# Two-color QCD phases and the topology at low temperature and high density

Etsuko Itou

Refs:

(1) K.Iida, EI, T.-G. Lee: JHEP2001(2020)181

(2) K.Iida, EI, T.-G. Lee: arXiv:2008.06322

(3) T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

(4) T.Hirakida, EI, H.Kouno: PTEP 2019 (2019) 033B01

YITP workshop

**Probing the physics of high-density and low-temperature matter with ab initio calculations in 2-color QCD**

online, 2020/11/6

(1) K.Iida, EI, T.-G. Lee: JHEP2001(2020)181

Phase diagram in $T-\mu$ plane for Nc=Nf=2 QCD

beta=0.8 (Iwasaki gauge + Wilson fermion)

16^4 : T=0.45Tc (~ 90MeV)

32^3x8: T=0.89Tc (~ 180MeV)

Cf.)Tc=200MeV

(2) K.Iida, EI, T.-G. Lee: arXiv:2008.06322

Scale setting of Nc=Nf=2 QCD at $\mu=0$

(3) T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

Anomaly matching and phase diagram Nc=Nf=2 QCD at massless point

Furusawa-san's talk, yesterday

(4) T.Hirakida, EI, H.Kouno: PTEP 2019 (2019) 033B01

Thermodynamics of pure SU(2) gauge theory

(Show Nc dependence)

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Phase diagram in $T-\mu$ plane for Nc=Nf=2 QCD

beta=0.8 (Iwasaki gauge + Wilson fermion)

16^4 : T=0.39Tc (~ 79MeV)

32^3x8: T=0.79Tc (~ 156MeV)

Cf.)Tc=200MeV

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# Plan of talk

1. Why 2color QCD?

   Sign problem and numerical-instability problem

2. Definition of phase

   Spontaneous flavor symmetry breaking in Nc=Nf=2

3. Simulation results

   Phase diagram at T=0.39Tc, 0.79Tc

   Topological susceptibility

4. Summary and discussion

   A role of nontrivial topology in the phase diagram

# Motivation

System of few quarks/hadrons has been well-understood!!

How about finite-density system....?

Although the real system exists, it is hard to obtain something theoretically.


日経サイエンス2020年1月号

LHCb, RHIC (mid-density, high-T)

Neutron star (high-density, low-T)

LIGO NICER

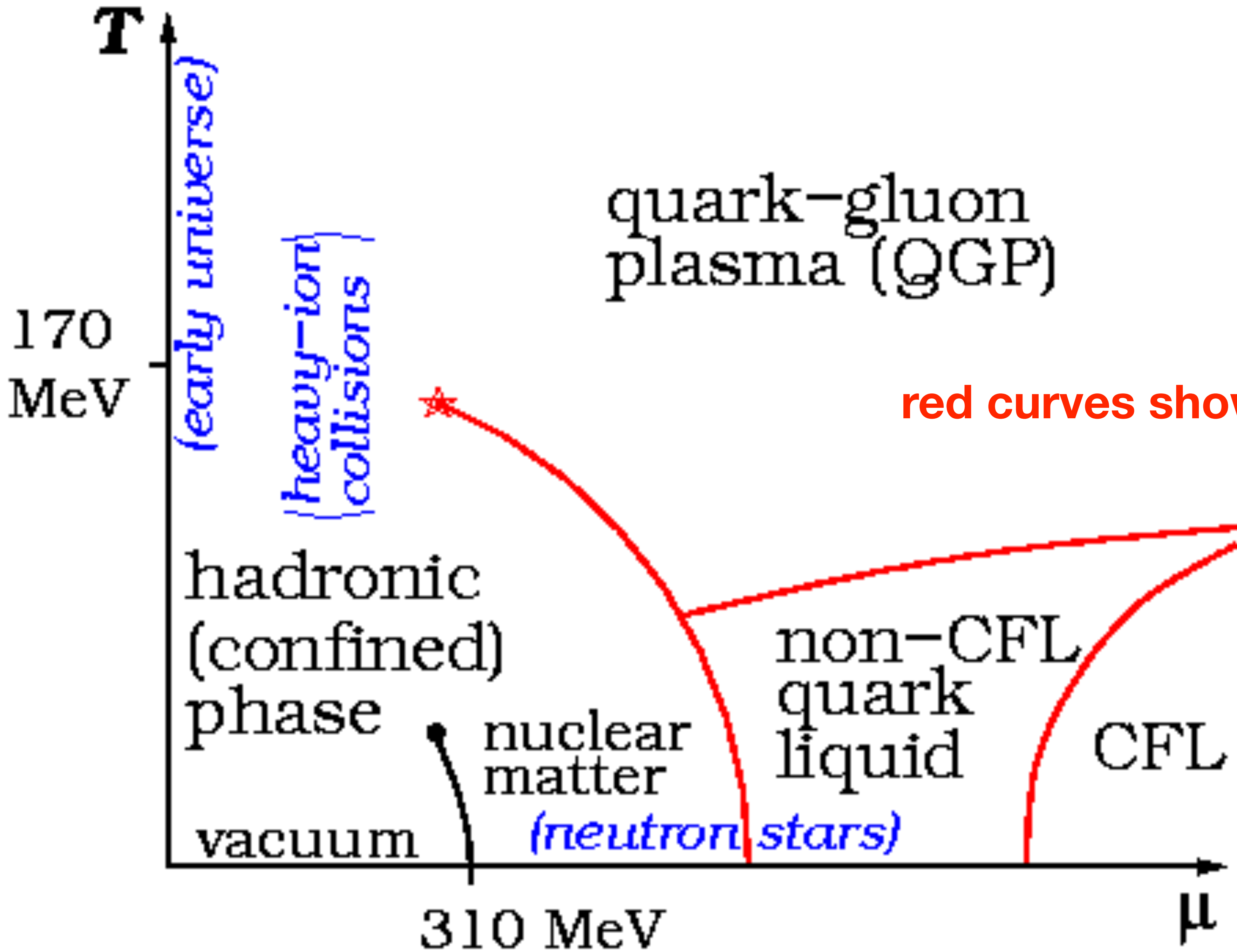
Naively we expect that quarks make a boson to avoid Fermi degeneracy, and bosons form some condensates

## What we want to know？

- Phase diagram on $T-\mu$ plane
- Nonperturbative objects (instanton, monopole)
- $\mu$ dependence of hadron masses and nuclear force
- Eq. of state (pressure, internal energy, entropy)
- Transport coeff. (Viscosity, superfluid density)

# Schematic picture

QCD phase diagram in Wikipedia

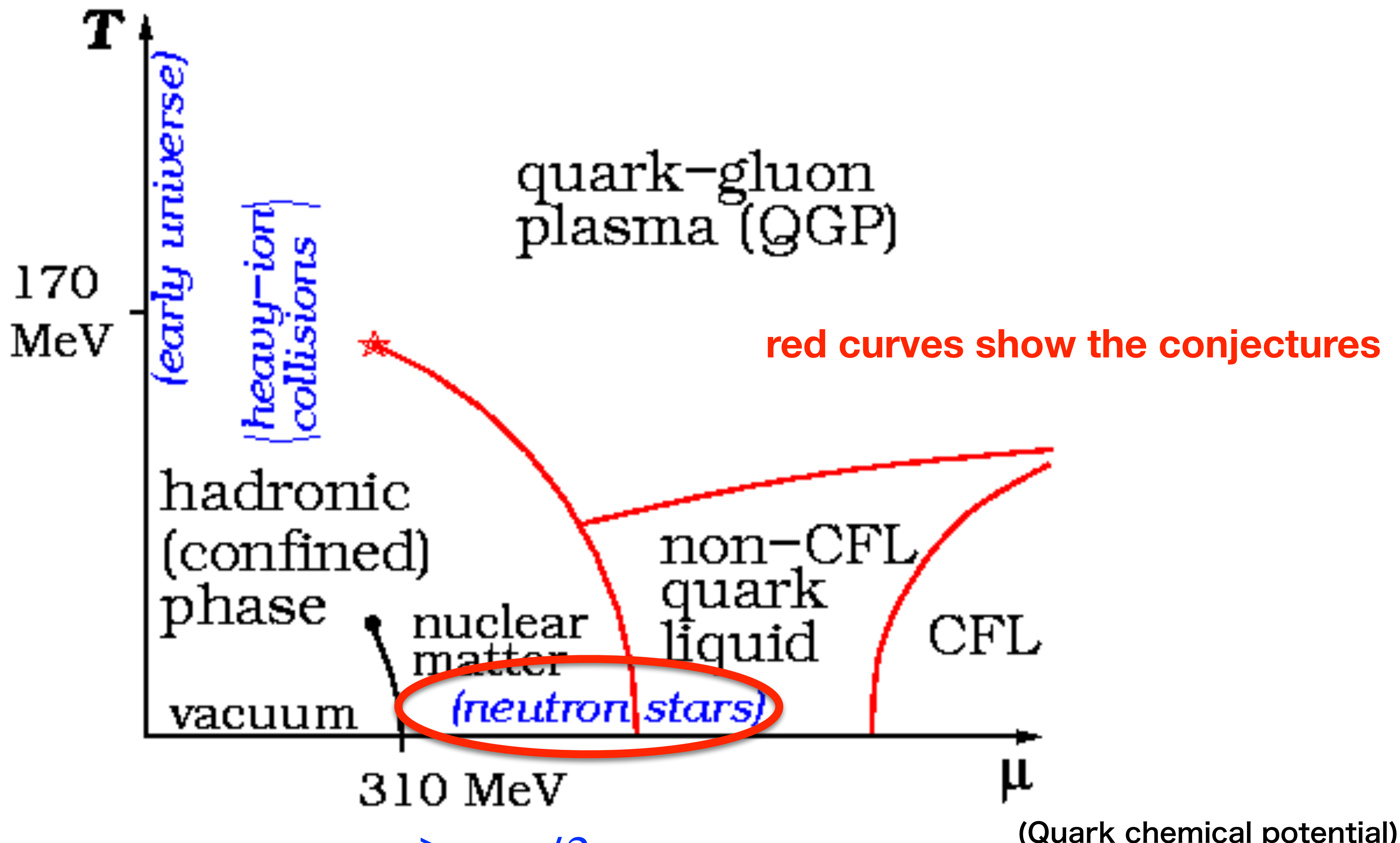



red curves show the conjectures

(Quark chemical potential)

# Schematic picture

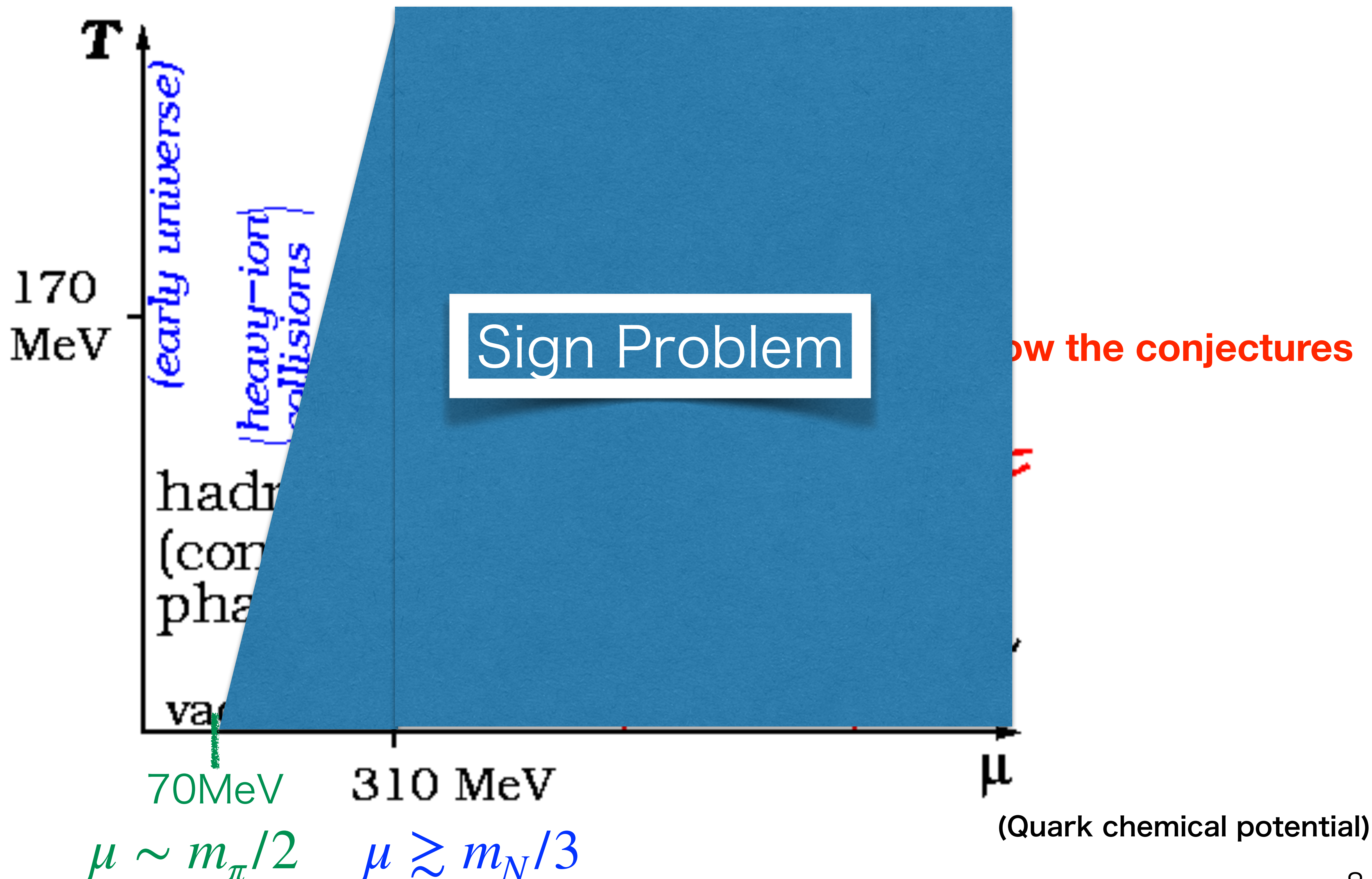
QCD phase diagram in Wikipedia




$\mu \gtrsim m_N/3$

# Schematic picture

QCD phase diagram in Wikipedia



# 2color QCD

A simple reduction of real finite-density QCD

## (1) sign problem

Avoid the sign problem (consider 2color 2flavor QCD)

## (2) Numerical instability $\mu/m_{PS} \geq 1/2$ in low-T

Introduce the diquark source in the action

cf.) diquark -> $\pi^-$ in 3-color QCD with isospin chemical

D. H. Rischke, D. T. Son and M. A. Stephanov, Phys. Rev. Lett.87(2001) 062001
D. T. Son and M. A. Stephanov, Phys. Atom. Nucl.64(2001) 83
B. B. Brandt, G. Endrodi and S. Schmalzbauer, Phys. Rev.D 97(2018) 05451

# Action with diquark source term

## Fermion action in continuum limit

$$S_F^{cont.} = \underbrace{\int d^4x \bar{\psi}(x)(\gamma_\mu D_\mu + m)\psi(x)}_{\text{QCD}} + \underbrace{\mu \hat{N}}_{\text{Number op.}} - \underbrace{\frac{j}{2}(\bar{\psi}_1 K \bar{\psi}_2^T - \psi_2^T K \psi_1)}_{\text{diquark source}}$$

QCD Number op. diquark source

Related works on Nc=2 with even # flavor

**Kogut et al.** NPB642 (2002)18, **Alles et al.** NPB752 (2006)124,
**Hands et al.** NPB752 (2006) 124, PRD81 (2010) 091502,, EPJ. A47 (2011) 60, PRD87 (2013) 034507, **Kotov et al**. PRD94 (2016) 114510, JHEP 1803 (2018) 161

The QCD phase diagram appears in the j->0 limit

## Fermion action on the lattice

$$\det[\mathcal{M}^\dagger \mathcal{M}]^{1/2} = \det[\Delta^\dagger(\mu)\Delta(\mu) + |\bar{J}|^2]^{1/2} \det[\Delta^\dagger(-\mu)\Delta(-\mu) + |J|^2]^{1/2}$$

j-source lifts the eigenvalue of Dirac op. up

# 2 color QCD vs 3 color QCD

(At least $\mu = 0$)qualitative properties are the same

Low temperature：

Confinement、SSB of chiral sym. (In 2color massless QCD it is possible no SSB chiral sym.), nontrivial topological background (instanton)

Order of meson spectra

High temperature：

Deconfinement (QGP phase) , it is consistent with RHIC experiment

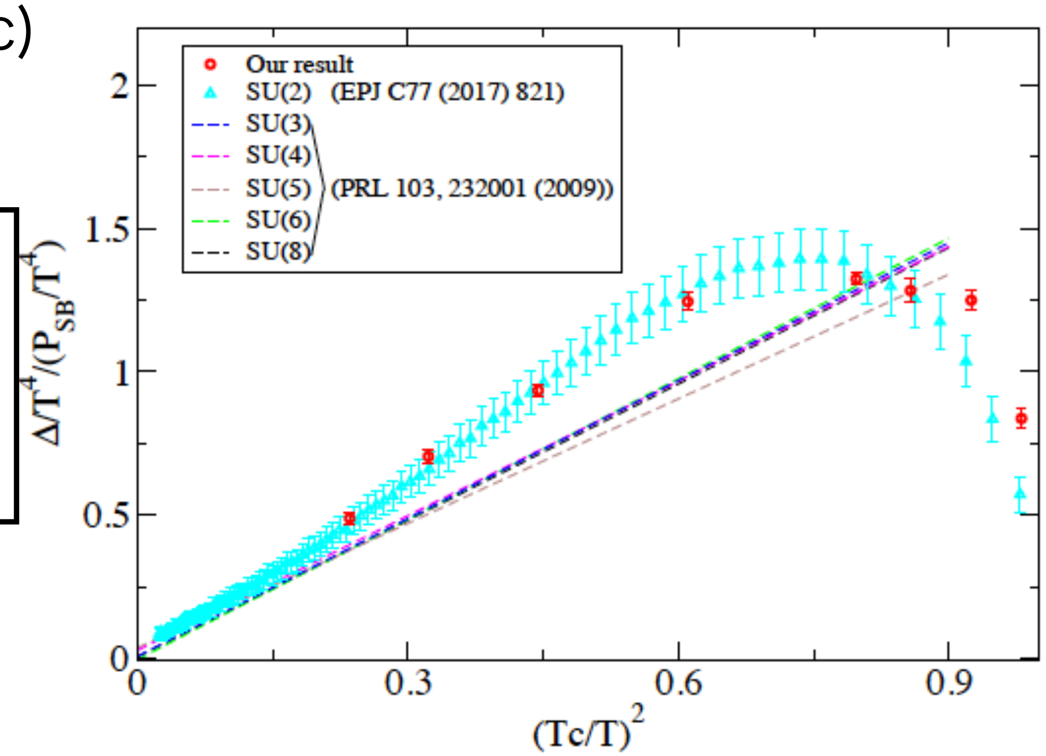
Restoration of chiral sym.

Equation of state and transport coefficients (shear viscosity) as a function of T

Quantitatively, two theories have a tiny difference…?

T. Hirakida, EI, H. Kouno, PTEP **2019** (2019) 033B01

(Ex.) Trace anomaly ($\Delta = (\epsilon - 3p)$) of pure SU(Nc) gauge theories with several Nc




(Comments)

To compare the quantities among different Nc (or mass) theories, taking dim-less quantities (T/Tc and $\mu/m_{PS}$) are better.

Cf.) Original QCD phase diagram : T and $\mu$ [MeV]

# Plan of talk

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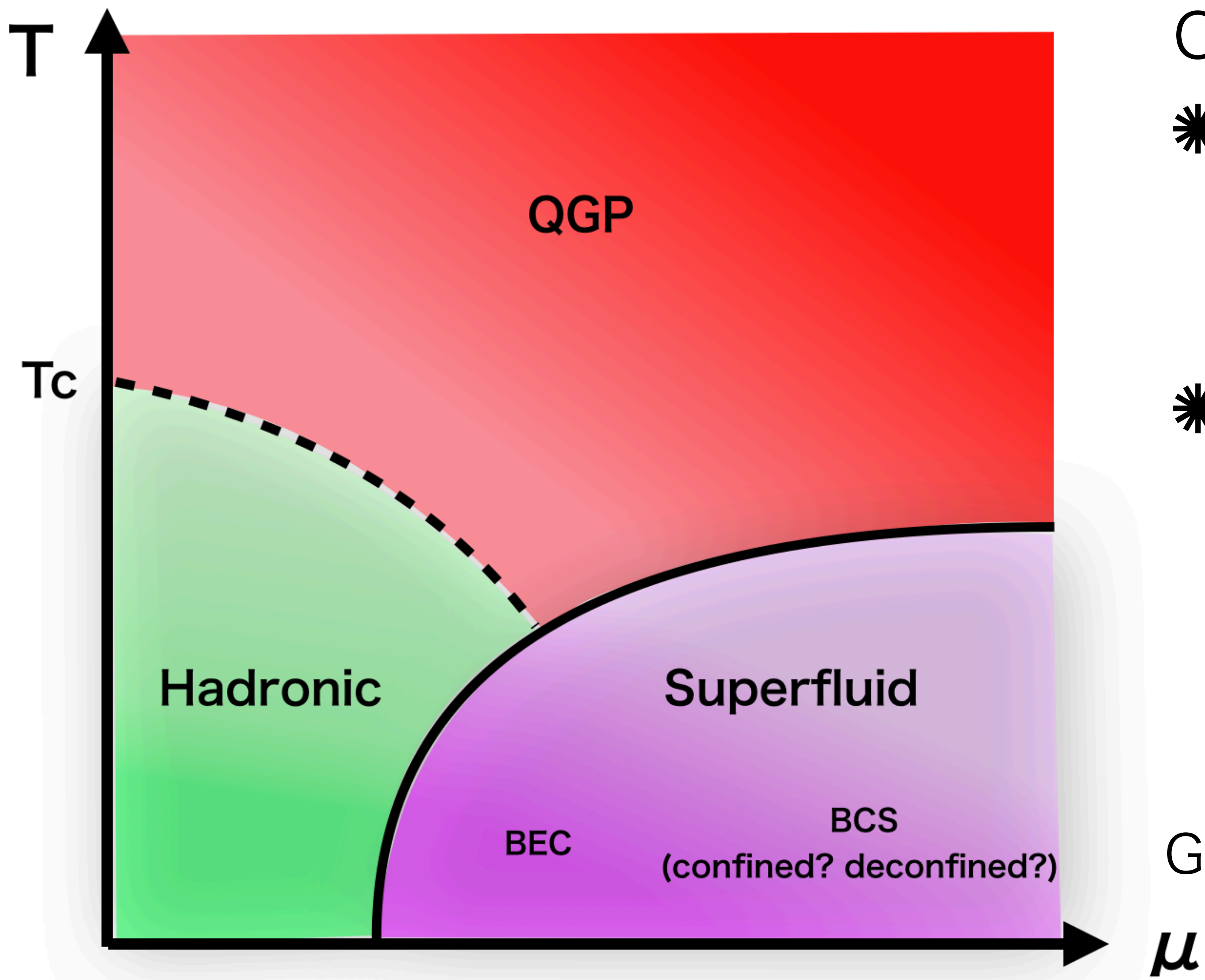
# Expected phase diagram in Two-color QCD




Order parameters

❋ Polyakov loop

$\langle |L| \rangle \sim 0$ confined

$\langle |L| \rangle \neq 0$ deconfined

❋ (Isoscalar) diquark cond.

(dynamical scale: diquark gap $\Delta(\mu)$)

$\langle qq \rangle = 0$ no superfluidity

$\langle qq \rangle \neq 0$ superfluidity

Goldstone mode of $U(1)_B$ sym. breaking

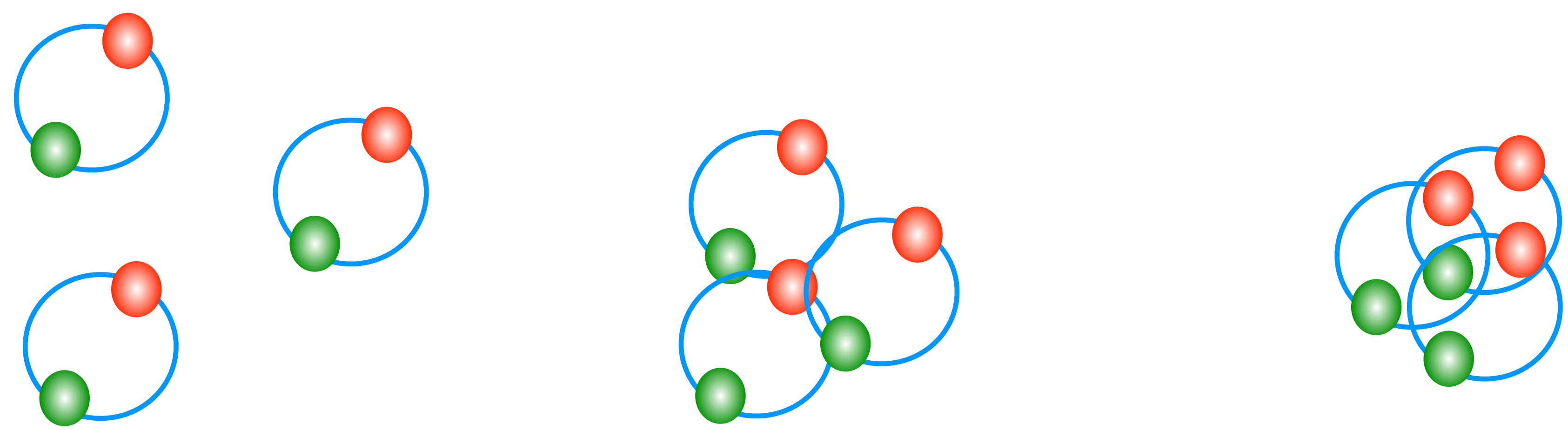
$$\psi \to e^{i\alpha}\psi \quad \bar{\psi} \to \bar{\psi}e^{-i\alpha}$$

| | Hadronic | QGP | Superfluid BEC | Superfluid BCS |
|---|---|---|---|---|
| $\langle \lvert L \rvert \rangle$ | zero | non-zero | | |
| $\langle qq \rangle$ | zero | zero | non-zero | $\propto \Delta(\mu)\mu^2$ |
| $\langle n_q \rangle$ | | | | |

# BEC / BCS crossover in superfluid phase

BEC phase

BCS phase

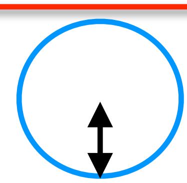


Distance between quarks $\gg \Delta^{-1}$

Distance between quarks $\ll \Delta^{-1}$

Quarks behave free particles

density

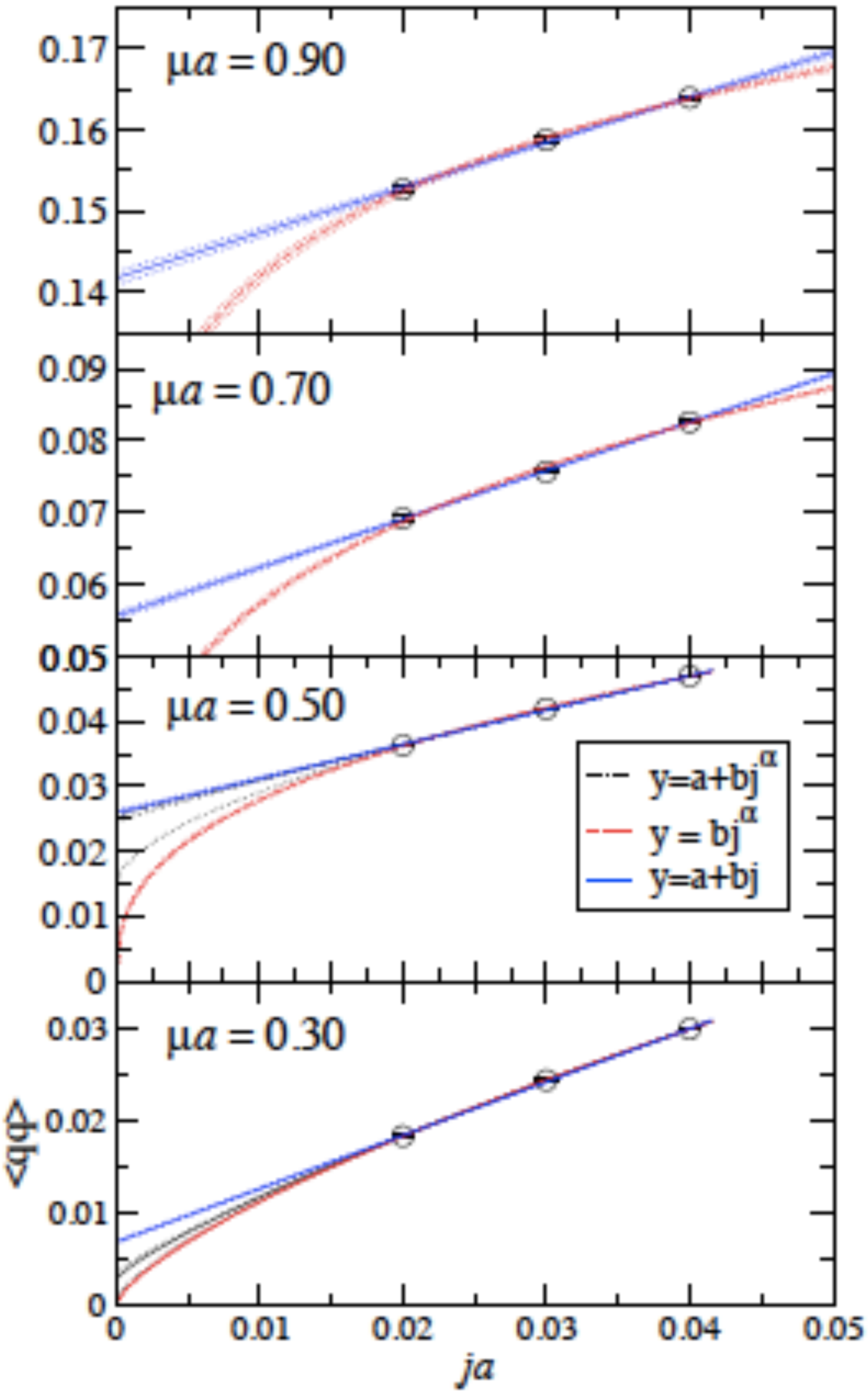
 $\Delta^{-1}$ Here, $\Delta$: gap energy

Number density of free particle $n_q^{\text{tree}}(\mu) = \frac{4N_cN_f}{N_s^3N_\tau}\sum_k \frac{i\sin\tilde{k}_0[\sum_i \cos k_i - \frac{1}{2\kappa}]}{[\frac{1}{2\kappa} - \sum_\nu \cos\tilde{k}_\nu]^2 + \sum_\nu \sin^2\tilde{k}_\nu}$

| | Hadronic | QGP | Superfluid BEC | Superfluid BCS |
|---|---|---|---|---|
| $\langle \lvert L \rvert \rangle$ | zero | non-zero | | |
| $\langle qq \rangle$ | zero | zero | non-zero | $\propto \Delta(\mu)\mu^2$ |
| $\langle n_q \rangle$ | | | non-zero | $n_q/n_q^{\text{tree}} \approx 1$ |

# Technical progresses in our work

# diquark cond. with j=0.02, 0.03, 0.04




S.Cotter et al. Phys.Rev. **D87** (2013) 034507

j -> 0 extrapolation is a hard task

Reweighing of j-parameter

- reweighing factor is almost unity, $(R_j - 1) \sim 10^{-3}$, in our calculations
- convergence of log-expansion is very well

Cf) B. B. Brandt, G. Endrodi and S. Schmalzbauer, Phys. Rev.D 97(2018) 05451

# To find diquark cond. in j=0 limit
# — reweighting —

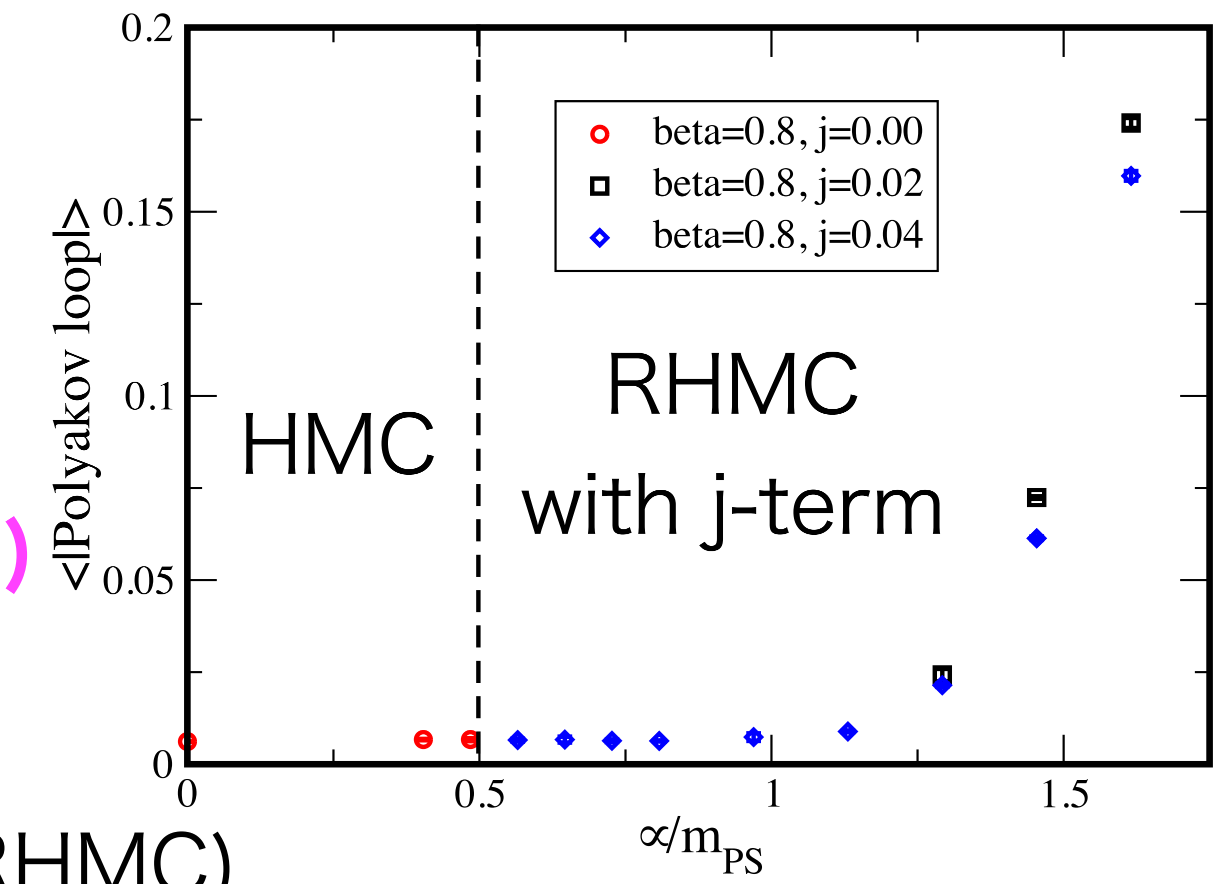
$\mu/m_{PS} \lesssim 0.5$

config. genration @j=0 (HMC)

measure by introducing

small j-source as a probe

J (measurement) > J0 (sampling)




$\mu/m_{PS} > 0.5$

config. genration @j=0.01- 0.04 (RHMC)

measure by introducing a probe j-source

@ j=0.001- 0.04

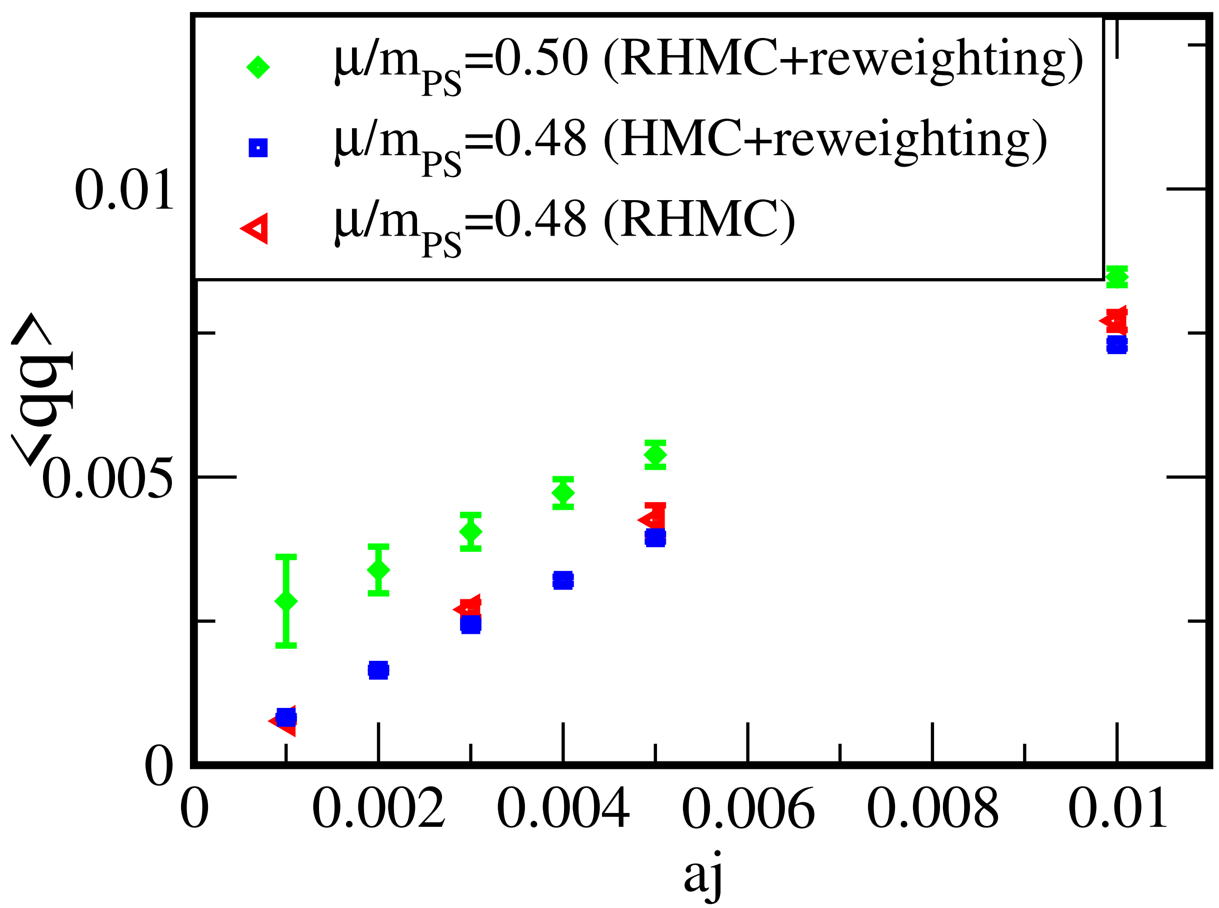
J (measurement) < J0 (sampling)

Both reweighting works very well.

(Reweighting factor is almost unity.)

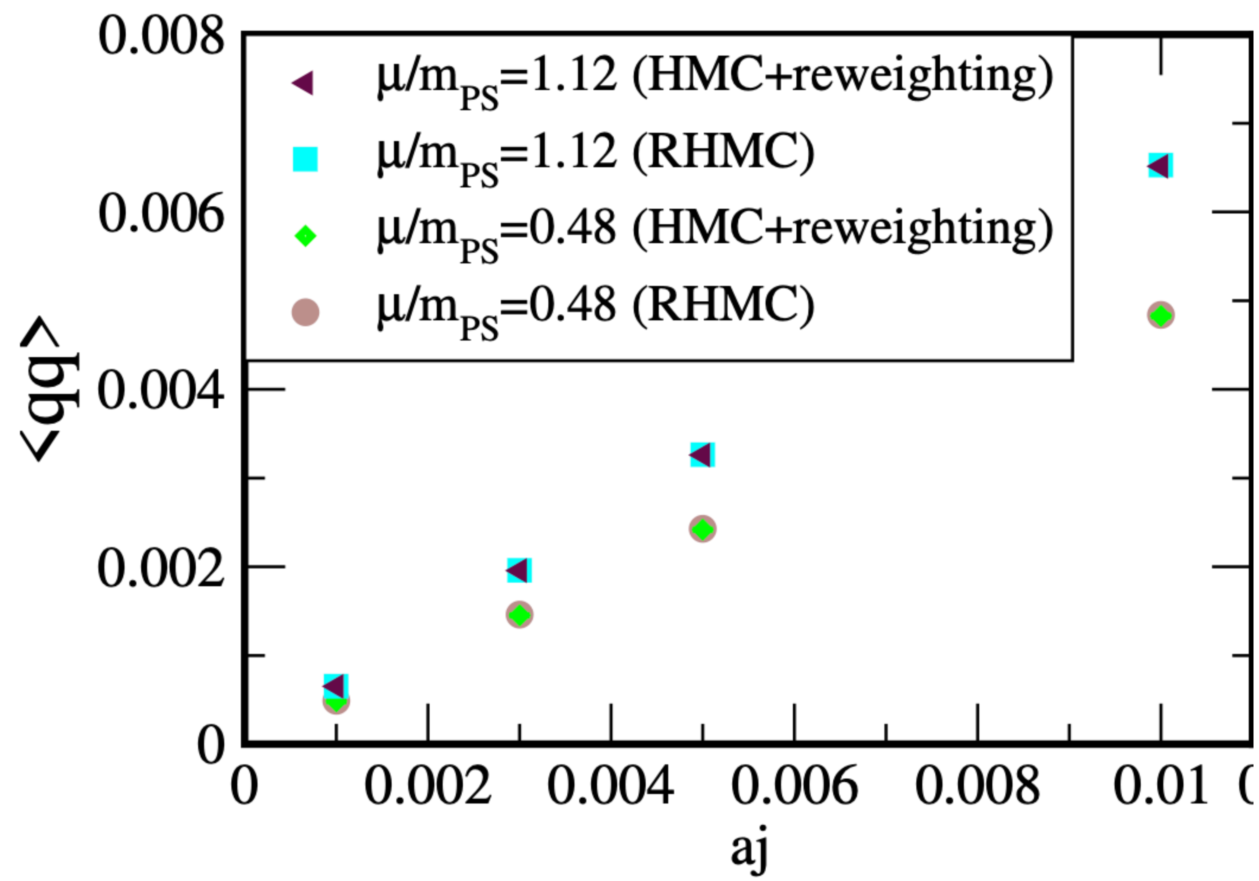
# Test of reweighting method

Lattice size: $16^4$




In j=0 limit, both blue and red data go to zero.

Lattice size: $32^3 \times 8$




Raw data is independent of the methodology.

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# Lattice setup

Lattice action:

Iwasaki gauge action + Nf=2 Wilson fermion

Include quark chemical potential + diquark source term

RHMC algorithm

Lattice parameter: beta=0.8

mass para. ($\kappa$) is tuned to be $m_{\mathrm{PS}}/m_{\mathrm{V}} = 0.823(9)$

at $\mu = 0$

Lattice size: 16^4 : T=0.39Tc (~ 79MeV)

32^3x8: T=0.79Tc (~ 158MeV)

By scale setting

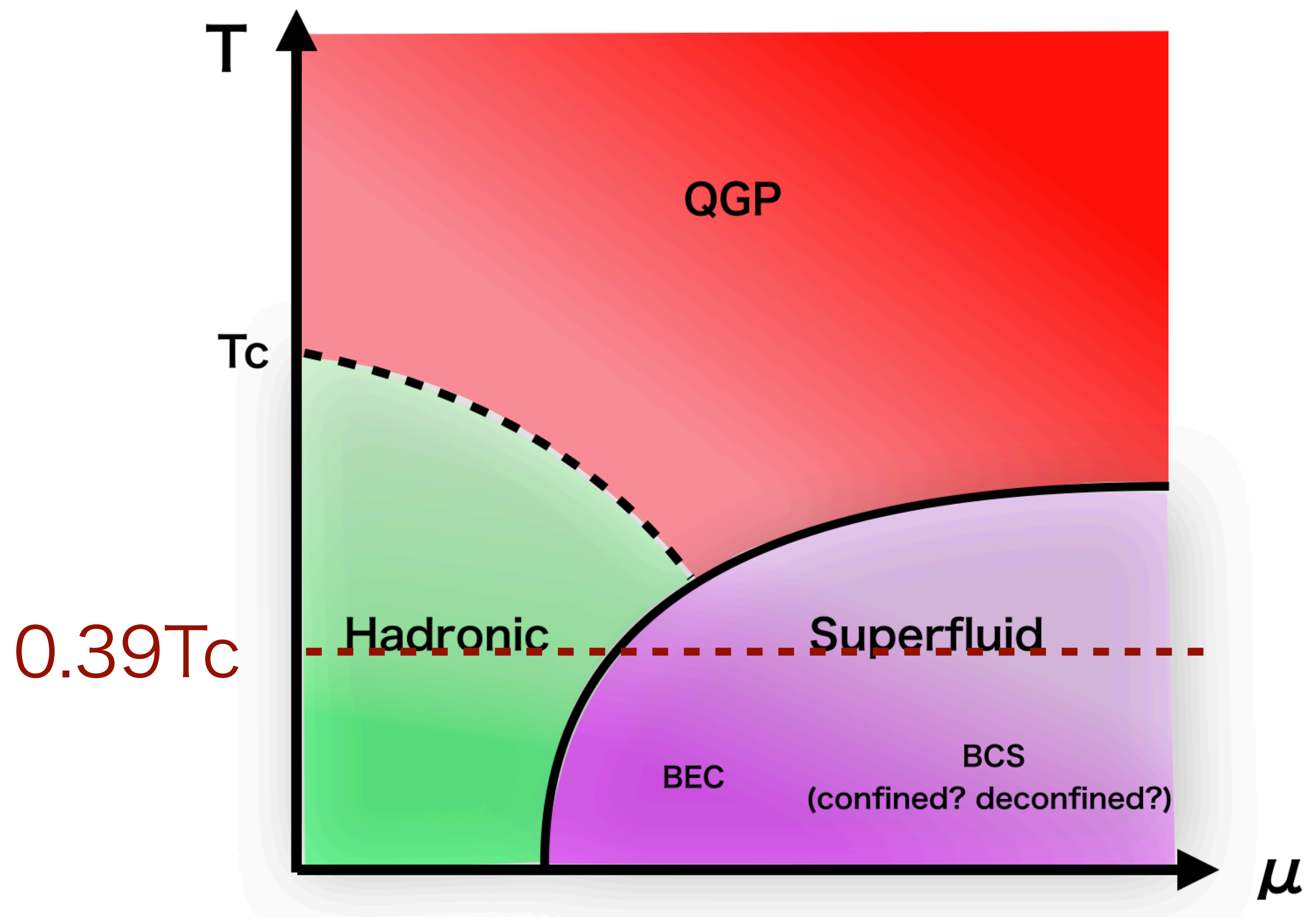
Tc: (chiral) critical temperature at $\mu = 0$

Parameter regime of chemical potential

$$\mu/T \leq 16, \quad \mu/m_{\mathrm{PS}} \leq 1.60$$

# Results

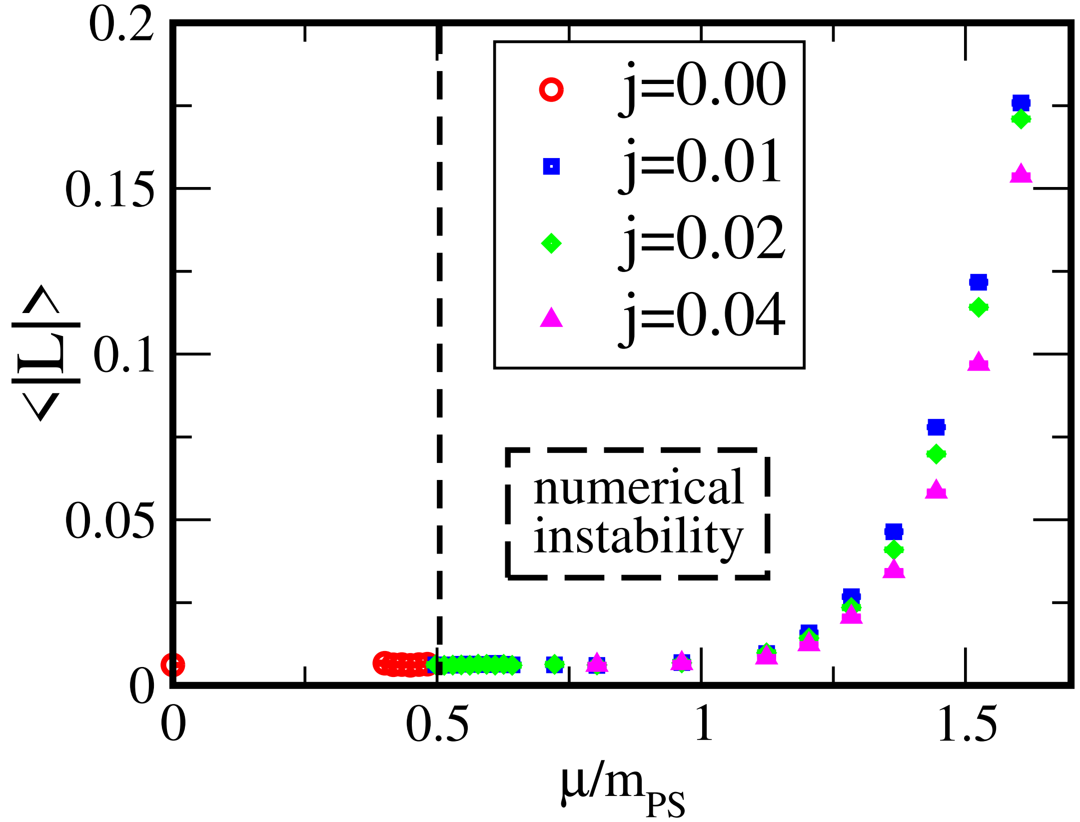
Lattice size: 16^4 :    T=0.39Tc (~ 79MeV)

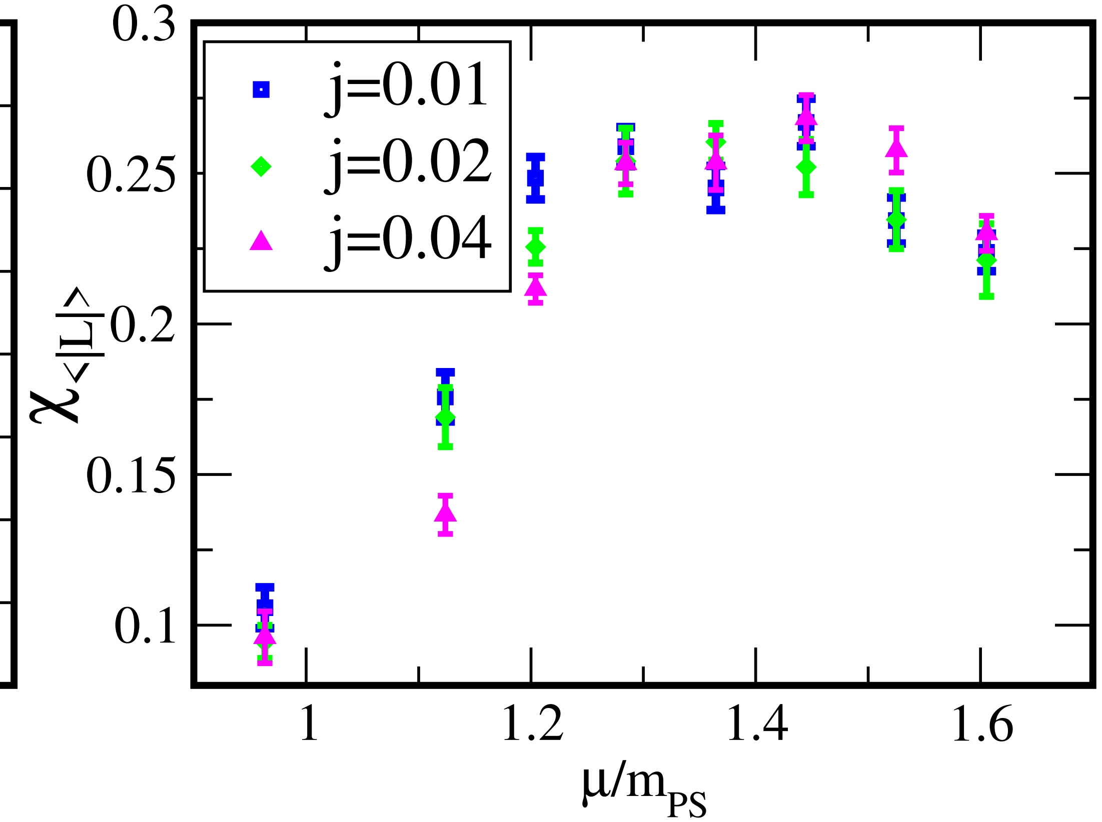


# Polyakov loop

$\langle L \rangle \approx 0 \ \ (F_q \approx \infty)$ : confinement

$\langle L \rangle \neq 0 \ \ (F_q \neq \infty)$ : deconfinement




Susceptibility has a peak

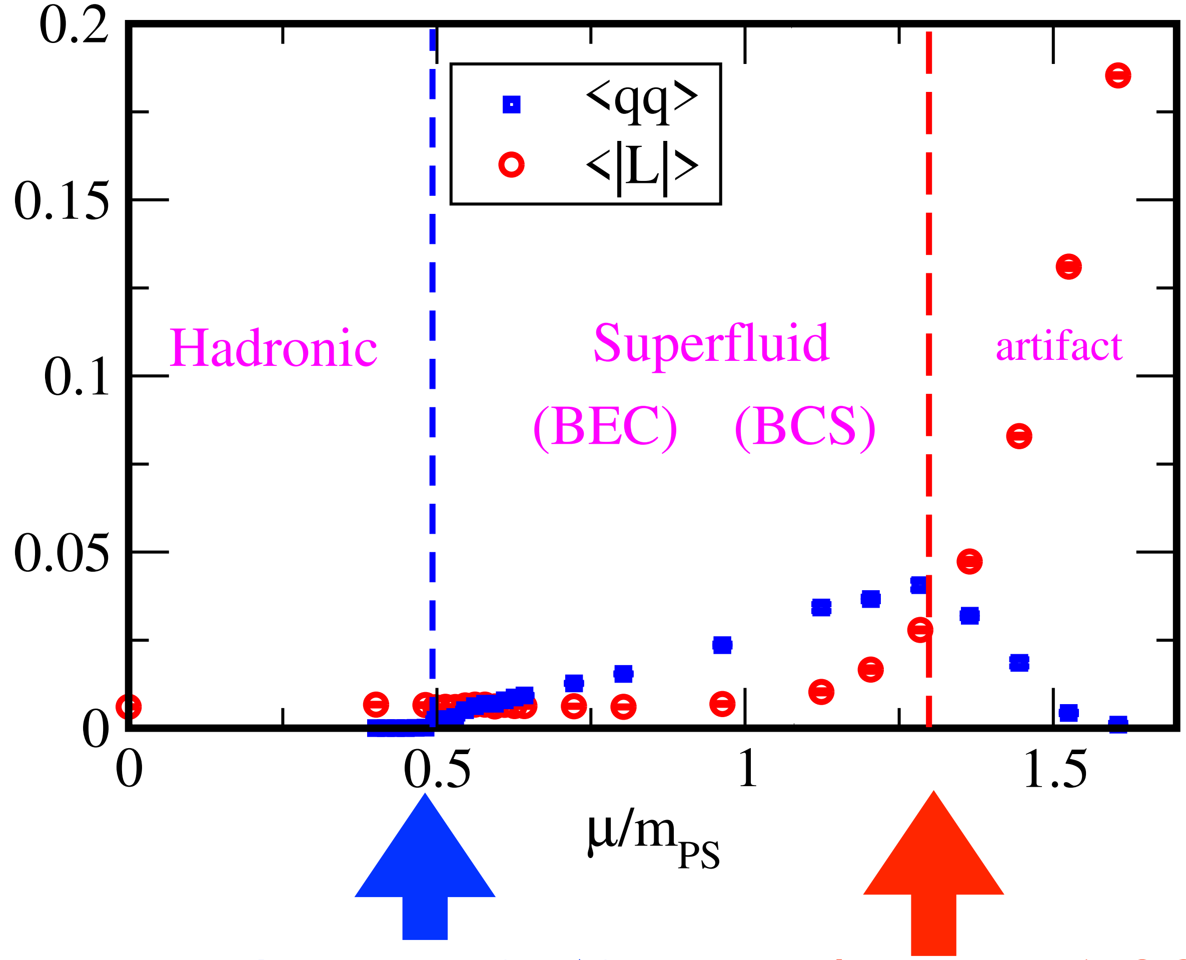
Peak position indicates the critical $\mu$

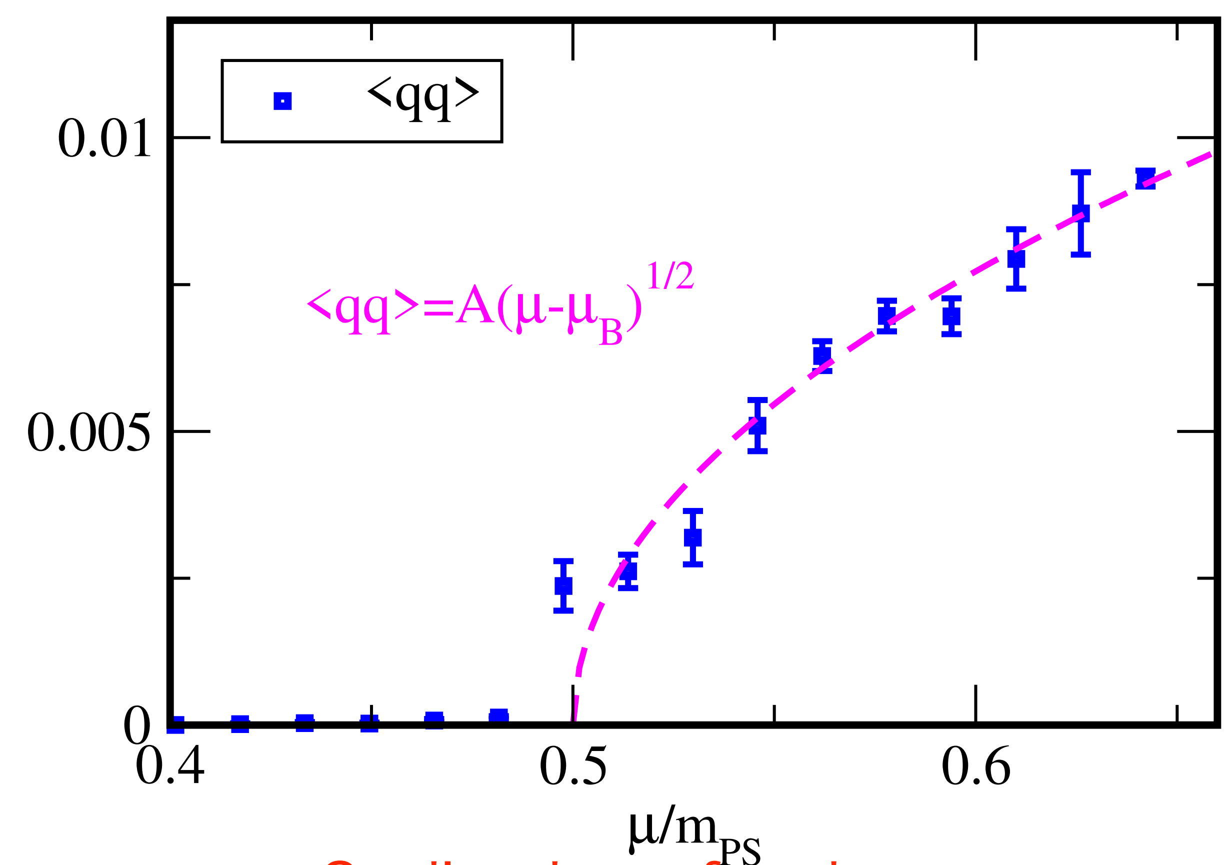
We found $\mu_D$ is independent of j

$\mu_D$ confined/deconfined transition

$\mu_D = 1.44 m_{\rm PS}$

# Phase diagram in j=0 limit




$\mu_B/m_{\rm PS} \simeq 0.50$

$\mu/m_{\rm PS} \simeq 1.28$

$(\mu_D/m_{\rm PS} \simeq 1.44)$

Scaling law of order param.

Is consistent with ChPT.

Ref.) Kogut, Stephanov, Toublan, Verbaarshot, Zhitnitsly NPB 582 (2000) 477

At T=0.39Tc, we find the BCS with confined phase until $\mu \lesssim 1152MeV$.

It is consistent with the study of string tension, yesterday's talk by K.Ishiguro.

Cf.) At $T \simeq 0.25Tc$, there was a contradiction when our paper submitted on arXiv:

Confined/deconfined transition at $\mu \approx$ 800MeV by Wilson fermion was artifact (Hands et al, 2011, arXiv:1912.10975)

Cannot find the transition $\mu \lesssim$ 1410MeV by rooted staggered (Braguta et al, 2016)

# Result of Kogut et al. (2002), Nf=4, rooting staggered fermion

16^4 (estimated mu_c~ 0.3 in lattice units)

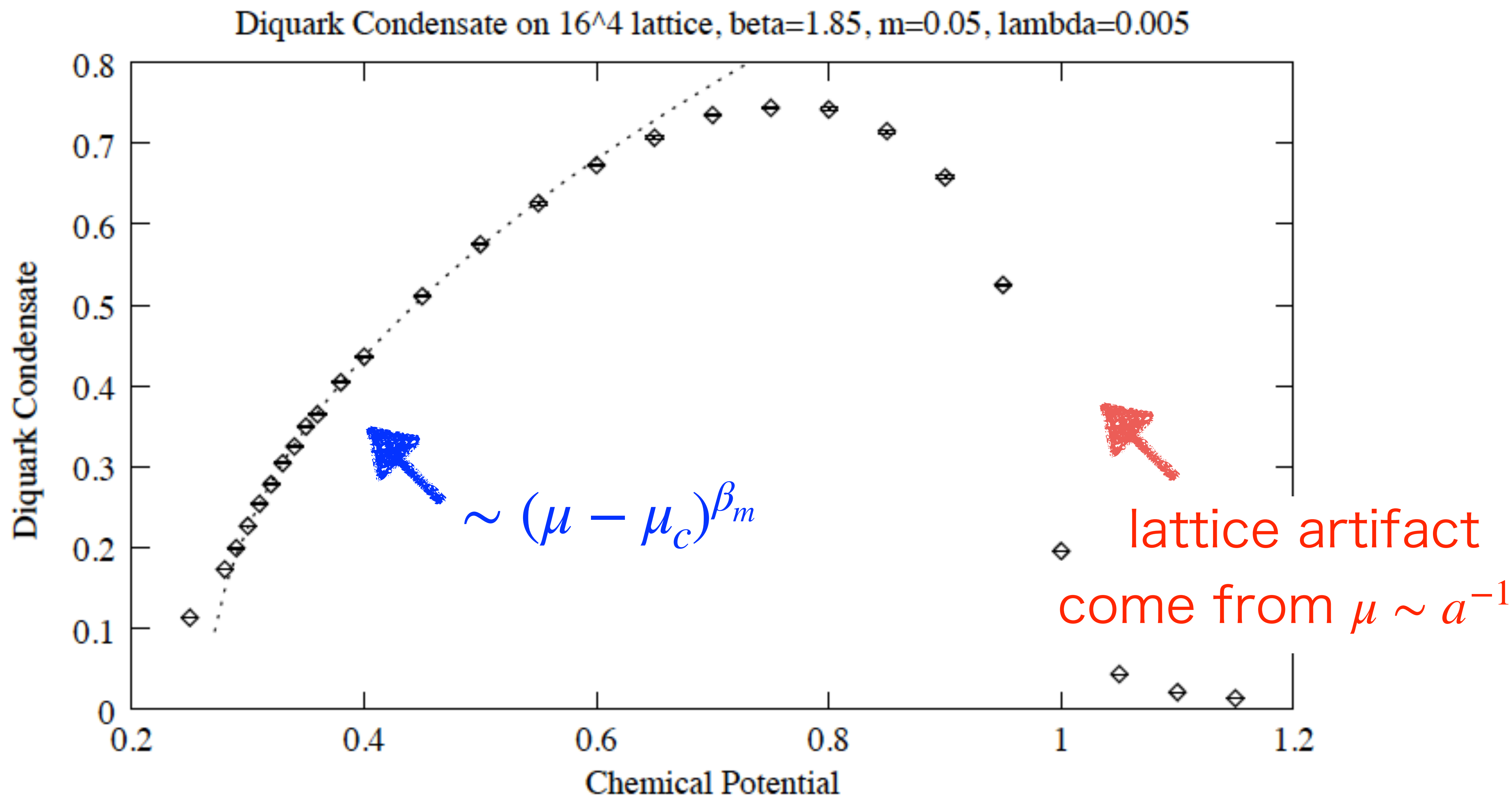
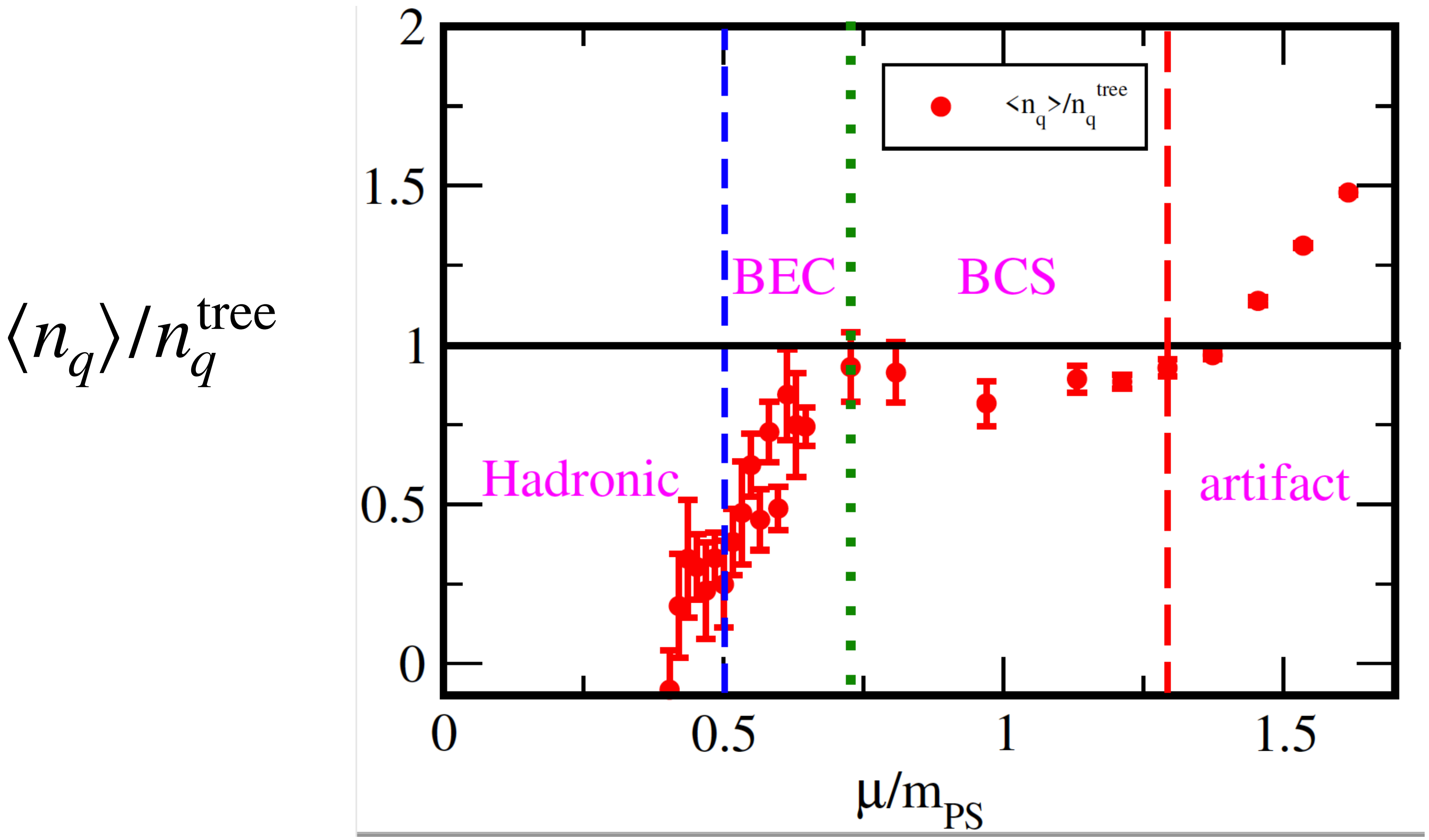



FIG. 7. Diquark Condensate vs. $\mu$.

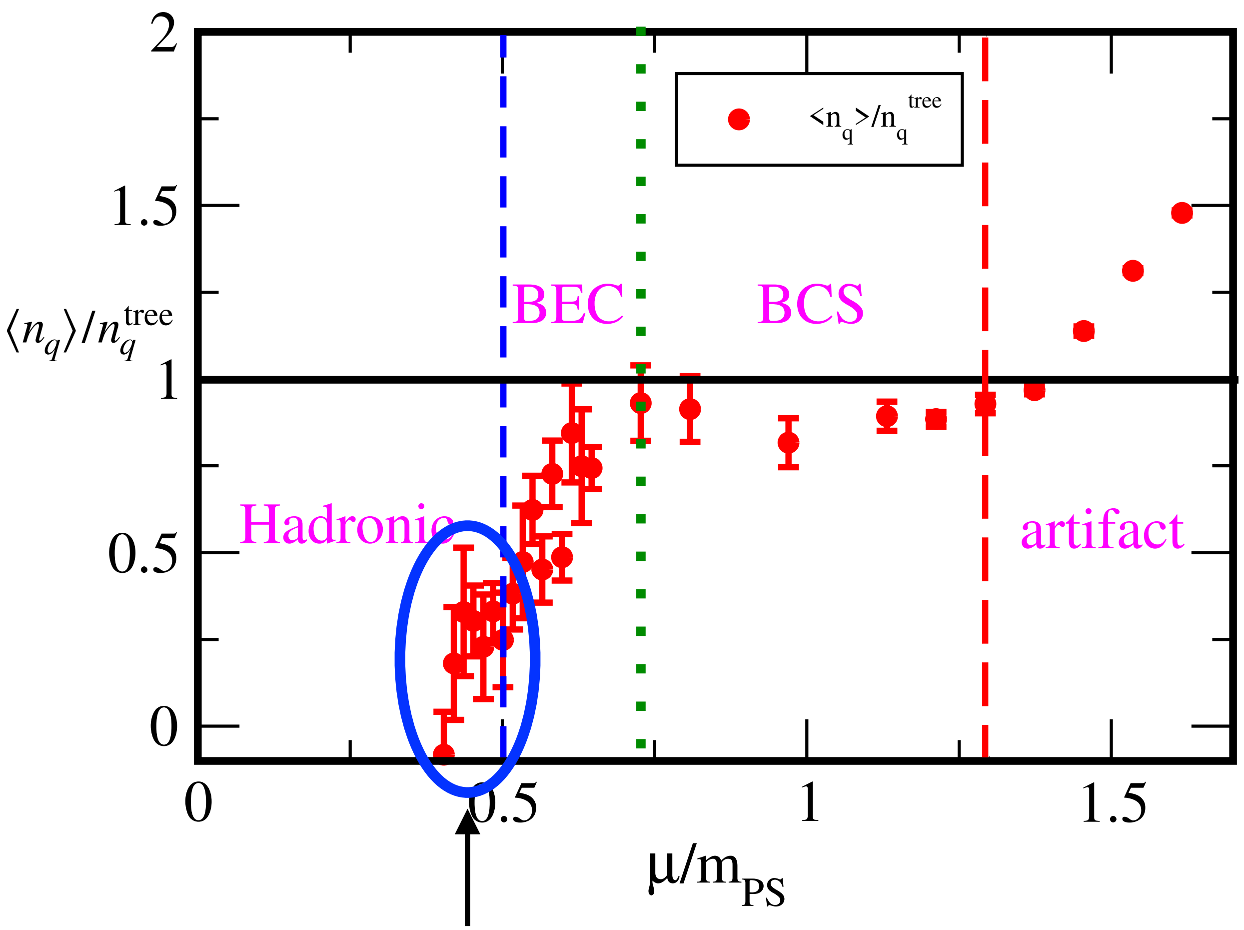
# quark number density

$$n_q = \sum_i \kappa \Big\langle \bar{\psi}_i(x)(\gamma_0 - 1)e^{\mu} U_t(x)\psi_i(x+\hat{t}) + \bar{\psi}_i(x)(\gamma_0 + 1)e^{-\mu} U_t^{\dagger}(x-\hat{t})\psi_i(x-\hat{t}) \Big\rangle$$

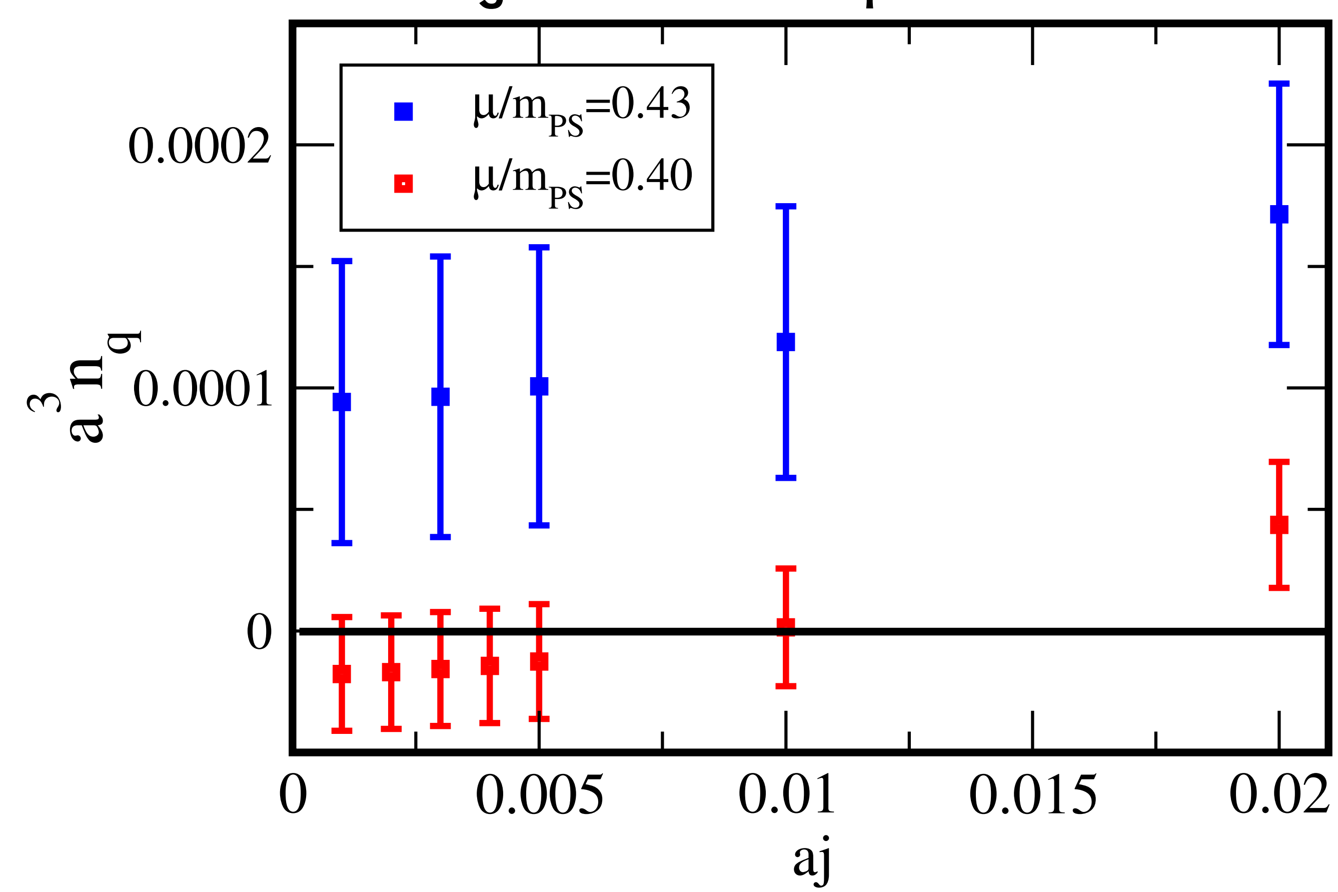



BEC-BCS crossover occurs at $\mu \approx 0.72 m_{\text{PS}}$

# quark number density




j->0 extrapolation




$\langle n_q \rangle \neq 0, \ \ \langle qq \rangle = 0$

Some quark d.o.f. exists

Superfluidity does not emerges

New!

Hadronic-matter phase (coexistence phase)

# Hadronic matter phase

Prediction of ChPT: $n_q$ becomes nonzero at $\mu = m_{PS}/2$.

In the present simulation,

$$am_{PS} \simeq 0.6229$$

$$T = 1/(aN_\tau) = 1/(16a)$$

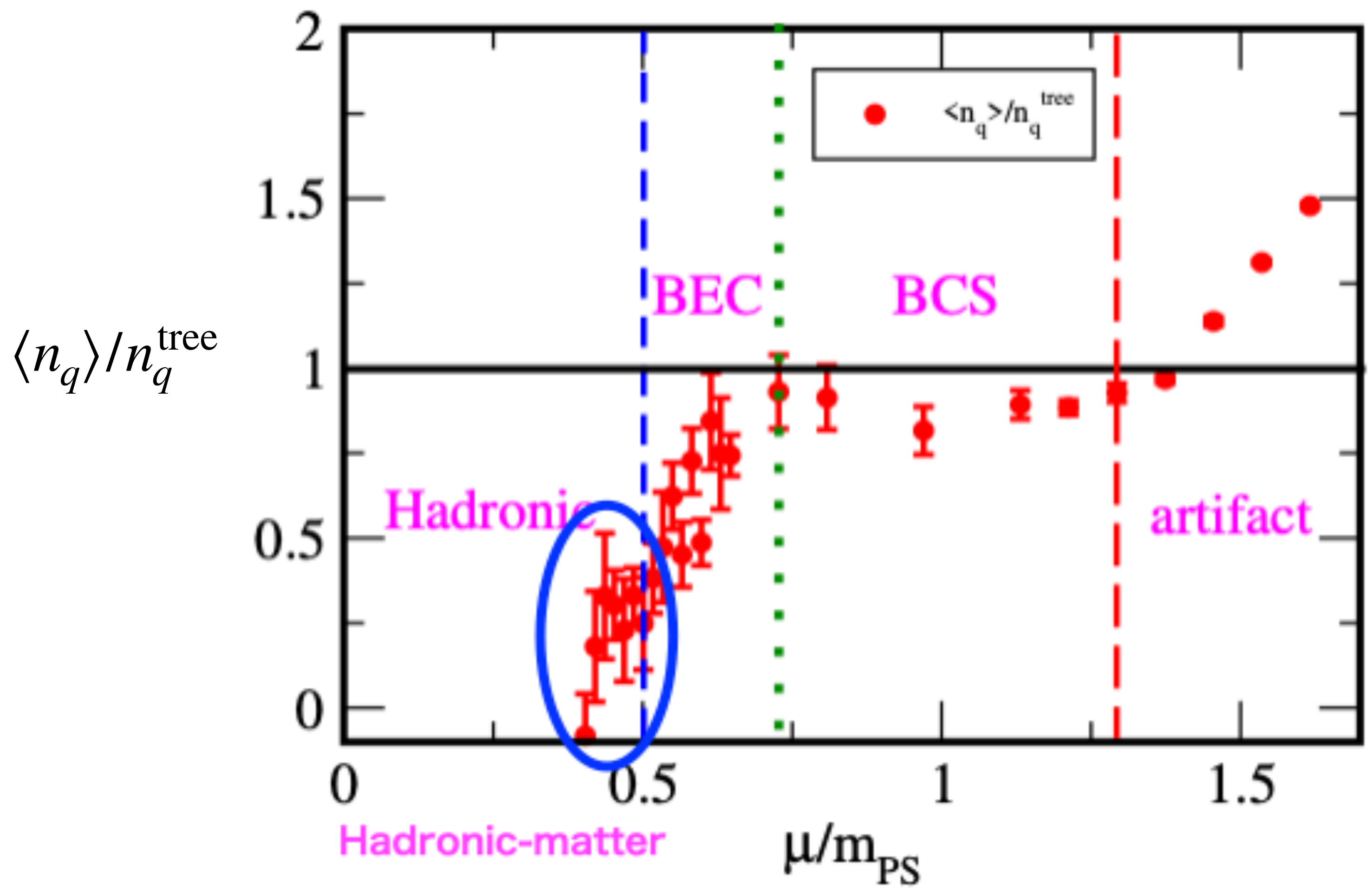
⇨ $T \simeq 0.1 m_{PS}$

According to ChPT, $m_{qq} \approx m_{PS} \mp 2\mu$.

At $\mu \sim 0.45 m_{PS}$, $T \simeq m_{qq}$, thus diquarks are thermally excited.

It is reasonable for the quark number density to start increasing at $\mu \simeq 0.45 m_{PS}$.

# Summary of phase diagram at T=0.39Tc




$\mu \leq 0.45 m_{\text{PS}}$ : Hadronic phase

$0.45 m_{\text{PS}} \lesssim \mu \lesssim 0.50 m_{\text{PS}}$: Hadronic-matter phase

@T=0.39Tc
(~79MeV)

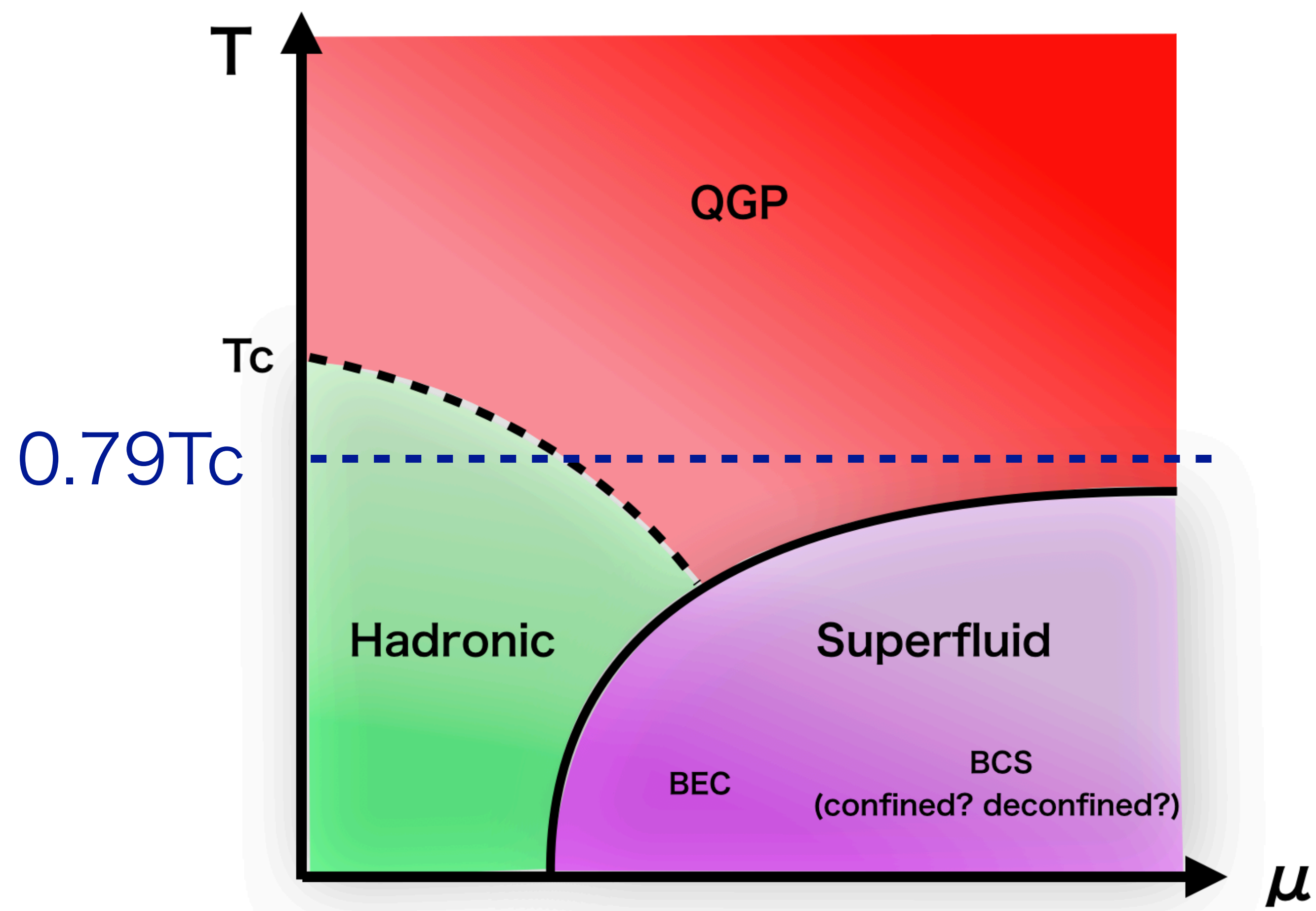
$0.50 m_{\text{PS}} \lesssim \mu \lesssim 0.72 m_{\text{PS}}$: BEC phase

$0.72 m_{\text{PS}} \lesssim \mu \lesssim 1.28 m_{\text{PS}}$: BCS phase

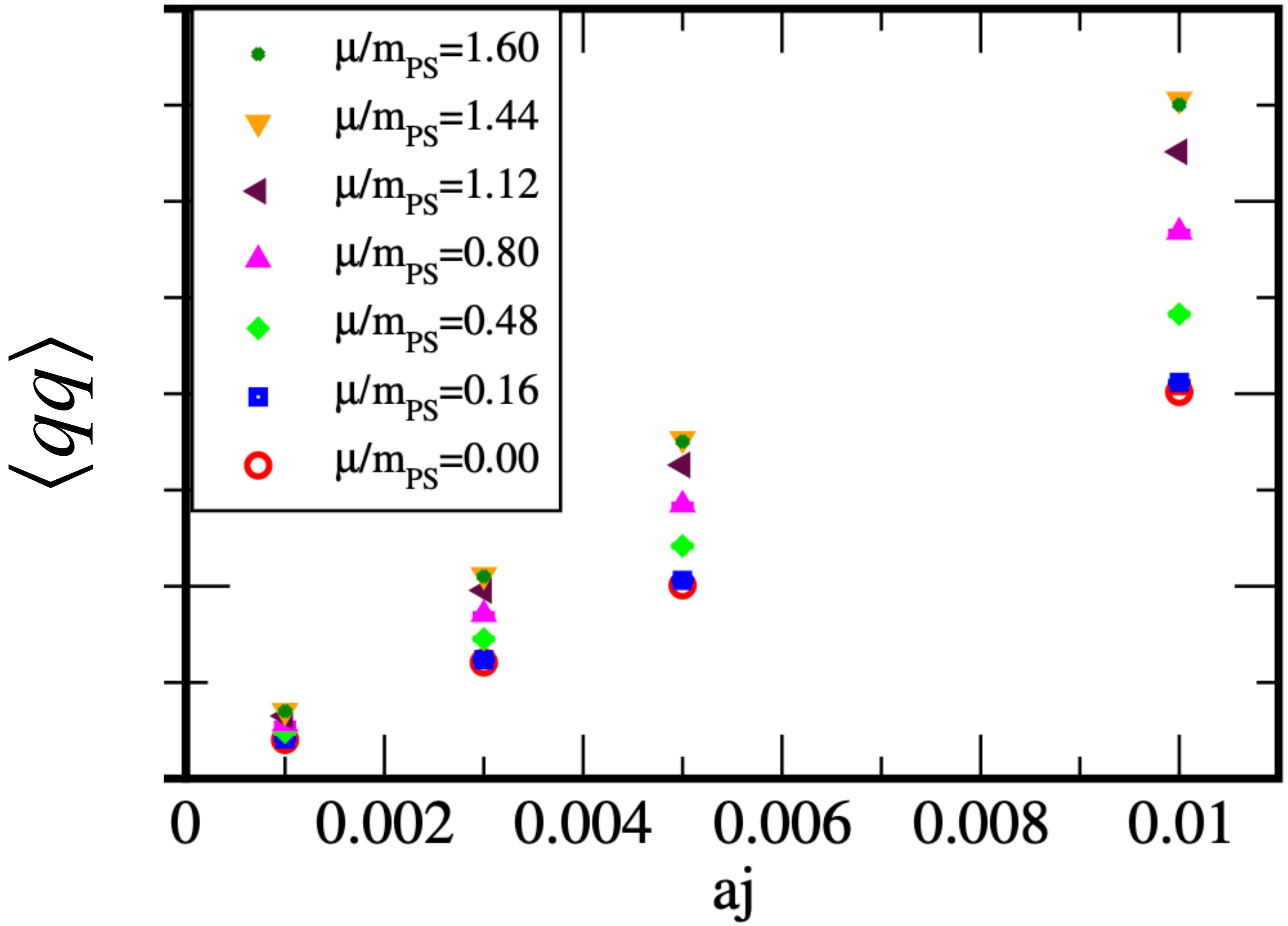
$1.28 m_{\text{PS}} \lesssim \mu$ : lattice artifact is strong

# Results

Lattice size: 32^3x8 :  T=0.79Tc (~ 158MeV)



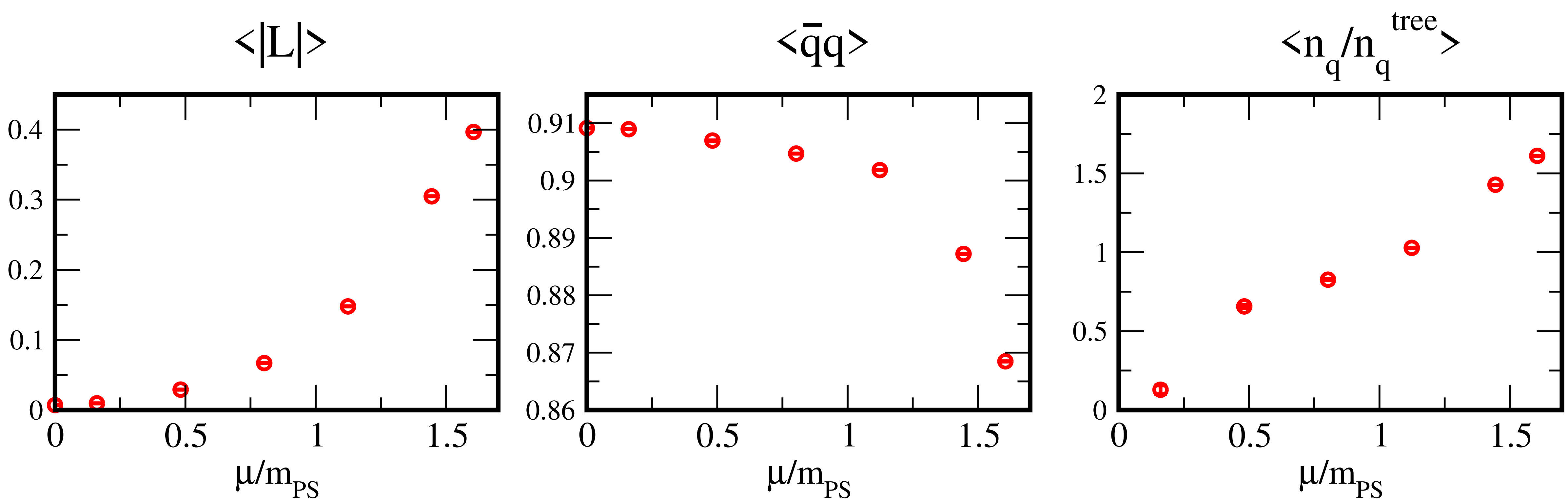
# Diquark condensate




No superfluidity in whole $\mu$ regime

Actually, we can generate the configurations using HMC without j-term.

# Polyakov loop, chiral condensate, number density




confined -> deconfined

chiral broken ->restored

non-zero even in $\mu \ll m_{PS}/2$

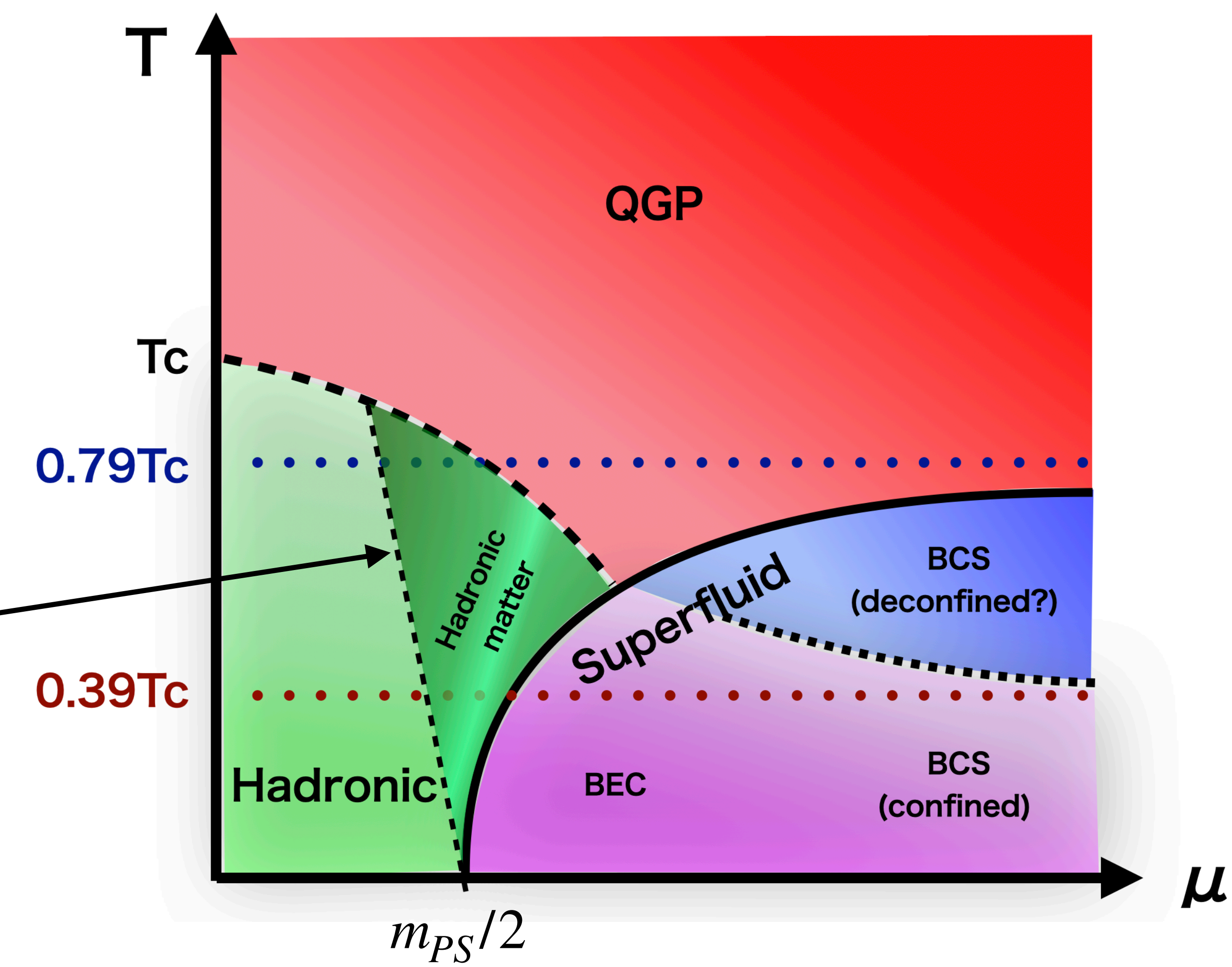
and no superfluidity

Hadronic -> QGP transition

# Summary of phase diagram

T=0.79Tc :158MeV, T=0.39Tc: 79MeV

$$T = m_{PS} - 2\mu$$




Below Tc, there is T dependence of phase structure.

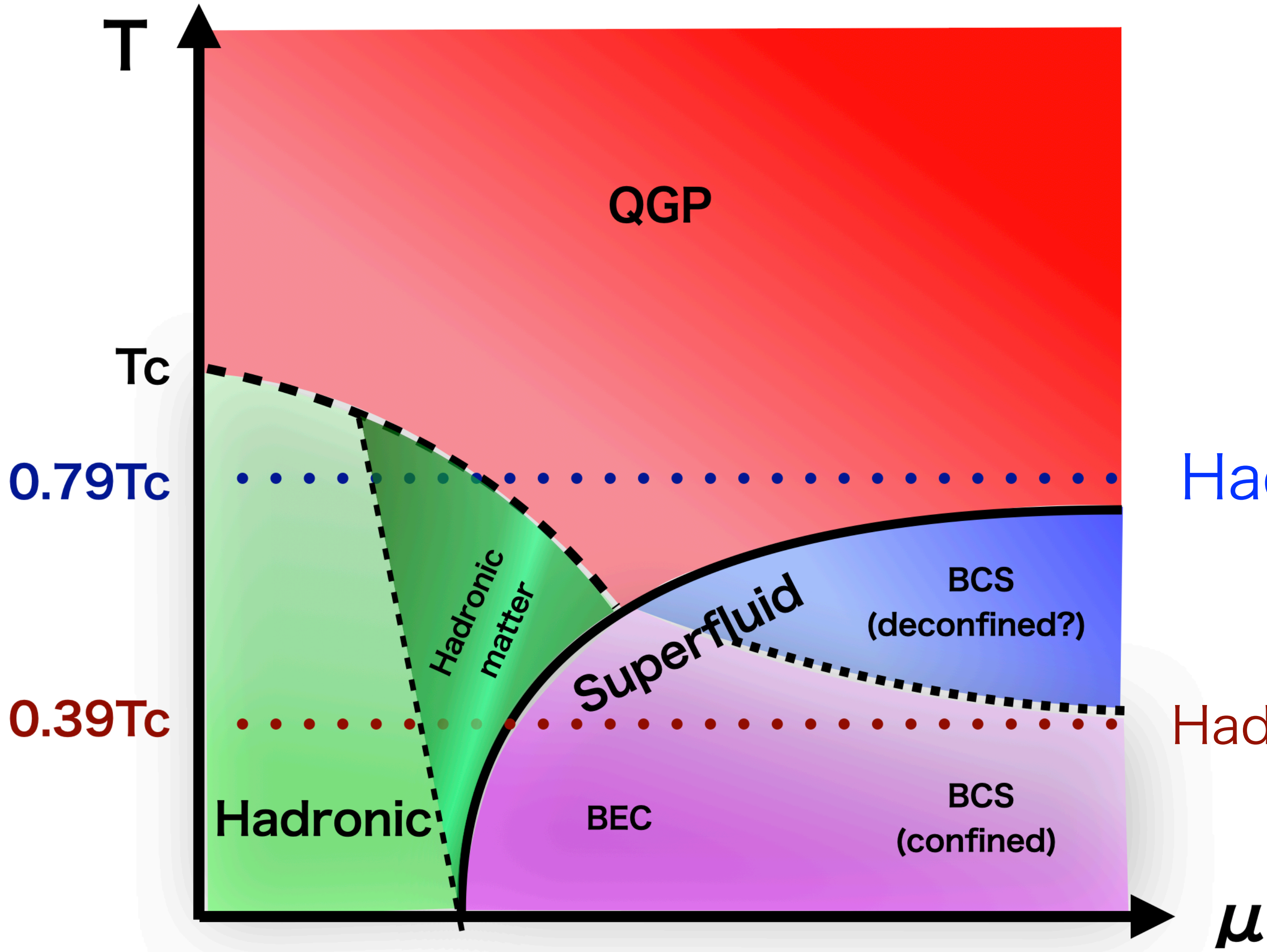
The QGP/SF transition must be below Tc.

In T~80MeV, the hadronic-matter phase emerges.

It comes from thermal excitation of hadrons.

# Topological susceptibility

Measure the topological charge using gradient flow




Hadronic -> QGP

Hadronic -> Hadronic-matter -> BEC-> BCS

# Earlier works

*Hands et.al. (arXiv:1104.0522)*

*Nf=4, T=0 (12^3x24)*

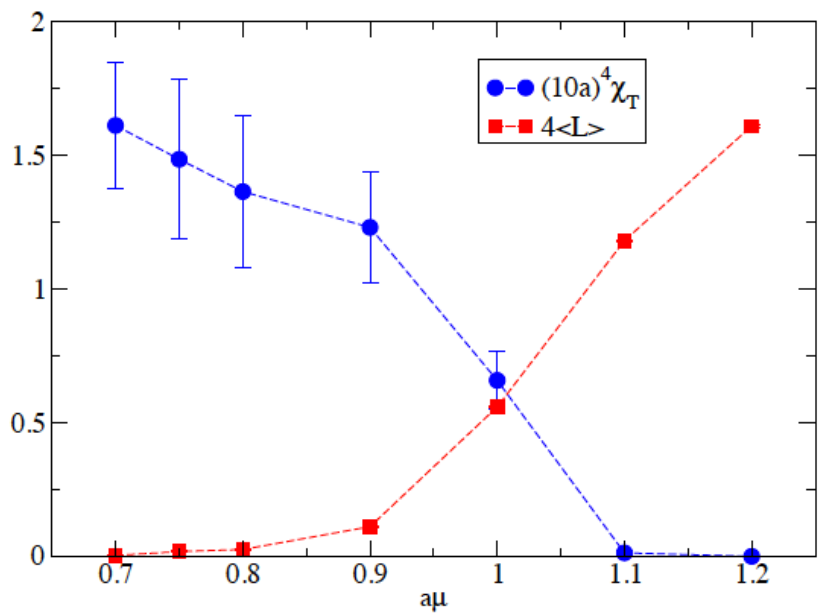
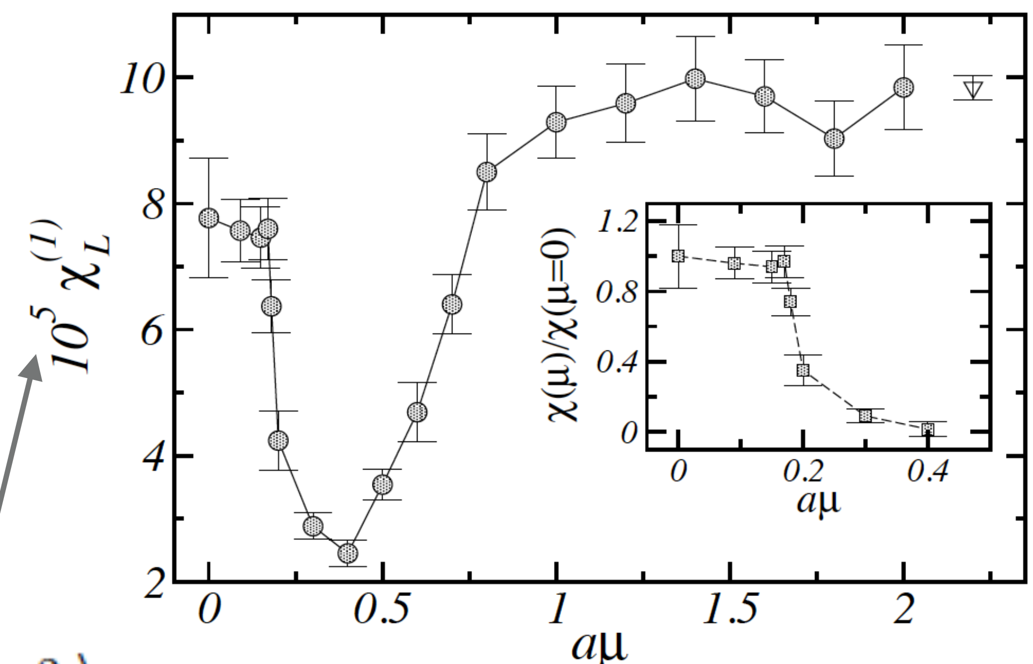



Figure 2: *The suppression of $\chi_T$ coinciding with the rise in $\langle L\rangle$ for $N_f = 4$. Note $\langle L\rangle$ has been rescaled for clarity.*

*Alles, D'elia, Lombardo(arXiv:0602022)*

*Nf=8 staggered, finite T (14^3x6)*




$$\chi_L \equiv \frac{\langle (Q_L)^2 \rangle}{V},$$

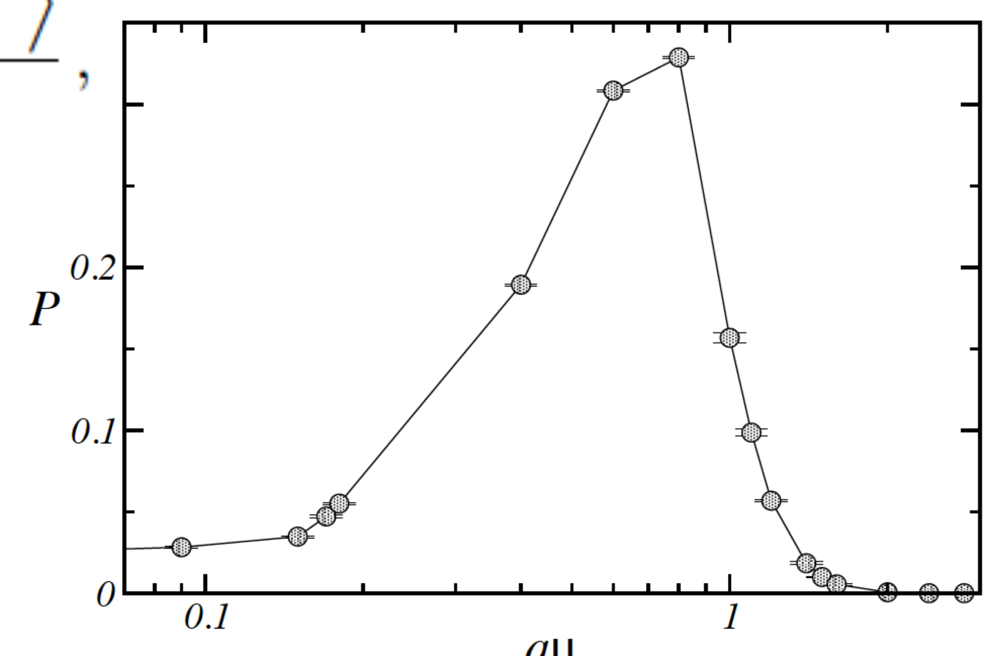



FIG. 2. Polyakov loop $P$ as a function of $a\mu$. The logarithmic scale allows to disentangle the data obtained in the vicinity of the transition point. Points are joined by a line to guide the eye.

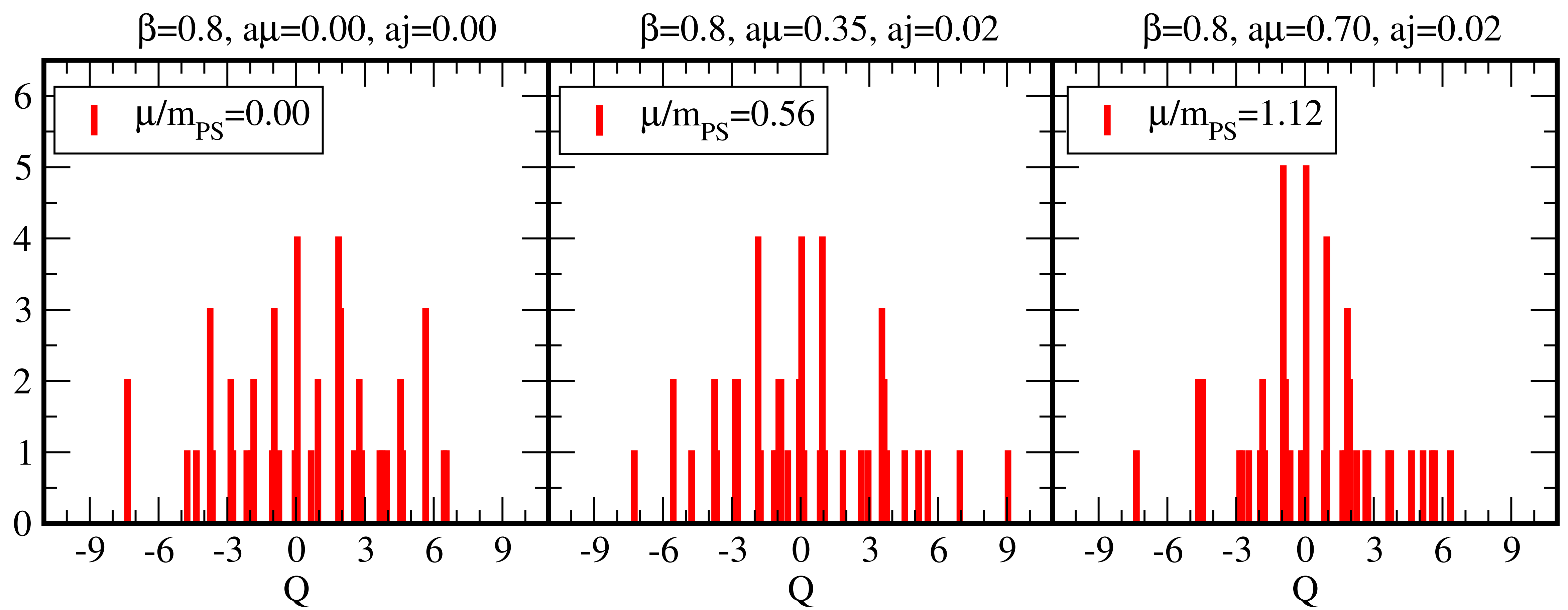
Polyakov loop increasing

|| ?

Topological suscep. decreasing

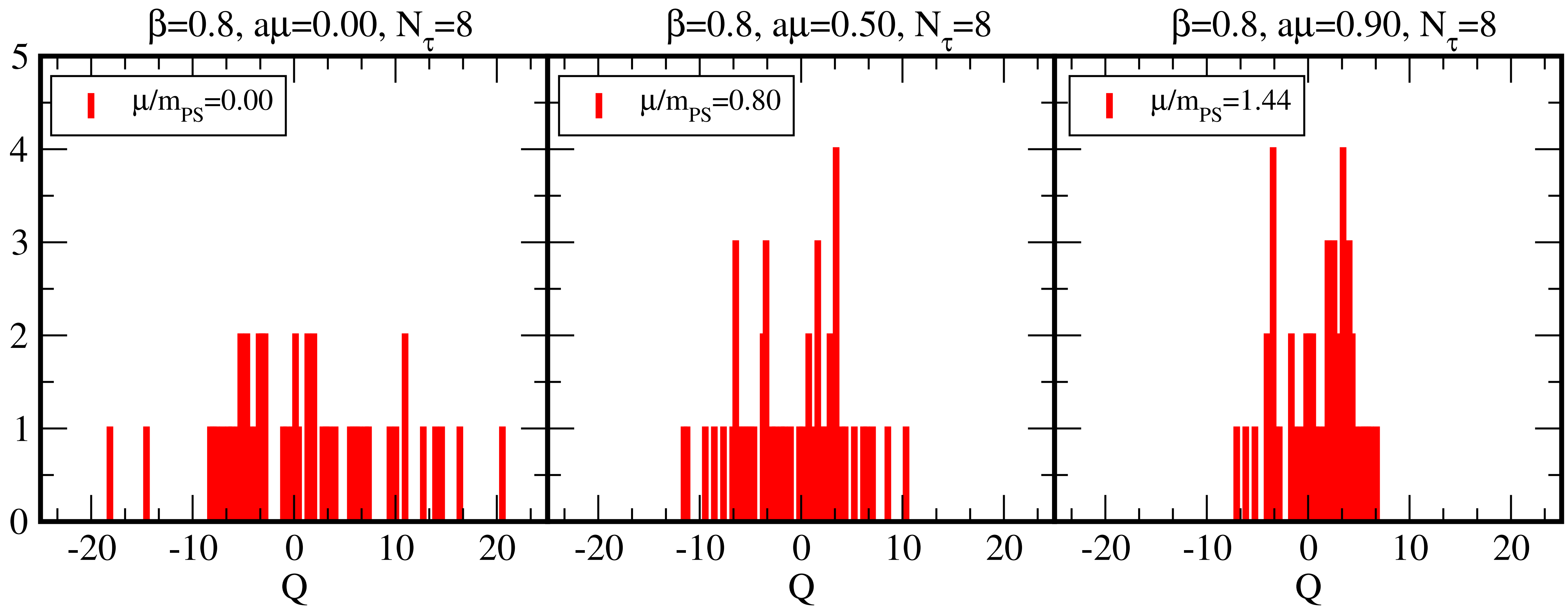
# Topological charge distribution
# T=0.39Tc

Hadronic phase BEC phase BCS phase



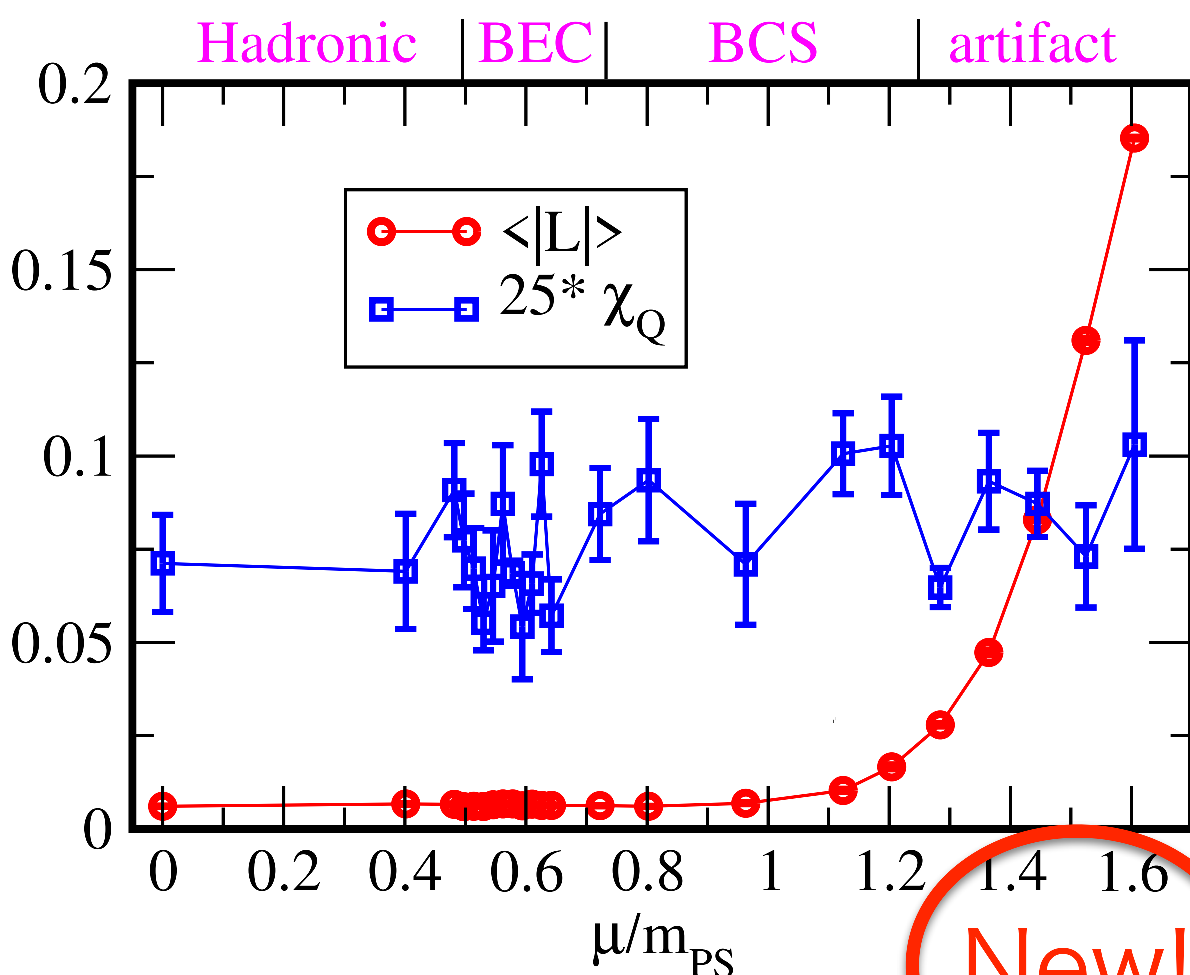
# Topological charge distribution
# T=0.79Tc

Hadronic phase QGP phase



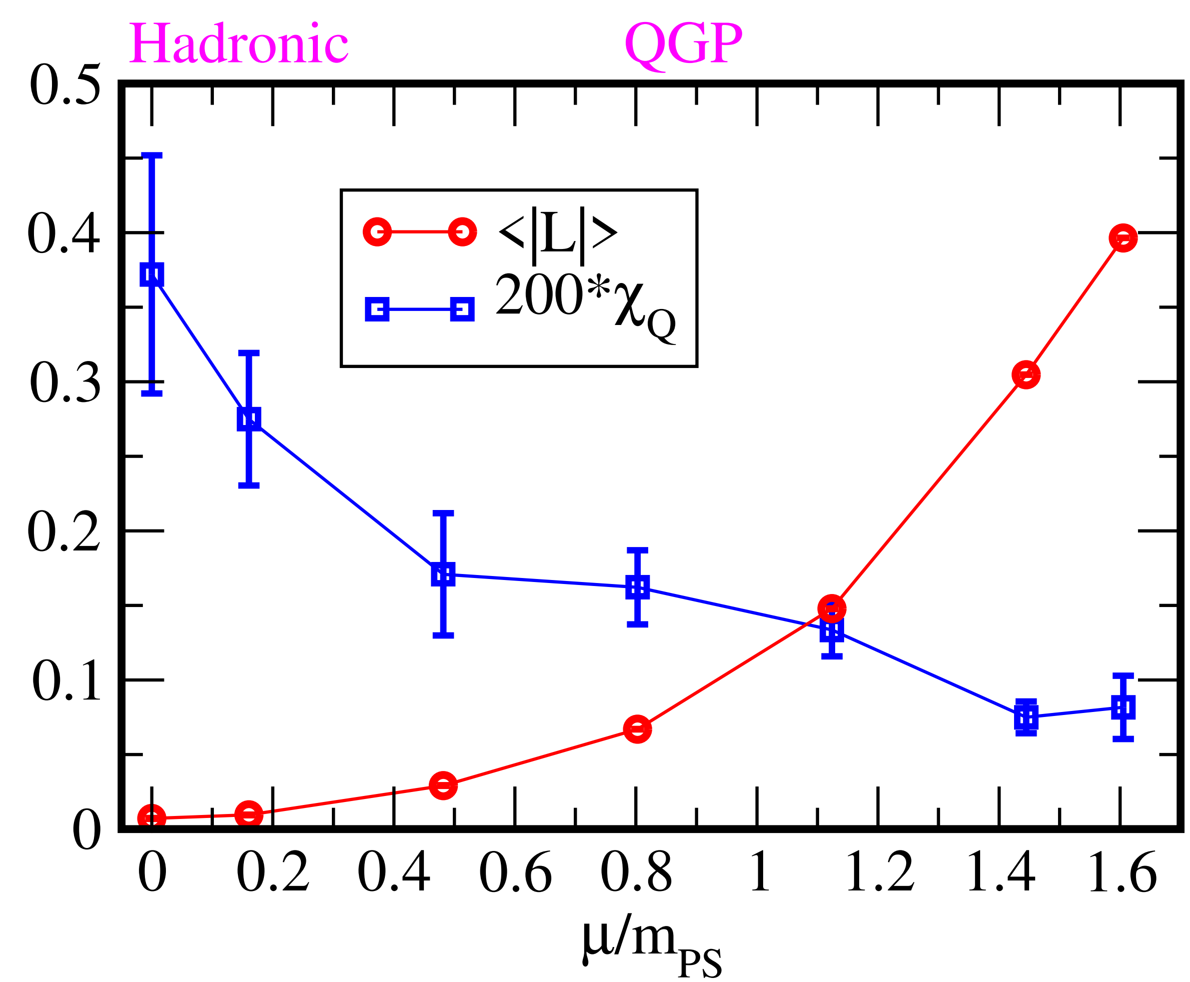
# Topological susceptibility and Polyakov loop

T=0.39Tc




New!

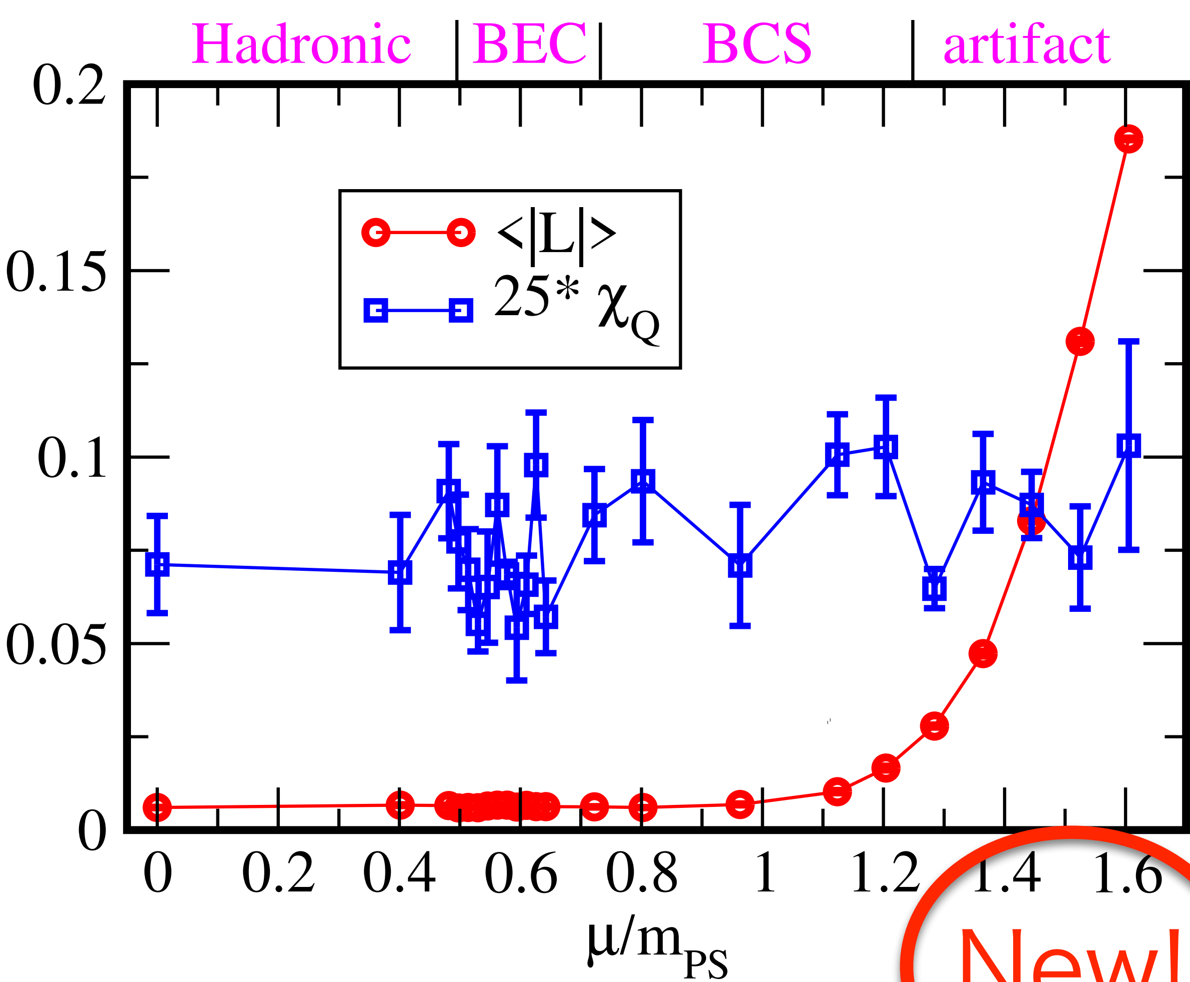
T=0.79Tc




This behavior is consistent with the earlier works.

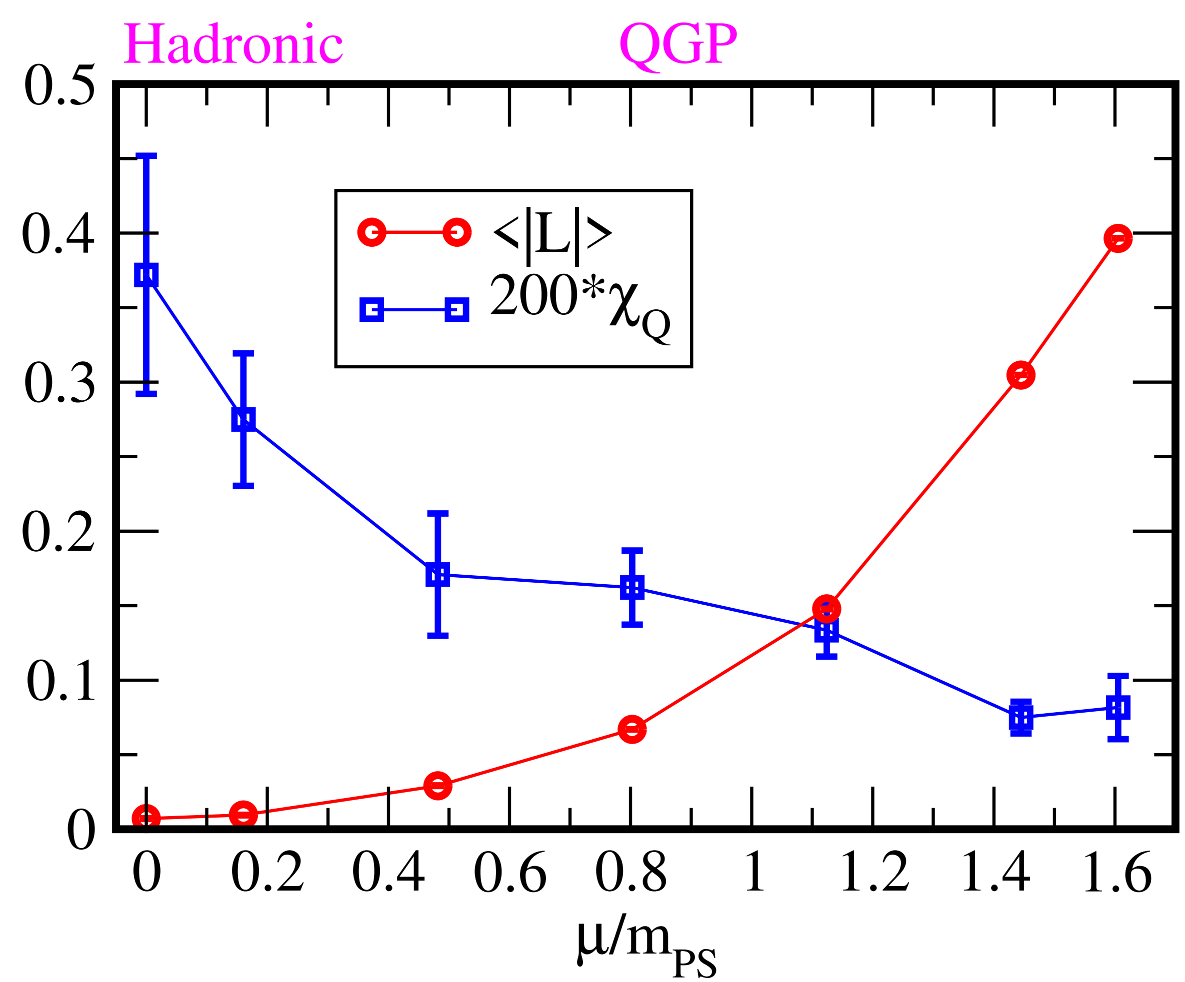
# Topological susceptibility and Polyakov loop

T=0.39Tc




New!

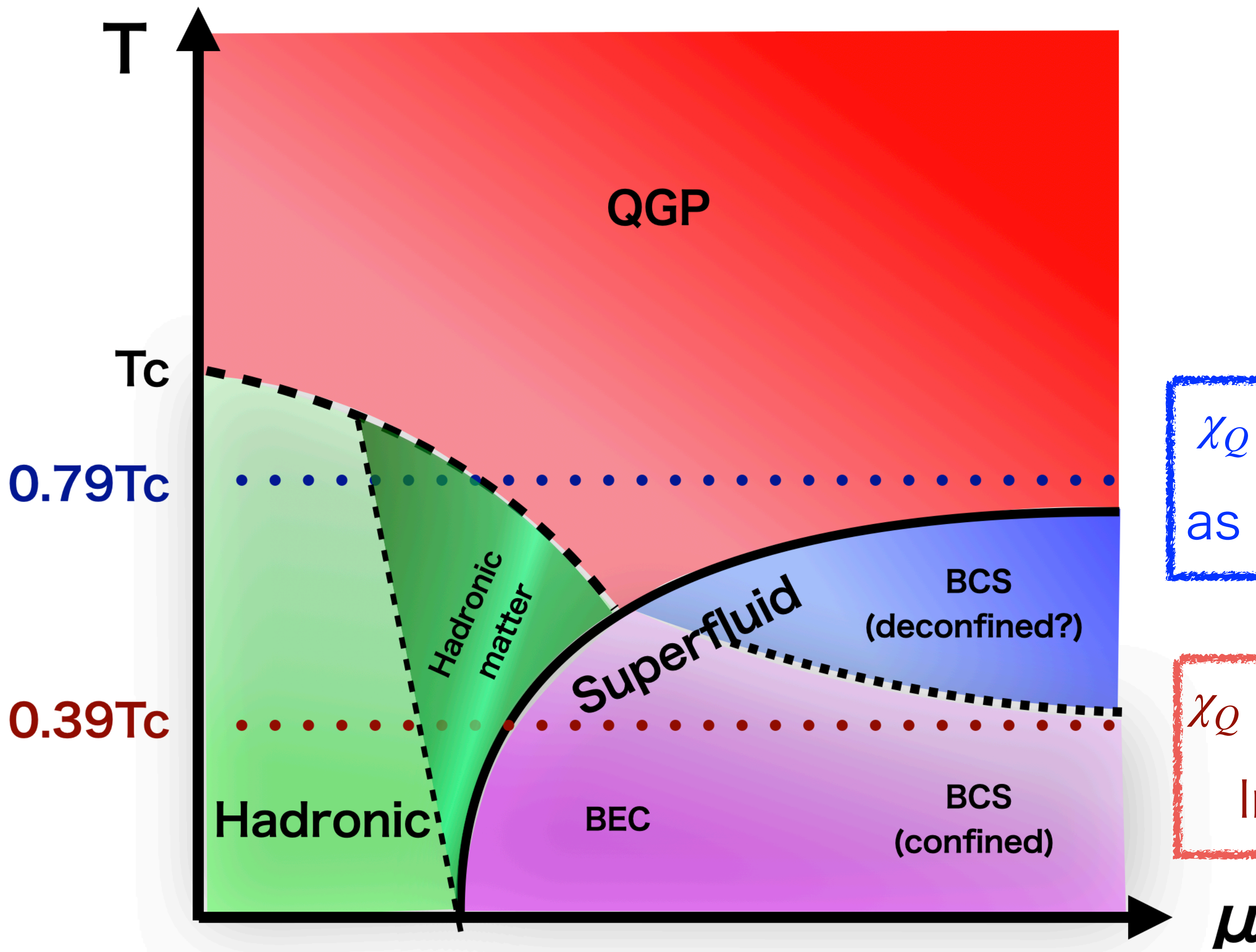
T=0.79Tc




This behavior is consistent with the earlier works.

Cf.) N. Astrakhantsev et al., arXiv:2007:07640
See a decreasing behavior of $\chi_Q$ in superfluid phase with deconfinement property

# Summary of our work




$\chi_Q$ decreases as $\mu$ increases

$\chi_Q$ is constant In whole $\mu$

New!

New insight: BCS phase with nontrivial topological backgrounds

# Plan of talk

1. Why 2color QCD?

    Sign problem and numerical-instability problem

2. Definition of phase

    Spontaneous flavor symmetry breaking in Nc=Nf=2

3. Simulation results

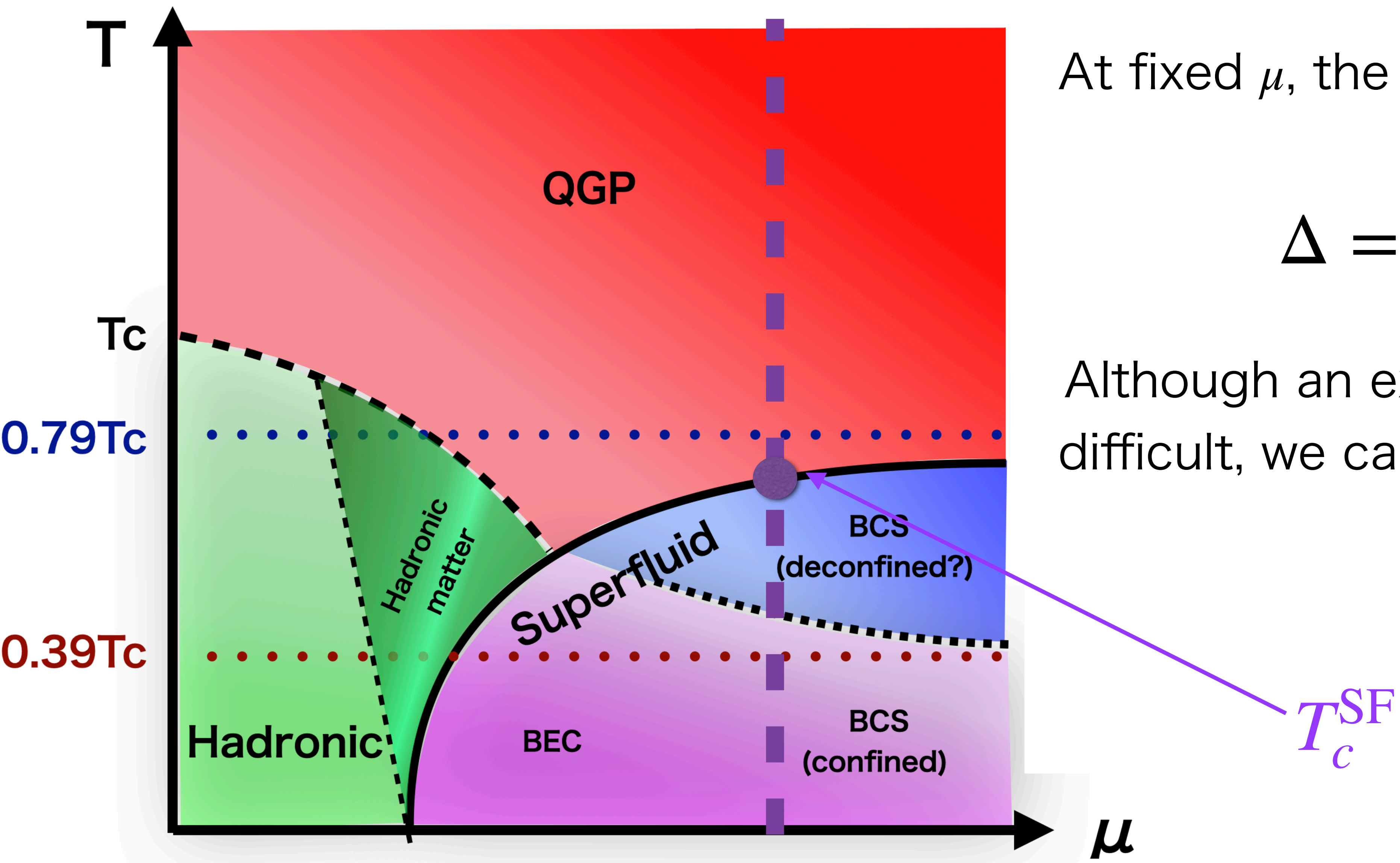
    Phase diagram at T=0.39Tc, 0.79Tc

    Topological susceptibility

4. Summary and discussion

    A role of nontrivial topology in the phase diagram

# A role of instanton in high density




At fixed $\mu$, the BCS relation is valid

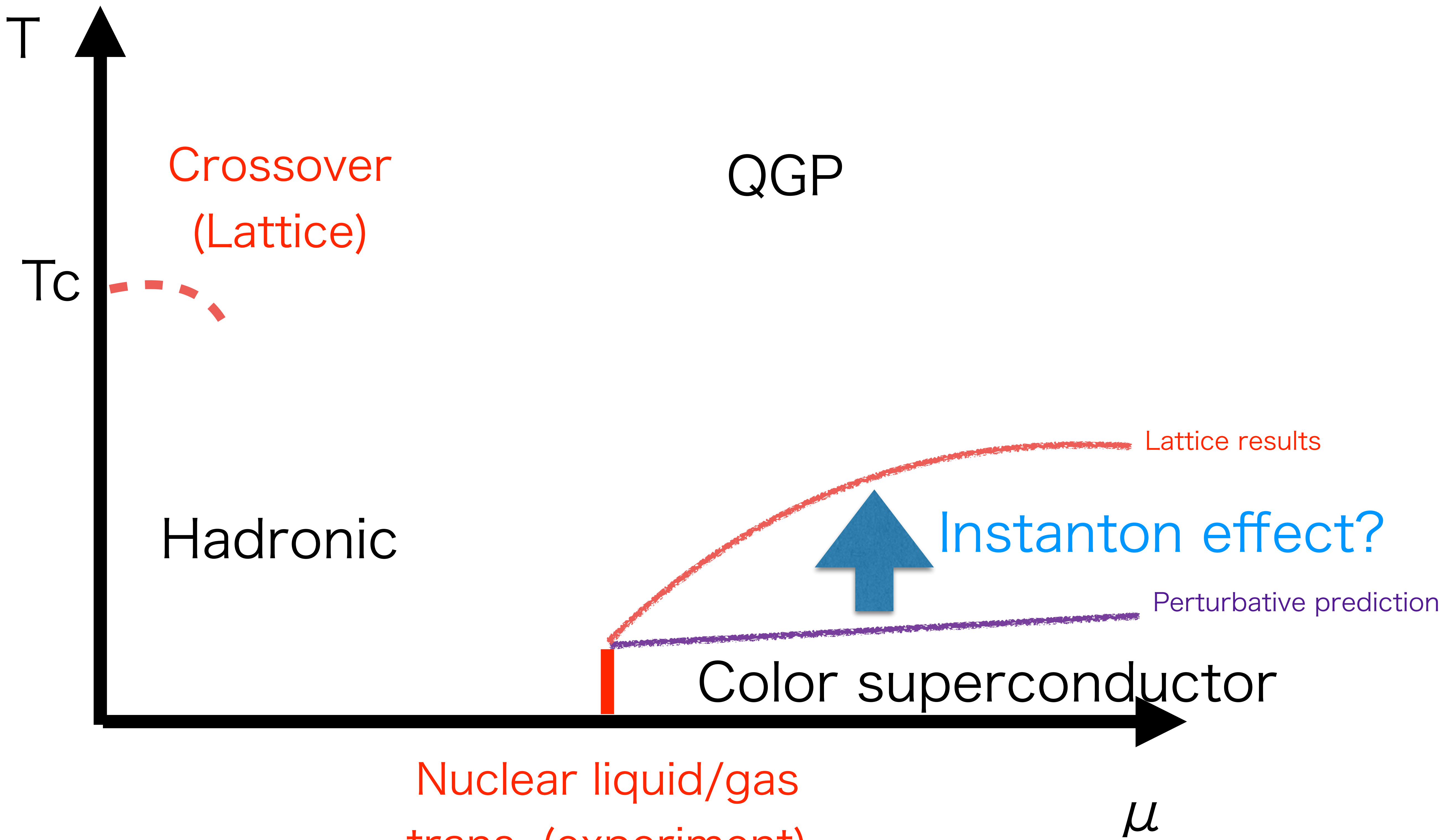
$$\Delta = \pi \frac{T_c^{\mathrm{SF}}}{e^{\gamma_E}}$$

Although an exact zero-T simulation is difficult, we can find a value of diquark gap.

**speculation: diquark gap may get fat because of the interaction via nontrivial topological objects.**

**$T_c^{\mathrm{SF}}$ may be higher than analytical prediction**

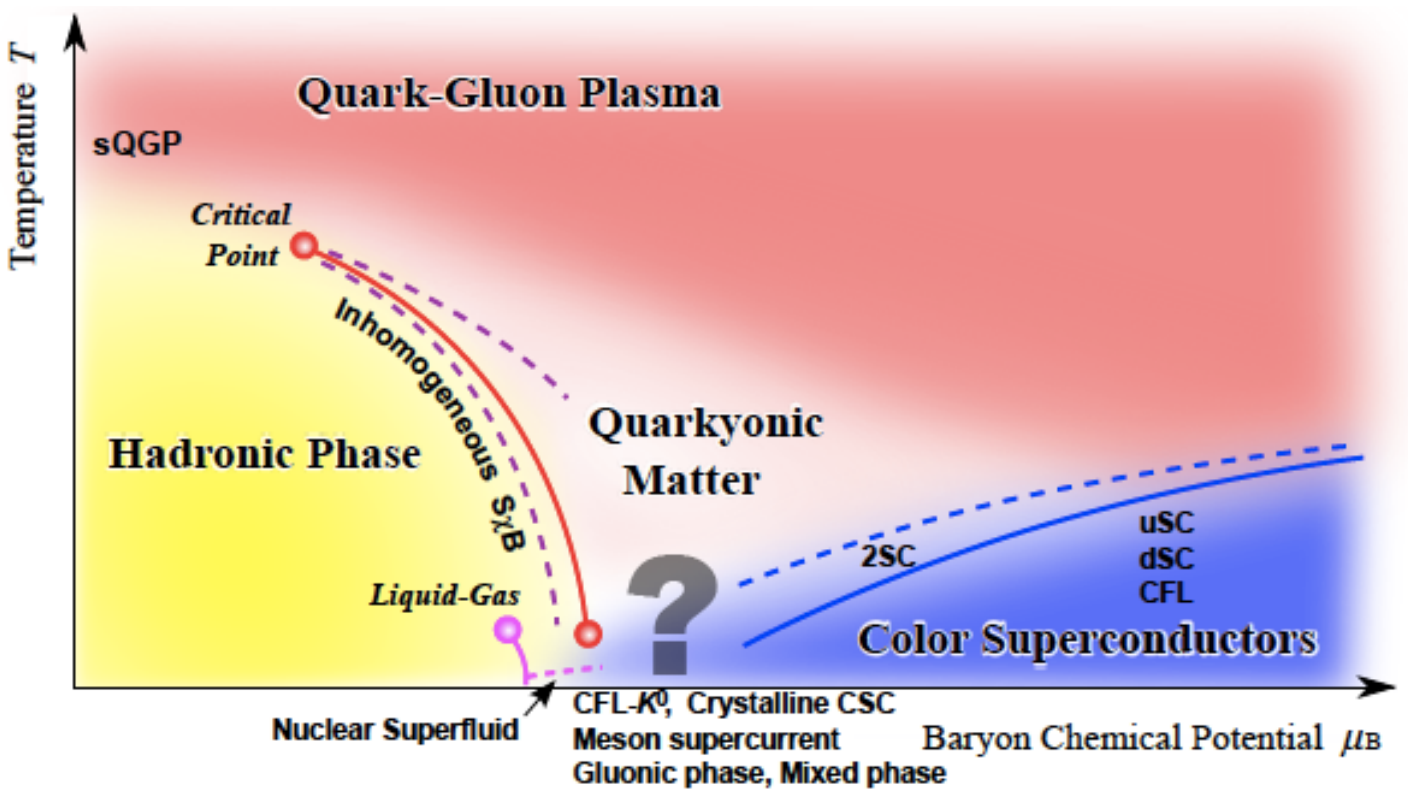
# What is really known…



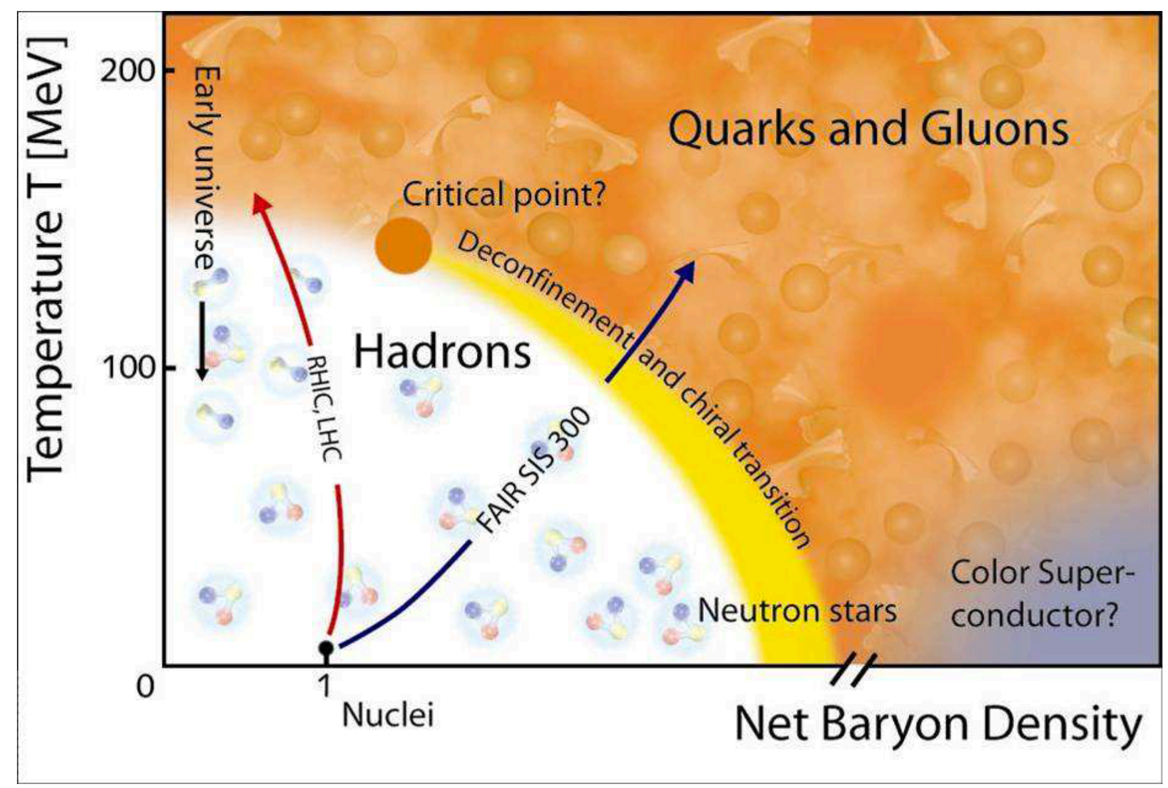

Pochodzalla et al. PRL75 (1995) 1040

# Schematic picture of QCD phase diagram

K.Fukushima and T.Hatsuda
RPR74(2011)014001

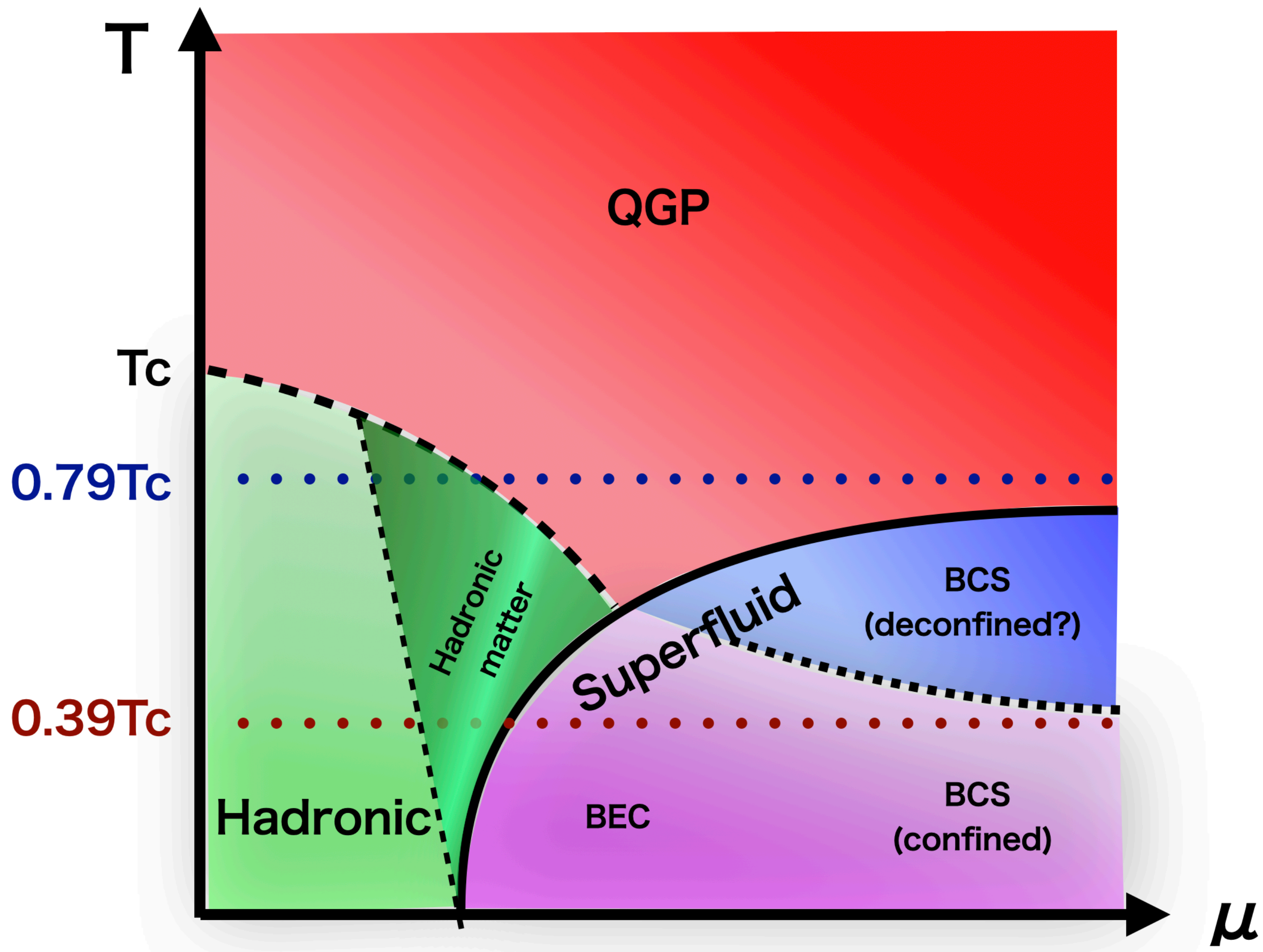



Gert Aarts
J.Phys.Conf.Ser.706(2016)022004




Instantons are classical configuration of gluons.
It suggests that the instanton exists also in CSC phase.
Also inside the neutron star (?).

# Confined or deconfined in high density




**BCS(deconfined)** does not appear in our simulations, but it is widely believed.

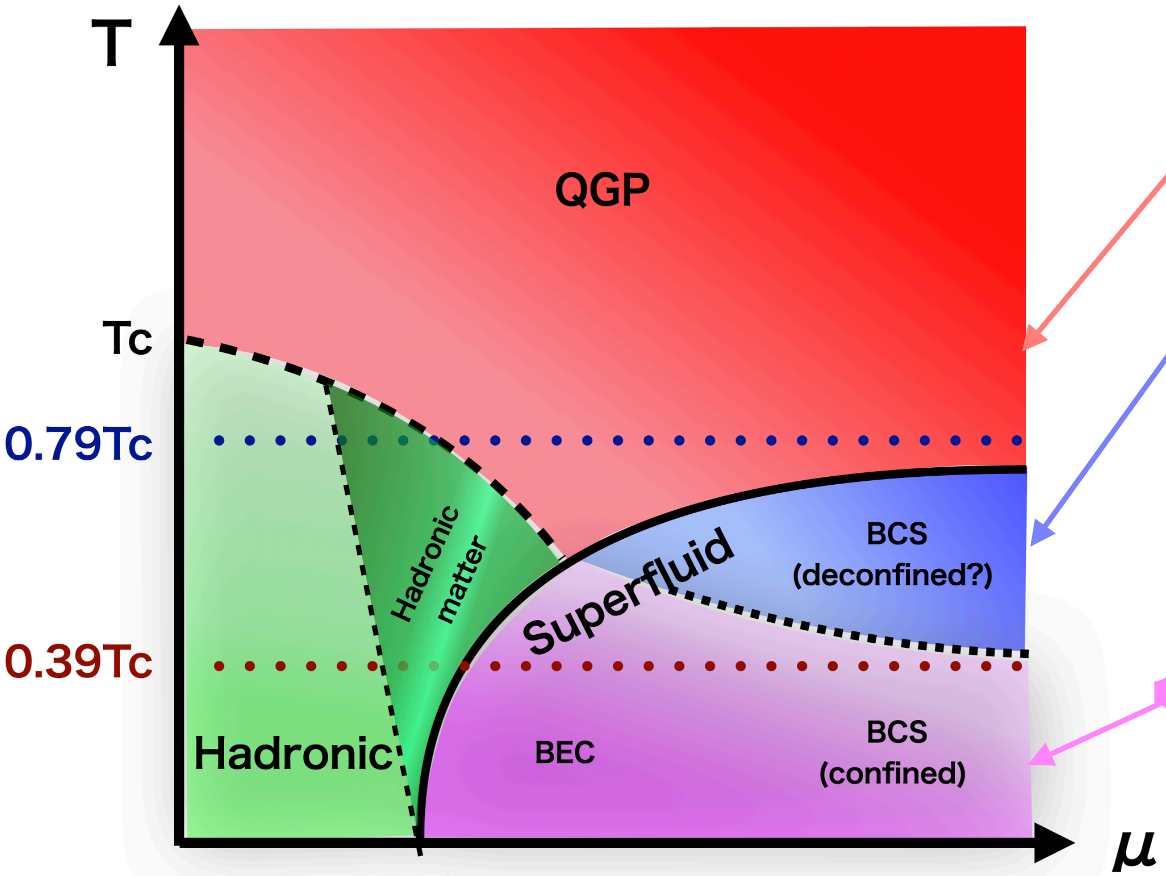
# Confined or deconfined in high density

**Three independent group' studies:**

**(1) Swansea group : Wilson-Plaquette gauge + Wilson fermion**

**(2) Moscow group : tree level improved Symanzik gauge + rooted staggered fermion**

**(3) Our group : Iwasaki gauge + Wilson fermion, Tc=200 MeV to fix the scale**




T=158 MeV (**deconfined**, hadron -> QGP phase transition occurs)

T=130 MeV (**deconfined? QGP phase?** , 2019)

T=140 MeV (**deconfined** in high mu, <qq> is not zero, 2017, 2018, 2020)

T= 93 MeV (**deconfined** in high mu ?, also <qq> is not zero?, 2017)

T=87 MeV (**confined** in 2019)

T=79 MeV (**confined** even in high mu)

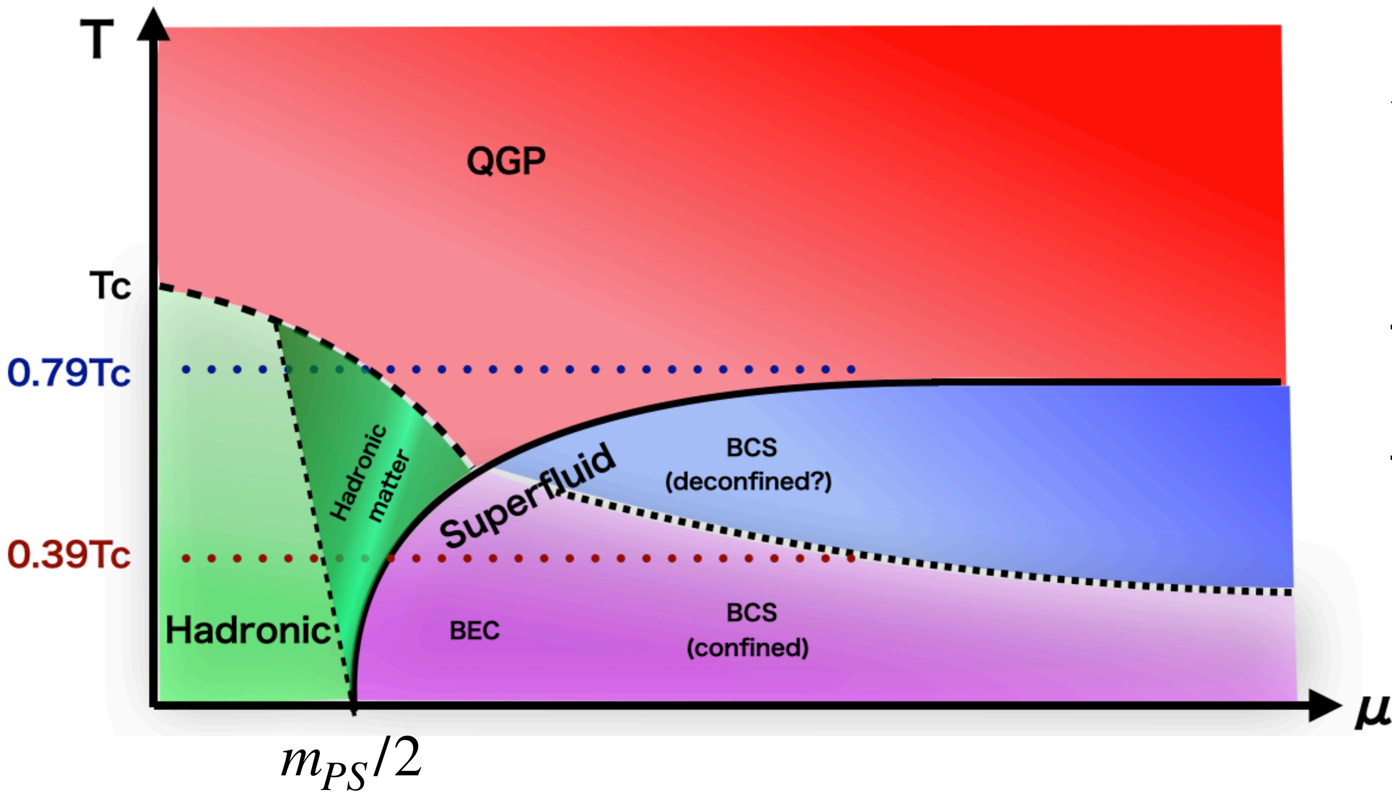
T=55 MeV (**confined** in high mu, 2016)

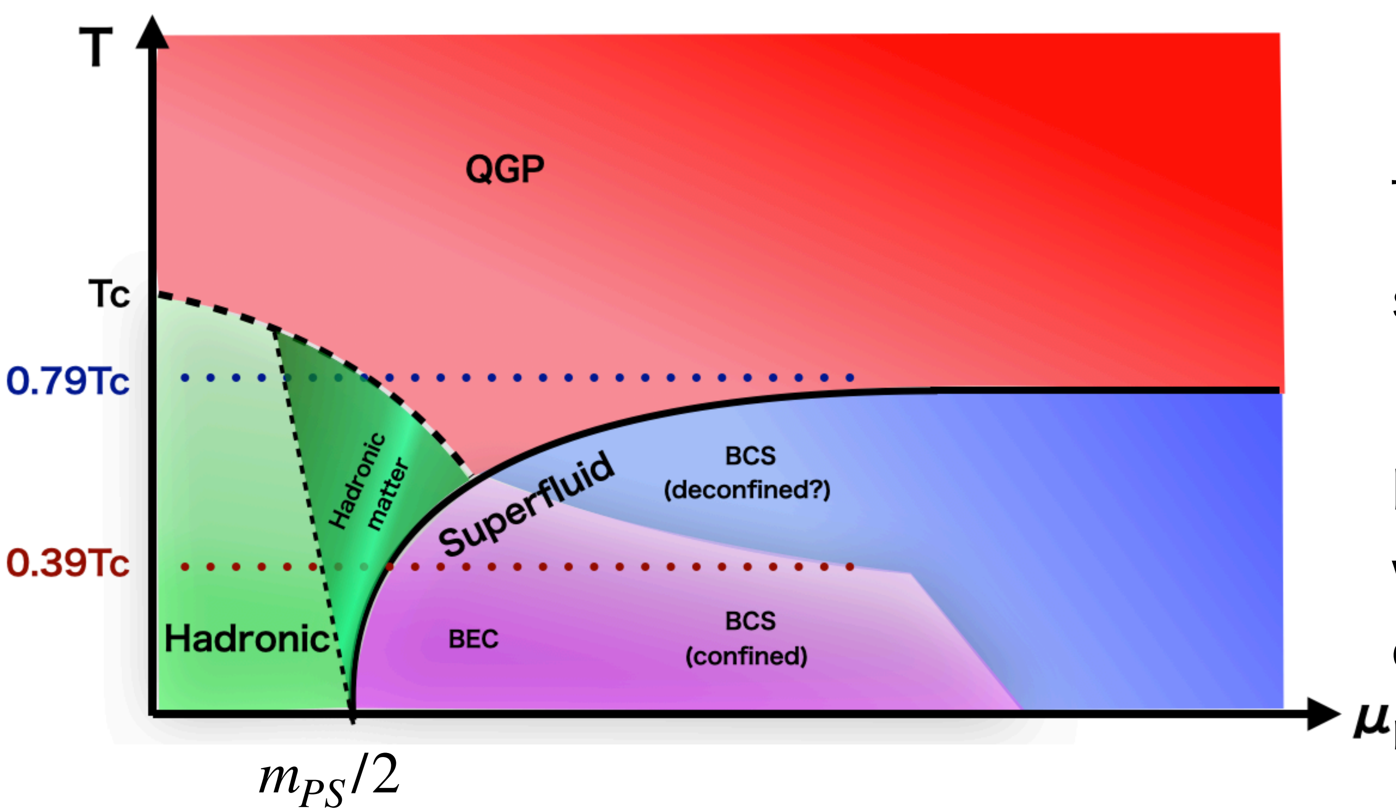
T=47 MeV (**deconfined** coarse lattice in 2012, but **confined** in 2019)

T=45 MeV (**confined** in 2019)

**All data seem to be in agreement with the phase diagram,**
**though all data are not taken in the continuum limit and the scale setting may not be seriously estimated**

# Confinement/deconfinement in extremely high-density?




A typical momentum of quarks is T.
If T is lower than the gap energy in SF phase, then quarks are quenched.
Thus, $\Lambda_{QCD} \gg T$ , the quenched QCD shows the confinement.

In $\mu \gg \Lambda_{QCD}$ , the deconfinement occurs since the distance between quarks is shorter than the confinement scale?

The $\mu$ regime inside of the neutron stars is the same order of the onset scale.
($m_{PS}/2 \leq \mu \lesssim (2 \sim 3) m_{qq}/2$ )

Both cases indicate that the study of the BCS with confinement will be important to qualitative understanding of the physics of neutron stars.

# Summary

- 2color-QCD phase diagram is being determined by first-principle calculations
- Instanton configurations exist at low-T and high-$\mu$

(it suggests that the phenomena may be different from a perturbative picture)

What we want to know?

- Phase diagram on $T-\mu$ plane
- Nonperturbative objects (instanton, monopole)
- $\mu$ dependence of hadron masses and nuclear force
- Eq. of state (pressure, internal energy, entropy)
- Transport coeff. (Viscosity, superfluid density)