Two color QCD as a laboratory : delineating gluon dynamics in dense quark matter & the zero point energy in quark-hadron continuity

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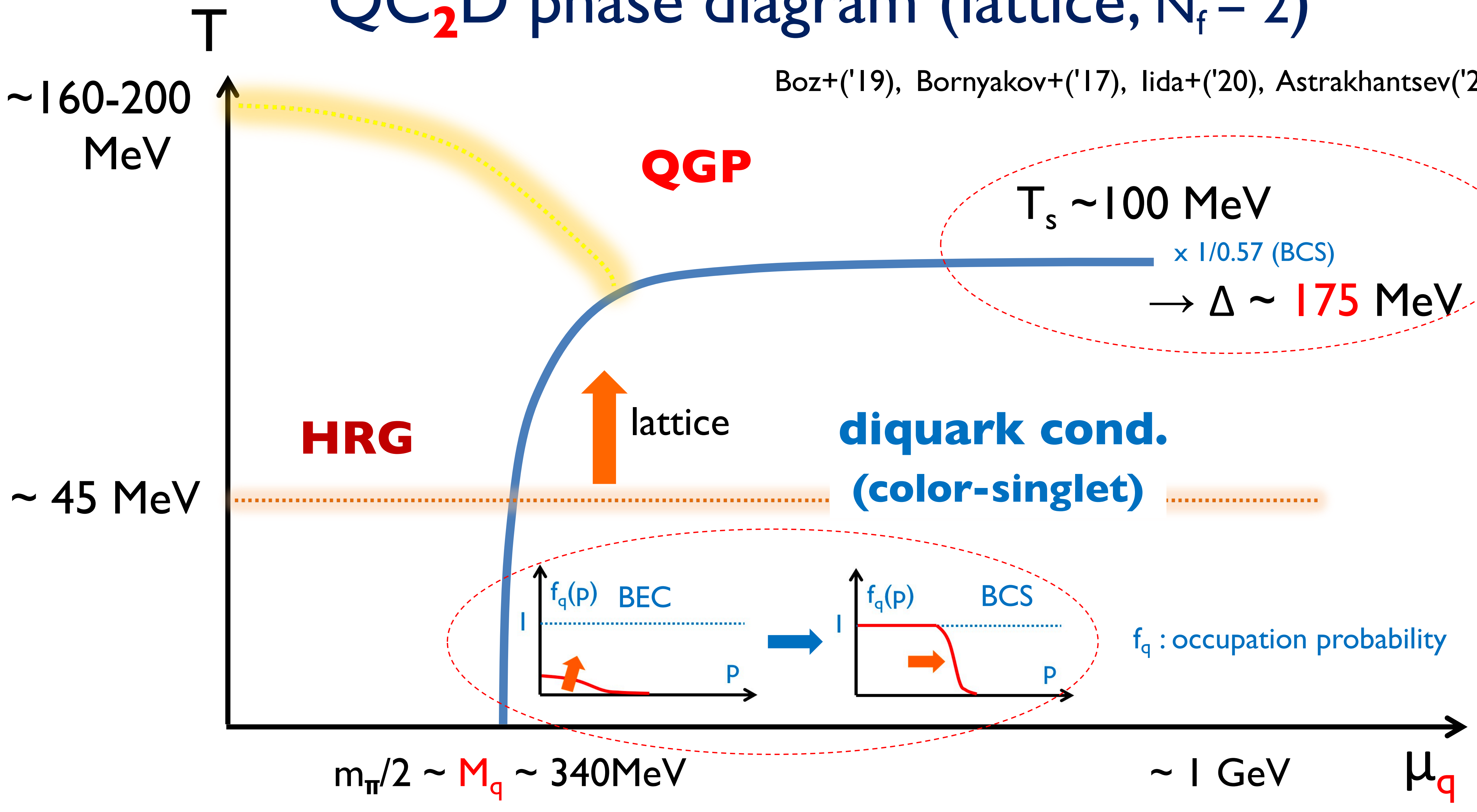
Refs) T.K., G. Baym, PRD89 (2014) no.12, 125008

D. Suenaga, T.K., PRD100 (2019) no.7, 076017

T.K. PRD101 (2020) 3, 036001

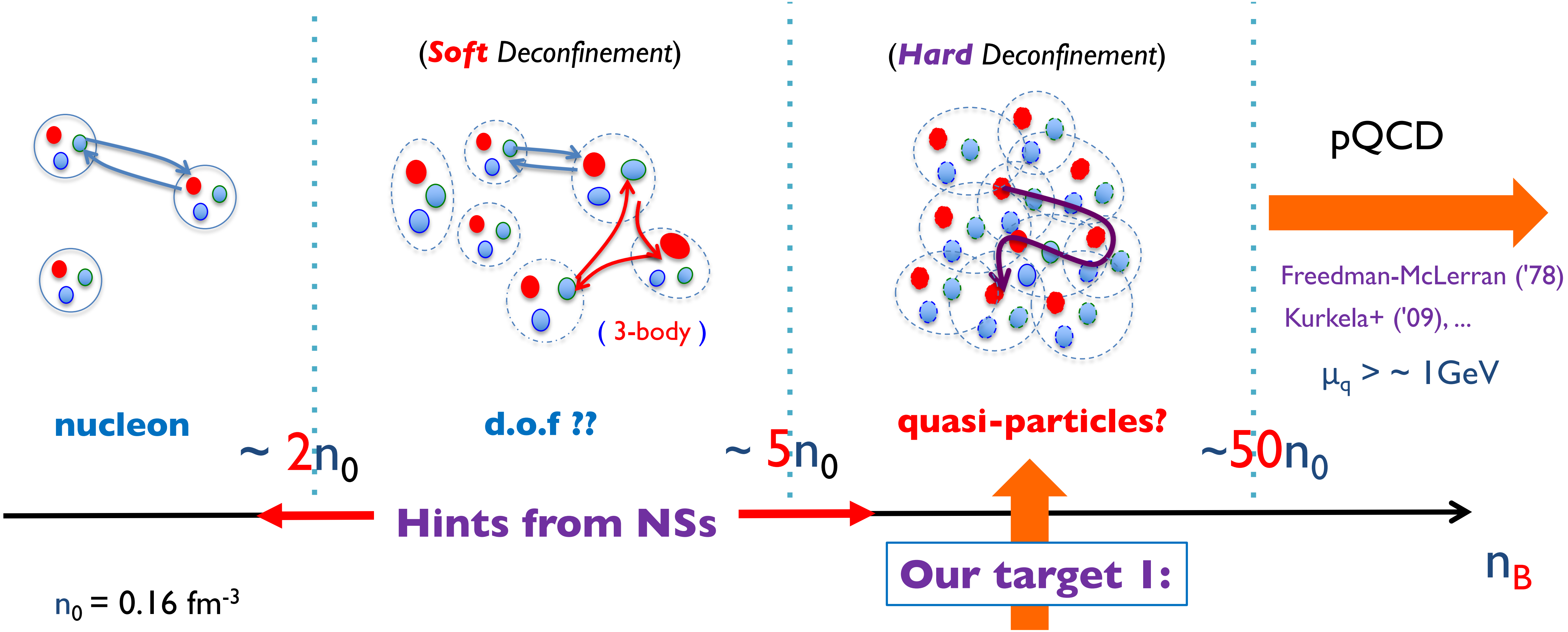
QC_2D phase diagram (lattice, $N_f = 2$)

Boz+('19), Bornyakov+('17), Iida+('20), Astrakhantsev('20)



What would we learn for QCD (Nc=3)?

A sketch for Nc = 3: Masuda+('12), TK+('14), Fukushima+('20),...

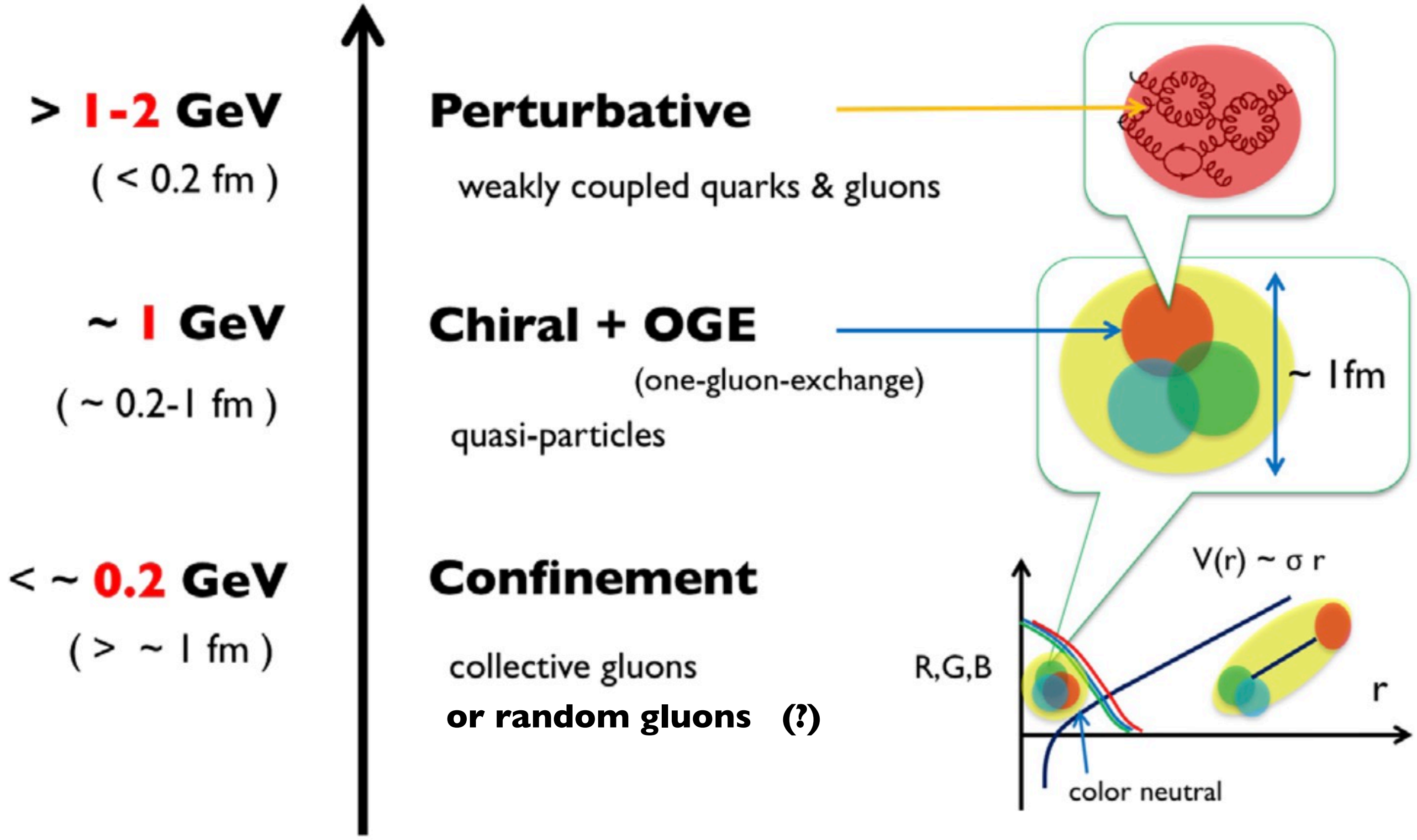


The difference b.t.w 2- and 3-colors may not be important

What would we learn for QCD (Nc=3)?

quasi-particles → the **residual** interactions get **under control**

e.g.) **a** hadron [Manohar-Georgi('83), Weinberg('10)]



[fig. from Suenaga+ ('19)]

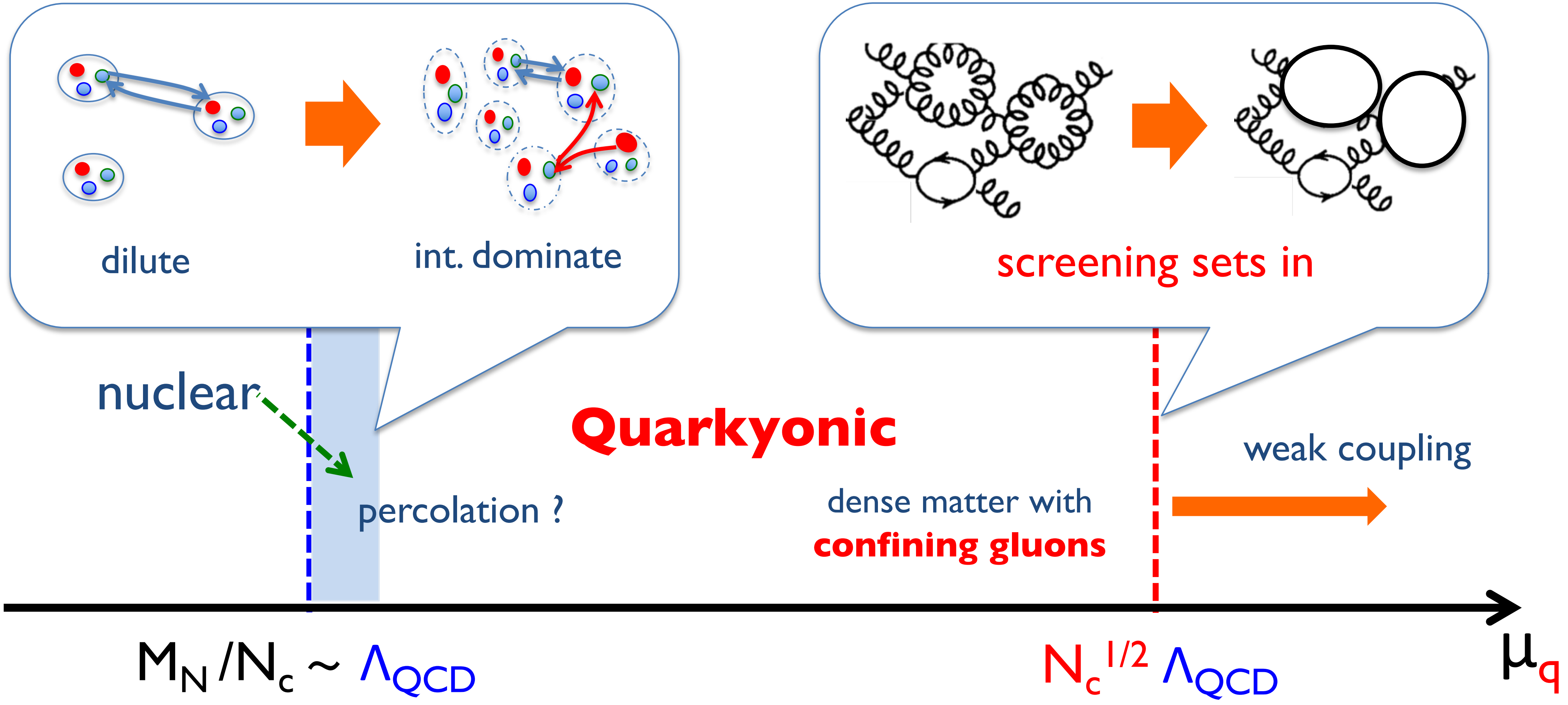
→ **target 1**

also helps to **sharpen** questions on the **deep IR** part

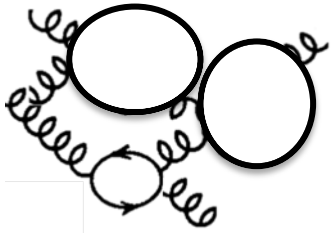
What would we learn for QCD (Nc=3)?

target 2 : the **interplay** between **gluons** & **quarks**

e.g.) **large N_c** : [McLerran-Pisarski '07]



What would we learn for QCD (Nc=3)?



sensitive to **soft** excitations (*phase structure*)

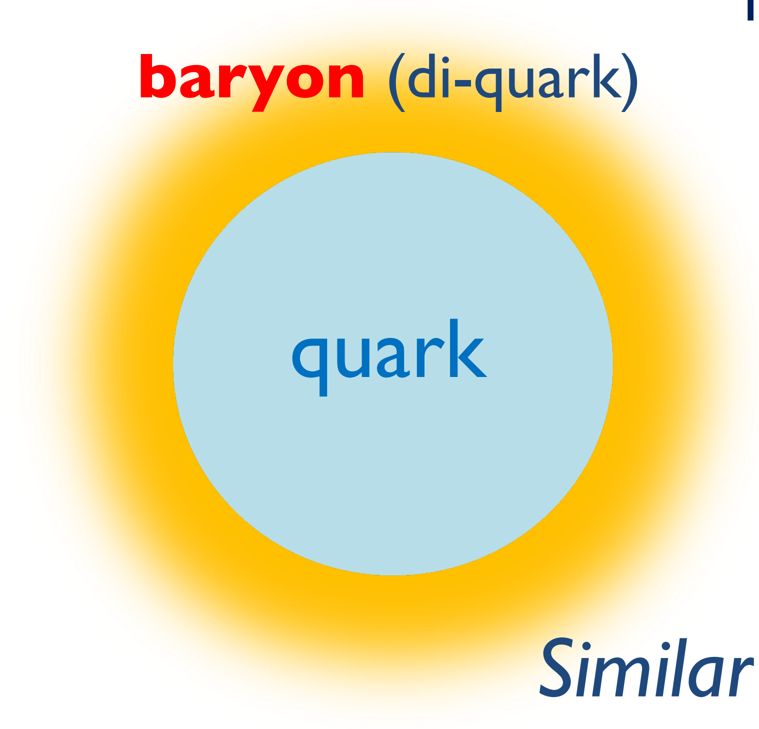
Fermi sea

N_c= **2**

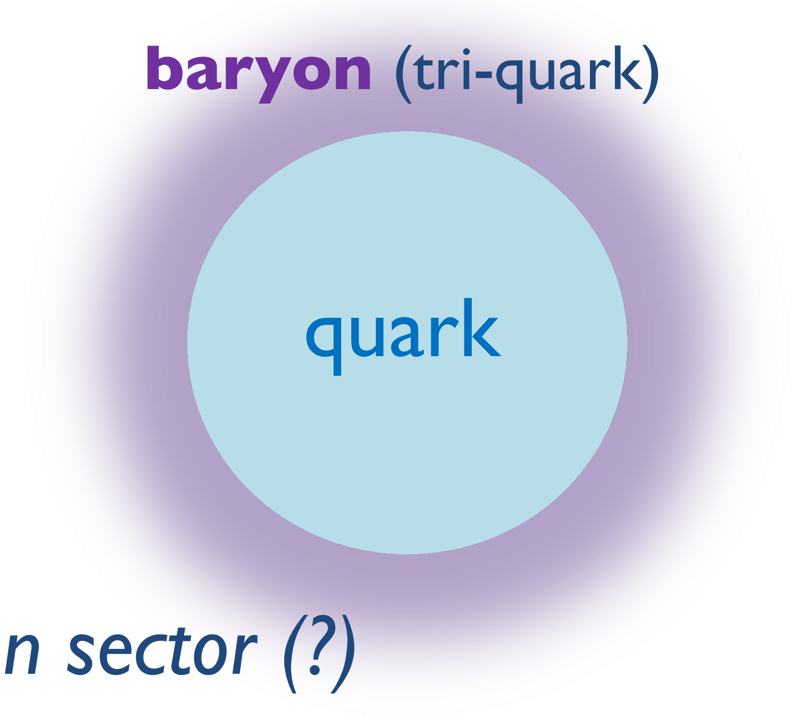
N_c= **3**

(*quarkyonic conjecture*)

baryon (di-quark)



baryon (tri-quark)



in both cases,
quarks *(near the edge of the FS)*
are ***trapped*** *into*
color-singlet composites

Similar medium effects on the gluon sector (?)

Contents

- 1, Motivations (DONE)
- 2, Color-screening in 2-color dense matter
- 3, Zero-point energy of composite particles

Target: in-medium gluon propagators

$$D_{\mu\nu}(k) = D_E(k) P^E_{\mu\nu} + D_M(k) P^M_{\mu\nu}$$

E/M = Electric/Magnetic

gluon **tree** gluon quark **medium** modification

$$D^{-1}_{E,M} = \left[D^g_{\rm tree}\right]^{-1} + \Pi^g_{\rm vac} + \Pi^q_{\rm vac} + \left(\Pi^q_{E,M} - \Pi^q_{\rm vac}\right)$$

$$= D^{-1}_{\rm vac} + \Delta\Pi_{E,M}$$

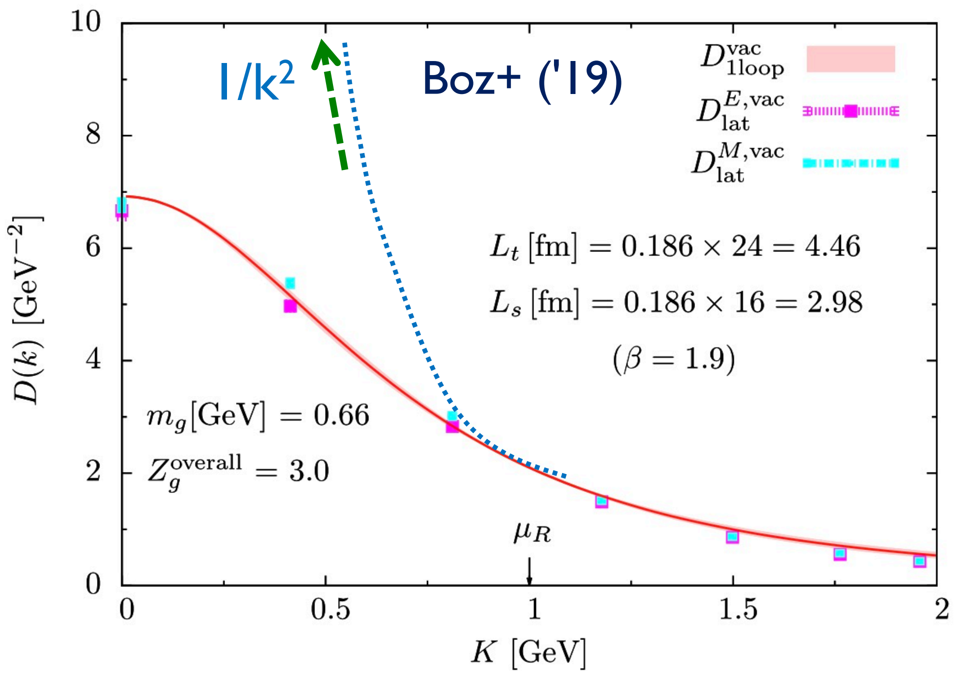
use **vac** lattice data as an **input**

to be **computed** **& compared** with **lattice's**

$D_{\rm vac}(k)$: we use **massive** YM as a guide (**Landau** gauge)

[Curci-Ferrari('76), Tissier-Wschebor('11-), Kondo+('16-), ...]

vac lattice data for $D_{\rm vac}$ (k)



Gribov copies & massive YM (**Landau** gauge)

Gribov ('78): Faddeev-Popov method doesn't fix the non-Abelian gauge :

$$A \neq A^{\theta} \quad \text{but} \quad \partial A = \partial A^{\theta} = 0 \quad \text{or} \quad \partial^2 \theta = \partial \left(\underline{g A}\, \theta \right)$$

(for QED; $\partial^2 \vartheta = 0$)

non-trivial ϑ for **large** gA (strong coupling and/or large amplitudes)

Some copies A^{ϑ} make the **FP-det.** **negative** → **spoil the FP method**

To **remove** problematic copies: **impose** $\partial \cdot D[A^{\theta}] > 0$ (**non-local** condition)

Gribov converted it into a *tractable* *(approximate)* form:

$$\frac{V_4}{N_c^2} \int_k \frac{g^2 \underline{A(k) A(-k)}}{k^2} \lesssim O(1)$$

tempers the **size** of A

"**saturation**" *(?)*

Gribov copies & massive YM (**Landau** gauge)

A more primitive way to **cutoff** **large amplitude** fields:

$$\mathrm{e}^{-S_{\mathrm{YM}}} \;\to\; \underline{\mathrm{e}^{-\frac{m_g^2}{2}\int_x A_\mu^2(x)}}\, \mathrm{e}^{-S_{\mathrm{YM}}} \;\equiv\; \mathrm{e}^{-S_{\mathrm{mYM}}}$$

massive YM

local, gauge-**variant** extension

(extra discussions needed)

or

$$\mathrm{e}^{-S_{\mathrm{YM}}} \;\to\; \mathrm{e}^{-\frac{m_g^2}{2}\int_x A_\mu(x)\left(g_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\underline{\partial^2}}\right)A_\nu(x)}\, \mathrm{e}^{-S_{\mathrm{YM}}}$$

(**to be used**)

non-local, but gauge-**invariant** extension

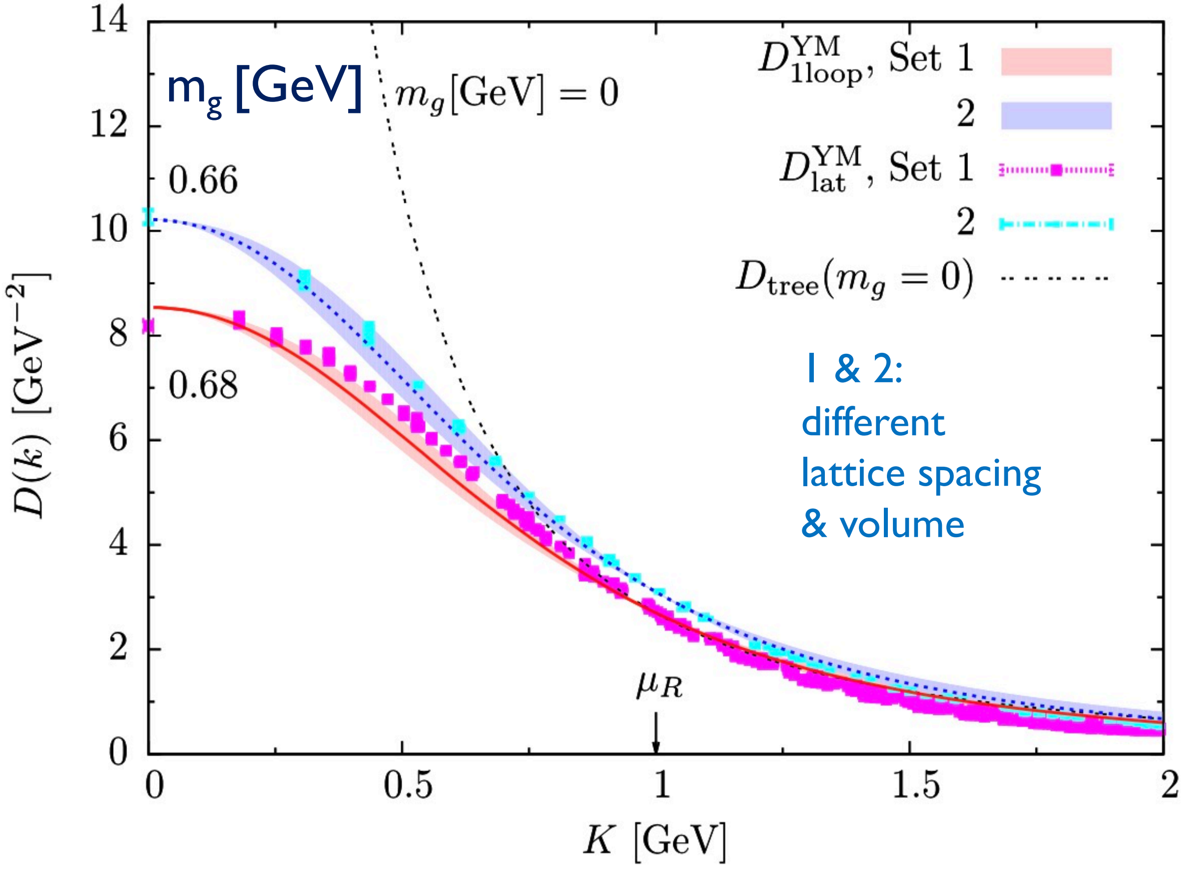
this massive extensions lead to qualitatively similar results as Gribov's.

(**except** the **deep** IR domain; i.e., *scaling* vs *decoupling*)

massive YM & perturbative stability

(see also Fukushima-Su '13 for hot QCD)

comparisons with pYM data (Boz+'19)



error bands for theory side

variations: α_s = **1 - 3** (!)

→ only < ~ 10 % corrections

at **small *k***

$$D^{-1}_{\mathrm{tree}}(k) \sim \underline{k^2} \qquad \ll \quad m_g^2$$

$$\Pi(k) \sim \alpha_s \times \underline{k^2}$$

once $\mathbf{m_g}$ is chosen, the **residual** corrections are **under control**

(better **systematics**)

in-medium propagators with Δ

Nambu-Gor'kov: $$\mathcal{S}^{-1} = \begin{pmatrix} i\partial\!\!\!/ + i\mu_q\gamma_4 - M_q & \bar{\mathbf{\Delta}} \\ \mathbf{\Delta} & i\partial\!\!\!/ - i\mu_q\gamma_4 - M_q \end{pmatrix}$$

color-flavor-Dirac
$$\mathbf{\Delta} \equiv \sigma^2\tau^2\gamma_5\Delta$$

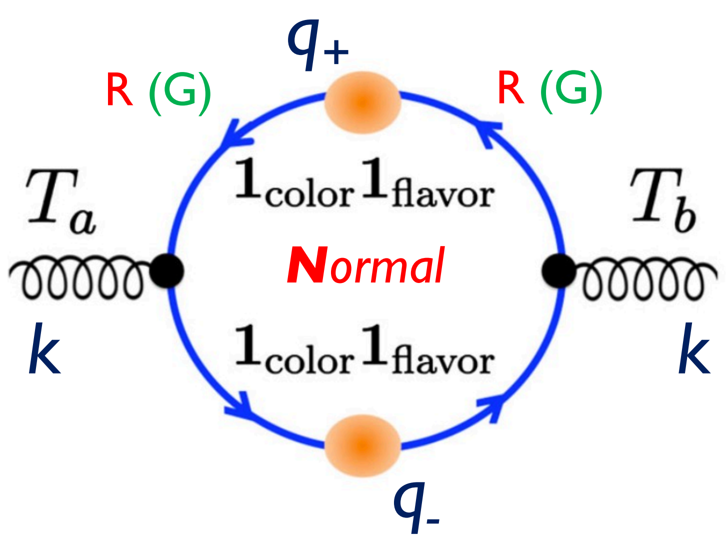
1) *particle*- & *antiparticle*- decomp. : $$\mathcal{S} = \mathcal{S}^{\mathrm{p}}\Lambda_{\mathrm{p}} + \mathcal{S}^{\mathrm{a}}\Lambda_{\mathrm{a}}$$

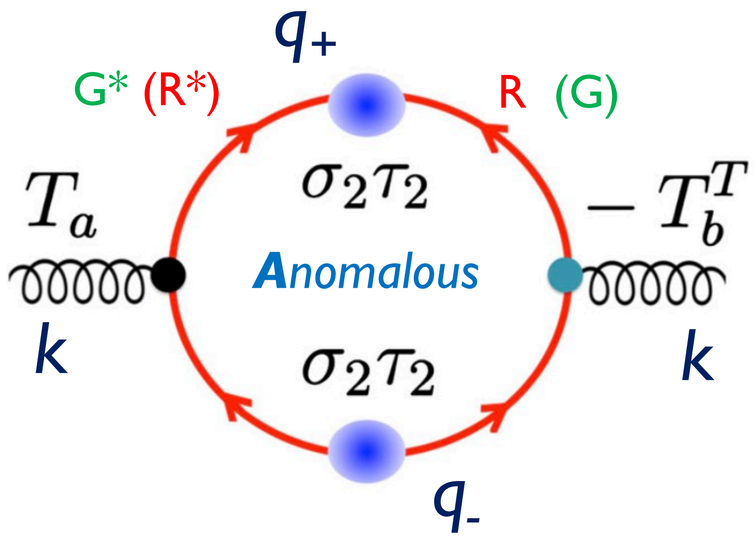
2) ***N**ormal*- & ***A**nomalous*- decomp. : $$\mathcal{S}^{\mathrm{p/a}} = \mathcal{S}_N^{\mathrm{p/a}} + \mathcal{S}_A^{\mathrm{p/a}}\sigma_2\tau_2$$

projectors
$$\Lambda_{\mathrm{p,a}} = \gamma_0 \frac{E_q\gamma_0 \pm (M_q + \vec{\gamma}\cdot\vec{q})}{2E_q}$$

- *polarization tensors:* $$\Pi_{\mu\nu}(k) = \Pi_E(k)P^E_{\mu\nu} + \Pi_M(k)P^M_{\mu\nu}$$ ← **target**

  [Rischeke+, Shovkovy+, many others]





pseudo-real

G* = R

Diagnosing screening effects

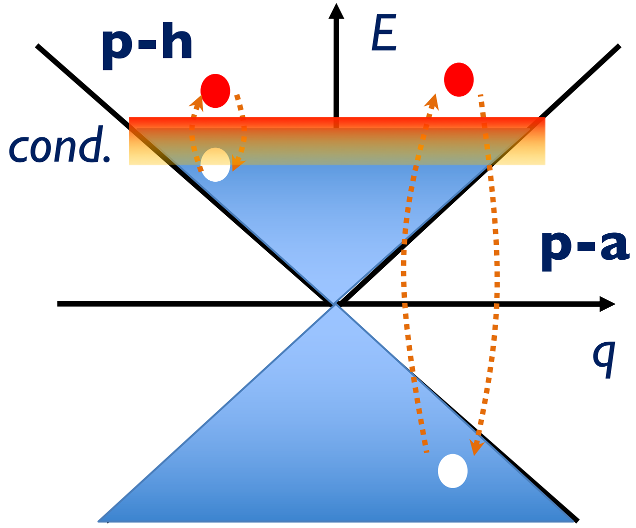
$$\Delta\Pi^{\rm phys}_{E,M} = g^2 \sum_{s,s'={\rm p,a}} \int_{\vec q} \mathcal{K}^{ss'}_{E,M}(\vec q_\pm)\, \mathcal{C}^{ss'}_{E,M}(\vec q_\pm)\, \mathcal{P}^{ss'}_{E,M}(\vec q_\pm) - (vac\ \&\ counter)$$

kin. factors ($\mathcal{K}$), coherence factors ($\mathcal{C}$), propagators ($\mathcal{P}$)

$\mathcal{C}^{ss'}_{E,M}(\vec q_\pm)$: from $\mathrm{Tr}_{color}[...]$, for **color-interference**

$\mathcal{P}^{ss'}_{E,M}(\vec q_\pm)$: sensitive to **gaps**

$\mathcal{K}^{ss'}_{E,M}(\vec q_\pm)$: from $\mathrm{Tr}_D[...]$, **doesn't depend on Δ**



	p-h	*p-a*	*k → 0*	
K_E	**+2**	$\sim k^2$	➡	***p-h* dominant**
K_M	$\sim q^2 \sin^2\theta$	−2	➡	*p-h*, *p-a* (& *a-ah*) **all important** (for gauge invariance)

Electric screening (p-h dominant, $K_E \sim 2$)

$$\Delta\Pi_E^{\text{phys}}(k) \propto \int_k \langle\, j_0^a(k) j_0^b(-k)\,\rangle \overset{k\to 0}{\sim} \langle\, \delta Q_a\, \delta Q_b \,\rangle$$ [x **IR** contributions]

· **p-h : sensitive to the phase structures**

$\delta Q_a \,|\text{normal/singlet}\rangle = 0$ *no fluct.*

$\delta Q_a \,|\text{Higgs}\rangle \neq 0$ *coherent state*

	condensates	***normal***	***singlet diquarks***	***colored diquarks***
coherence factors:	$\mathcal{C}_E\vert_{\vert\vec{q}\vert\simeq p_F}$	$\sim k^2 \to 0$	$\sim k^2 \to 0$	$\sim \Delta^2$ *finite for k->0*
propagators:	$\mathcal{P}_E\vert_{\vert\vec{q}\vert\simeq p_F}$	$\sim 1/k^2$ *gapless (**IR singular**)*	$\sim 1/\Delta^2$ *gapped*	$\sim 1/\Delta^2$ *gapped*
⇨	*Debye mass* *k->0*	$\sim g\, p_F$	0 ***vanishing***	$\sim g\, p_F + (\Delta/p_F)^2$

Magnetic screening (p-h, p-a, a-ah)

more tricky than the electric case

- *p-a* : **dia**-magnetic → screening
- *p-h* : **para**-magnetic → **anti**-screening
- *a-ah* : **small** (but **must be kept** to maintain the **gauge inv.**)

for k → 0	$\Delta\Pi^{M}_{\rm pa}$	$\Delta\Pi^{M}_{\rm ph}$	$\Delta\Pi^{M}_{\rm aah}$		***mag. mass***	
normal	> 0	< 0 (!)	$= 0$	⇒	0	*as expected*
***singlet** diquark*	> 0	< 0 (!)	$\sim(\Delta/\mu)^2$	⇒	0	*indep. of Δ*
***colored** diquark*	> 0	$= 0$ (!)	$\sim(\Delta/\mu)^2$	⇒	$\sim g\, p_F$ (**NOT** $\sim g\Delta$)	*Meissner mass*

Spurious contributions (correcting errors in literatures)

calculations include the vacuum subtraction : tricky if quark bases in med. & vac. are different

e.g. magnetic sector)

$$\Delta\Pi_M(k) = \Pi_M(k;\underline{\mu,\Delta,M_*}) - \Pi_M(k;\underline{0,0,M_{\rm vac}}) \overset{(k\sim 0)}{\sim} O(\underline{\Delta^2, M_*^2 - M_{\rm vac}^2})$$

(dim 2) — med. values — vac. values — IR quantities !!

magnetic mass without Meissner effects ?

Essentially the same problems were found in 2SC calculations:

- Rischke ('00): neglected higher order terms would cancel the artifacts (?).
- Alford-Wang ('05): found this is not the case, and proposed a subtraction scheme,

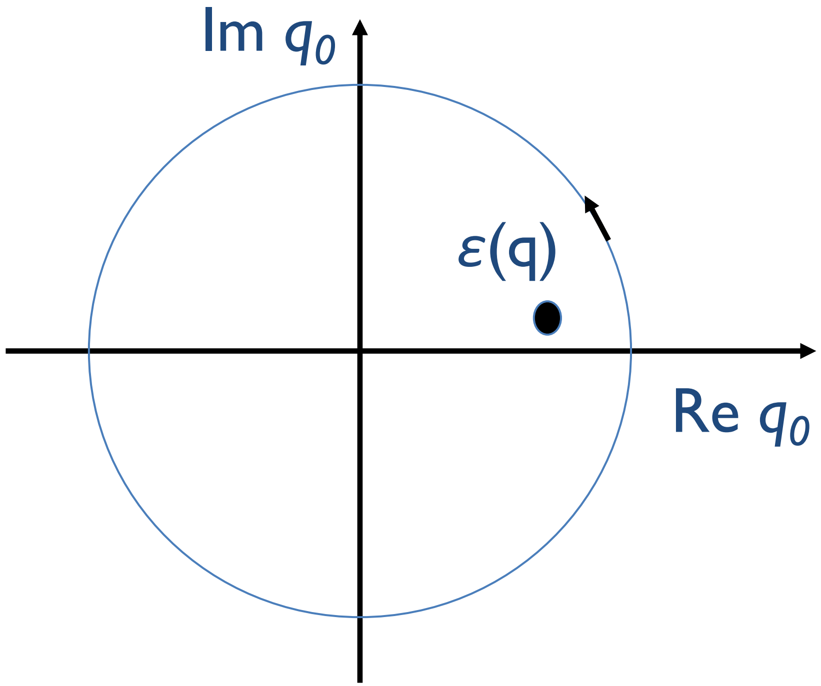
$$\Delta'\Pi_M(k) = \Pi_M(k;\mu,\underline{\Delta}) - \Pi_M(k;0,\underline{\Delta})$$

at work, but no theoretical justification.

Identifying the origin of artifacts

In medium calculations,

we first pick up poles in the q_0 - complex plane (dim reg. is complicated when **μ ≠ 0**)



For this computations to be unambiguous,

q_0 must be always greater than ε(**q**) :

Regularization

we regulate |**q**| < Λ, pick up residues,
and take **Λ** → ∞ at the **end** of calculations

-> ***gauge variant terms***

(vac. subtraction cancels in textbook examples)

Ward-identities to identify the gauge variant terms

WTI $\rightarrow$ $$k_\mu \Pi^{1\mathrm{loop}}_{\mu\nu}(k) \propto \int_{\vec{q}} \int_{q_0} \theta(\Lambda^2 - \vec{q}^2)\, \mathrm{tr}_D \Big[\big(\mathcal{S}^D(q_+) - \mathcal{S}^D(q_-) \big) \gamma_\mu \Big]$$

should be zero

the origin of 1-loop artifacts

$$\propto \; k_j \delta_{j\nu} \times \Lambda^3 \times \left(\frac{C_{\mathrm{univ}}}{\Lambda} + \frac{C_{\mathrm{dim2}}[\mathcal{S}]}{\Lambda^3} + \cdots \right)$$

phase space

asymp. behavior of S

after vacuum subtraction:

IR quantities!

$$k_\mu \big(\Pi[\mathcal{S}_{\mathrm{med}}] - \Pi[\mathcal{S}_{\mathrm{vac}}] \big)^{1\mathrm{loop}}_{\mu\nu} \;\propto\; k_j \delta_{j\nu} \big(C_{\mathrm{dim2}}[\mathcal{S}_{\mathrm{med}}] - C_{\mathrm{dim2}}[\mathcal{S}_{\mathrm{vac}}] \big)$$

Remarks :

- **UV artifacts couple to IR quantities** (gap functions, etc).
- The electric sector is safe (ν=0); but the magnetic sector violates the WTI.
- If $S_{\mathrm{med}} = S_{\mathrm{vac}}$ (e.g. as in pert. theories), no violation of the WTI.

Eliminating artifacts

- introduce gauge **variant** counter terms [T.K. and Baym ('14)]

Demand: gauge variant (regularization + counter terms) → gauge invariant results

$$\delta\Pi^{\rm counter}_{\mu\nu}[\mathcal{S}] = -g_{\mu i} g_{\nu j} \delta_{ij} \underline{C_{\rm dim2}[\mathcal{S}]}$$

depends on bases *S (!)*

This cancels the longitudinal (gauge variant) contributions. For the EM sectors

$$\Delta\Pi^{\rm phys}_E \equiv \Delta\Pi^{\rm 1loop}_E \big|_{\rm 3d\,reg.} - \frac{k_0^2}{k^2}\left(C_{\rm dim2}[\mathcal{S}_{\rm med}] - C_{\rm dim2}[\mathcal{S}_{\rm vac}] \right)$$

$$\Delta\Pi^{\rm phys}_M \equiv \Delta\Pi^{\rm 1loop}_M \big|_{\rm 3d\,reg.} - \left(C_{\rm dim2}[\mathcal{S}_{\rm med}] - C_{\rm dim2}[\mathcal{S}_{\rm vac}] \right)$$

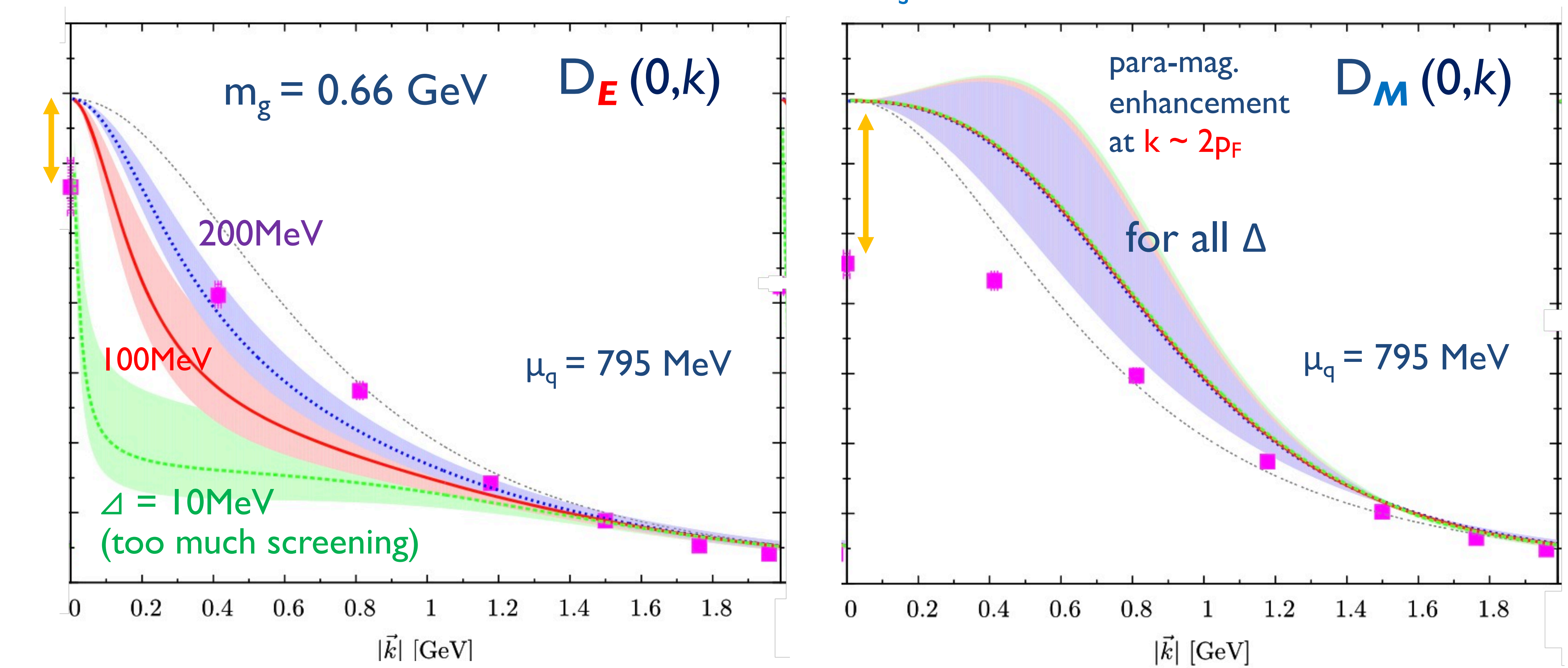
The extra contributions **precisely** cancels the **spurious** magnetic masses.

[see also Suenaga-T.K. ('19) for the elaborated version]

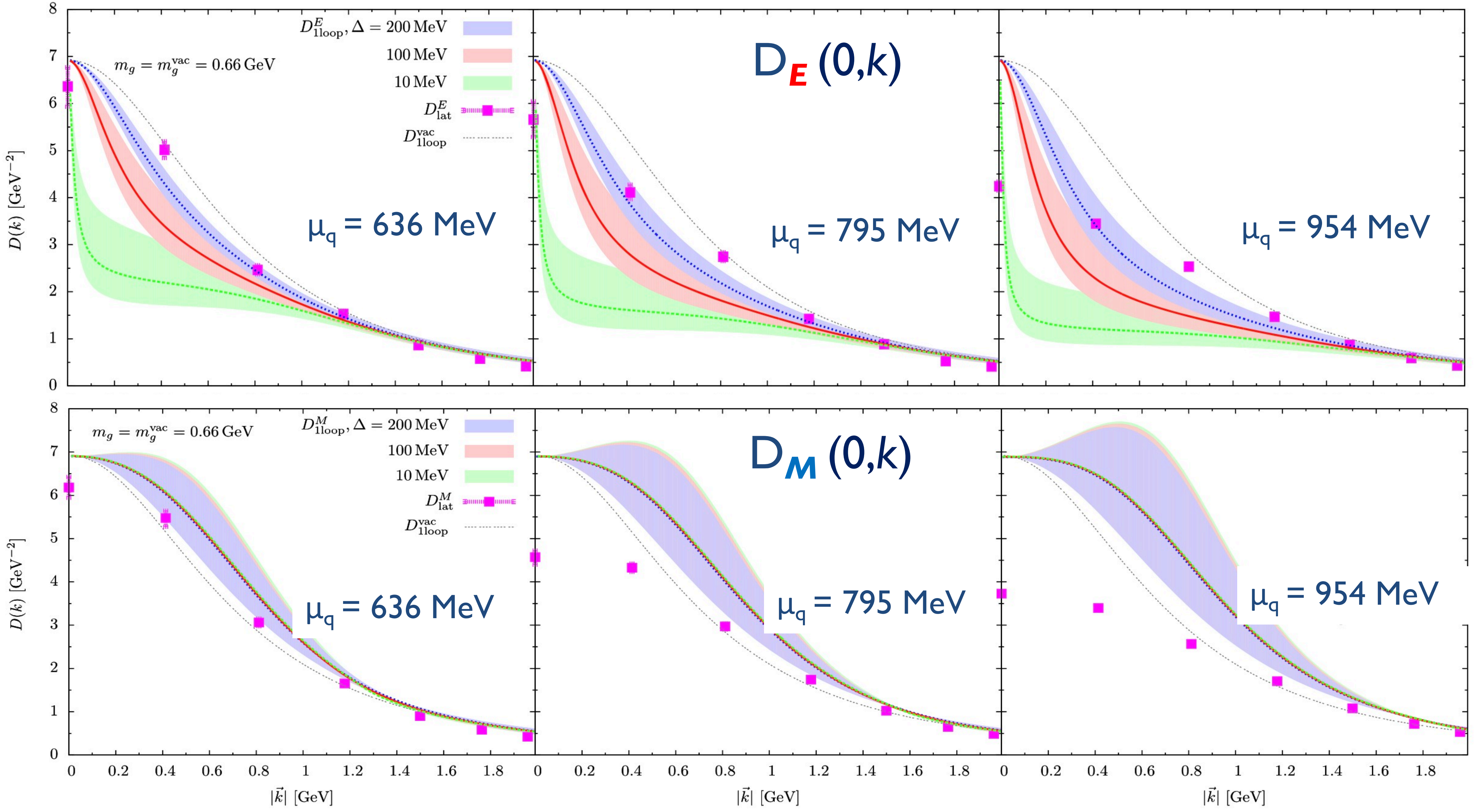
Comparisons with Lattice results (Boz+'19)

[figs. from Suenaga+ ('19)]

error bands: for α_s = 1-3



difference in the IR, on the lattice the suppressions in both E & M


$D_E\,(0,k)$
$D_M\,(0,k)$
$m_g = m_g^{vac} = 0.66$ GeV
$D^E_{1loop}, \Delta = 200$ MeV
100 MeV
10 MeV
D^E_{lat}
D^{vac}_{1loop}
$D^M_{1loop}, \Delta = 200$ MeV
D^M_{lat}
$\mu_q = 636$ MeV
$\mu_q = 795$ MeV
$\mu_q = 954$ MeV
$D(k)$ [GeV^{-2}]
$|\vec{k}|$ [GeV]

Summary on the comparisons

- massive YM : perturbative stability
- Δ = 100 -200 MeV reasonable, consistent with $T_s \sim$ 100 MeV
- discrepancy; IR gluons are protected in theory; mild modifications on the lattice

obvious things to test: [Suenaga-T.K., work in progress]

- **RG improvement**: medium effects are already important at momenta k ~ 1GeV

  (still in 1-loop regime) p-h (Overhauser channel) with ~ $2k_F$

- During such computations, E-sector enters internal loops of M-gluons, and vice virca.

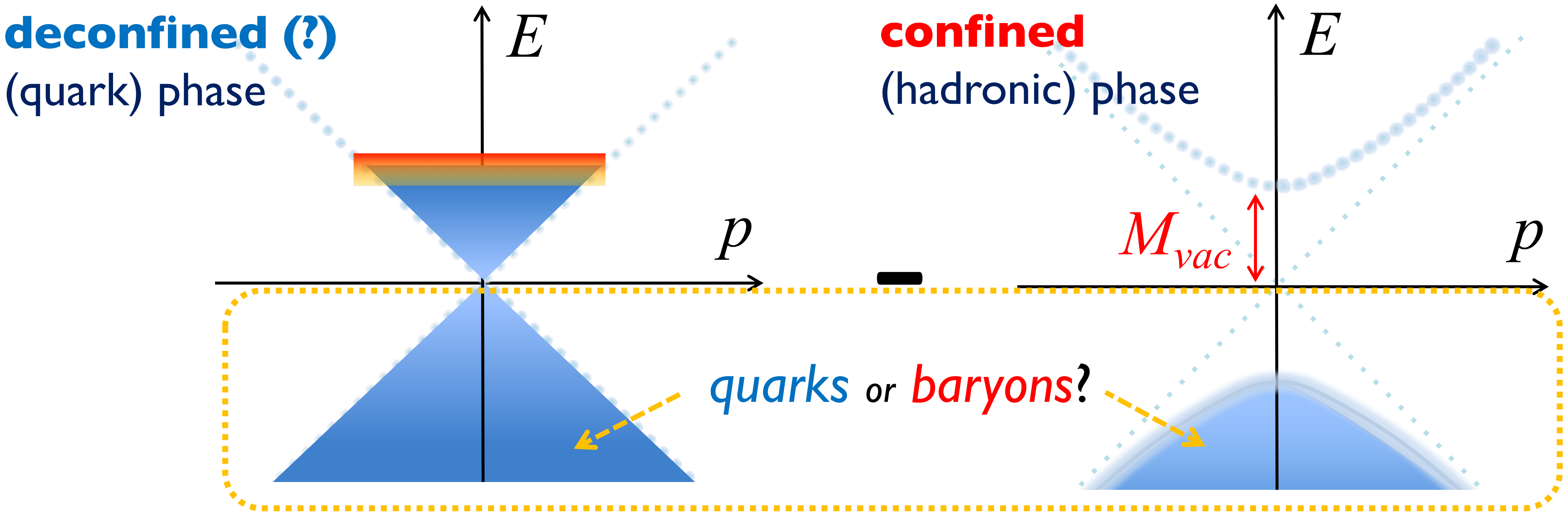
  neutralize the disparity b.t.w E and M sectors (?)

3, Zero point energy of composite particles:

in quark-hadron continuity

zero-point energy: quarks vs baryons

when we compute EoS at finite density, we need to subtract P_{vac}



d.o.f to compute the vac. energy? impacts of the **phase structure**?

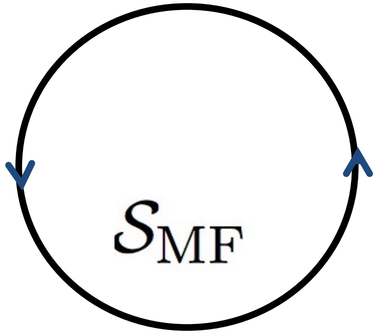
(or even well-defined question?)

Gaussian *pair-fluctuation* theories (**2**-color)

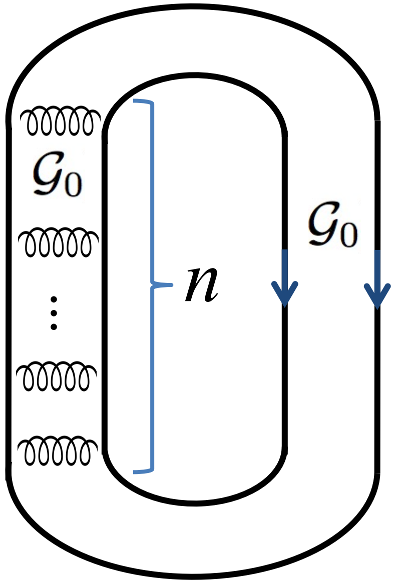
[**relativistic** ver. : Abuki ('07), He+ ('10), ...]

quark (MF) + **baryon**

$$\Omega_{\rm GF} = $$



$$+ \sum_{n=1} \frac{1}{2n}$$



$$\Omega_q = {\rm TrLn}\mathcal{S}_{\rm MF}$$

$$\Omega_{2q} = -\frac{1}{2}{\rm TrLn}\left(\mathcal{G}/\mathcal{G}_0\right)$$

At μ = 0

$$\sim \; -\int_{\mathbf{k}} \sqrt{\mathbf{k}^2 + M_q^2}$$

$$\sim \; -\int_{\mathbf{K_B}} \sqrt{\mathbf{K_B}^2 + M_B^2} + \cdots$$

pole contributions

UV divergences couple to IR

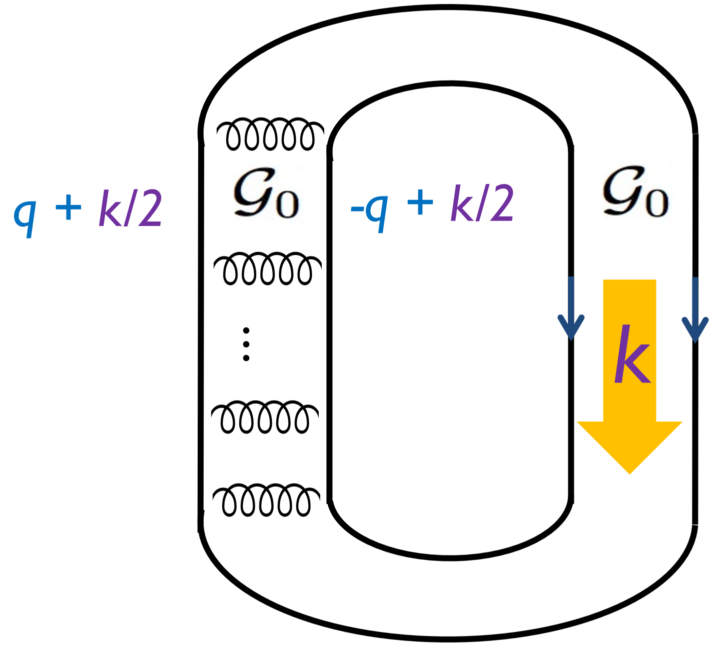
after **vac. subtraction**

dim 4

$$\Omega_{\rm GF}(\mu) - \Omega_{\rm GF}(\underline{\mu = 0}) \sim \underline{\Lambda_{\rm UV}^2} \,\underline{\left(\mathcal{C}_{\rm dim2}[\mathcal{S}_{\rm med}] - \mathcal{C}_{\rm dim2}[\mathcal{S}_{\rm vac}] \right)}$$

IR quantities

e.g. diquark fluct. part : no natural cutoff for **total** momentum ***k***



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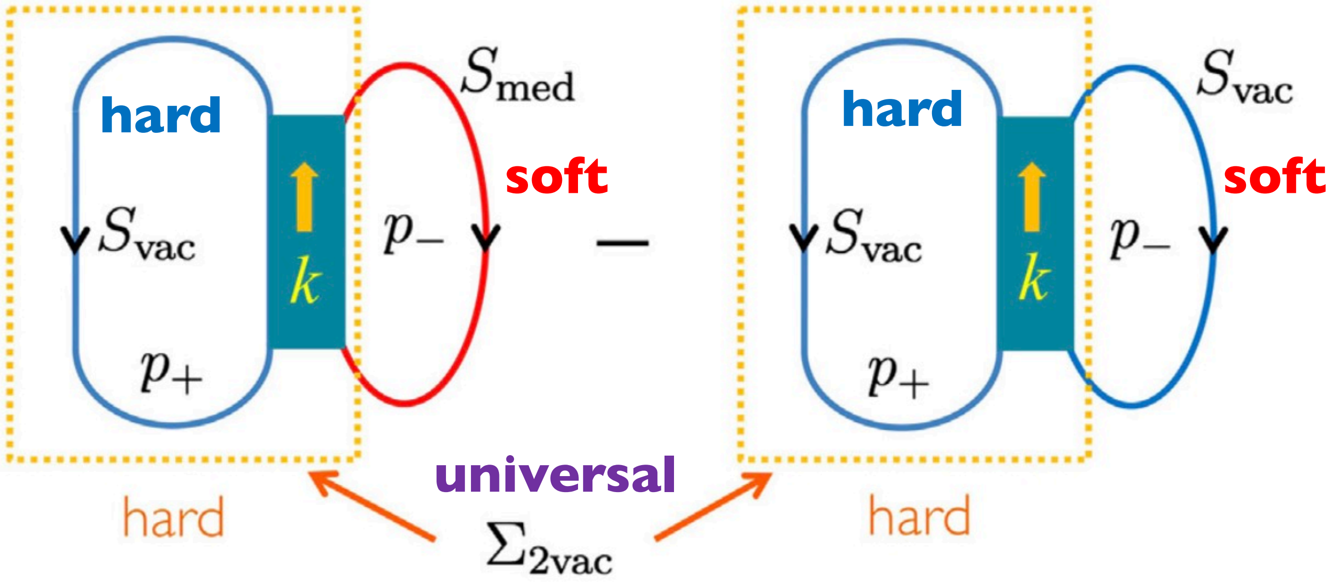


2PI (Φ-derivable) effective action

[Luttinger-Ward ('60), Baym ('62), Cornwall+ ('74), ...]

The *GPF*-theory did not correctly handle the **double-counting** problem; to fix, we use

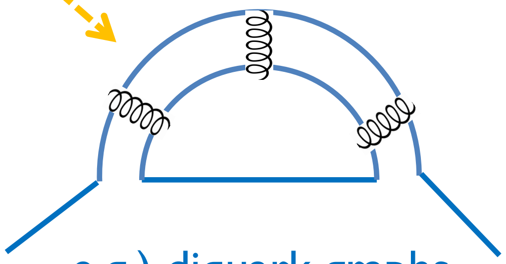
$$I[S] = \mathrm{TrLn}S + \underline{\mathrm{Tr}[S\tilde{\Sigma}]} + \underline{\Phi[S]} \qquad \tilde{\Sigma} = S^{-1} - S^{-1}_{\mathrm{tree}}$$

absent in the *GPF* theory   e.g.) diquark graphs

The *extrema:* $$\left.\frac{\delta I[S]}{\delta S}\right|_{S=S_*} = 0 \quad \Rightarrow \quad S^{-1} = S^{-1}_{\mathrm{tree}} - \frac{\delta \Phi}{\delta S}$$ Schwinger-Dyson eq.

The *thermodynamic potential:* $$\Omega = I[S_*]$$

[**Renormalizability**: vanHees+ ('01), Blaizot+ ('04), Reinosa+('10),...]



e.g.) diquark graphs

UV divergences & quark bases

[TK ('19)]

$$I_R[S] \equiv I^{\mu}[S] - I^{\mu=0}[S_{\rm vac}]$$

vac bases except $p_0 \to p_0 - \mathrm{i}\mu$

$$= \left(I^{\mu}[S] - I^{\mu}[S_{||}] \right) - \left(I^{\mu}[S_{||}] - I^{\mu=0}[S_{\rm vac}] \right)$$

at same μ, but different bases — *same bases, but at different μ*

$$\equiv I^{\mu}_{\Delta S}[S] + I_{\Delta\mu}$$

our target

(textbook example; UV div. can be cancelled term by term)

Expand in powers of $(S\text{-}S_{||})$:

$$S - S_{||} \sim \frac{\Delta\Sigma}{\not p} \sim \frac{1}{\not p} \frac{\mathcal{M}^2}{p^2}$$

IR quantities

extra suppression in UV

$$I^{\mu}_{\Delta S}[S] = \mathrm{Tr}\left[\frac{\delta I_{\Delta S}}{\delta S}\bigg|_{S_{||}} (S - S_{||}) \right] + \frac{1}{2}\mathrm{Tr}\left[\frac{\delta^2 I_{\Delta S}}{\delta S_1 \delta S_2}\bigg|_{S_{||}} (S - S_{||})_1 (S - S_{||})_2 \right] + \cdots$$

quadratic div (?) — logarithmic div (?)

consistency $\longrightarrow$ cancel divergences

quark : $\mathrm{TrLn}S - \mathrm{TrLn}S_{||} = \underline{-\mathrm{Tr}\left[S_{||}\Delta\Sigma\right]} + O(\Delta\Sigma^2)$

↕ cancel

double counting corrections $\mathrm{Tr}\left[S\tilde{\Sigma}^{[S]}\right] - \mathrm{Tr}\left[S_{||}\tilde{\Sigma}^{[S_{||}]}\right] = \underline{\mathrm{Tr}\left[S_{||}\Delta\Sigma\right]} + \underline{\mathrm{Tr}\left[(S - S_{||})\tilde{\Sigma}^{[S]}\right]} + O(\Delta\Sigma^2)$

absent in the *GPF* theory

↕ combined to yield ~ $(\Delta\Sigma)^2$

diquark graphs : $\Phi[S] - \Phi[S_{||}] = \underline{-\mathrm{Tr}\left[(S - S_{||})\Sigma^{[S_{||}]}\right]} + O(\Delta\Sigma^2)$

assembling quark & diquark terms ➡ quadratic divergences **all** cancel

(log div. needs more arguments)

To avoid divergences :

when we include diquark d.o.f., quark self-energies ***must*** *include diquark loops.*

Summary

- QC_2D is a great laboratory

  as important as cosmic laboratories such as NSs

- may be very similar to QC_3D at high density

  a regime from $n_B \sim 5n_0$ to $\sim 50\, n_0$ has not been well explored

- a good testing ground for hadron-quark transitions