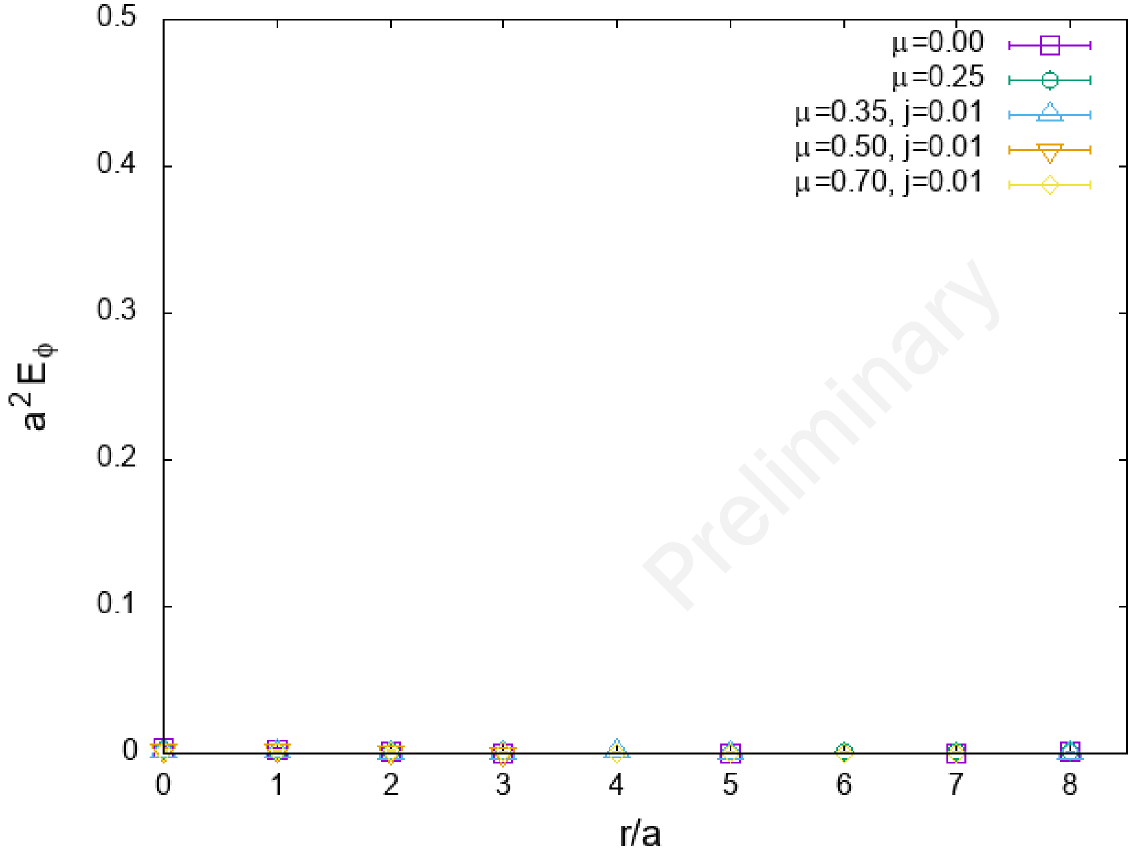
- μ dependence of color electric fields
- Penetration length
  - Exponential form
  - $K_0$ form
  - Clem form
- GL parameter
- Static potential between quark and antiquark

# Ez (color electric field)



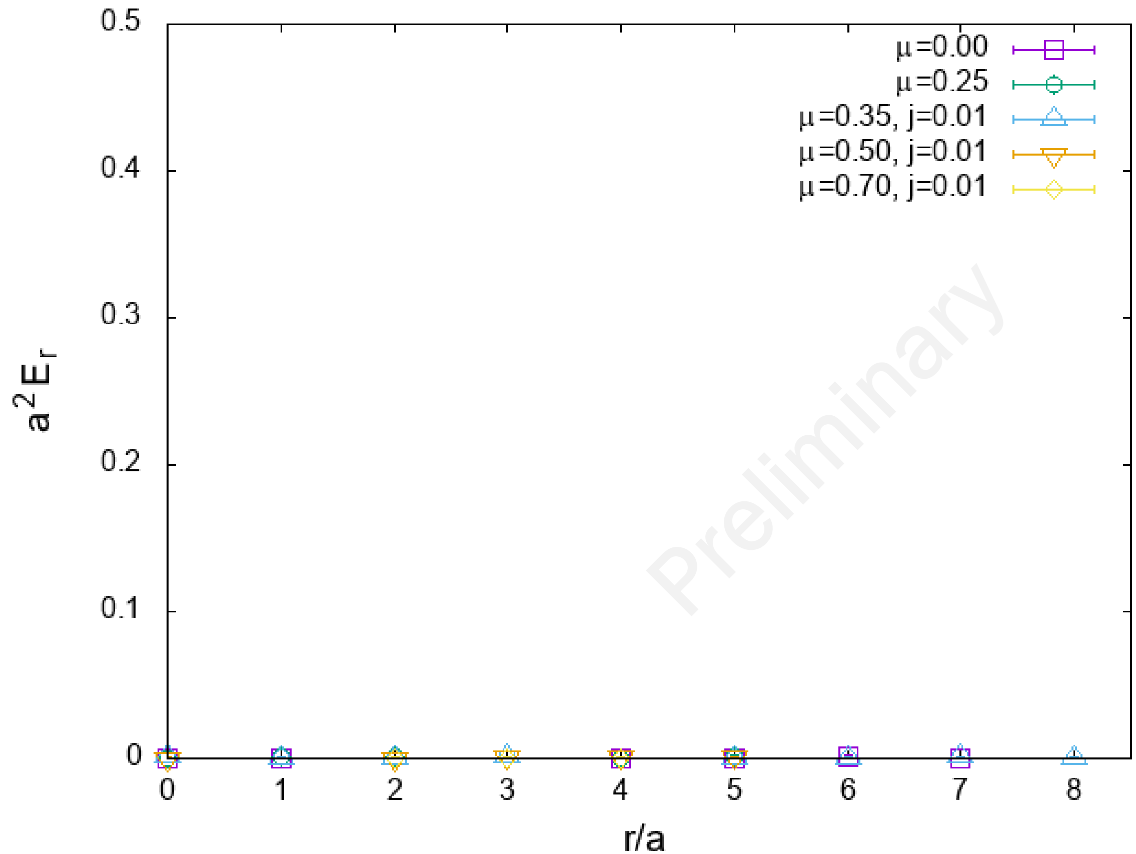

- Flux tubes are formed and there is a slight density dependence.

## $E_\phi$




- The azimuthal components of the color electric field are almost zero.

# Er



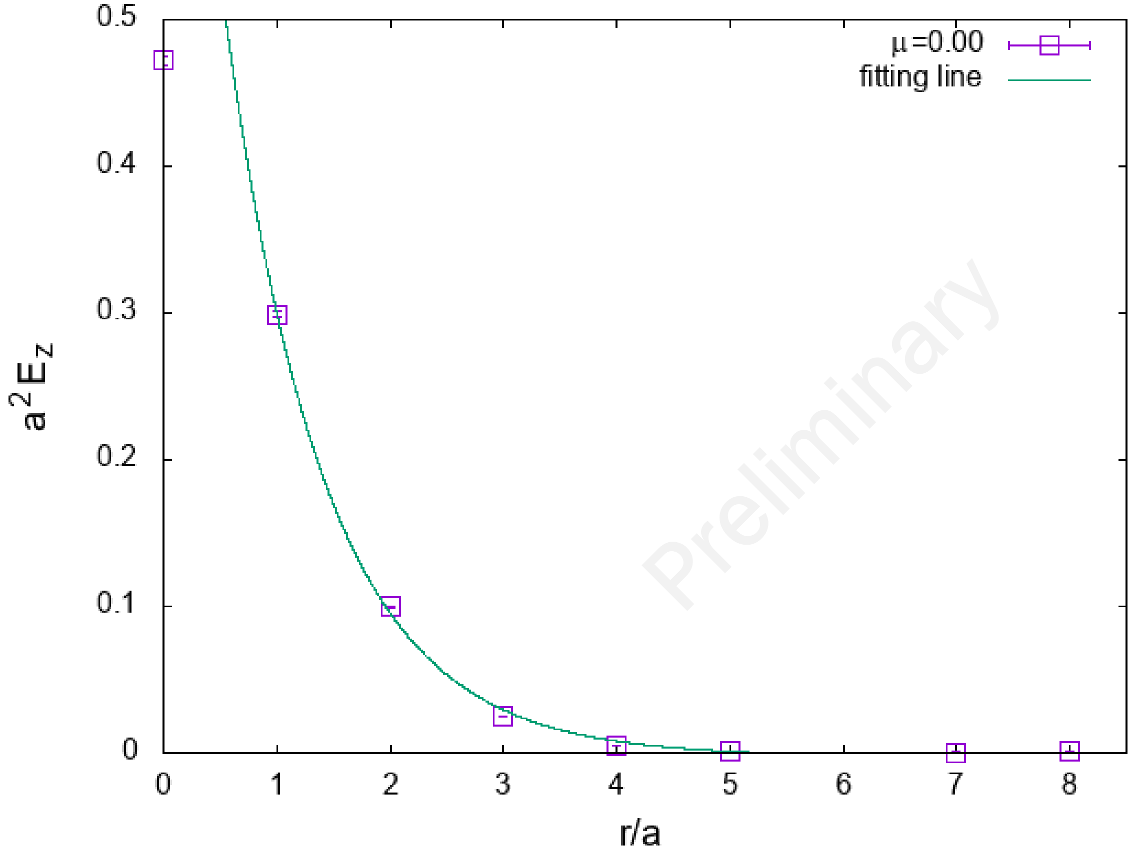

- The radial components of the color electric field are almost zero.

- $E_\phi$ and Er are zero and only Ez has a non-zero value in all μ region.
- The color electric field is squeezed into a linear shape to form a flux tube.

↓

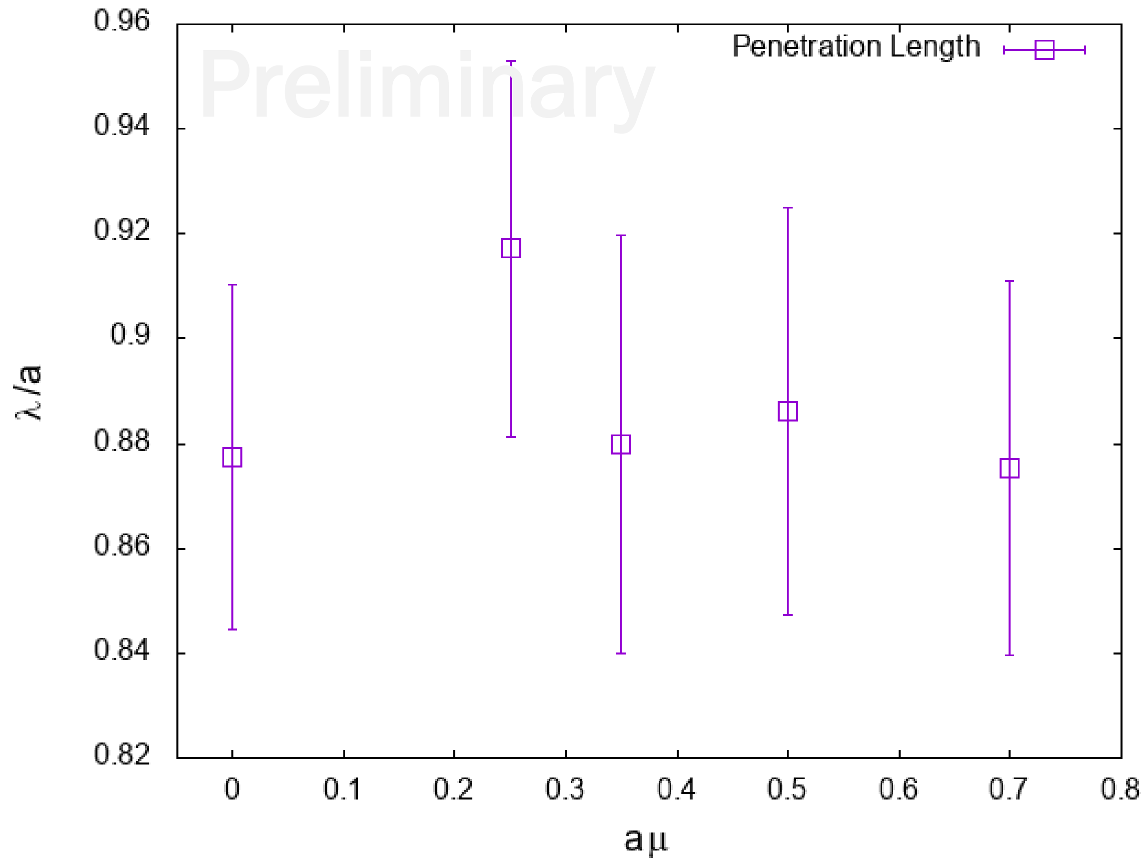
- Investigate the electric field Ez by doing a fit using the three forms described above.

# Exponential fitting of Ez $E_z(r) = E_z(0) \exp\left(-\frac{r}{\lambda}\right)$



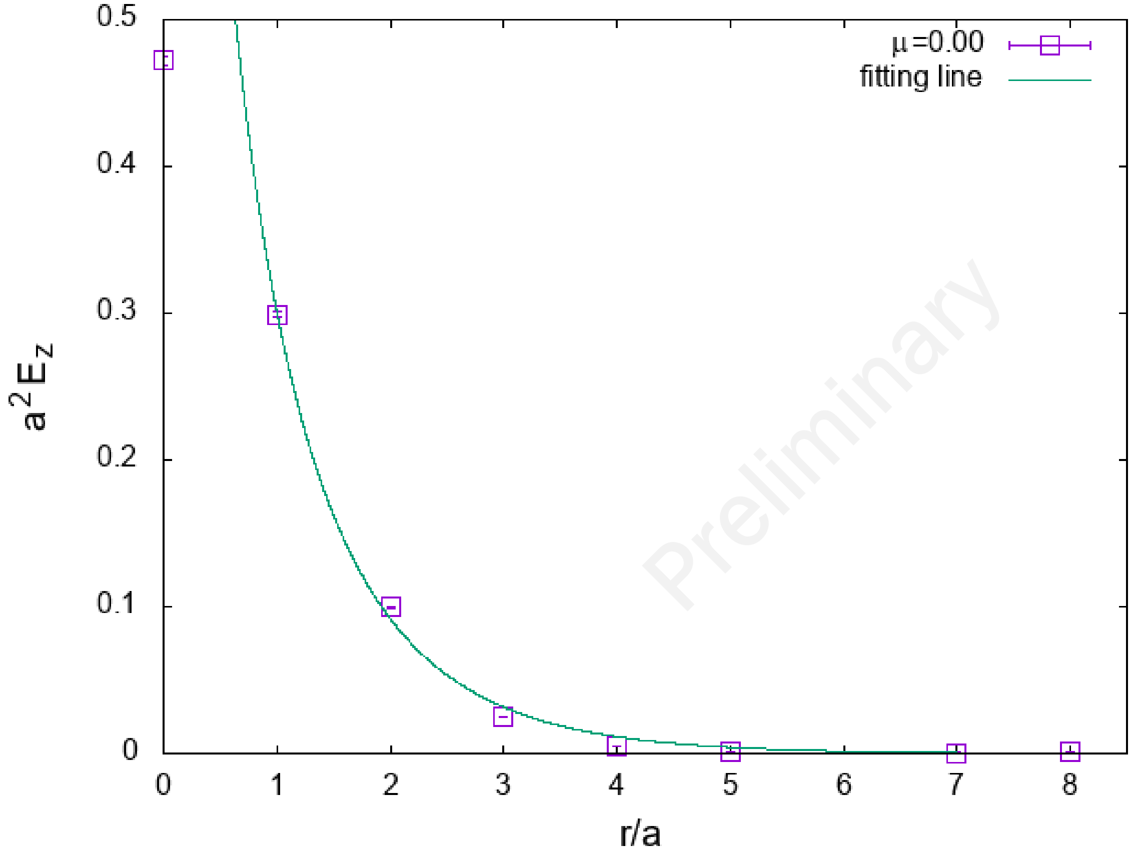

- The exponential fitting works, except for r=0.

# Penetration length (Exponential fitting)



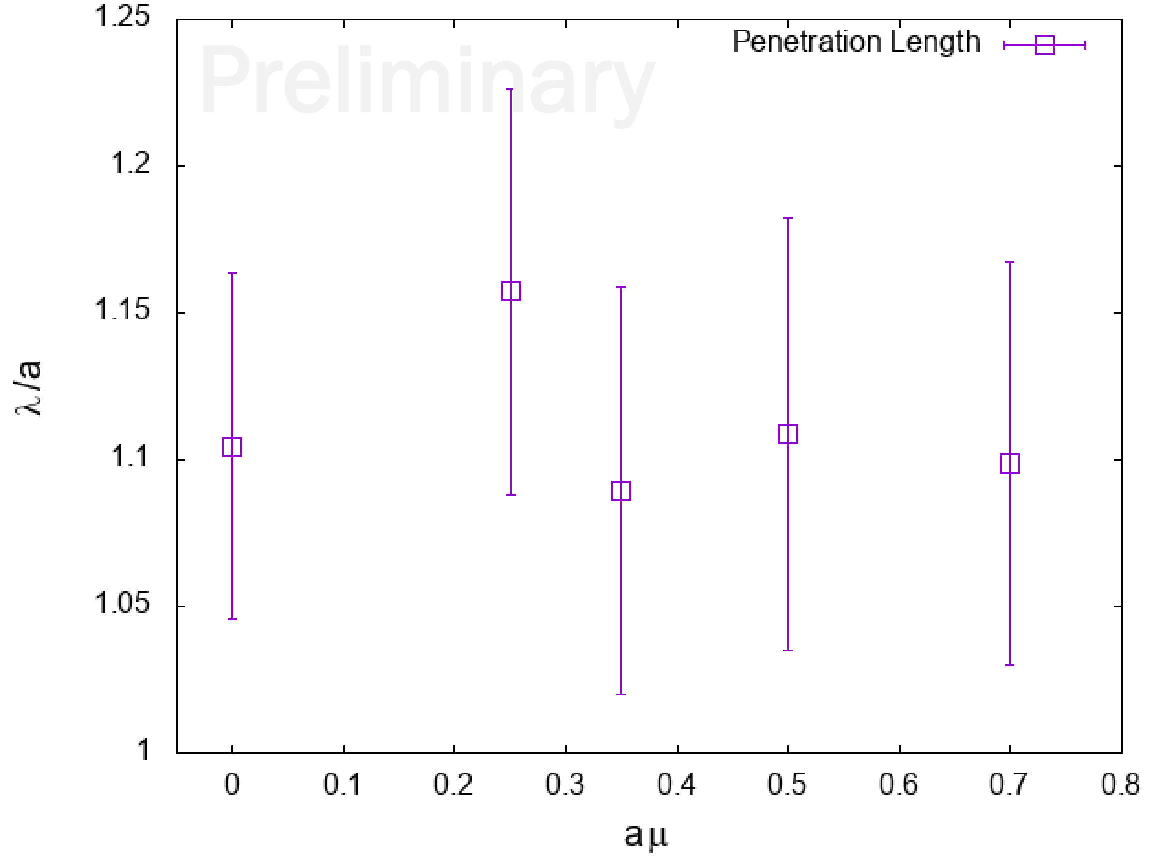

- No density dependence on penetration length

# $K_0$ form fitting $E_z(r) = \frac{\phi}{2\pi\lambda^2} K_0\left(\frac{r}{\lambda}\right)$




- The $K_0$ form fitting works, except for r=0.

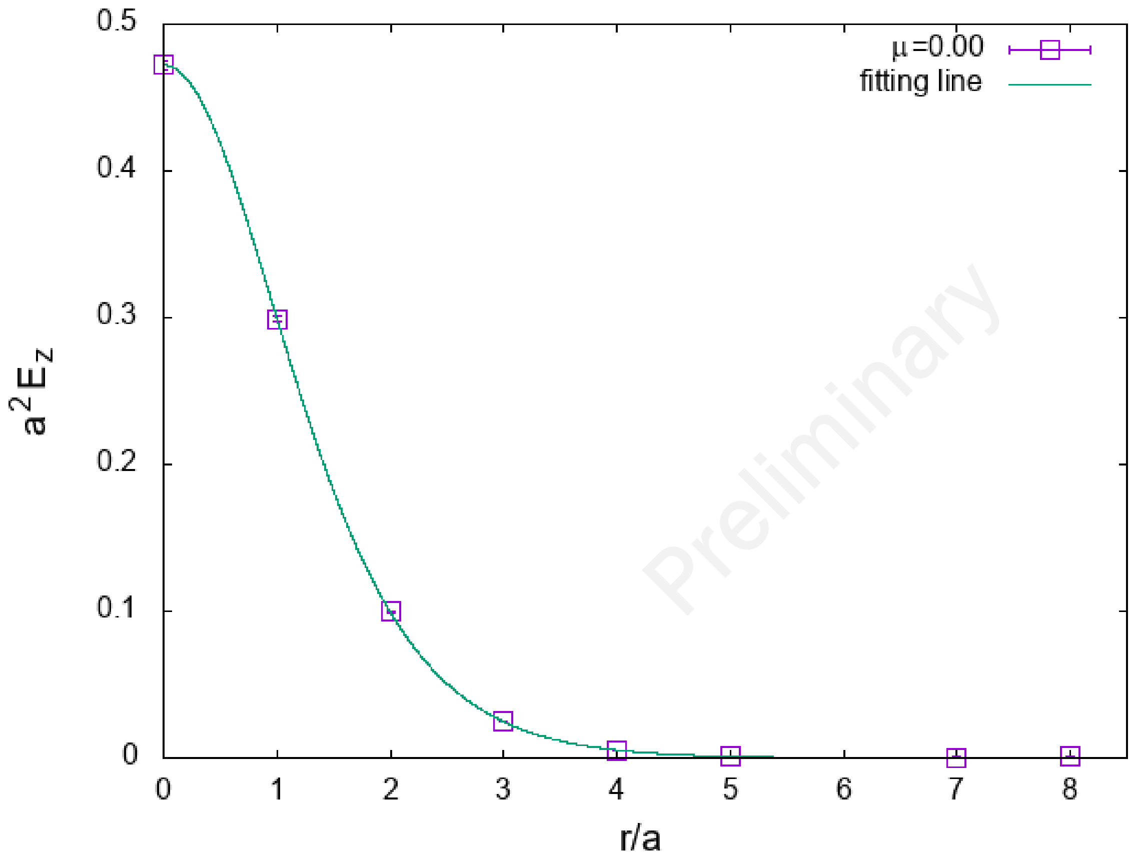
# Penetration length ($K_0$ form fitting)




- No density dependence on penetration length

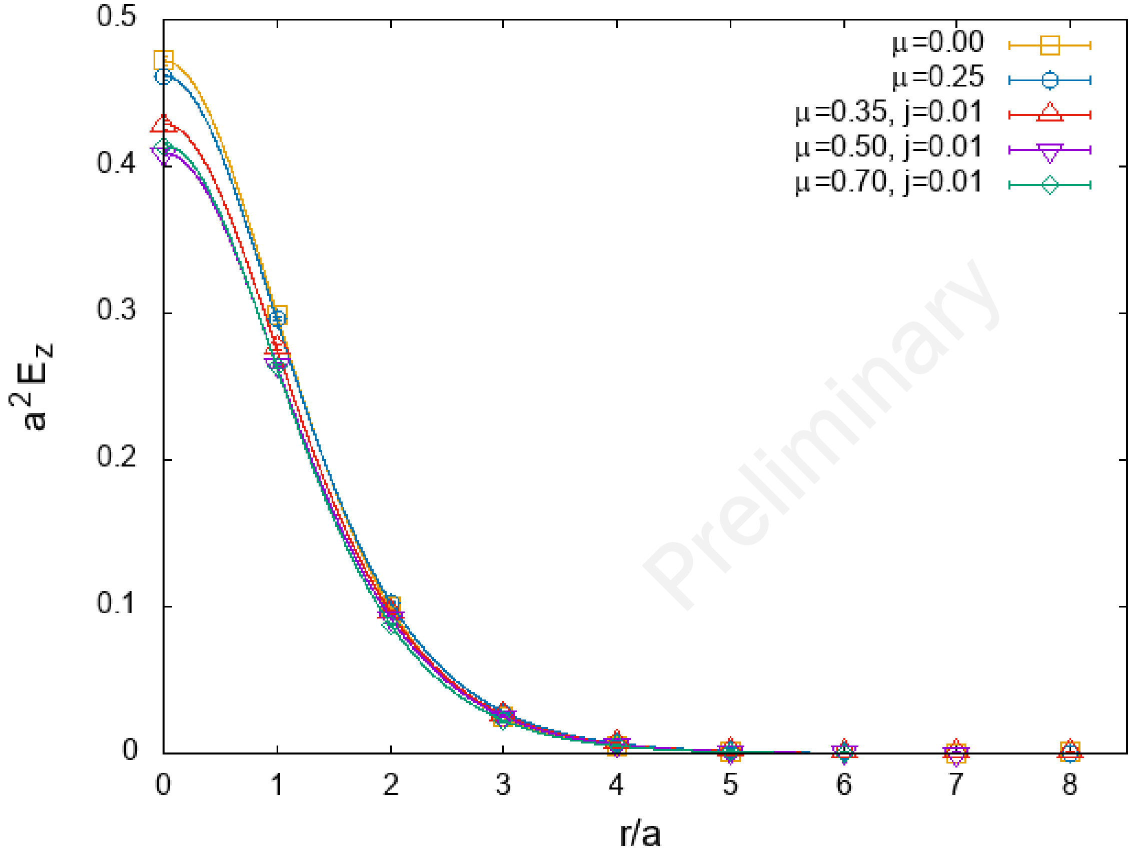
# Clem form

$$E_z(r) = \frac{\phi}{2\pi\lambda^2\alpha}\frac{K_0[(r^2/\lambda^2+\alpha^2)^{1/2}]}{K_1[\alpha]}$$



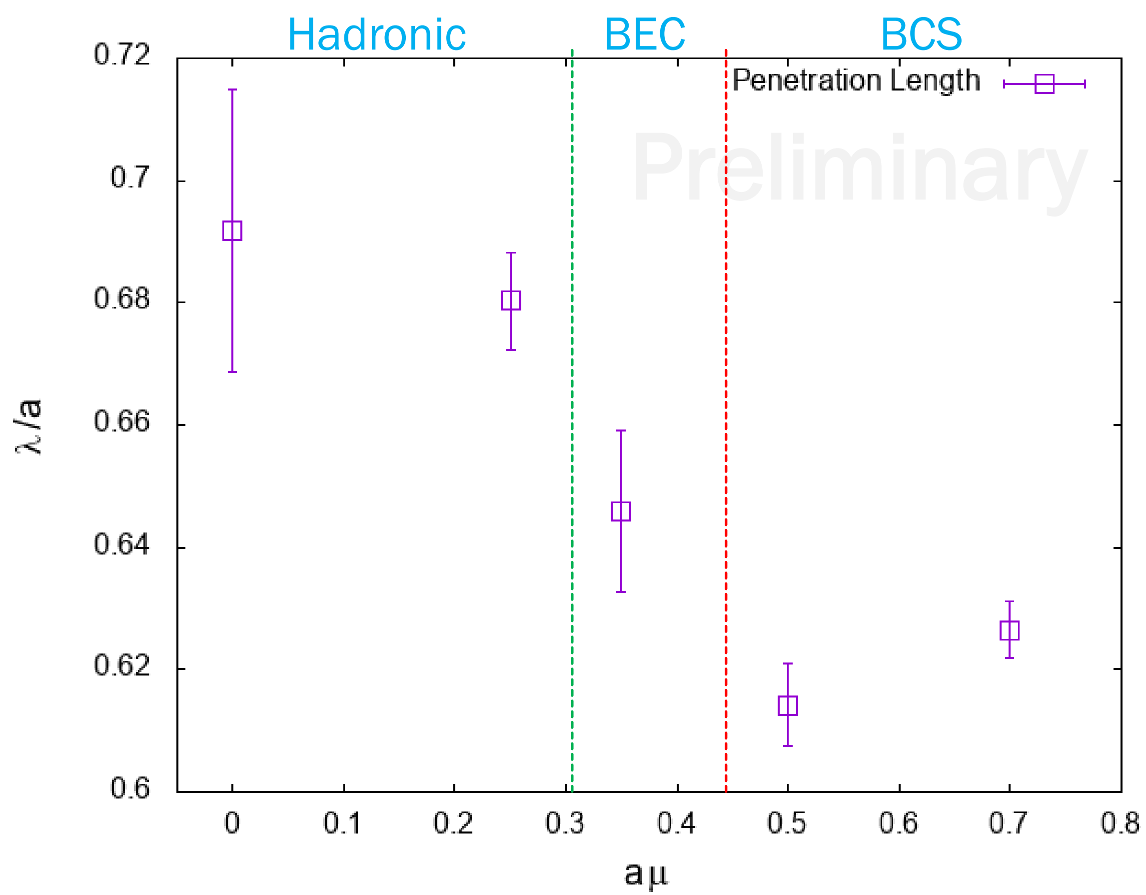

- Fitting by the Clem form is working, including r=0.

# Density dependence of Ez



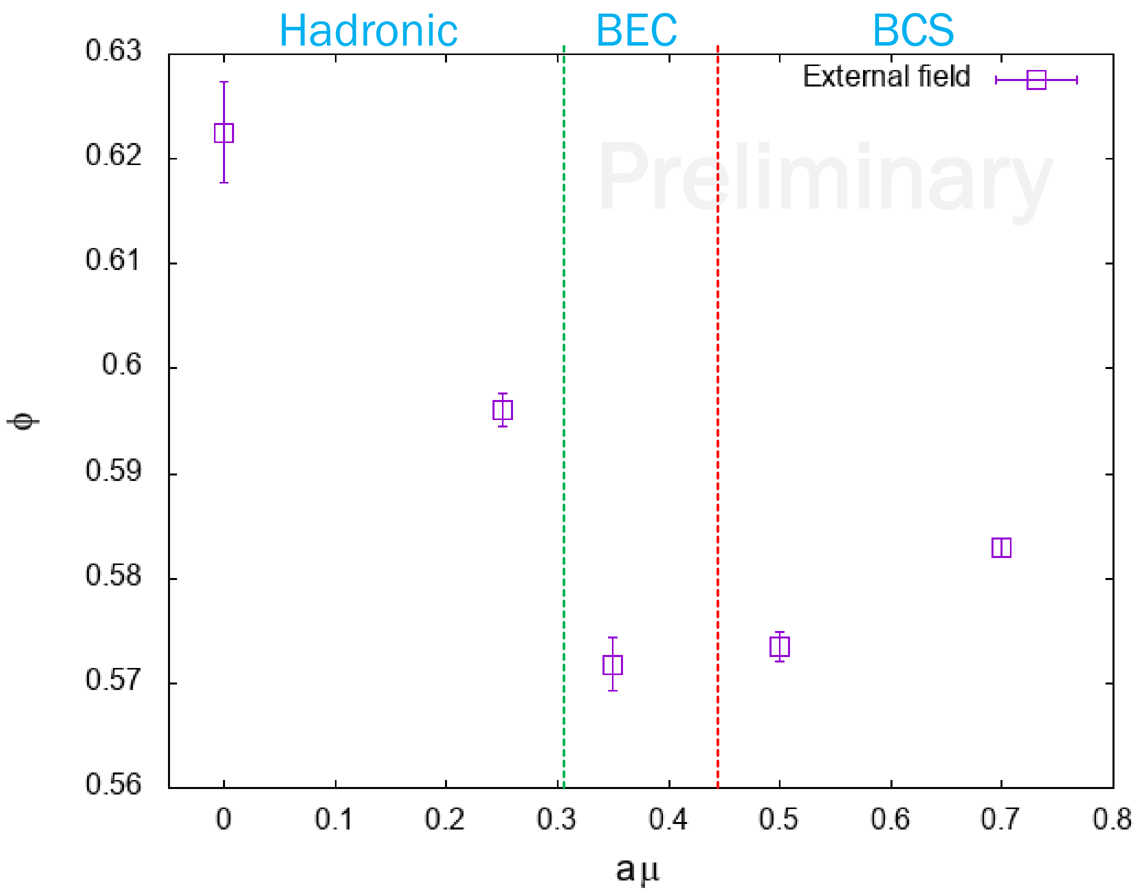

- Fittings by the Clem form is working in all μ region.

# Penetration length



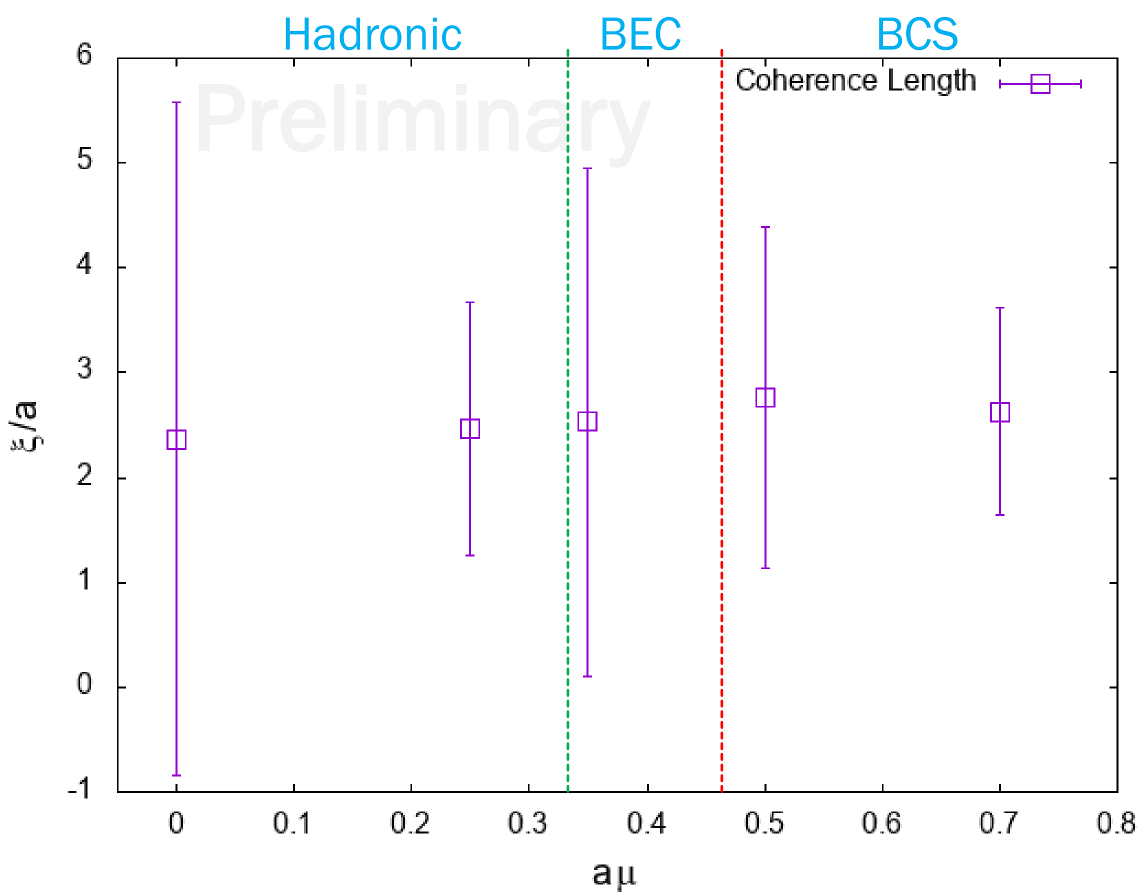

- The penetration length fitted by the Clem form shows a density dependence The penetration length gradually decreases with μ.

# External field Φ



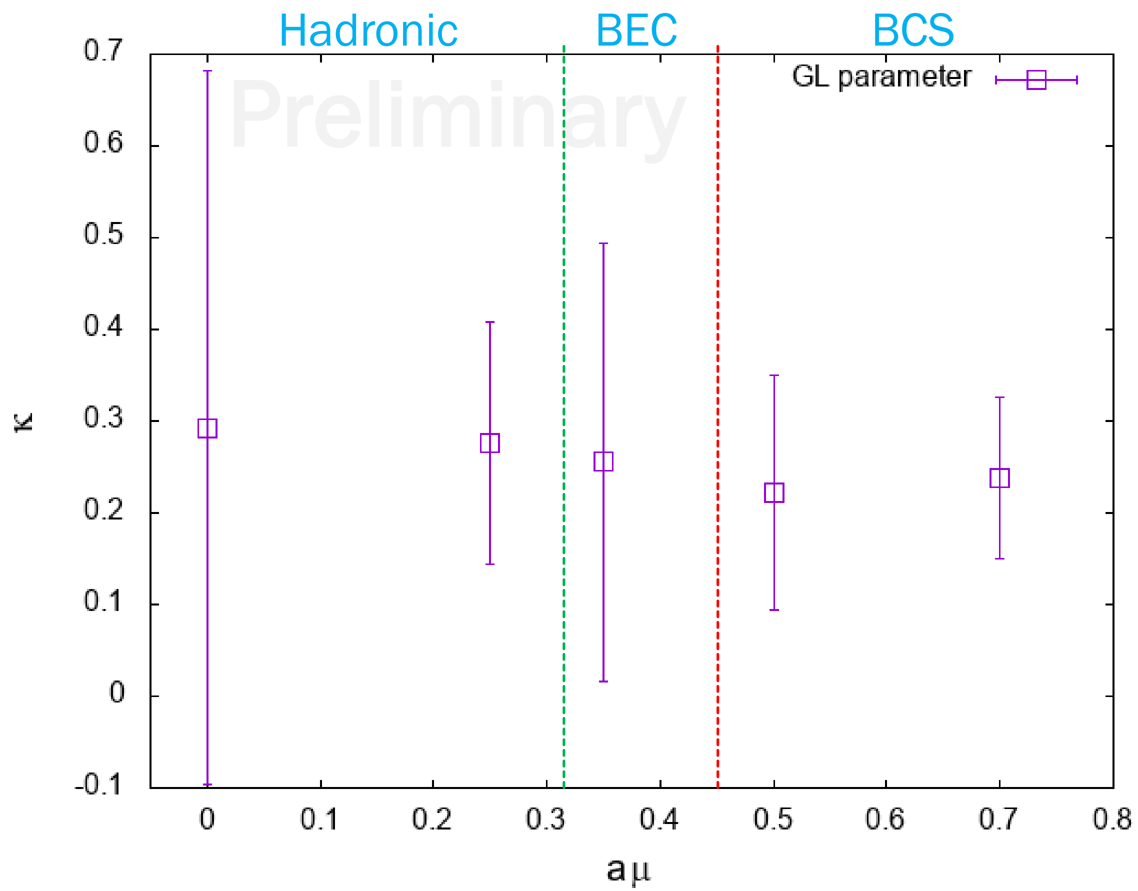

- External field fitted in the Clem form shows a density dependence. Φ decreases until μ=0.35 and appears to remain constant thereafter.

# Coherence length



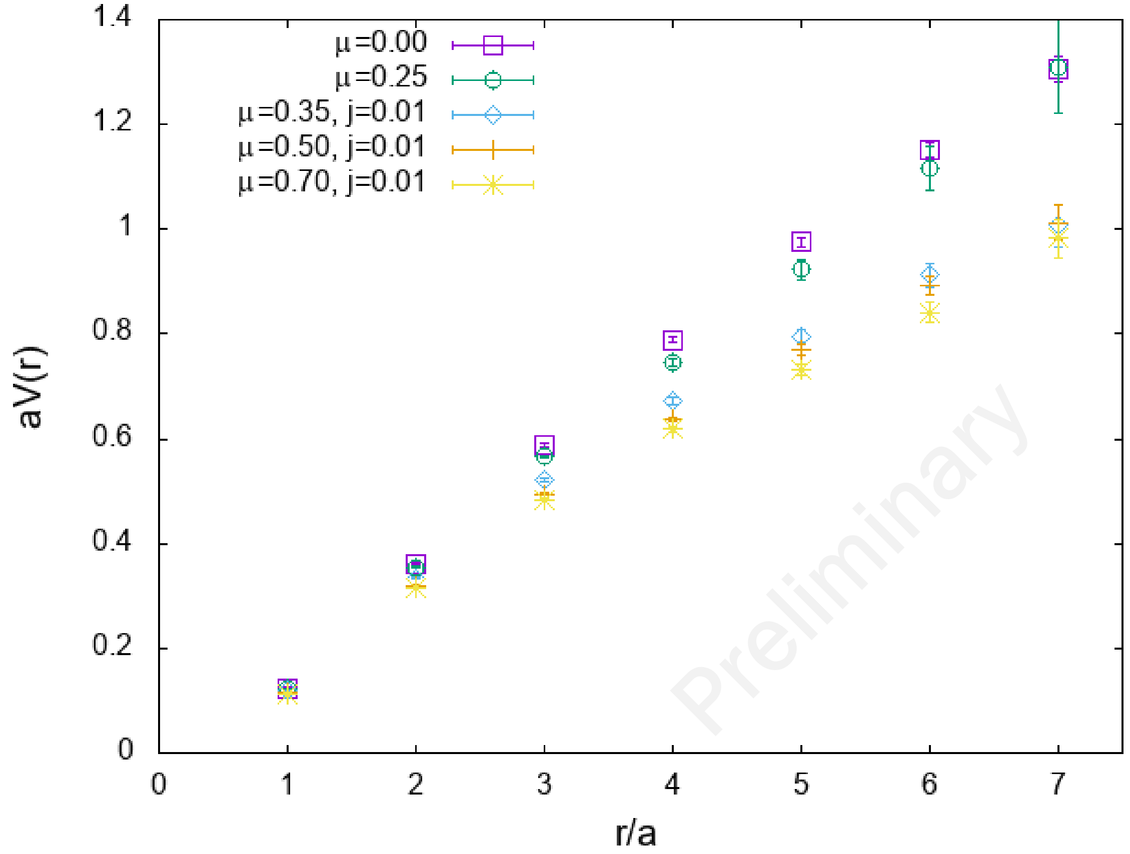

- The values of coherence length are almost constant in all μ region, although the error bars are large.

# Ginzburg-Landau parameter $\kappa = \lambda/\xi$




- Although the error bars are large, the values of Ginzburg-Landau parameter show almost constant or slightly decreasing with μ. This result suggests that the vacuum of two-color QCD is a type 1 dual superconductor.

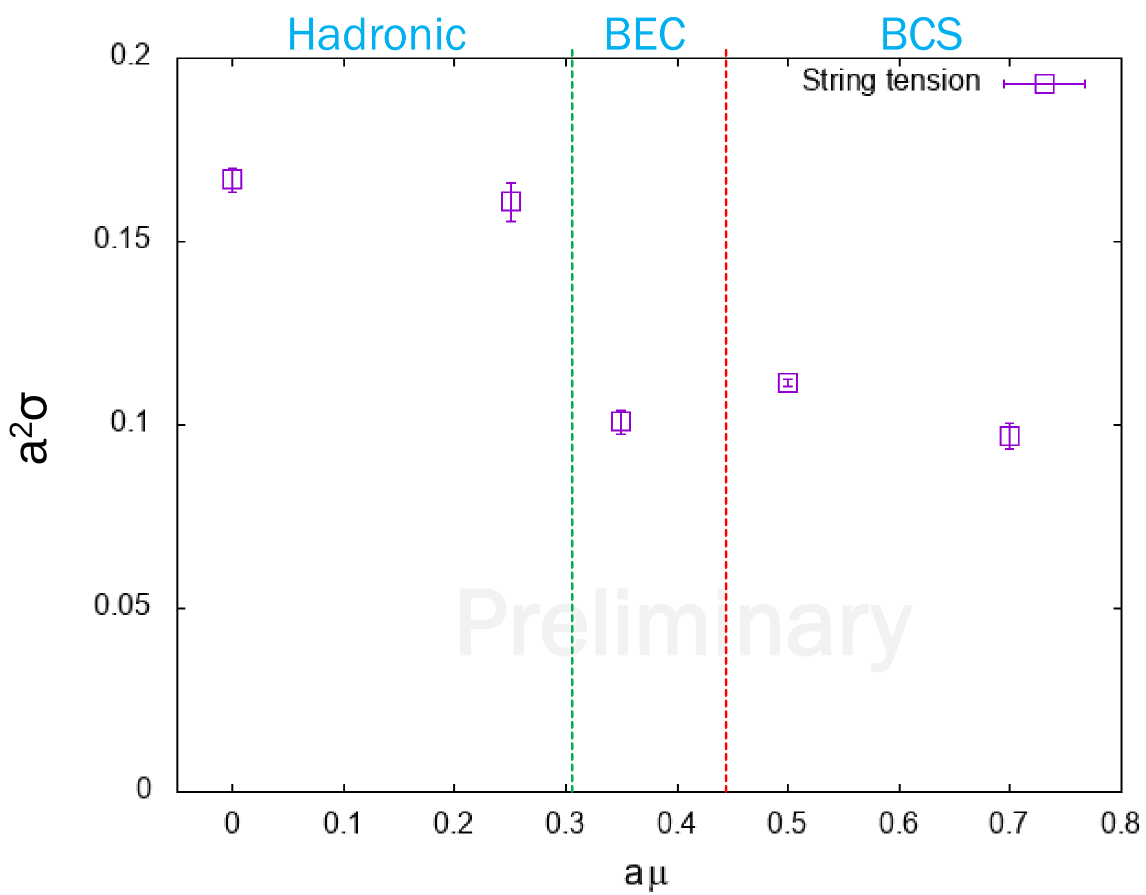
# Static potential




- Static potentials from Wilson loop operator and their density dependence .

# String tension

$$V(r) = \sigma r + \alpha / r + c$$




- The value of string tension σ varies around μ=0.35 and has a non-zero value even in the high μ region. This indicates that the system at T=0.45Tc is in a confined phase for all μ region within the scope of this study.

# Summary

- Calculate the μ dependence of the color electric fields in Nf=2 two-color QCD at T=0.45Tc.
- Penetration length of Ez
  - No μ dependence in exponential and $K_0$ form
  - Slightly μ dependence in the Clem form
- External flux
  - μ dependence
- GL parameter
  - No μ dependence or slightly μ dependence
  - Suggest type 1 dual superconductor

- Remain flux tube squeezing in all μ region at T=0.45Tc
- Non-zero string tension in all μ region

- The system at T=0.45Tc is in a confined phase for all μ region within the scope of this study.
- Consistent for the previous results of Polyakov loop and topological susceptibility

- In future works
  - Large lattice
  - Different lattice spacings
  - Different temperatures (0.89Tc, ···)