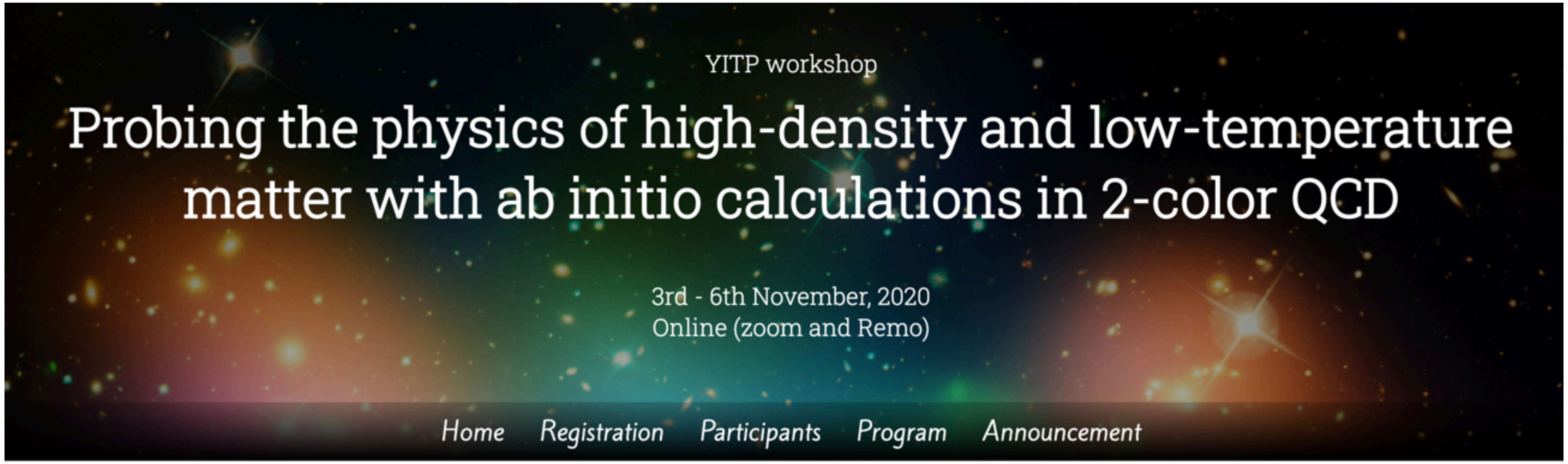
Two Color QCD

Past, Present and Future



Atsushi Nakamura
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RCNP, Osaka Univ.

Why SU(2) ?

1. No Sign problem
2. Less Computer time, and memory
3. Simple structure
4. Nc dependence
   Nc=2, 3 —> then estimate any Nc !

1. No Sign problem inTwo-Color

$$Z = \mathrm{Tr}\, e^{-\beta(H-\mu N)} = \int \mathcal{D}U \mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-\beta S_G - \bar{\psi}\Delta\psi}$$

$$= \int \mathcal{D}U \prod_f \det \Delta(m_f)\, e^{-\beta S_G}$$

$$\Delta(\mu) = D_\nu \gamma_\nu + m + \mu\gamma_0$$

$$\Delta(\mu)^\dagger = -D_\nu\gamma_\nu + m + \mu^*\gamma_0 = \gamma_5 \Delta(-\mu^*)\gamma_5$$

$$\Rightarrow (\det \Delta(\mu))^* = \det \Delta(\mu)^\dagger = \det \Delta(-\mu^*)$$

$(\det \Delta(\mu))^* = \det \Delta(\mu)^\dagger = \det \Delta(-\mu^*)$

For $\mu = 0$

$$(\det \Delta(0))^* = \det \Delta(0)$$

$\det \Delta$ ➡ *Real*

For $\mu \neq 0$ (in general)

$\det \Delta$ ➡ *Complex*

$$Z = \int \mathcal{D}U \prod_f \det \Delta(m_f, \mu_f) e^{-\beta S_G}$$

⬆

Complex ➡ Sign Problem

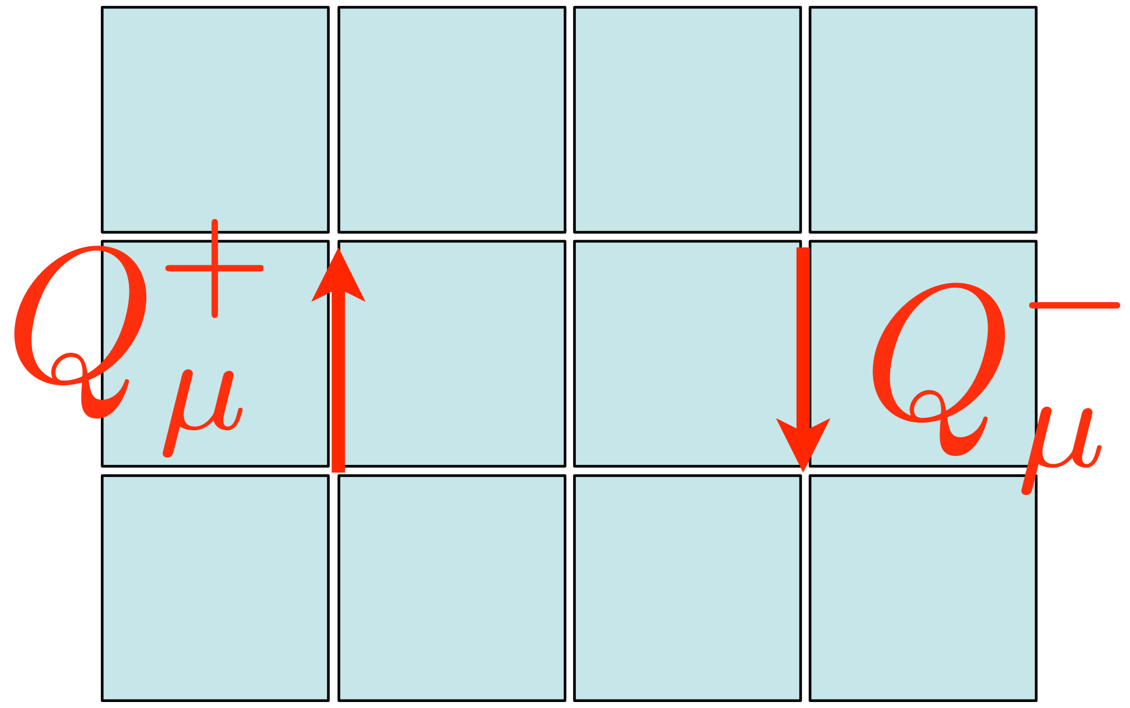
Physical Origin of Sign Problem

Wilson Fermions $\Delta = I - \kappa Q$

KS(Staggered) Fermions $\Delta = m - Q'$

$$= m(I - \frac{1}{m}Q)$$

$$Q = \sum_{i=1}^{3} (Q_i^+ + Q_i^-) + (e^{+\mu} Q_4^+ + e^{-\mu} Q_4^-)$$



$$Q_\mu^+ = **U_\mu(x)\delta_{x',x+\hat{\mu}}$$

$$Q_\mu^- = **U_\mu^\dagger(x')\delta_{x',x-\hat{\mu}}$$

$$\det\Delta = e^{\mathrm{Tr}\log\Delta} = e^{\mathrm{Tr}\log(I-\kappa Q)}$$
$$= e^{-\sum_n \frac{1}{n}\kappa^n \mathrm{Tr}Q^n}$$

Only closed loops survive.

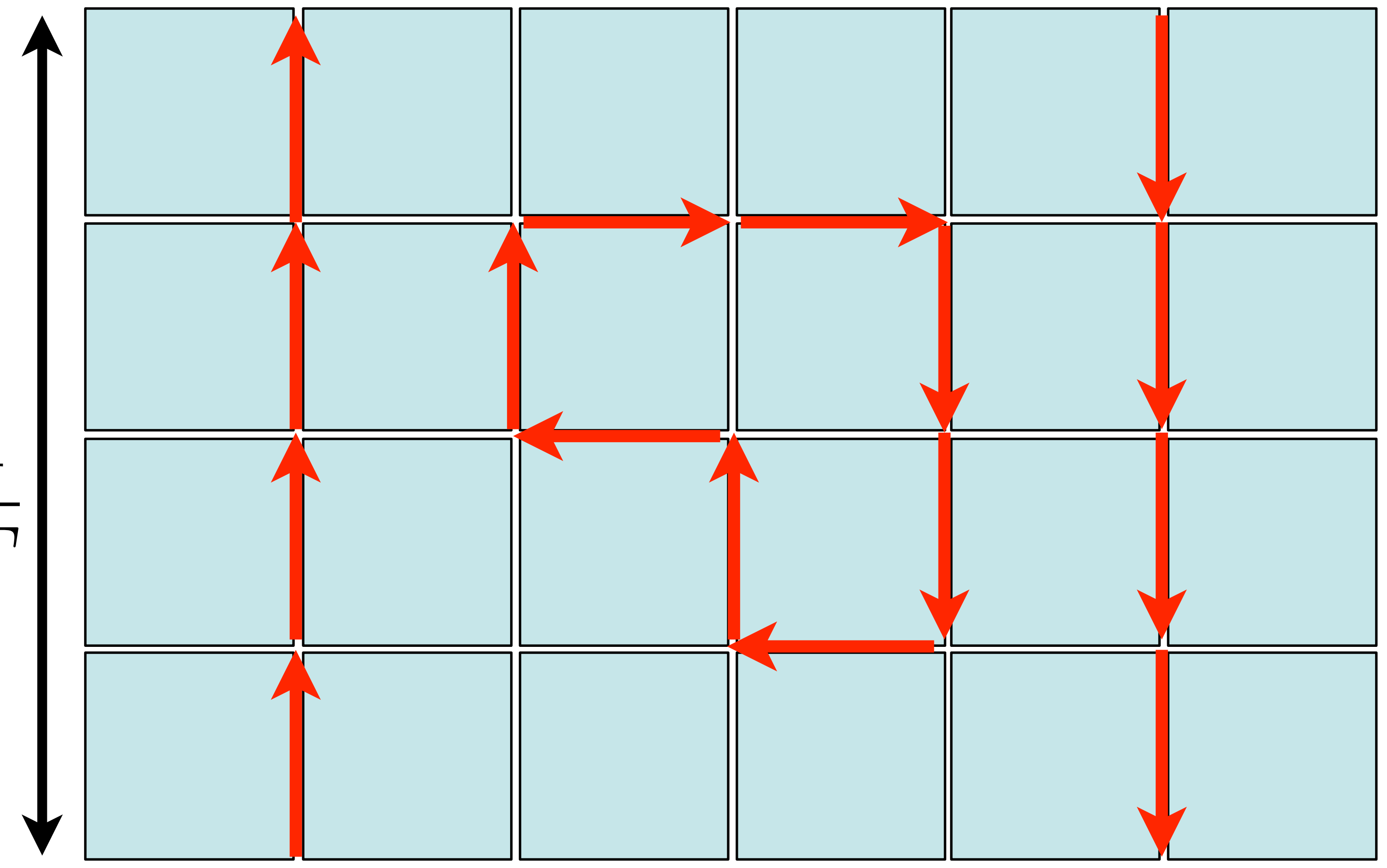
Lowest

μ -dependent terms

$$\kappa^{N_t} e^{\mu N_t}\mathrm{Tr}(Q^+\cdots Q^+)$$
$$= **\,\kappa^{N_t} e^{\mu/T}\mathrm{Tr}L$$
$$\kappa^{N_t} e^{-\mu N_t}\mathrm{Tr}(Q^-\cdots Q^-)$$
$$= **\,\kappa^{N_t} e^{-\mu/T}\mathrm{Tr}L^\dagger$$

$\mathrm{Tr}L$: Polyakov Loop

$N_t = \frac{1}{T}$



Add both terms

$$**\,\kappa^{N_t}\left(\cosh\frac{\mu}{T}\Re\mathrm{Tr}L + i\sinh\frac{\mu}{T}\Im\mathrm{Tr}L\right)$$

0 for SU(2)

SU(2) Case

$$U_\mu^* = \sigma_2 U_\mu \sigma_2$$

$$\det \Delta(U, \gamma_\mu)^* = \det \Delta(U^*, \gamma_\mu^*) = \det \sigma_2 \Delta(U, \gamma_\mu^*) \sigma_2$$

$$= \det \Delta(U, \gamma_\mu)$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad U = \begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix} \quad \sigma_2 U \sigma_2 = \begin{pmatrix} \alpha^* & \beta^* \\ -\beta & \alpha \end{pmatrix}$$

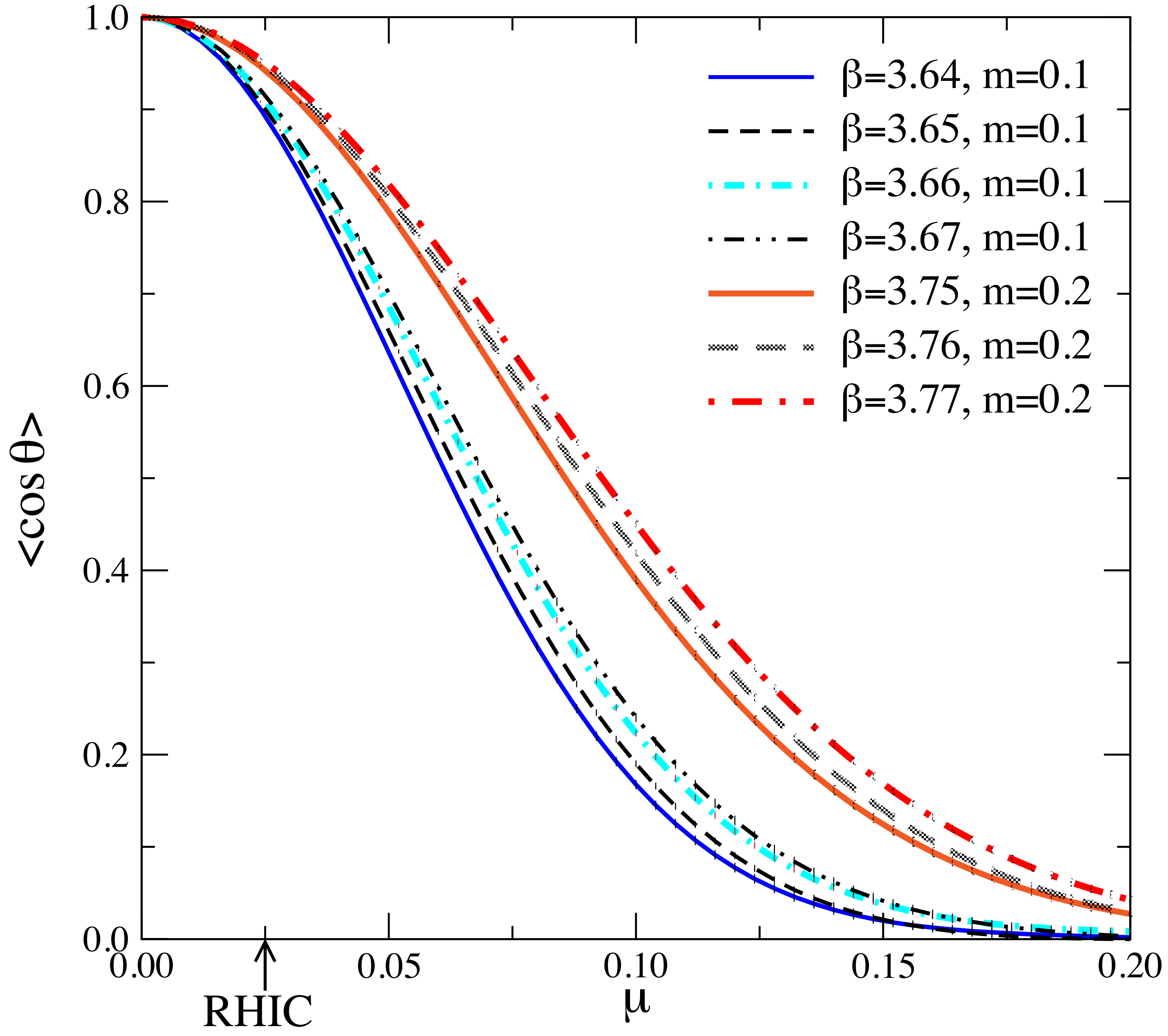
Sign Problem is sever

when μ is large

when T is low

Allton et al., Phys.Rev.D.66. 074507
(arXiv:hep-lat/0204010)

$$\det D = |\det D| e^{i\theta}$$



Why SU(2) ?

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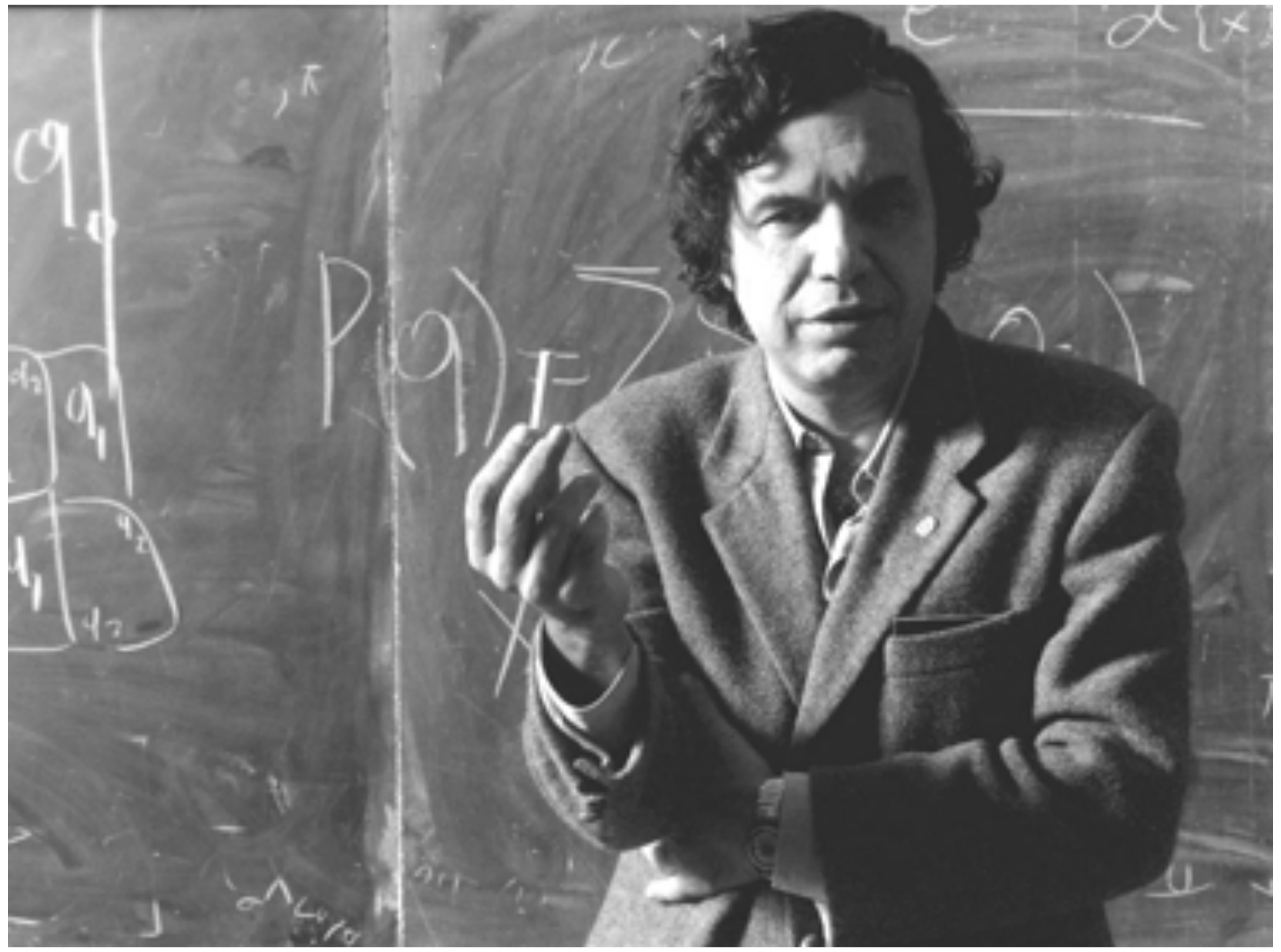
3. Simple structure
4. Nc dependence (Nc=2, 3 —> then estimate any Nc !

- 1982. Atsushi went to Italy
- He quitted the job at a university, and applied Italian Government Fellow.

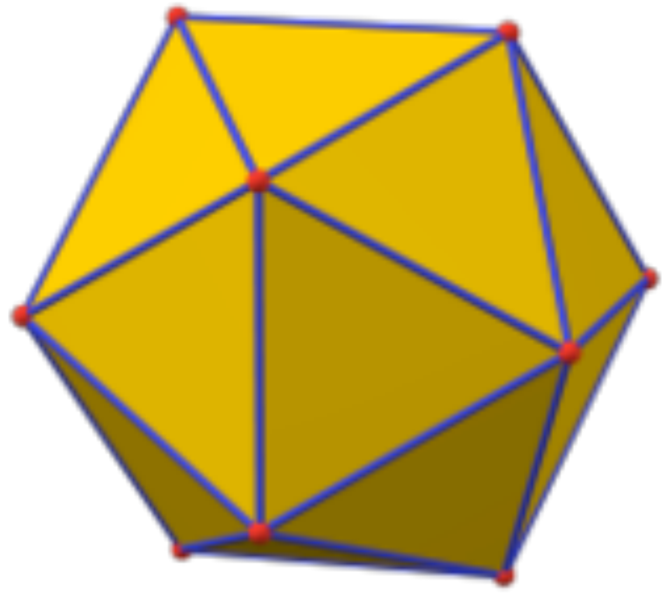
  Salary: 300 dollars/Month
- Italian embassy gave him a ticket to Roma.

- At Frascati, Atsushi asked Giorgio Parisi to work in Hadron physics by Lattice QCD.
- Only computer he could use was Vax 11, 1 MIPS machine (around 0.1MFLOPS) and 8MB memory.
- Giorgio suggested Atsushi to work for SU(2)



SU(2) ~ O(3)/Z2

O(3) ~ Icosahedron



120 elements

o	g1	g2	g3	…	g120
g1					
g2					
g3					
g120					

BEHAVIOR OF QUARKS AND GLUONS AT FINITE TEMPERATURE AND DENSITY IN SU(2) QCD

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Received 9 August 1984

We have run a computer simulation in SU(2) lattice gauge theory on a $8^3 \times 2$ lattice including dynamical quark loops. No rapid variation is observed in the value of the Polyakov line, while the energy densities of quark and gluon show a strong indication of a second order phase transition around $T \simeq 250$ MeV. In order to reduce finite size effects, the results are compared with those of a free gas on a lattice of the same size. The quark and gluon energy densities overshoot the free gas values at high temperature. The effect of the chemical potential is also studied. The behaviors of the energy densities and of the number density are far from the free gas case.

It has been conjectured that systems of quarks and gluons at high temperature and density show a completely different behavior from those at zero temperature and normal density [1–3]. Above some temperature and/or chemical potential, quarks and gluons are expected to be liberated in a deconfined quark–gluon plasma.

ments, we may develop and study models of the quark–gluon system. MC simulation of lattice QCD probably provides the most fundamental information for such an analysis. For the study of hadronic matter, it is important to include quark loops in the calculation since they play a crucial role in screening. The phase transition observed in the pure gauge cal-

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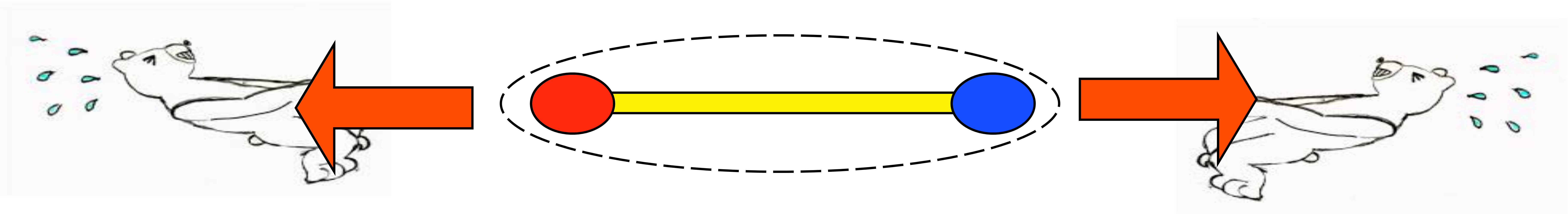
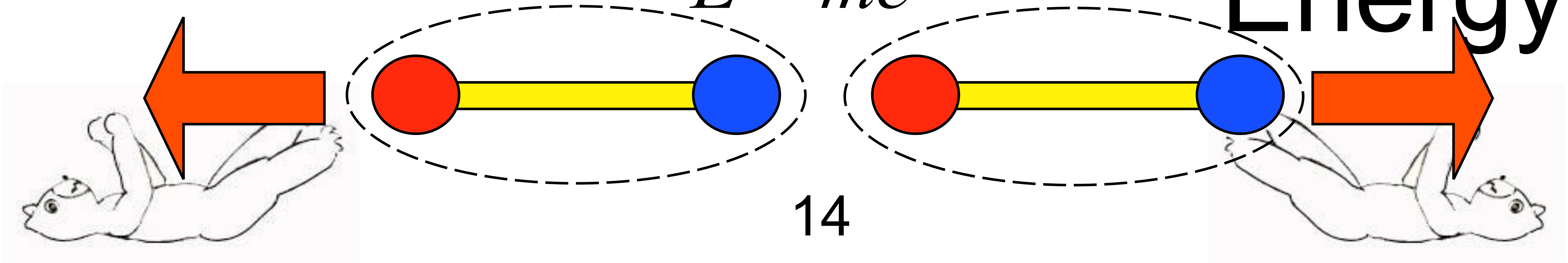
Magnetic Degrees of Freedom ?

- M. Chernodub and V. Zakharov
  - hep-lat/0611228
- J. Liao and E. Shuryak
  - hep-ph/0611131

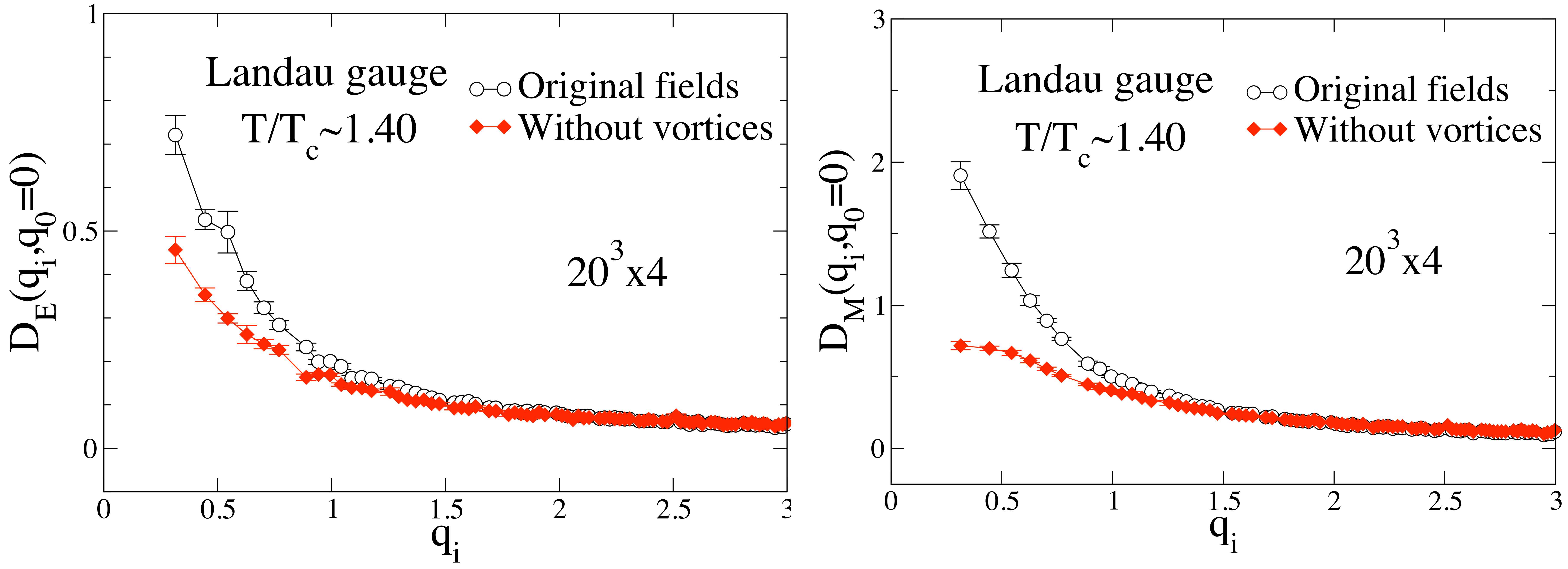
What is Confinement (Deconfinement) Mechanism ?





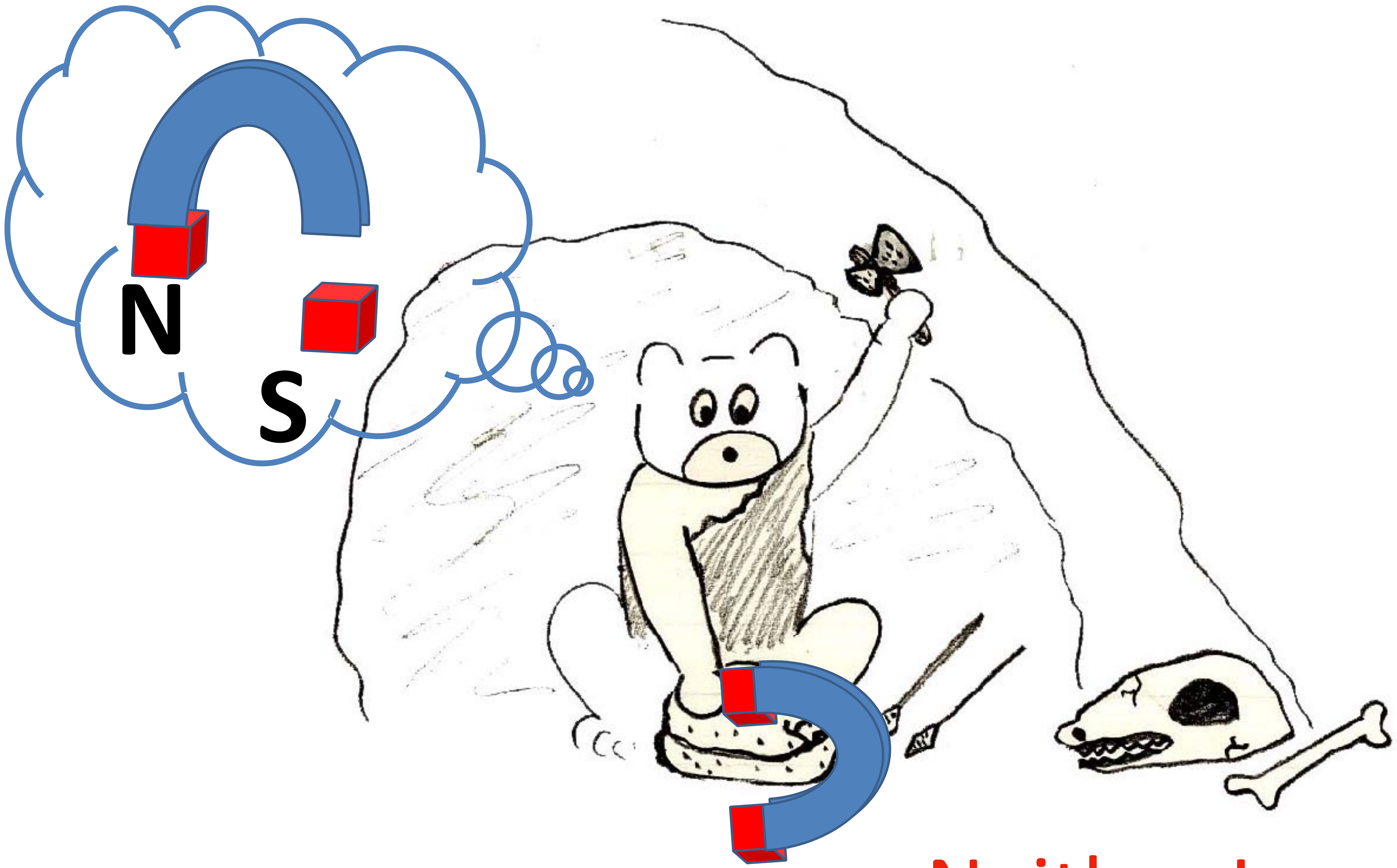
$E = mc^2$

Effects of Vortex (SU(2))



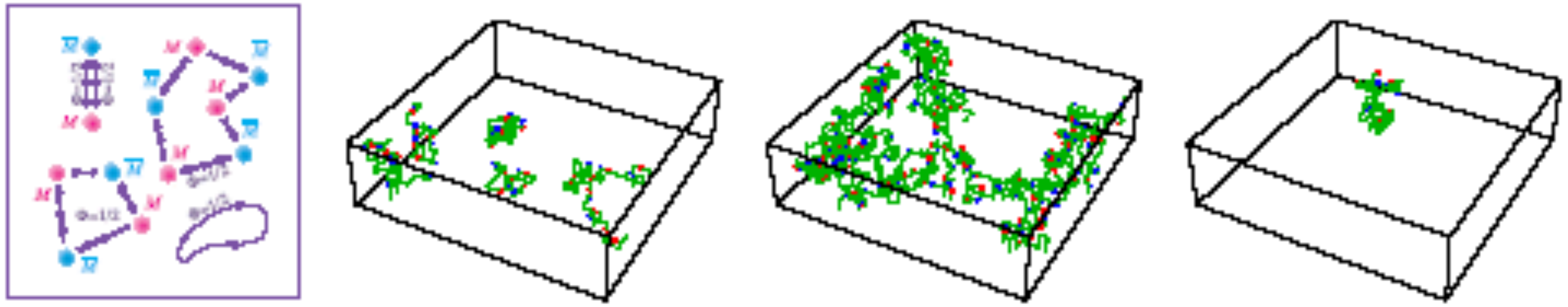
Infra-Red Regions of Magnetic Propagator is supressed after Vortex Removal

Who has seen the Mag. Monopole ?



Neither I nor you

- Confinement is due to monopole condensation
- **Center vortex mechanism**
  - Del Debbio, Faber, Greensite, Olejnik, ’97
- a realization of spaghetti (Copenhagen) vacuum
- Center strings are classified with respect to the center Z_N of the $SU(N)$ gauge group
- Confinement is due to vortex percolation

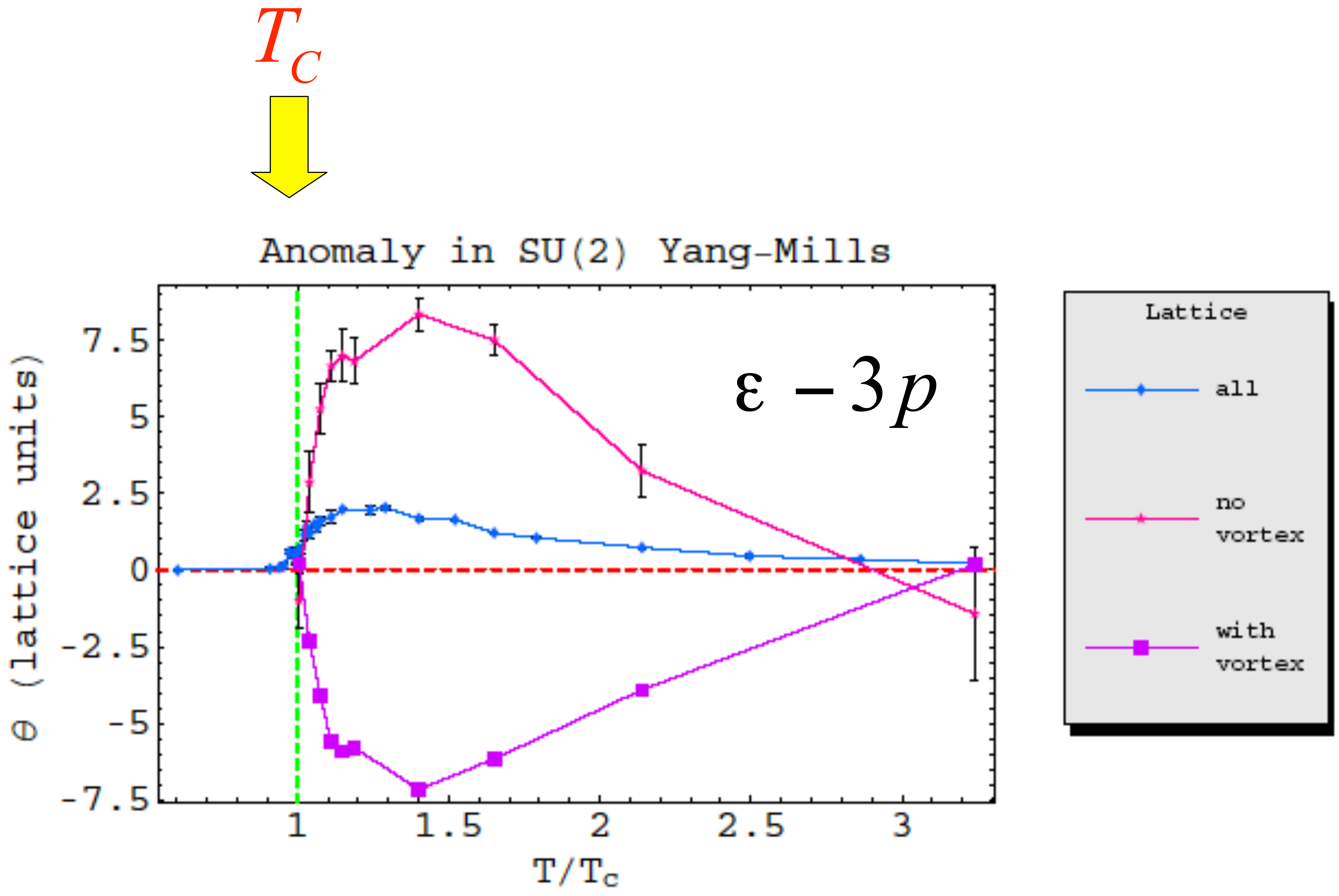


[results of numerical simulations are taken from Feldmann, Ilgenfritz, Schiller & M.Ch. '05]

- Observation of monopoles in the vortex chains:
- monopole is a defect, at which the flux of the vortex alternates.

Chernodub, AN, Zakharov

$$SU(2)\ \ 12^3\times 4$$



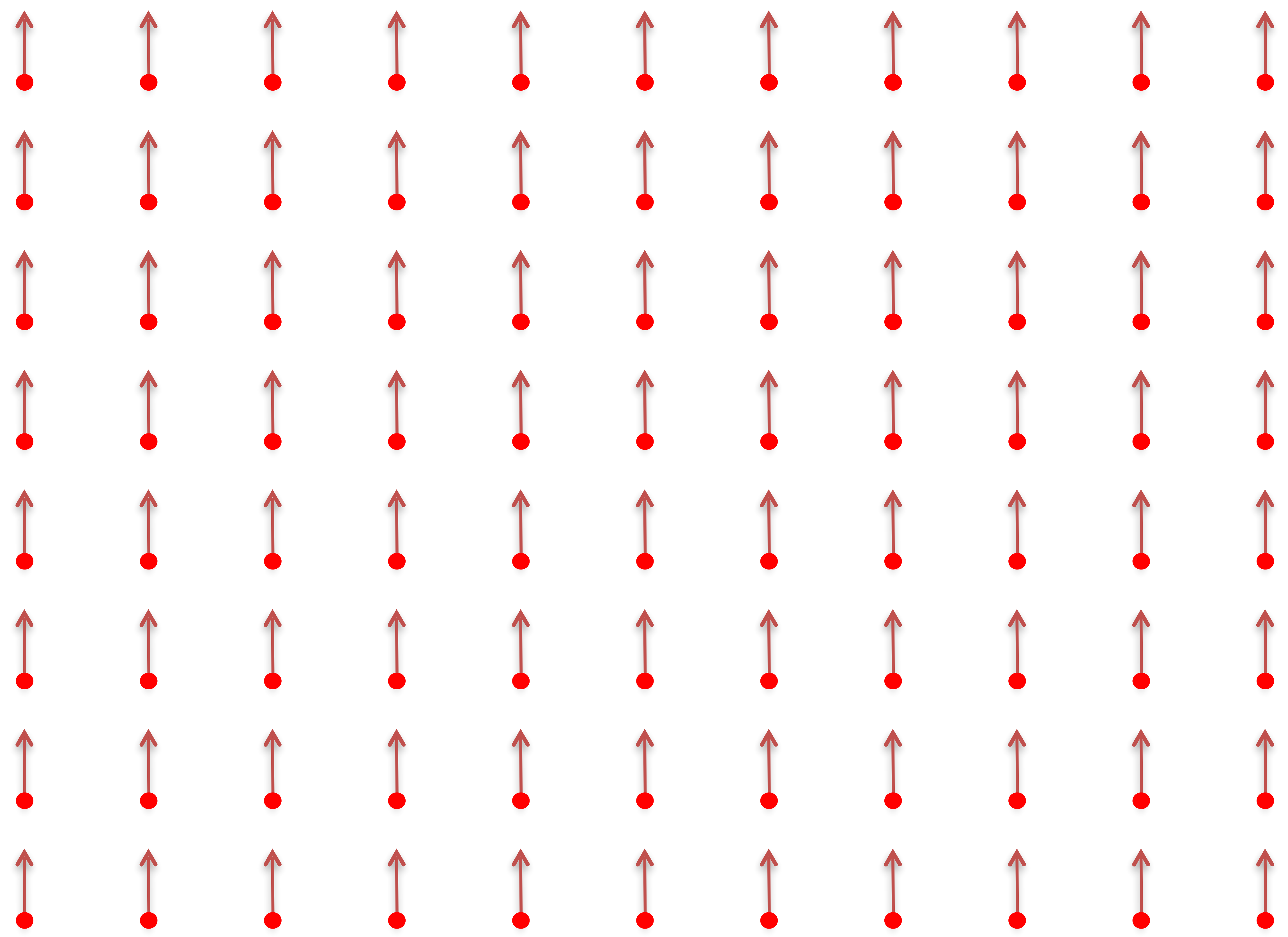
Vortex, Confinement, Transport Coefficient

Chernodub, Zakharov and A.N.

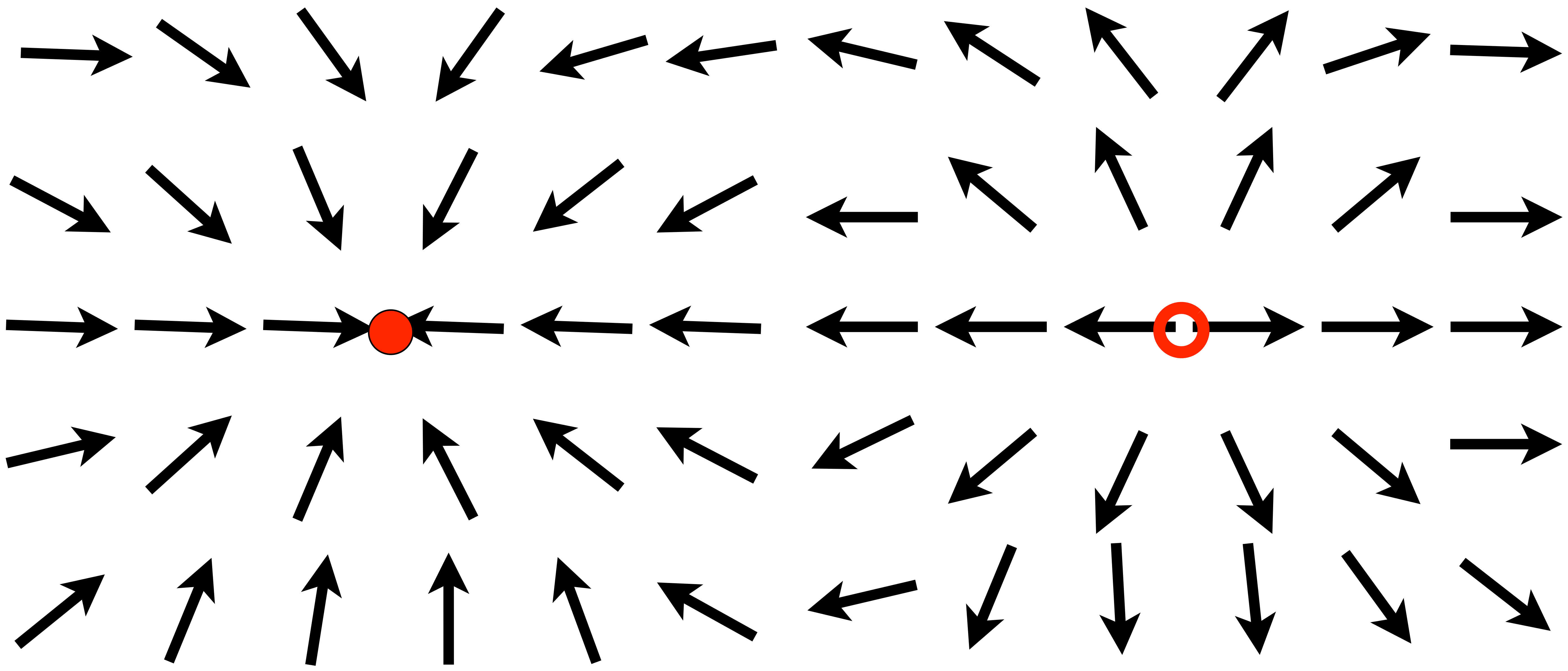
Magnetic Degrees of Freedom and the Confinement

- G.t'Hooft, Nucl.Phys. B190 (1981) 455
- H Shiba and T Suzuki. Phys. Lett. B (1994) 461
- A. Di Giacomo and G. Paffuti, Phys.Rev.D56,6816 (1997)
- Kei-ichi Kondo, Phys.Rev.D58,105019 (1998)
- …..
- J. Liao and E. Shuryak, Phys.Rev.Lett.,101, 162302 (2008)
- M.N. Chernodub and V.I. Zakharov, Phys. Rev. Lett.98, 082002 (2007)
- M.N. Chernodub, A. Nakamura and V.I. Zakharov Phys.Rev.D78:074021,2008
- M.N. Chernodub and V.I. Zakharov, Phys.Atom.Nucl. 72:2136-2145,2009 (arXiv:0806.2874)

Spin System



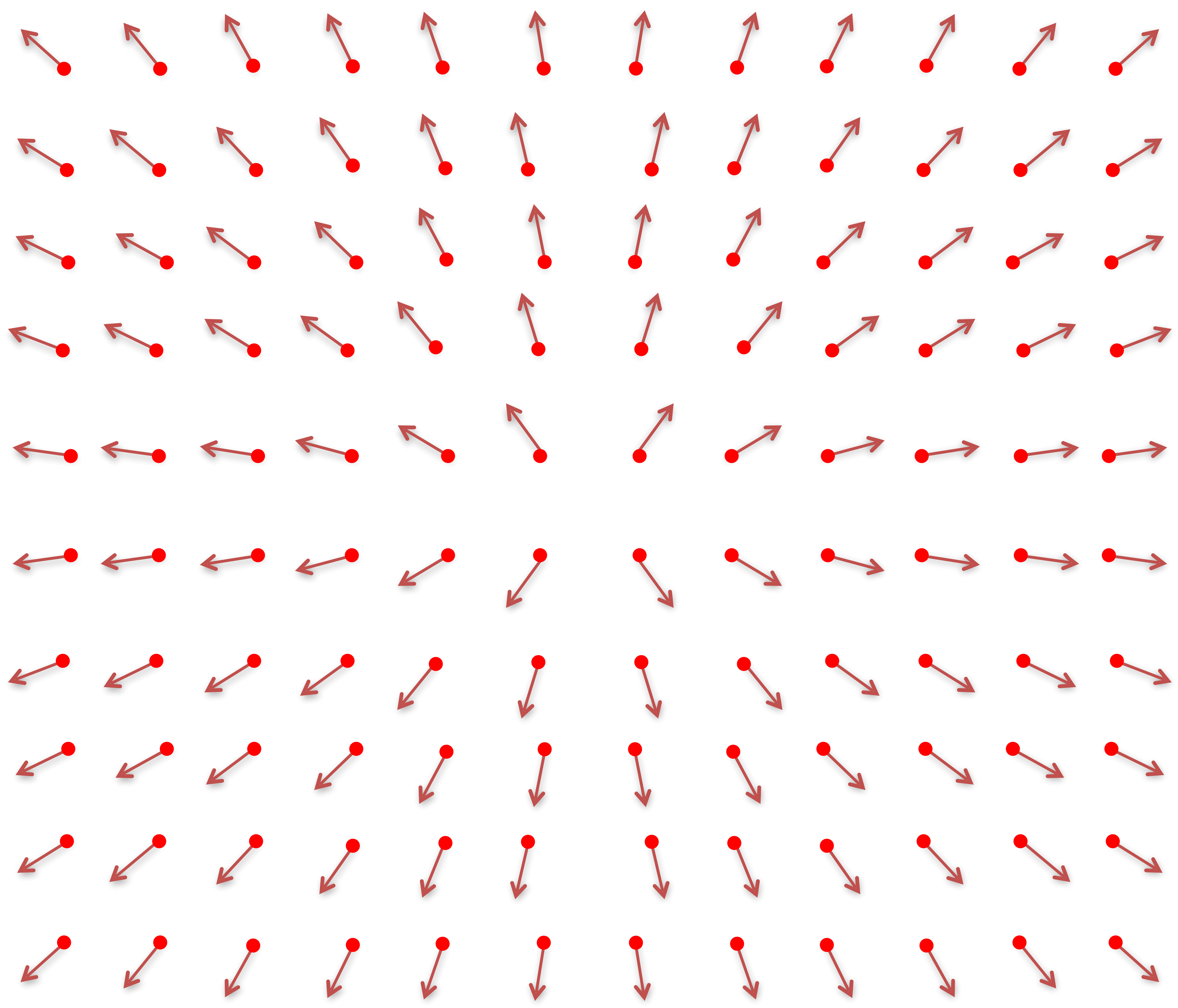
Who have seen Magnetic Monopole ?



Monopole/Anti-Monopole ?

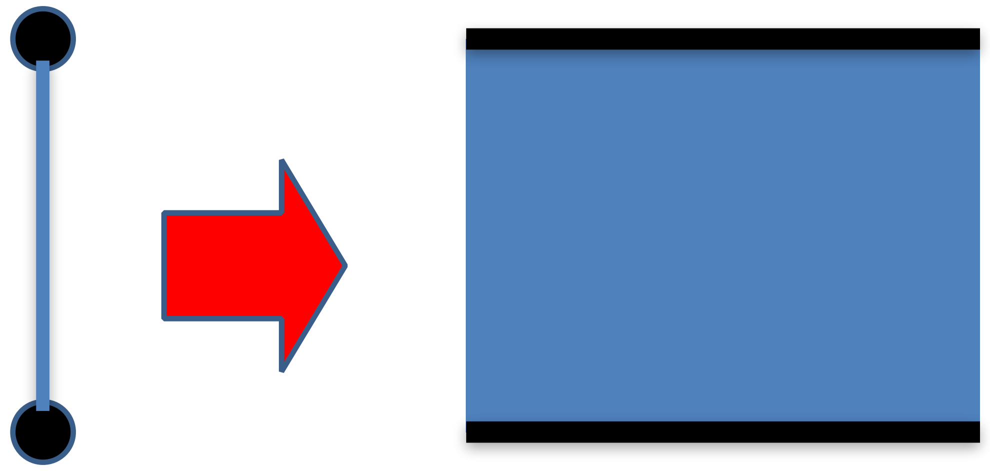
Topological Singularity

Singular Configuration, or Vortex



Vortices related to the Confinemen are a 2-d Object and we need Surface Operator

- S.Gukov and E.Witten, hep-th/0612073. E.Wittten, Fortsch. Phys. 55 (2007) 545.
- A.DiGiacomo and V.I.Zakharov, hep-th/0806.29382



For a Point Charge, Wilson invented Line Operators.

$$\alpha \int d\sigma_{\mu\nu} F_{\mu\nu}^{3} + \beta \int d\sigma_{\mu\nu} \widetilde{F}_{\mu\nu}^{3}$$

$$\widetilde{F}_{\mu\nu} = \varepsilon_{\mu\nu\lambda\rho} F_{\lambda\rho}$$

$$\Rightarrow$$

$$\alpha \int d\sigma_{\mu\nu} \left(\sqrt{FF} \frac{d\sigma_{\mu\nu}}{\left| d\sigma_{\mu\nu} \right|} \right) + \beta \int d\sigma_{\mu\nu} \left(\frac{F\widetilde{F}}{\sqrt{FF}} \frac{d\sigma_{\mu\nu}}{\left| d\sigma_{\mu\nu} \right|} \right)$$

$$FF = \sum_{\mu\nu} \mathrm{Tr}\, F_{\mu\nu} F_{\mu\nu} \qquad F\widetilde{F} = \sum_{\mu\nu} \mathrm{Tr}\, F_{\mu\nu} \widetilde{F}_{\mu\nu}$$

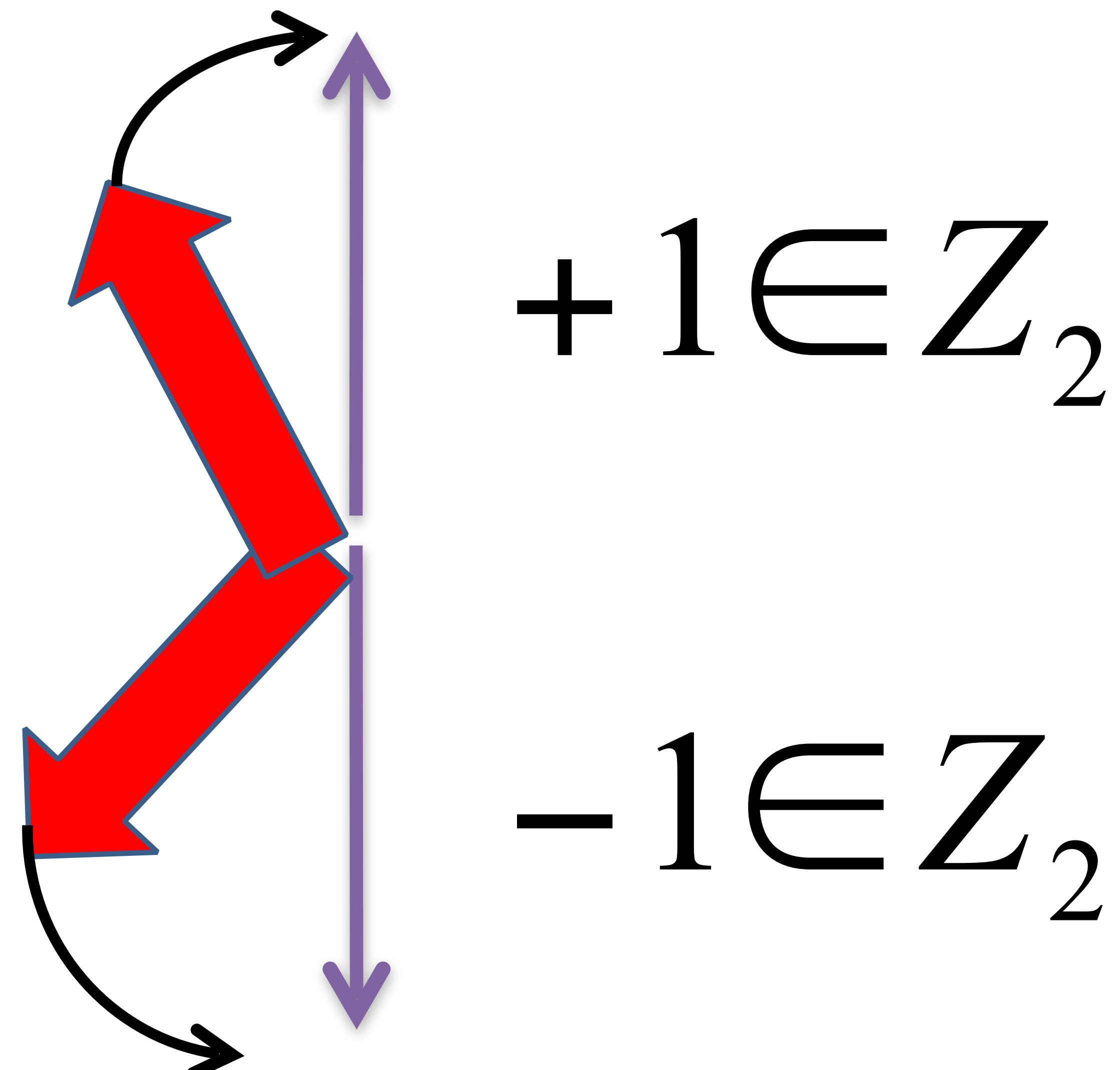
Center Projection

Del Debbio, Faber, Giedt, Greensite, Olejnik
Phys.Rev. D58, 1998, 094501

$$\mathrm{Max} \sum_{x,\mu} \mathrm{Tr}\, U_\mu(x) \;\Rightarrow\; \mathrm{Max} \sum \mathrm{Tr} \left(U_\mu \right)^2$$

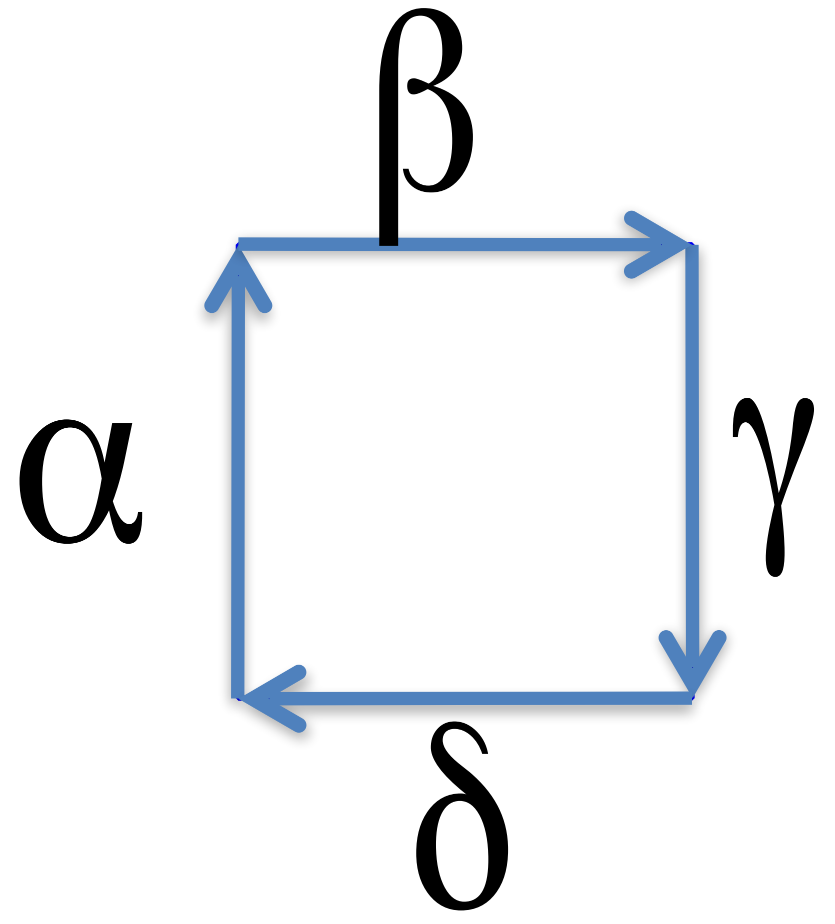
Landau gauge or
Coulomb Gauge

$$Z_\mu(x) \equiv \mathrm{sign}\, \mathrm{Tr}\, U_\mu(x) = +1 \text{ or } -1$$



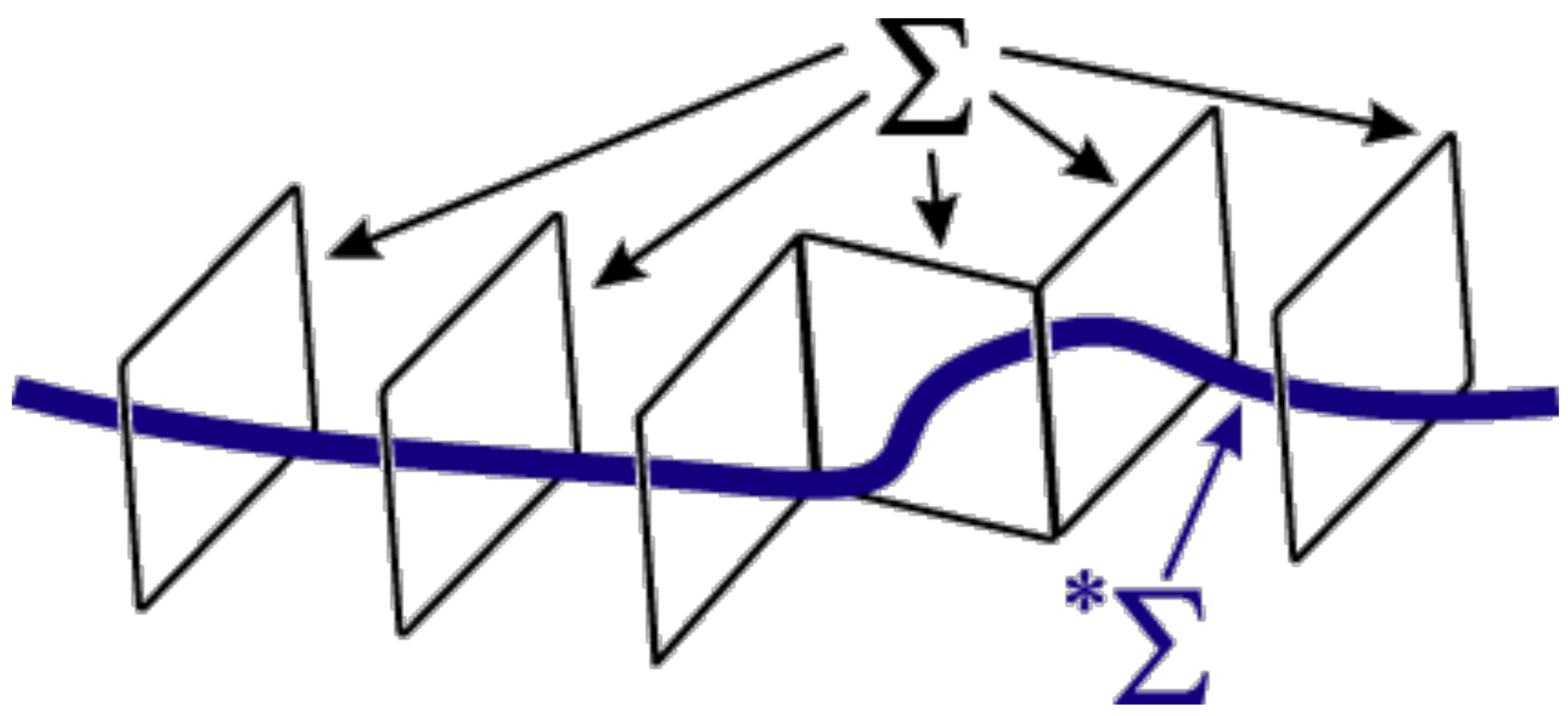
Gauge Rotation.
Therefore non-local

Plaquette pierced by a Vortex



If $Z_\alpha \times Z_\beta \times Z_\gamma \times Z_\delta = -1$

A Vortex pierces the Plaquette.



1-d Object (Charge) → Wilson Loop

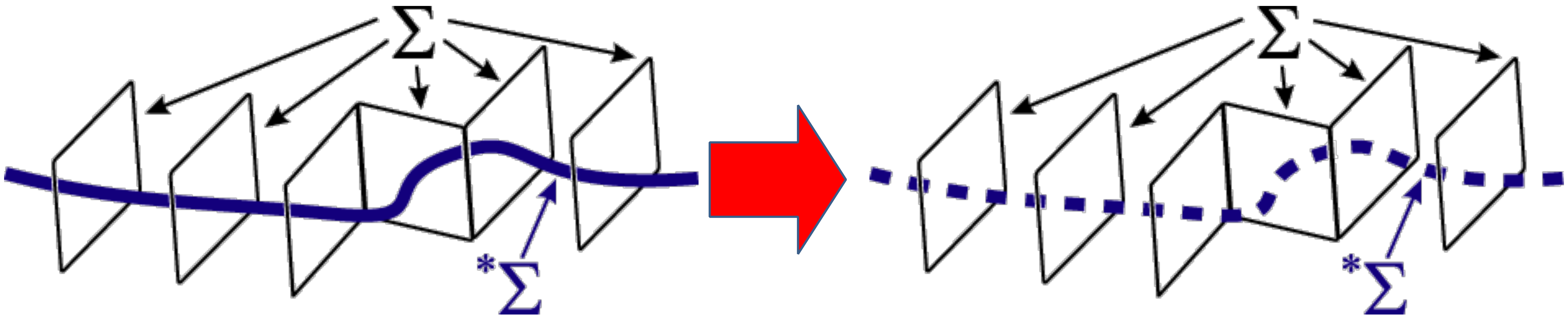
2-d Object (Vortex Line) → Surface Op.

Vortex Removing

$$U_\mu(x) \Rightarrow Z_\mu(x) \times U_\mu(x)$$

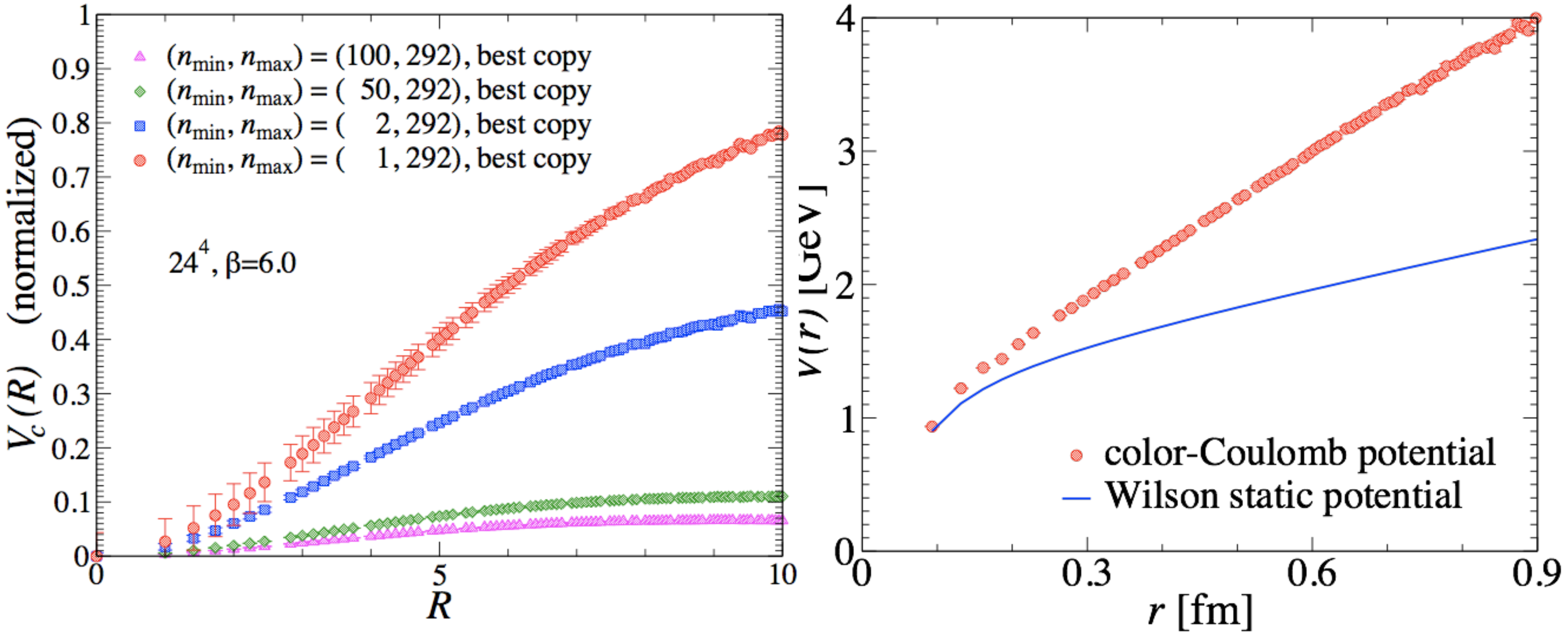
Remember that $Z_\mu(x) \equiv \text{sign}\,\text{Tr}\,U_\mu(x)$

By definition, now $\text{sign}\,\text{Tr}\,U_\mu(x) = +1$ for all links.



All vortices are fading out (by definition).

Color Coulomb Potential

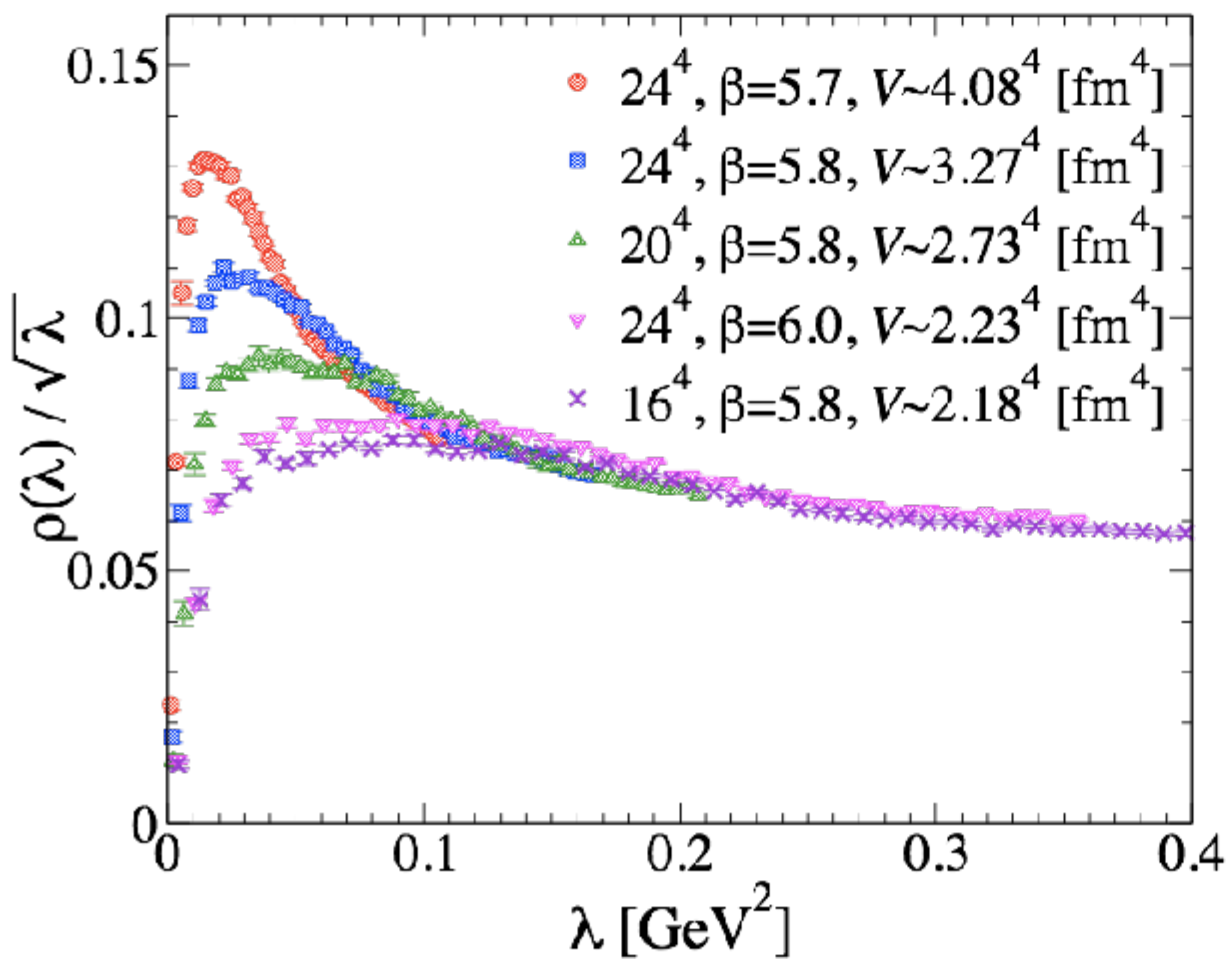


$$V(x,y) = \int d^3 z \frac{1}{M(x,z)} \left(-\partial^2_{(z)}\right) \frac{1}{M(z,y)}$$

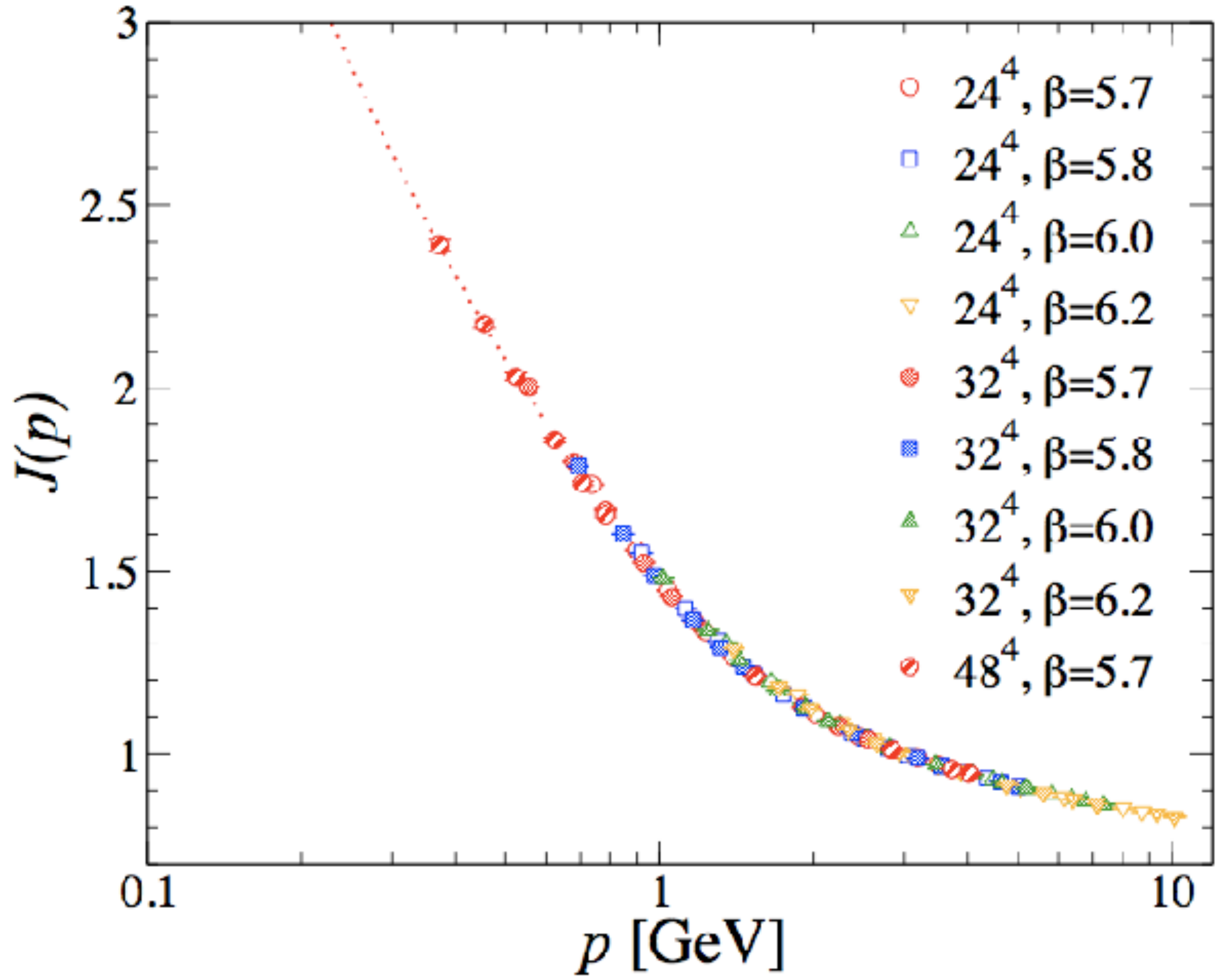
$$g^2 \langle A_0(x) A_0(y) \rangle = V(x-y) + P(x-y)$$

$$g^2 \langle \mathbf{V}(\vec{x},\vec{y}) \rangle \delta(x_4 - y_4)$$

$$V_{phys}(R) \leq V_{coul}(R)$$



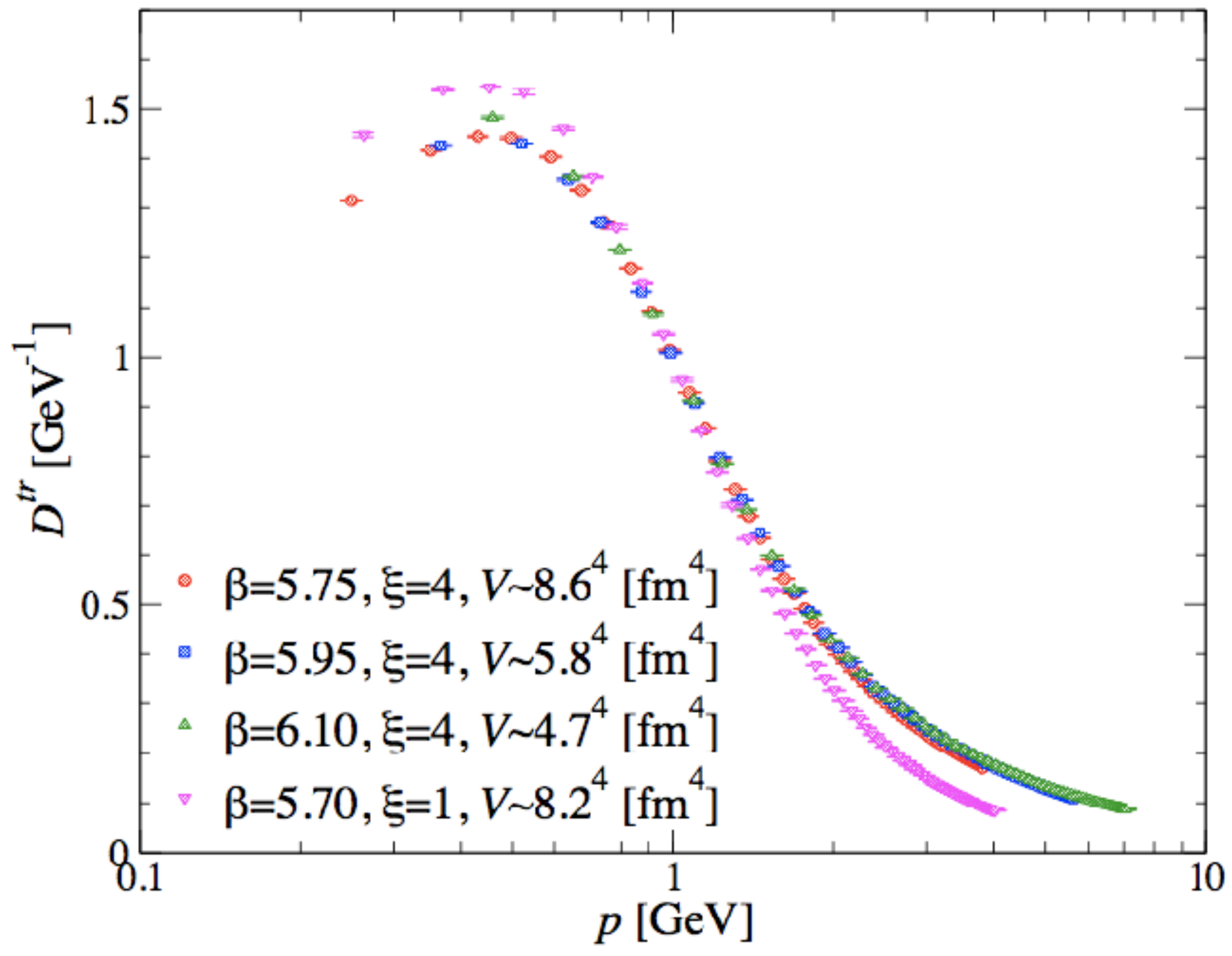
Eigen-Values of FP Operator accumulate near zero, i.e., Gribov boundary.



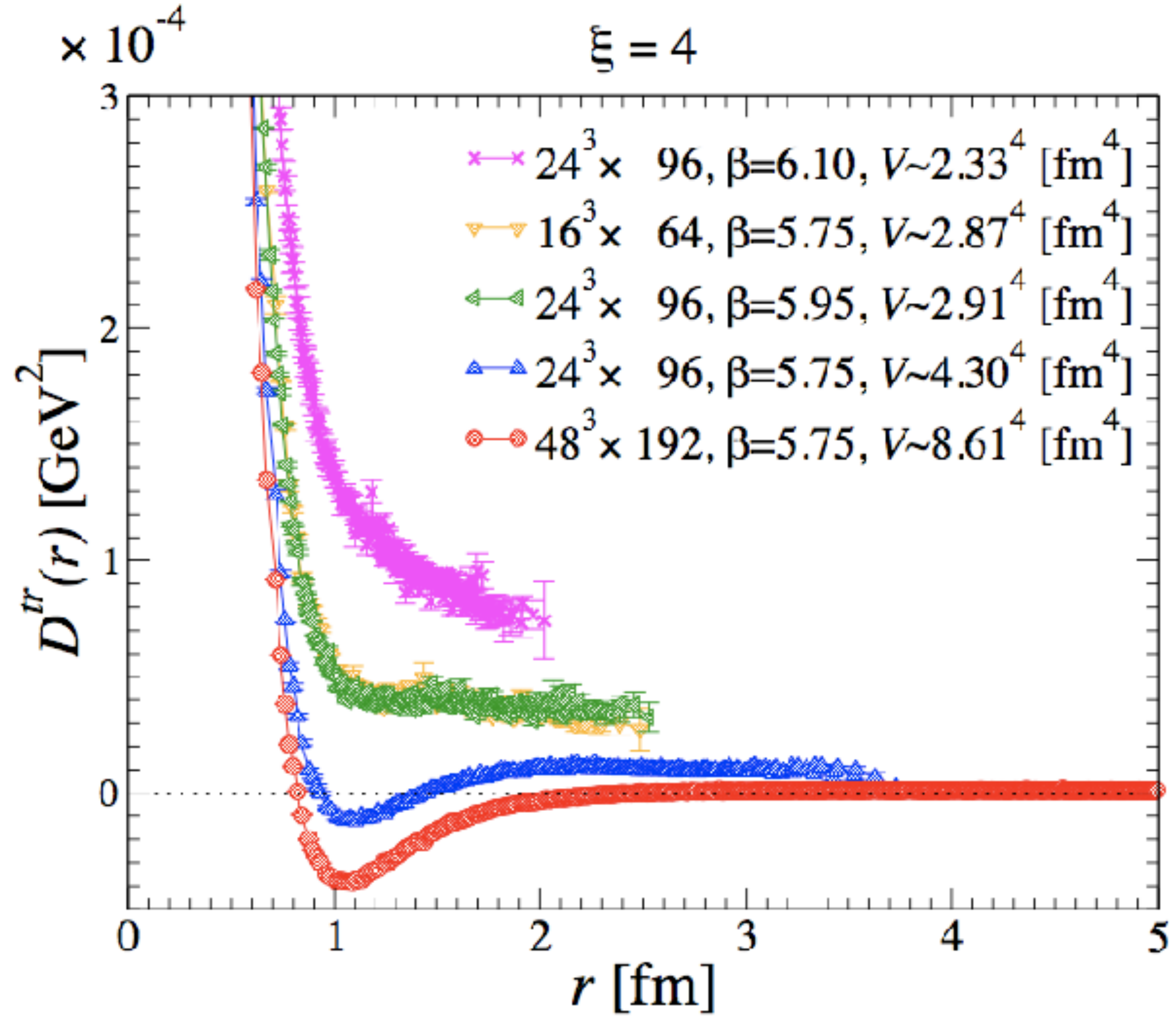
Ghost Dressing Function. Scaling Test

$$J(p^2) \equiv \vec{p}^{\,2} \langle M^{-1} \rangle (\vec{p})$$

Transverse Gluon Propagators



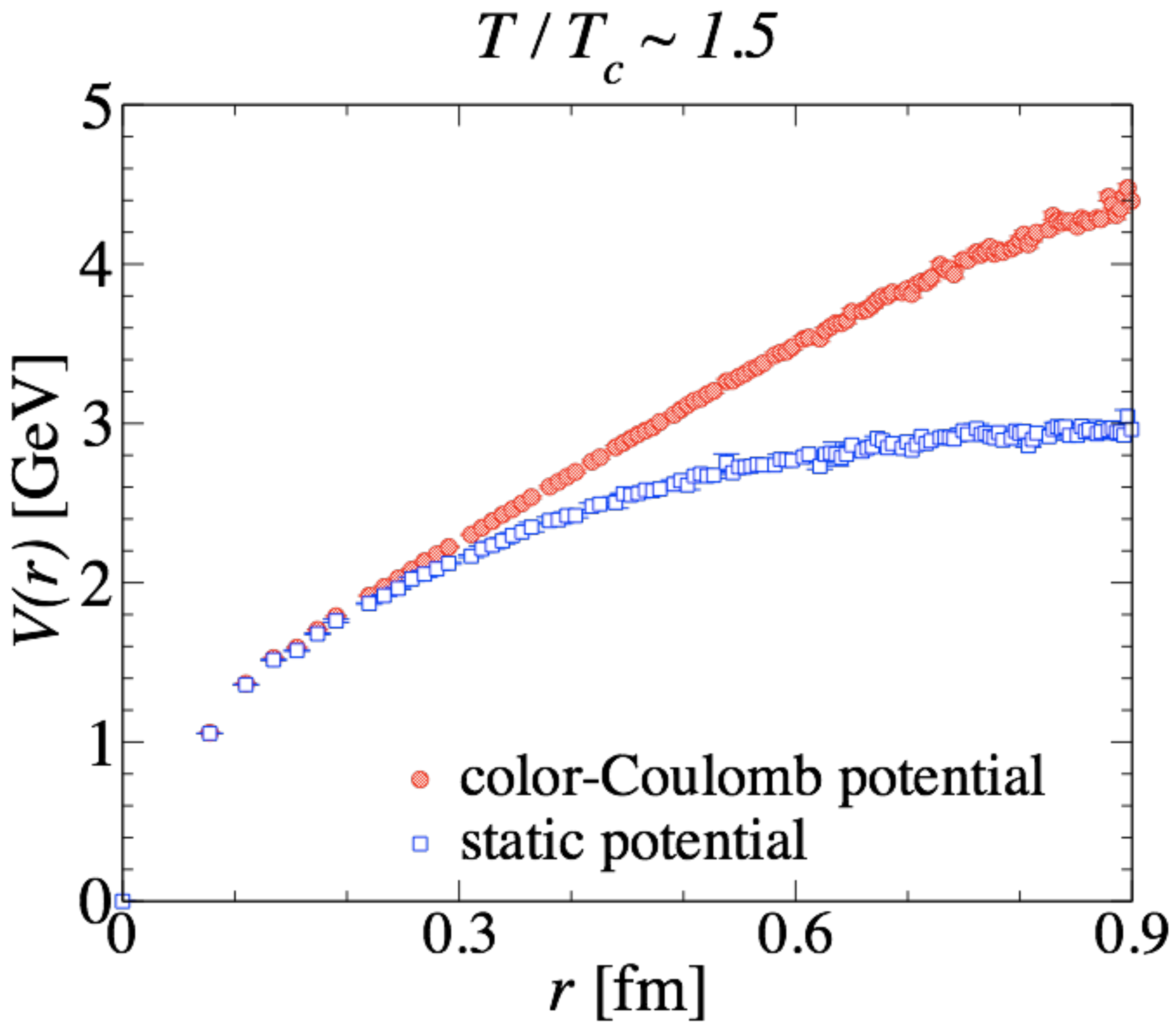
Momentum Space



Co-ordinate Space

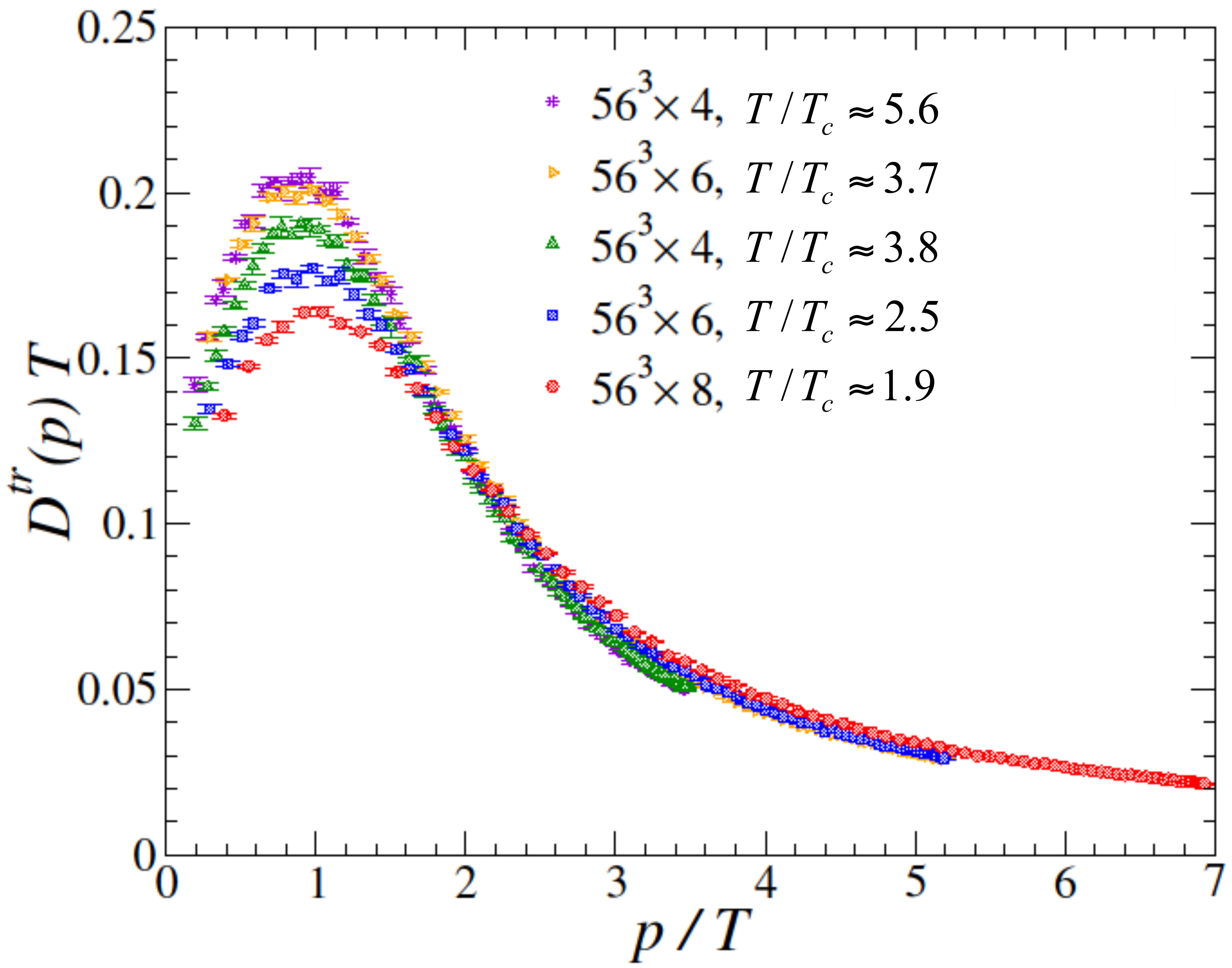
Phys.Rev.D81:054509,2010, Phys.Rev.D79:114504,2009, Phys.Rev.D75,014508,2007, Prog.Theor.Phys.115,189,2006)

Color Coulomb Potential at T>Tc



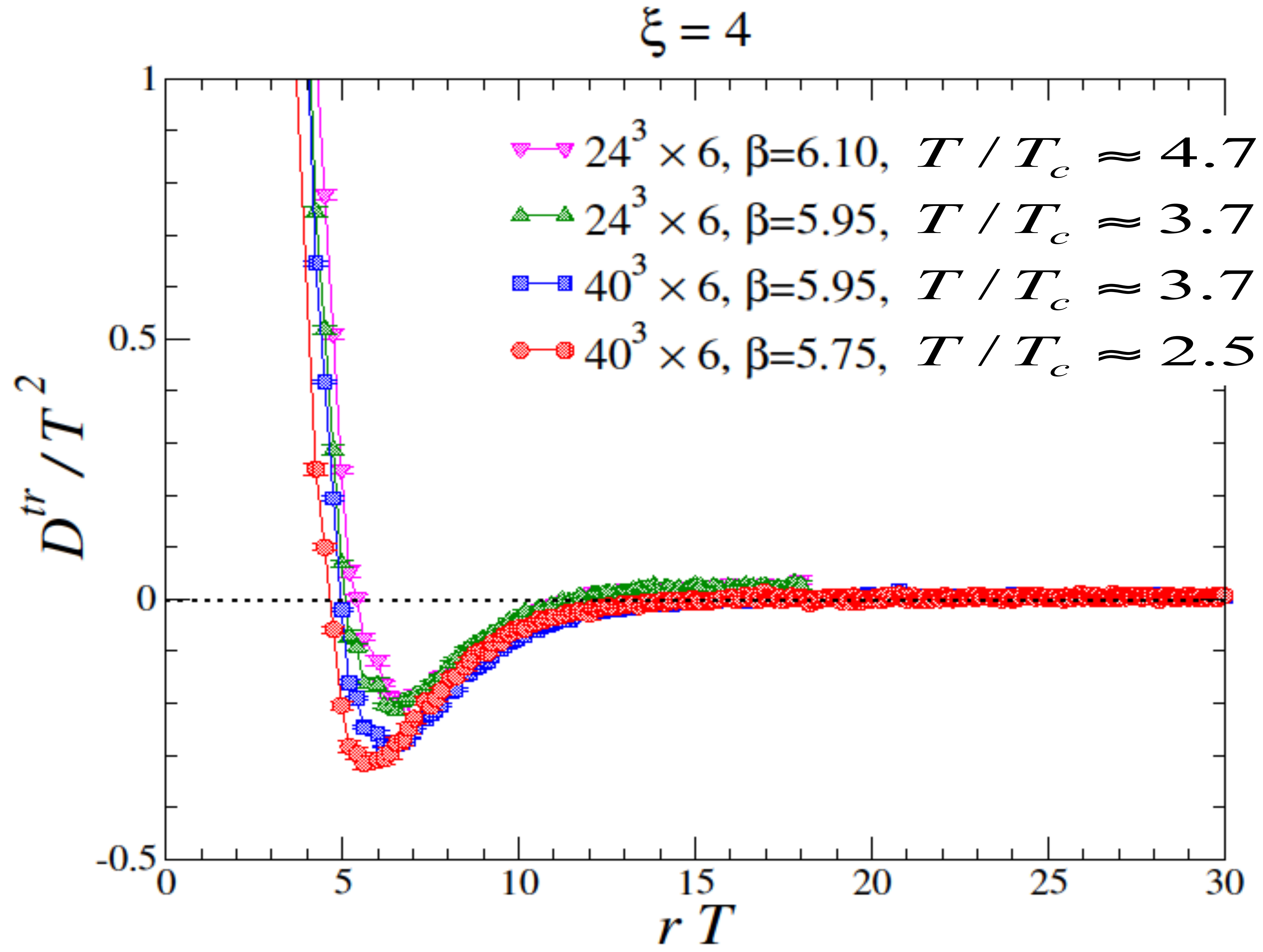
Y. Nakagawa, A. Nakamura, T. Saito, H. Toki, D. Zwanziger, Phys.Rev.D73, 014508 (2006))

Transverse Gluon Propagators at T>Tc



Y. Nakagawa, A. Nakamura, T. Saito, H. Toki

Transverse Gluon Propagators at T>Tc
Co-ordinate Space



Y. Nakagawa, A. Nakamura, T. Saito, H. Toki (in preparation)

Gluon Behavior with/without Vortex

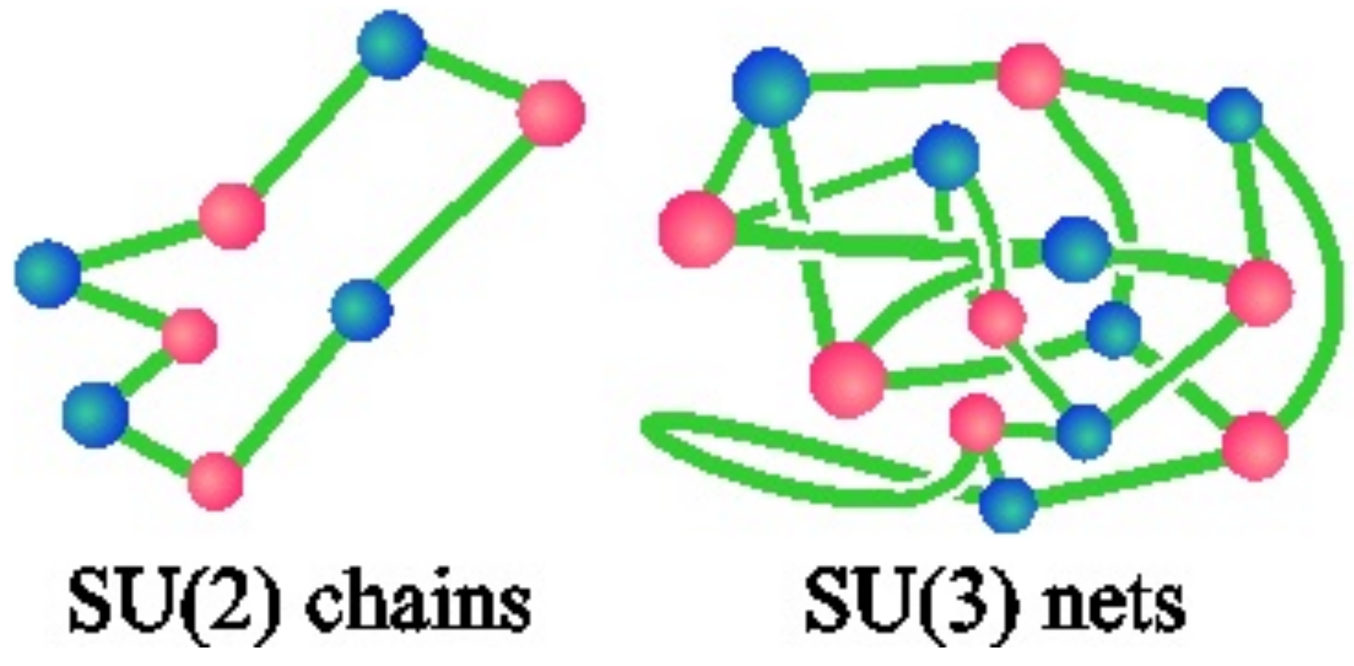
◆Lattice simulations

□Removing center vortices eliminates confinement and restores chiral symmetry (*de Forcrand , D'Elia, PRL82,4582(1999)*)

□Vortex density shows asymptotic scaling(*Langfeld,PRD69,(2004)014503;Langfeld, et.al PLB419(1998)317*)

□Phase transition and spatial string tension (*Langfeld,et.al PLB452(1999)301;Gattnar,et.al PLB489(2000)251;Engelhardt,et.al,PRD61(2000)054504*)

Illustration of vortex-monopole chain; *Chernodub, et. Al, PRD78:074021,2008*

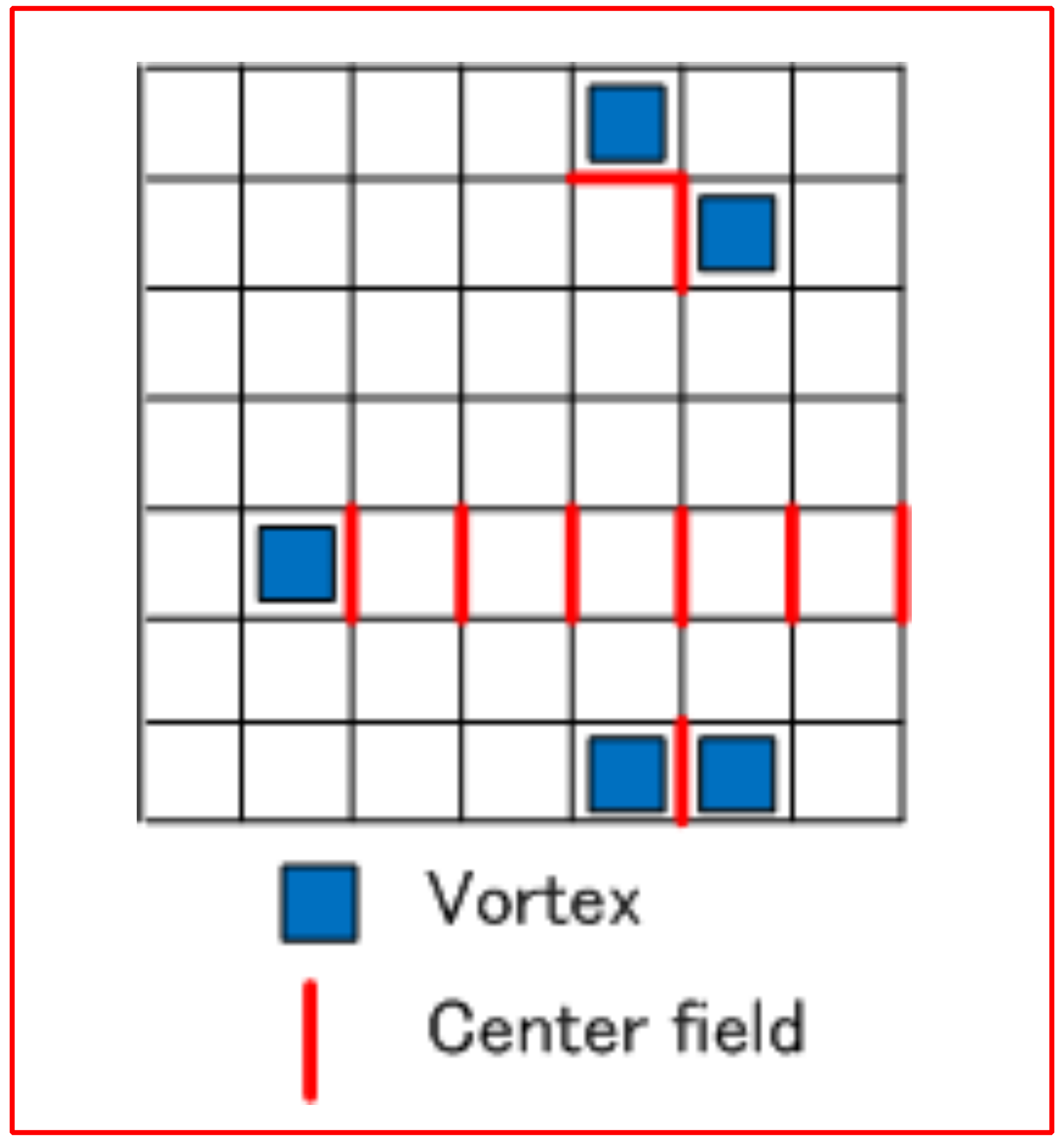


Maximal center projection

- Numerical technique
  - Direct Maximal Center Projection (MCP) by *Debbio, et. al, PRDv58,094501*
- We apply the MCP to all configurations of the SU(2) gauge field

$$\text{All the Us} \Rightarrow \pm \mathrm{I} \quad \text{Maximize } R = \frac{1}{VT}\sum_{x,t} \mathrm{Tr}\left[U_\mu(x,t)\right]$$

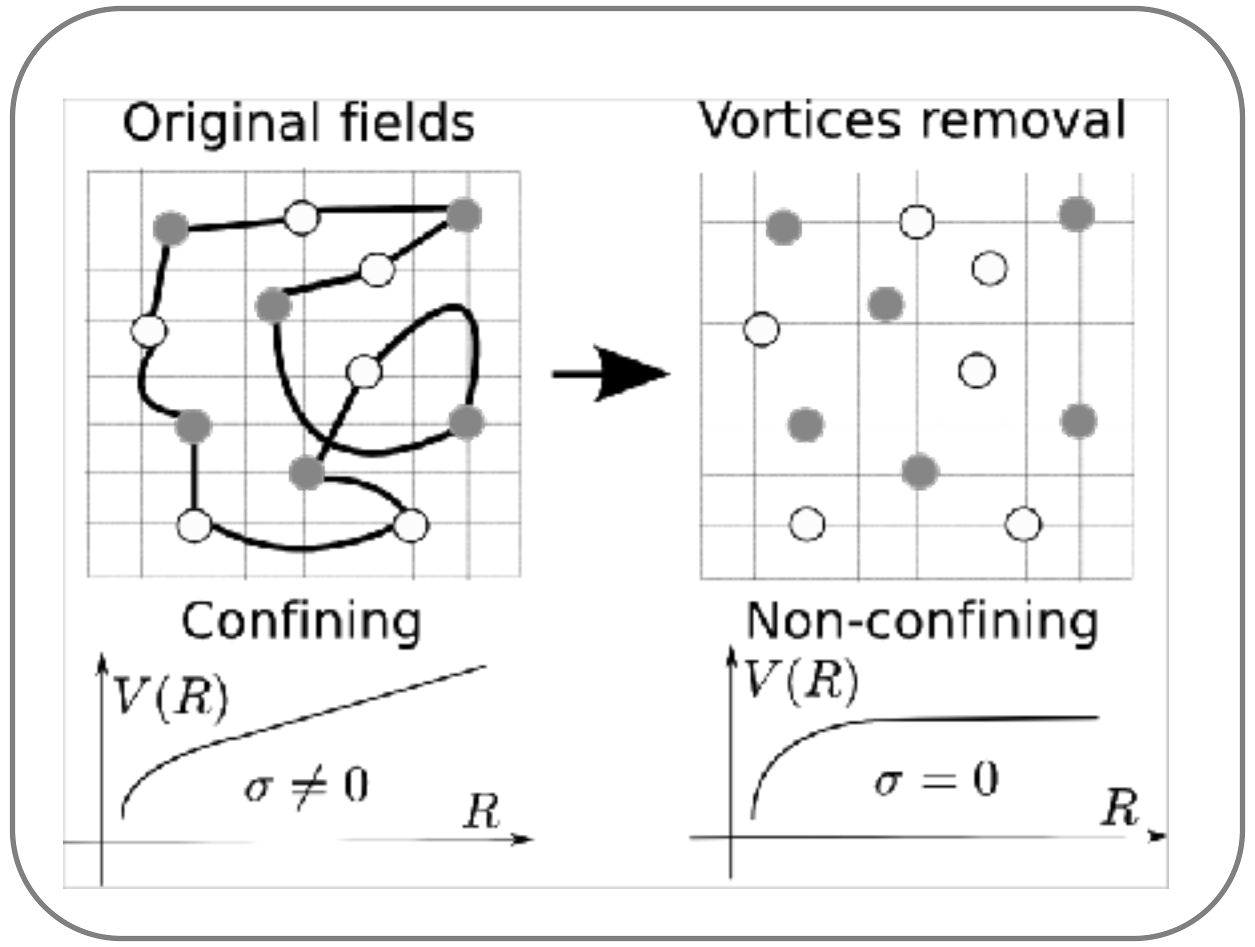
$$Z_\mu(x) = \mathrm{sgn}\,\mathrm{Tr}\left[U_\mu(x)\right]$$



- Removing center vortex (via *de Forcrand – D'Elia procedure,PRL82,4582(1999)*):

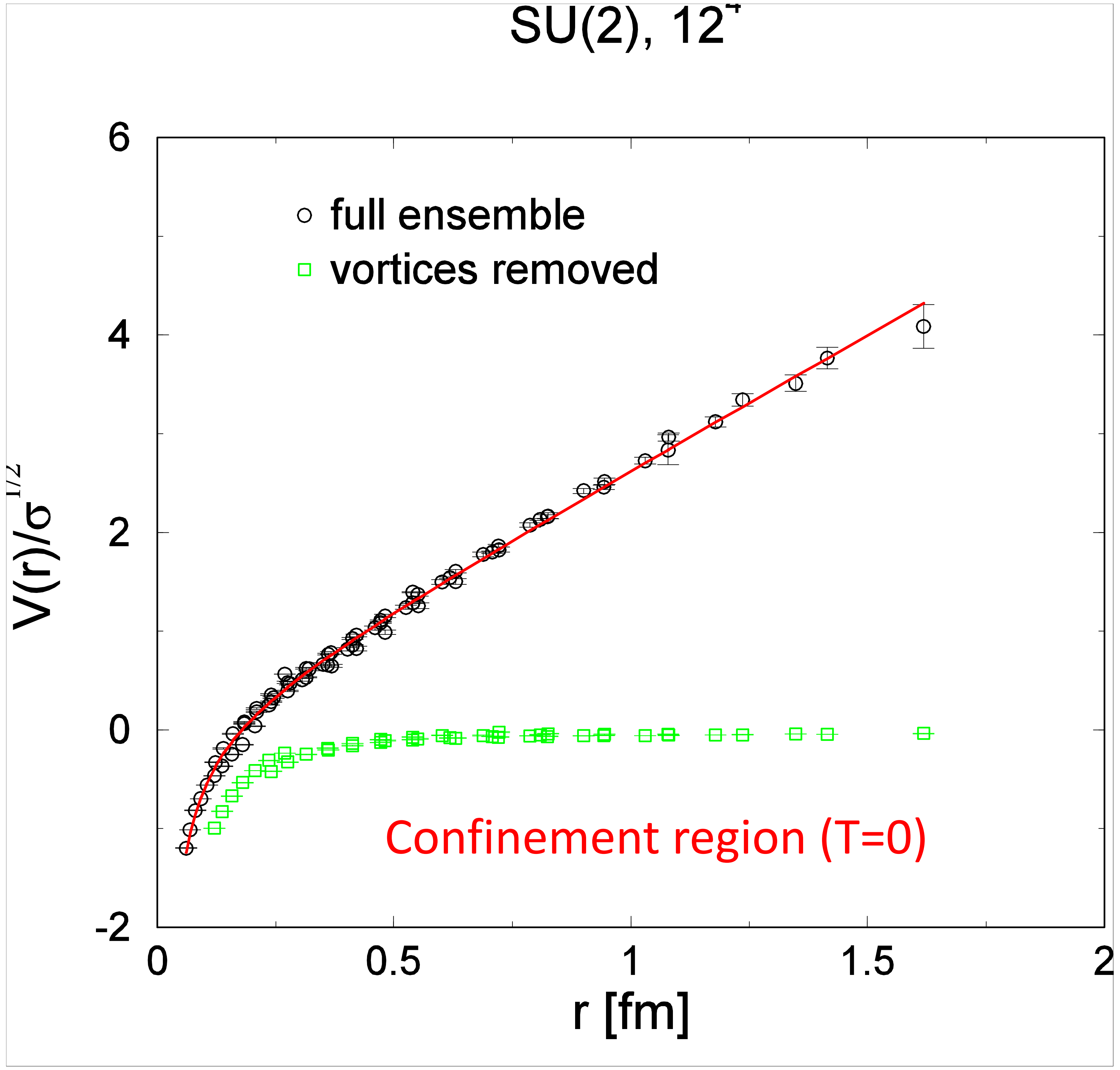
$$U_\mu(x) \rightarrow U'_\mu(x) = Z_\mu(x) U_\mu(x)$$

→ *Color confinement disappears and chiral symmetry restores.*



Center removal for quark potential

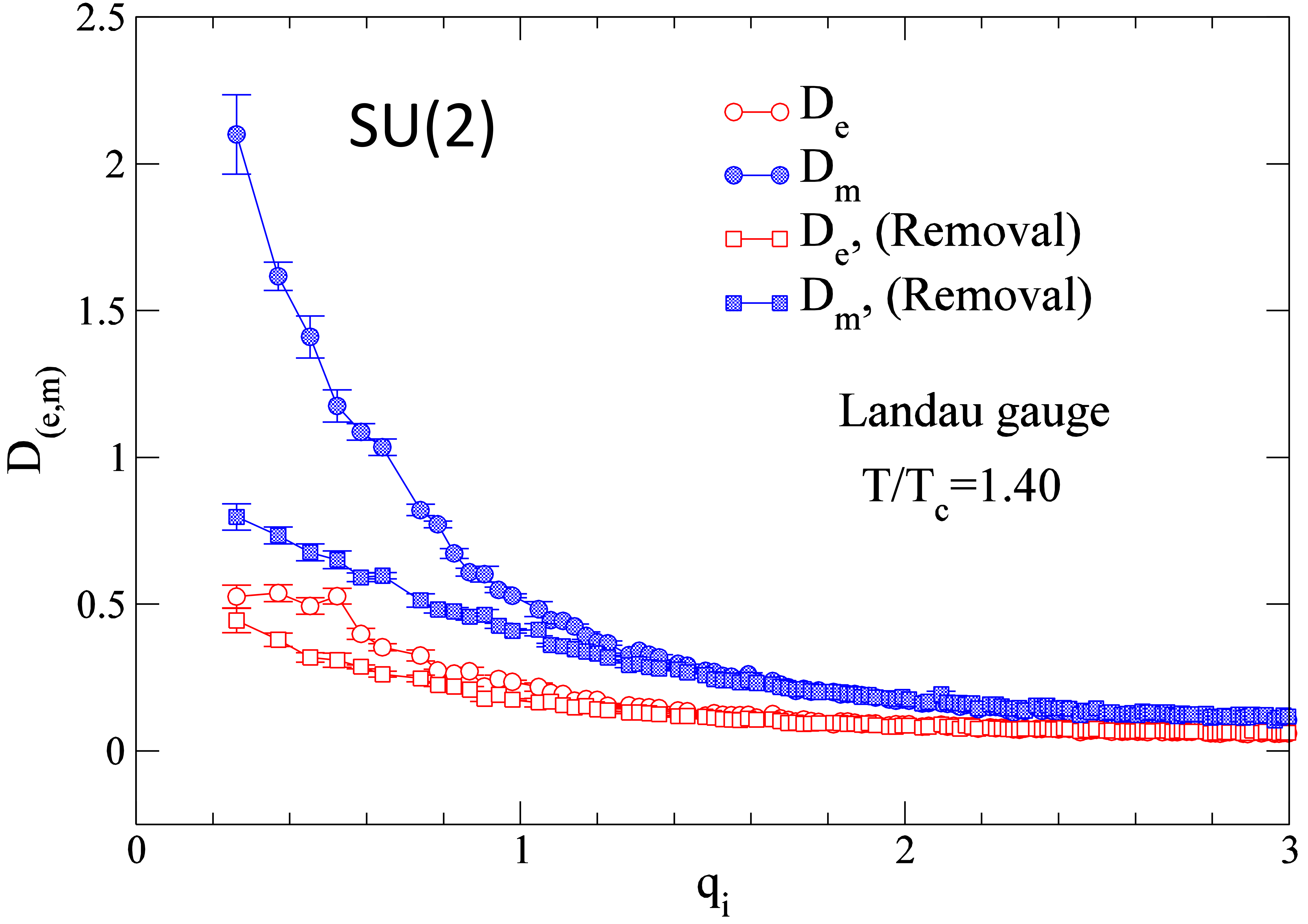
◆Removing vortices eliminates confinement.

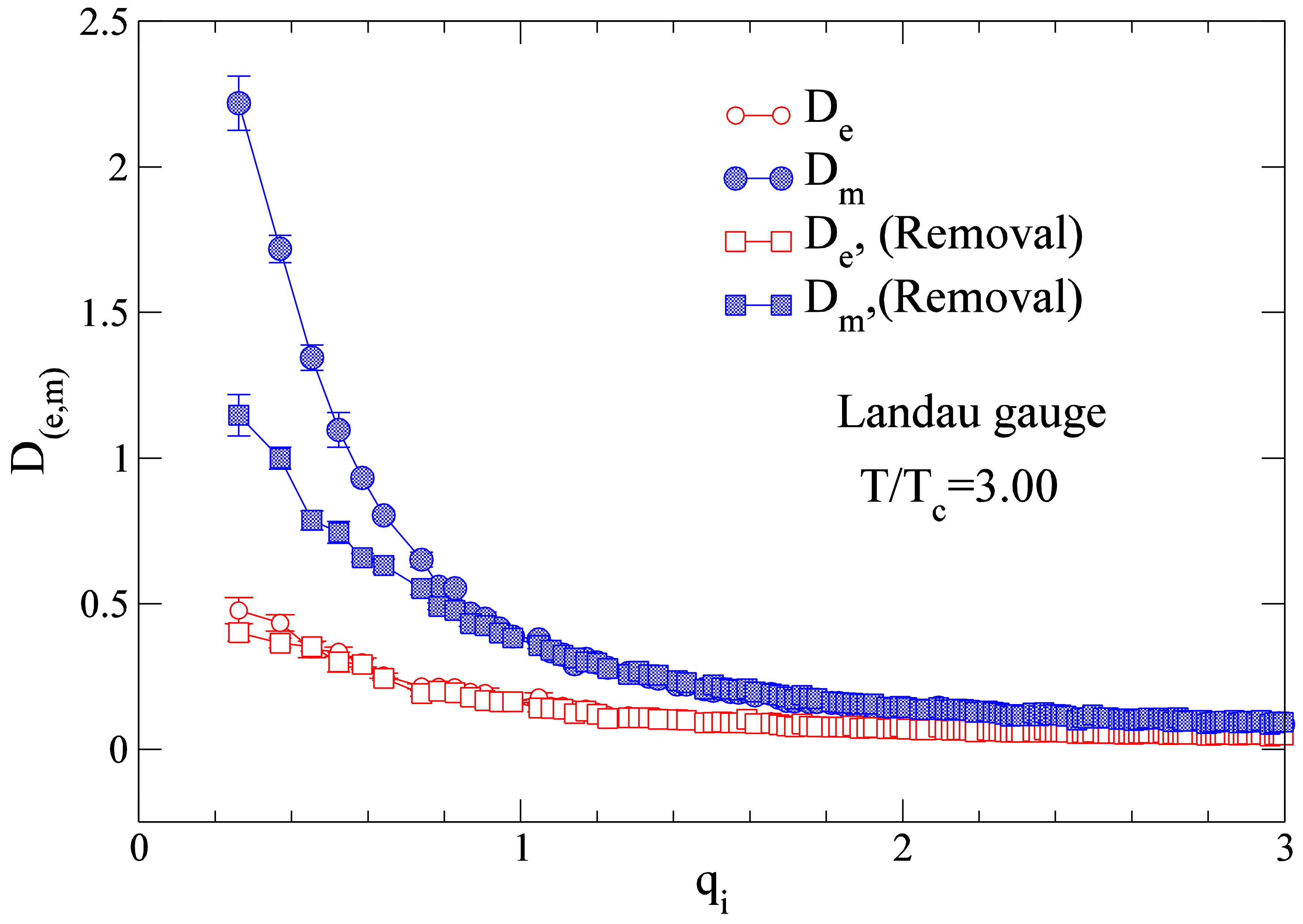


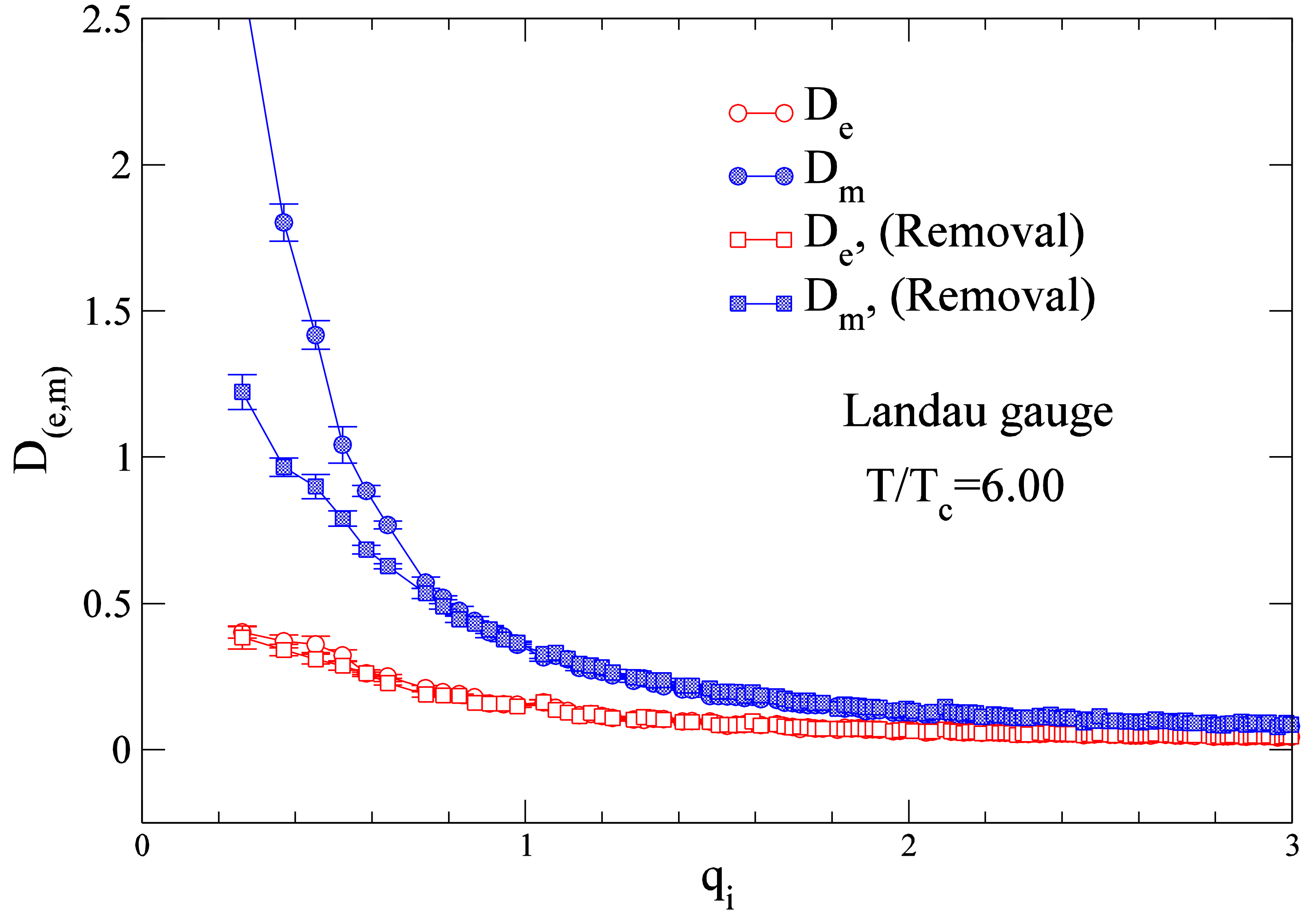
□"SU(2) gluon propagators from the lattice – a preview", *hep-lat/ 0104003, Kurt Langfeld*

□Removing center vortices eliminates confinement and restores chiral symmetry (*de Forcrand , D'Elia, PRL82,4582(1999)*)

Gluon propagators in the Landau gauge





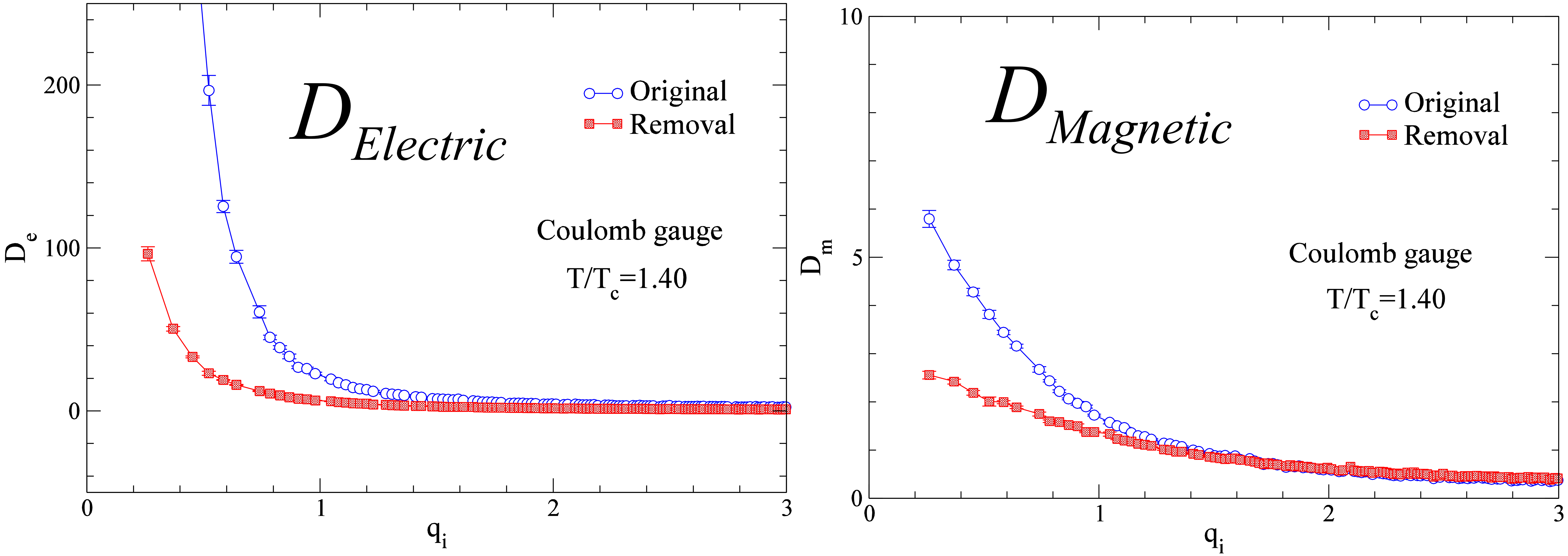


$$T/T_c = 3.0$$

$$T/T_c = 6.0$$

SU(2)

Gluon propagators in the Coulomb gauge



◆Time-time (electric) correlator diverges in the infrared limit.

→ Instantaneous linearly rising potential and non-zero thermal string tension that depends on magnetic scaling

◆Spatial-Spatial (magnetic) correlator is suppressed in the infrared limit.

SU(2)

Gluon Propagators
with and without Vortex
Summary

- Gluons at T>Tc have contribution of Vortex.

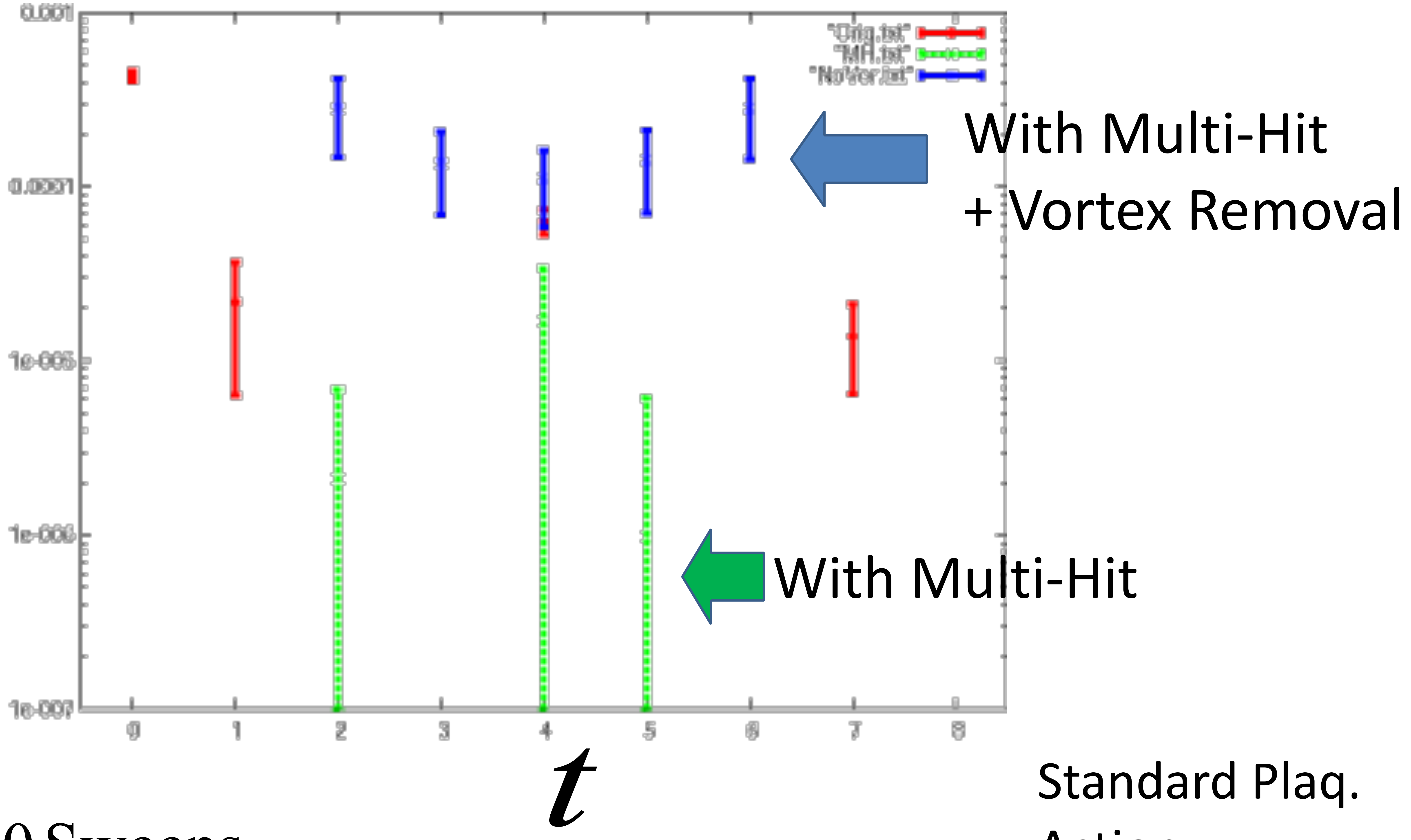
Transport Coefficients

$$\langle T_{12}(0) T_{12}(t) \rangle$$

$\beta = 3.0 \ (T/T_c \approx 5.1)$

$16^3 \times 8$

$SU(2)$



60×1000 Sweeps

Standard Plaq. Action

Manifestations of magnetic vortices in the equation of state of a Yang-Mills plasma

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The vacuum of Yang-Mills theory contains singular stringlike objects identified with center (magnetic) vortices. Percolation of magnetic vortices is known to be responsible for the color confinement in the low-temperature phase of the theory. In our work we study properties of the vortices at finite temperature using lattice simulations of $SU(2)$ gauge theory. We show that magnetic vortices provide a numerically large contribution to thermodynamic quantities of the gluon plasma in Yang-Mills theory. In particular, we observe that in the deconfinement phase at temperatures $T_c < T \lesssim 3T_c$ the magnetic component of the gluon plasma produces a negative (ghostlike) contribution to the anomaly of the energy-momentum tensor. In the confinement phase the vortex contribution is positive. The thermodynamical significance of the magnetic objects allows us to suggest that the quark-gluon plasma may contain a developed network of magnetic flux tubes. The existence of the vortex network may lead to observable effects in the quark-gluon plasma because the chromomagnetic field of the vortices should scatter and drag quarks.

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I. INTRODUCTION

Studies of properties of thermal plasma became a major development in QCD in recent years, for a review see, e.g. [1, 2]. Properties of the plasma are studied both directly, at RHIC and via lattice simulations. On the theoretical side, novel ideas, like AdS/CFT correspondence are being invoked [3], to say nothing of traditional approaches based on various quasiparticle models [4] and on field theory at finite temperature.

The traditional approach to the thermal plasma treats it, in zero approximation, as gas of free gluons and quarks and, then, takes into account perturbative corrections. An outcome of such calculations is a representation of the energy and pressure densities as perturbative series

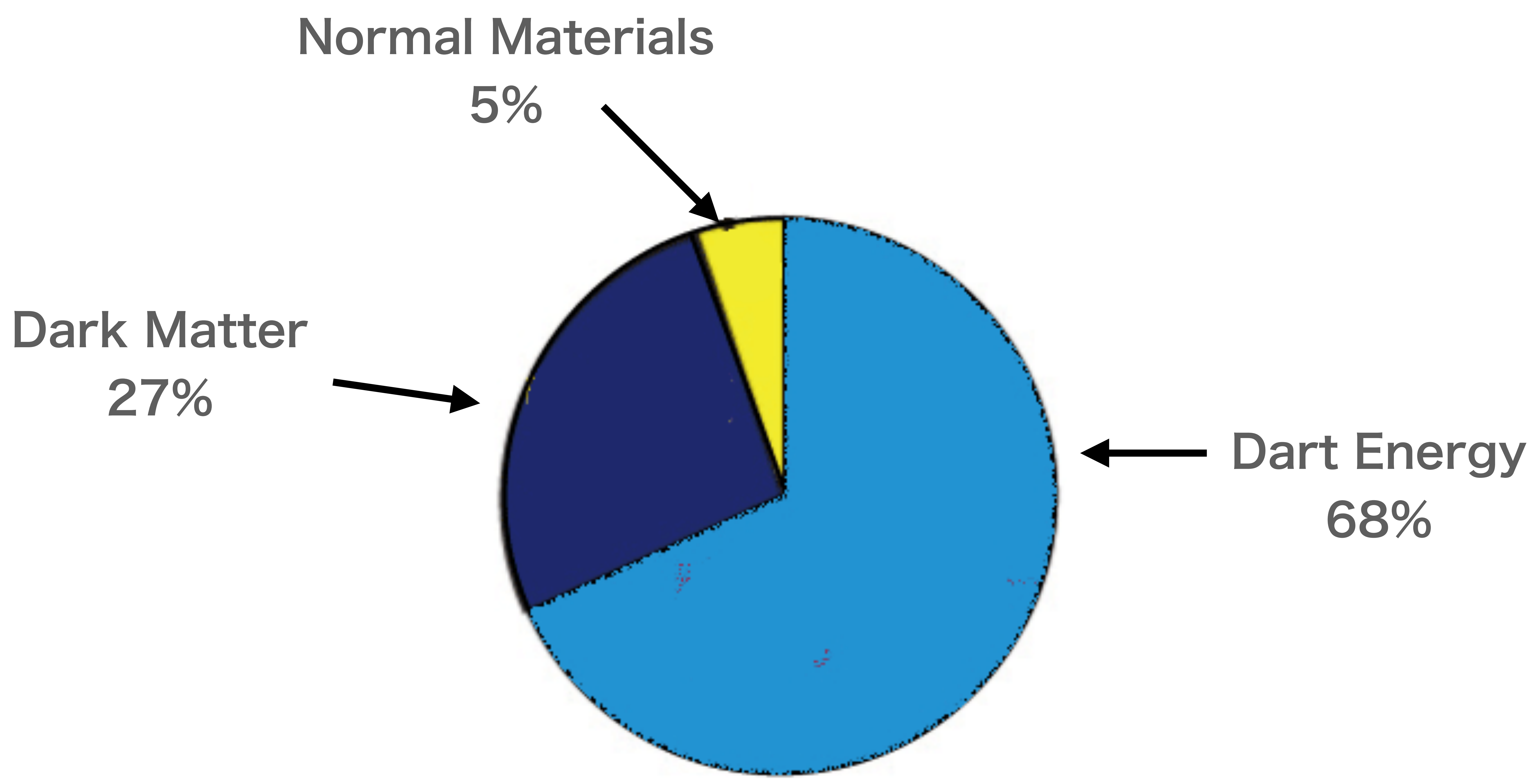
T_c is crucial for the plasma properties. In Refs. [7, 8] constituents of the magnetic component are thought to be classical magnetic monopoles. In Ref. [6] the magnetic component is identified with so-called magnetic strings related to magnetic monopoles. The properties of the strings, or center vortices and their role in confinement have been discussed in the lattice community for more than a decade, for review and references see Ref. [9].

According to the vortex picture the quark confinement emerges due to spatial percolation of the magnetic vortex strings which lead to certain amount of disorder. The value of the Wilson loop changes by a center element of the gauge group if the magnetic vortex pierces the loop. Therefore, very large loops receive fluctuating contributions from the vortex ensembles. These fluctuations make

Why SU(2) ?

1. No Sign problem
2. Less Com puter time, and memory
3. Simple structure
4. Nc dependence (Nc=2, 3 —> then estimate any Nc !

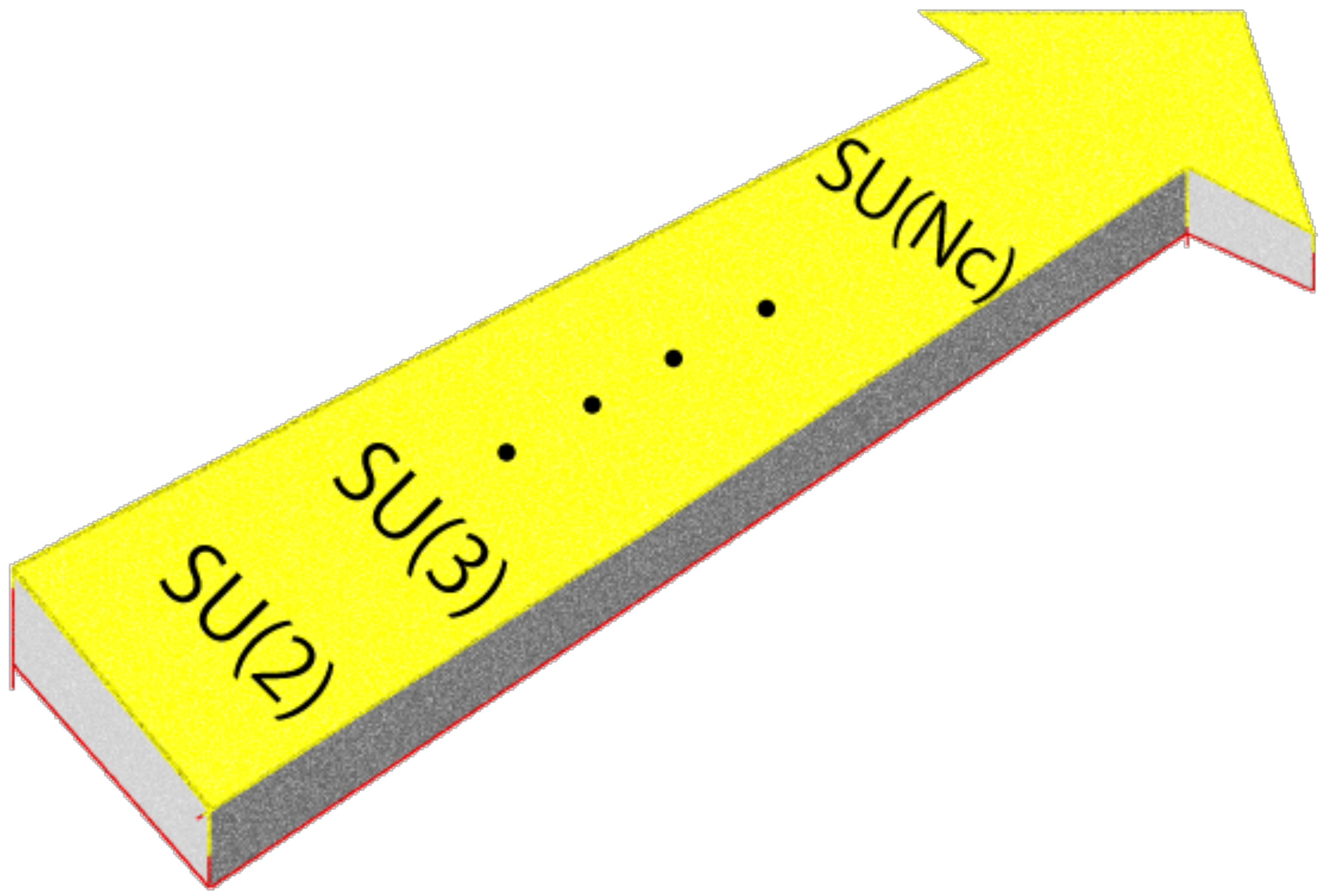
Our Universe is made from



What is the Dark Matter ?

Yamanaka, Iida, Nakamura, and Wakayama
Phys. Rev. D**102**, 054507
"Glueball scattering cross section in lattice SU(2) Yang-Mills theory"

Yamanaka, Iida, Nakamura, and Wakayam
arXiv:1911.03048 [hep-lat]
"Interglueball potential in SU(N) lattice gauge theory"



One Candidate : SU(N) Glue-ball

Gluon-bound state

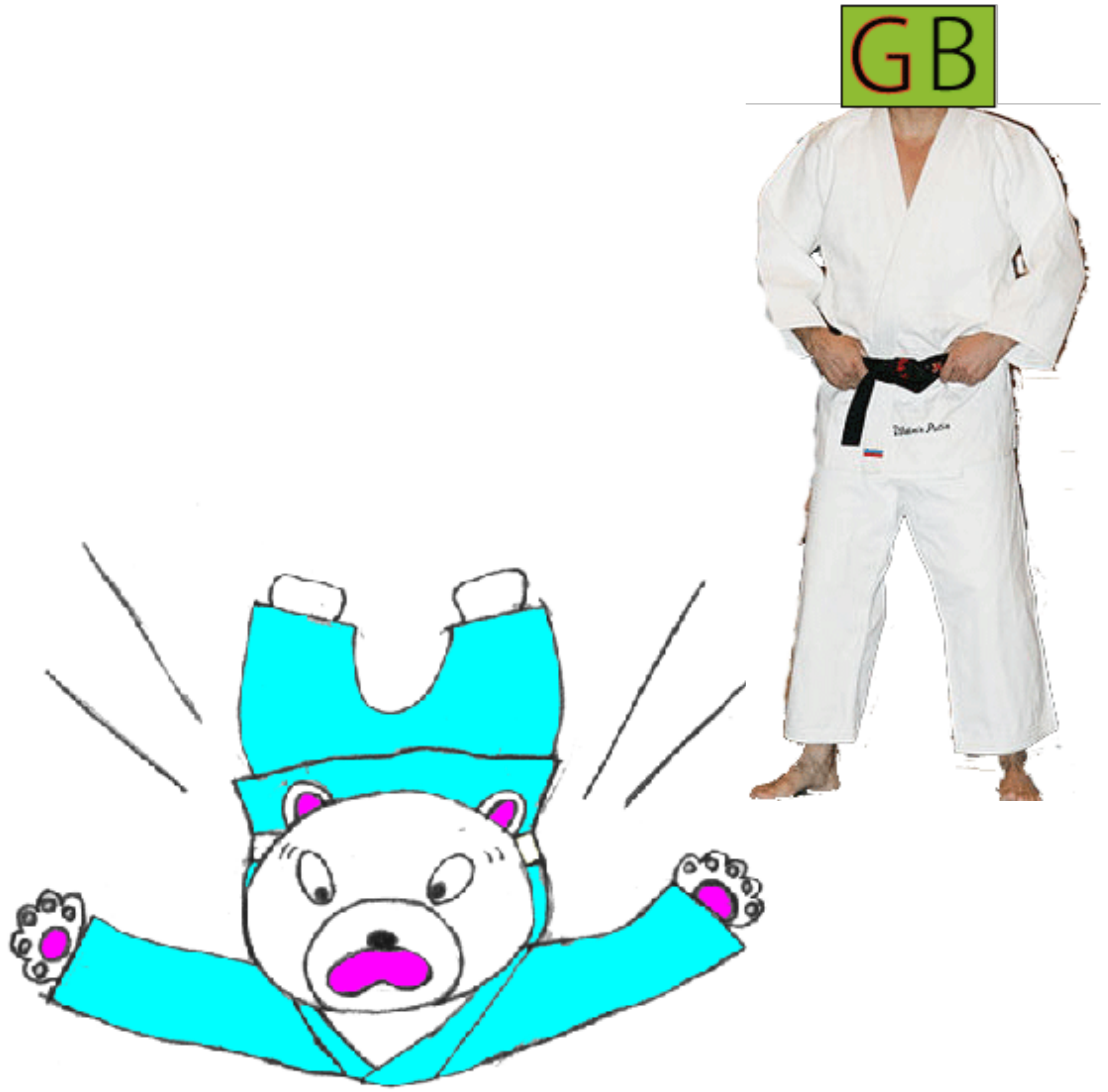
The lightest

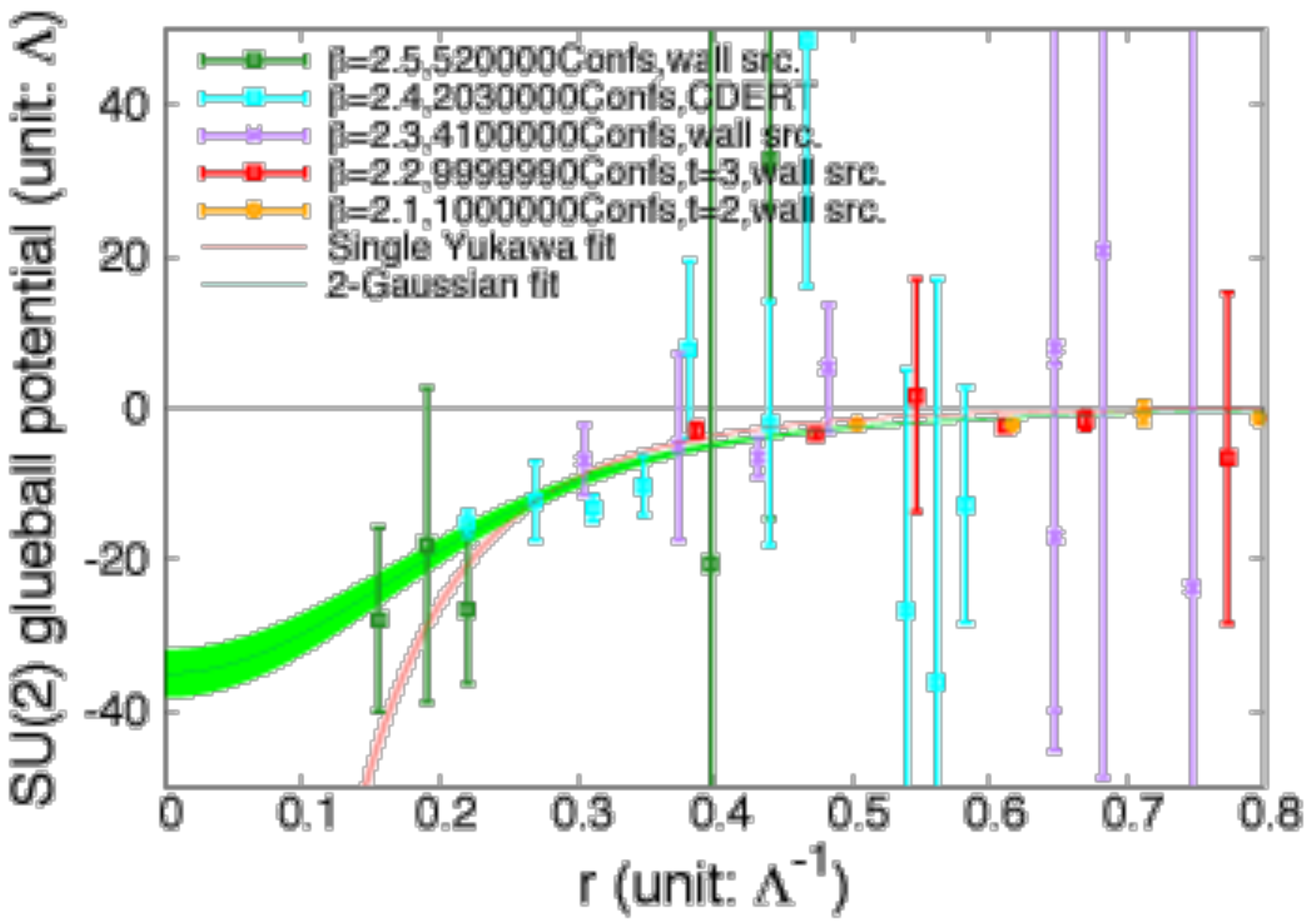
We calculate Glueball-Glueball scattering, which controls the galaxy structure such as the Hallo.

This is Not the QCD glueball.

We employ HAL method
(Aoki, Hatsuda, Ishii)
to extract Potential V.

Glueball is a tough object:





CDERT: Using “Variance Reduction and Cluster Decomposition”
by K-F.Liu, J. Liang, Y-B. Yang, Phys. Rev. D 97, 034507 (2018)

Future
is starting from this Workshop !

Work harder.
Go, Go !

Many Thanks organizers:

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