Lattice study of quark-antiquark interactions in dense quark matter

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Probing the physics of high-density and low-temperature matter with ab initio calculations in 2-color QCD

$SU(3)$ and $SU(2)$ QCD

- $SU(2) + \mu$: no sign problem!
- Symmetry:
  - $SU(3)$: $SU_R(N_f) \times SU_L(N_f) \to SU_V(N_f)$
  - $SU(2)$: $SU(2N_f) \to Sp(2N_f)$
- Goldstone bosons ($N_f = 2$): $\pi^+, \pi^-, \pi^0, d, \bar{d}$
- Baryons are diquarks
- At $\mu = 0$: $m_\pi = m_d$

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However, in dense medium:

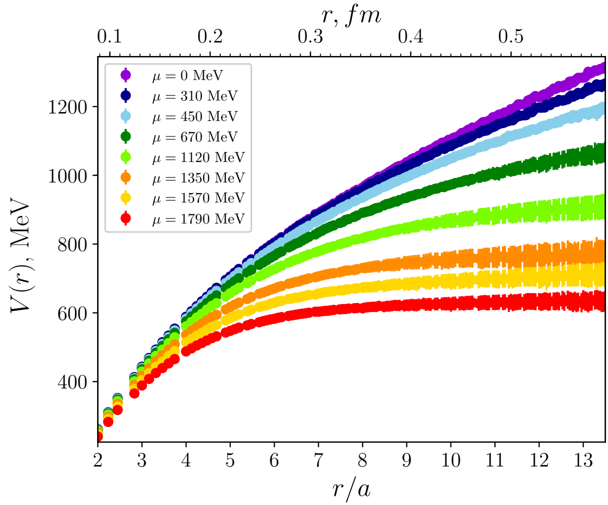
- **Chiral symmetry is restored**
  symmetry breaking pattern is not important
- **Relevant degrees of freedom are quarks and gluons**
  rather than Goldstone bosons

$SU(2)$ **QCD provides a perfect opportunity to study nonperturbative phenomena at large densities on the lattice!**

Lattice parameters

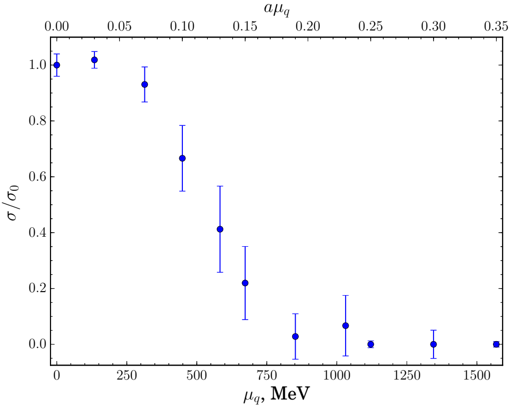
- Staggered fermions
- Rooting to get $N_f = 2$
- Symanzik tree-level improved gauge action
- $a = 0.044$ fm
  $\Rightarrow$ **close to continuum limit,**
  large accessible density $\mu \sim 2000$ MeV
- Lattice size: 32^4, $(1.4\ \text{fm})^4$
- $m_\pi = 740(40)$ MeV, $m_\pi L_s \approx 5$

Interaction potential between static quark-antiquark pair



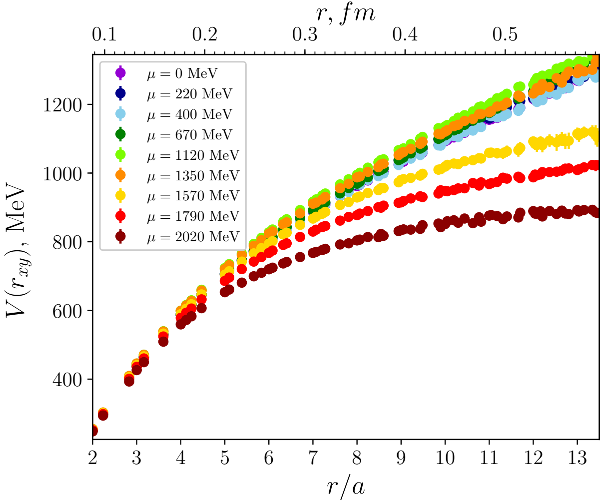
Deconfinement in dense medium!

String tension

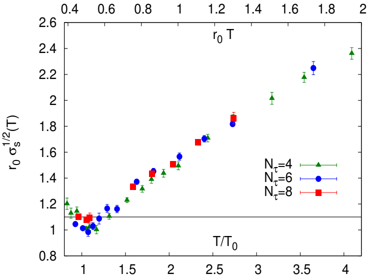


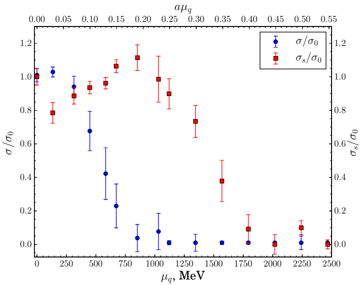
- Cornell potential: $V(r) = A + \frac{B}{r} + \sigma r \quad \mu \leq 1100$ MeV
- Debye potential: $V(r) = A + \frac{B}{r} e^{-m_D r} \quad \mu \geq 850$ MeV
- Confinement/deconfinement transition in $\mu \in (850, 1100)$ MeV

Spatial quark-antiquark potential in dense medium



Spatial string tension

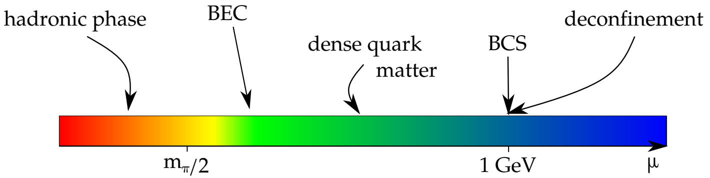




[M. Cheng et al., 2008]

- Deconfinement at $\mu > 900 - 1100$ MeV
- Spatial string tension disappears at $\mu \geq 1800$ MeV ($a\mu > 0.4$)
- Different behaviour of spatial string tension at finite temperature and finite density

Tentative phase diagram



Grand potential of a static quark-antiquark pair

$$
\begin{aligned}
\frac{\Omega_{\bar{q}q}(r,\mu)}{T} &= -\log\left[\frac{1}{4}\left\langle \mathrm{Tr}L(\vec{r})\ \mathrm{Tr}L^{\dagger}(0)\right\rangle\right] + c(\mu) \\
\frac{\Omega_{\infty}}{T} &= \frac{1}{T}\lim_{r\to\infty}\Omega_{\bar{q}q}(r,\mu) = -\log|\langle L\rangle|^2 + c(\mu)
\end{aligned}
$$

Renormalization of the Polyakov loop:

$$
L^{ren}(\mu) = \exp\left(-\frac{\Omega_{\infty}(\mu)}{2T}\right)
$$

Color singlet and triplet potential

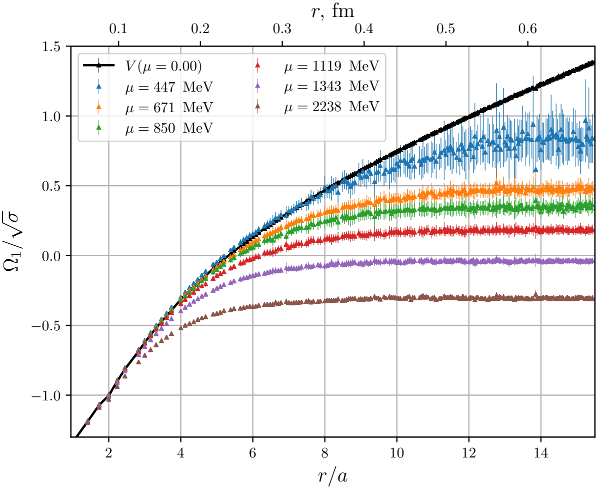
$$
\begin{aligned}
\frac{\Omega_1(r,\mu)}{T} &= -\log\left[\frac{1}{2}\left\langle \mathrm{Tr}\left(L(\vec{r})L^\dagger(0)\right)\right\rangle\right] + c'(\mu) \\
\exp\left(-\Omega_{\bar{q}q}/T\right) &= \frac{1}{4}\exp\left(-\Omega_1/T\right) + \frac{3}{4}\exp\left(-\Omega_3/T\right)
\end{aligned}
$$

Renormalization:

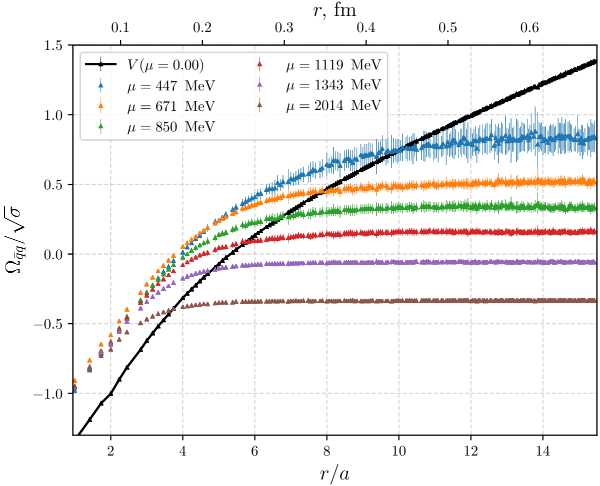
$$
\begin{aligned}
\Omega_{\bar{q}q}(r\to\infty) &= \Omega_1(r\to\infty) \\
\Omega_1(r\to 0) &= V^{ren}(r\to 0)
\end{aligned}
$$

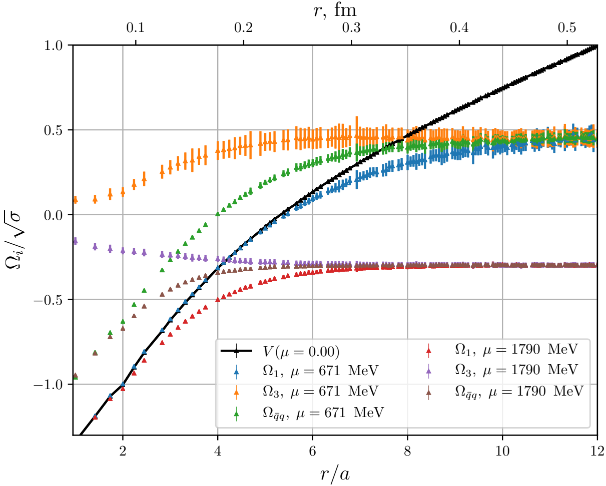
V^{ren} is determined from $\mu=0,\ T=0$ simulations.

Color singlet grand potential



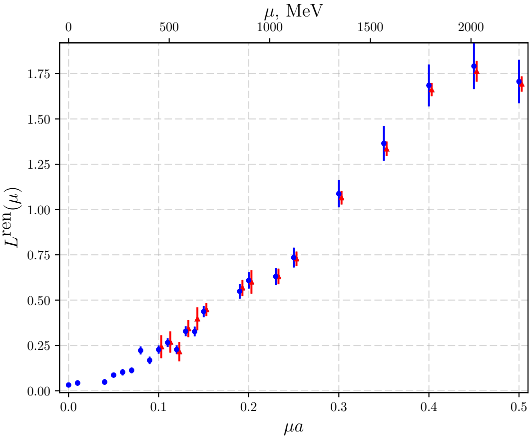
Color average grand potential




r, fm
0.1
0.2
0.3
0.4
0.5
1.0
0.5
0.0
−0.5
−1.0
$\Omega_i/\sqrt{\sigma}$
2
4
6
8
10
12
r/a
$V(\mu = 0.00)$
$\Omega_1,\ \mu = 671$ MeV
$\Omega_3,\ \mu = 671$ MeV
$\Omega_{\bar{q}q},\ \mu = 671$ MeV
$\Omega_1,\ \mu = 1790$ MeV
$\Omega_3,\ \mu = 1790$ MeV
$\Omega_{\bar{q}q},\ \mu = 1790$ MeV

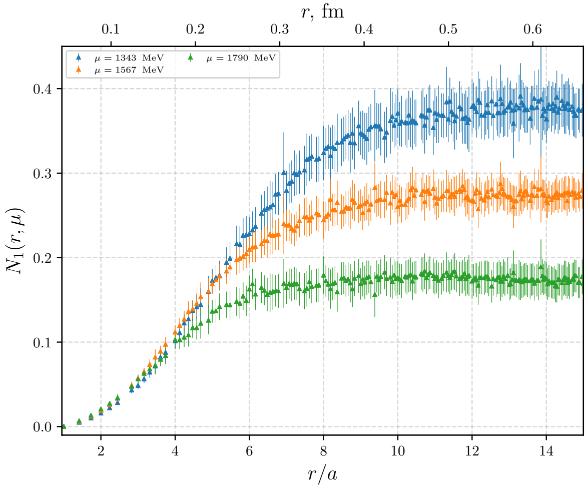
Renormalized Polyakov loop

- $L^{ren}(\mu) = \exp\left(-\Omega_\infty(\mu)/2T\right)$
- $L^{ren}(\mu) = L^{bare}(\mu)Z, \; Z = \frac{L^{ren}(\mu{=}1030\ \text{MeV})}{L^{bare}(\mu{=}1030\ \text{MeV})}$



Quark number by static quark-antiquark pair

$$\delta N = -\frac{\partial \Omega(r,\mu)}{\partial \mu}$$



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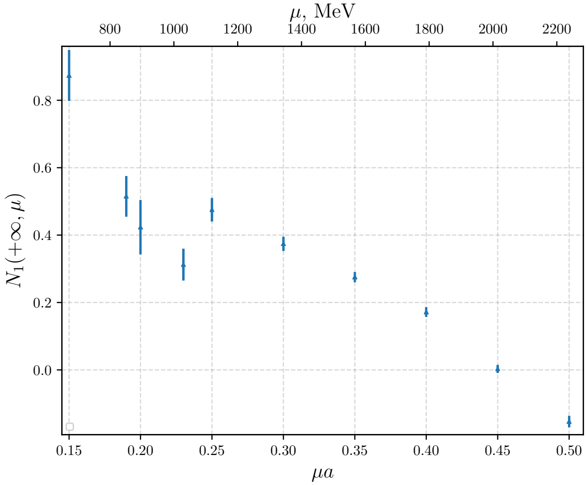
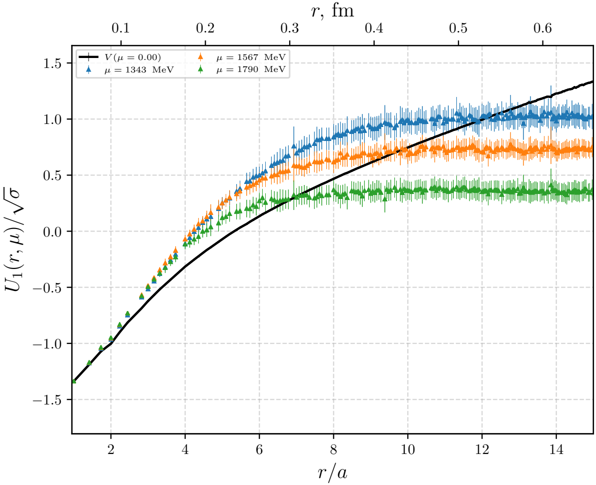


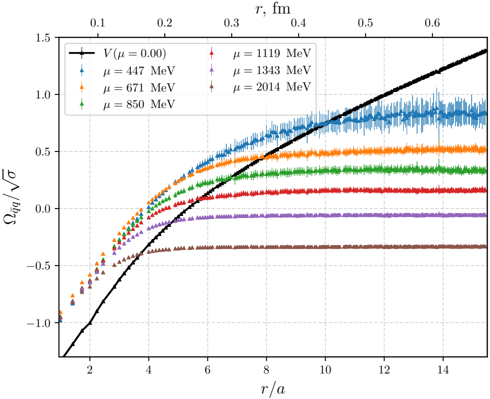
Internal energy U_1 of quark-antiquark pair

Ignoring the entropy contribution S:

$$U(r,\mu) = \Omega(r,\mu) + \mu N(r,\mu)$$



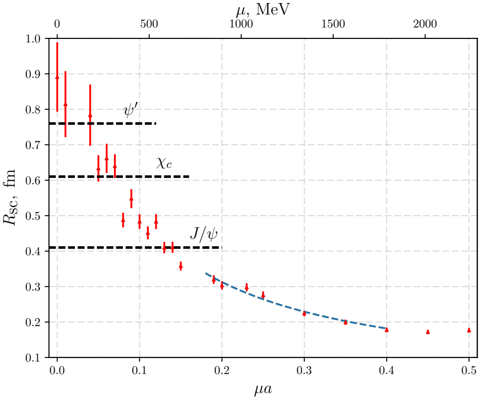
String breaking in cold dense quark matter



- Plateau in Ω: the string breaking
- Large baryon density $\Leftrightarrow$ small string breaking distance
- Quantitative: the screening length

$$V_{\mu=0}(R_{scr.}) = \Omega_{\bar{q}q}(\infty, \mu)$$

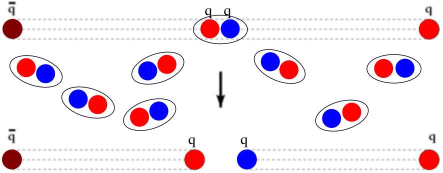
Screening length and quarkonia dissociation



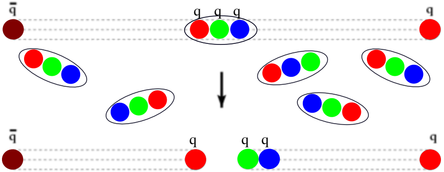
- Quarkonia in QC_2D: non-relativistic Schrödinger eq. + $V(r)$
- Quarkonia dissociation (in confinement!)
- Confinement: $R_{scr.}$ is described by string breaking
- Deconfinement: $R_{scr.}$ is described by Debye screening (curve: $R_{SC} = 1/[Am_D(\mu)]$, $m_D^2(\mu) = (4/\pi)\alpha_s(\mu)\mu^2$)

String breaking in dense medium

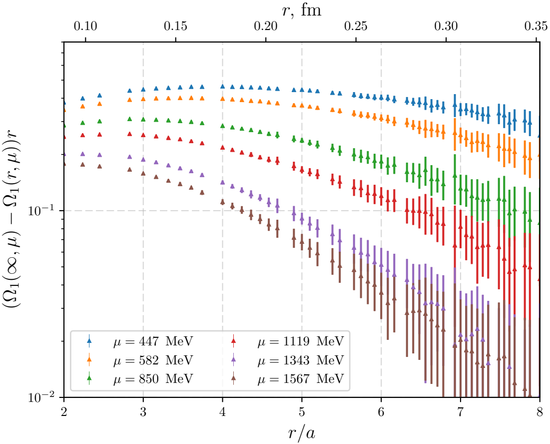
In SU(2) QCD:



Analogous mechanism may be proposed in SU(3) QCD:



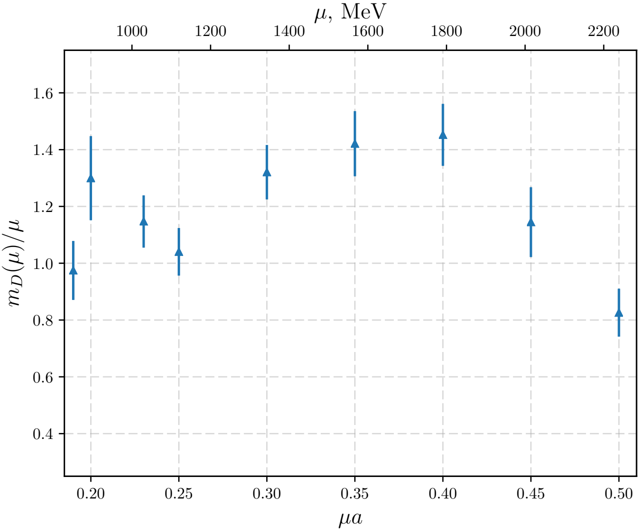
Debye screening in dense medium



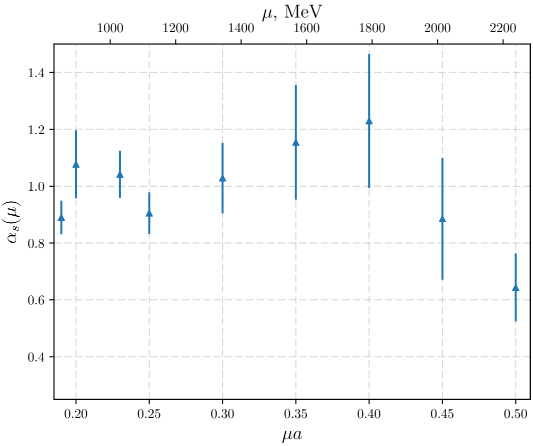
Debye screening in dense cold quark matter

- $\Omega_1(r,\mu) = \Omega_1(\infty,\mu) - \frac{3}{4}\frac{\alpha_s(\mu)}{r} exp(-m_D r)$
- We observe exponential Debye screening
- From the fit we determine the $m_D(\mu)$ and $\alpha_s(\mu)$

Debye mass in cold dense matter

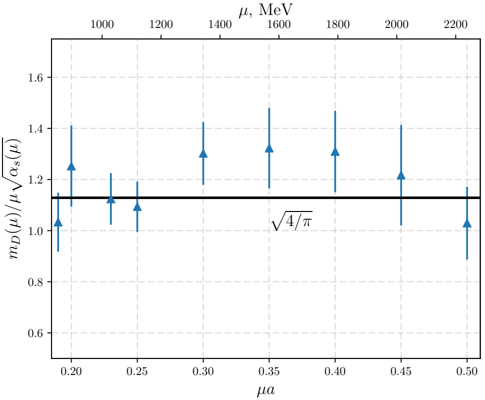


Effective coupling constant in cold dense matter



$\alpha_s \sim 1$ i.e. even at high density QCD is strongly correlated

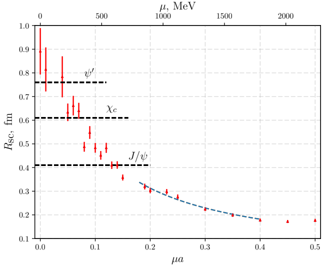
One-loop formula for the Debye mass



- The one-loop formula:
  $m_D^2(\mu) = \frac{4}{\pi}\alpha_s(\mu)\mu^2 \;\Rightarrow\; \frac{m_D(\mu)}{\mu\sqrt{\alpha_s(\mu)}} = \sqrt{\frac{4}{\pi}}$
- The one-loop formula works well even for the $\alpha_s \sim 1$

Conclusions

- QC_2D:
  lattice study of nonperturbative effects in dense medium



- Static quark-antiquark pair in dense medium is studied
- String breaking distance decreases with density
- Heavy quarkonia dissociate at moderate densities due to string breaking
- Debye screening in deconfinement phase
  $m_D^2(\mu) = \frac{4}{\pi}\alpha_s(\mu)\mu^2 \;\Rightarrow\; \frac{m_D(\mu)}{\mu\sqrt{\alpha_s(\mu)}} = \sqrt{\frac{4}{\pi}}$