Phase structure of QC_2D with Wilson fermions

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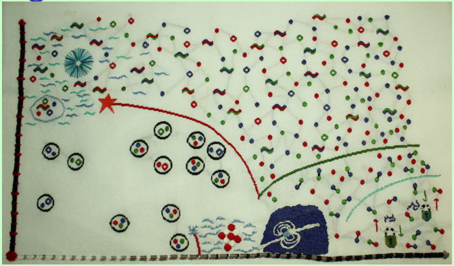
Outline

Background



- A plethora of phases at high μ, low T
- Based on models and perturbation theory

Indirect approach

Study QCD-like theories without a sign problem

- Generic features of strongly interacting systems at $\mu \neq 0$
- Check on model calculations, functional methods

QC_2D vs QCD

- Baryons are bosons (diquarks); superfluid ‘nuclear matter’
- Scalar diquark is pseudo-Goldstone (degenerate with pion)
- Onset transition at $\mu_q = m_\pi/2$, not $m_N/3$

Phase diagram

- Superfluid phase for $\mu > m_\pi/2$: BEC $\longrightarrow$ BCS?
- Exotic phases: quarkyonic, spatially varying?
- Deconfinement at high density, shape of deconfinement line?

Global symmetries of QC_2D

Quarks and antiquarks are in the same representation

Anti-unitary symmetry: $KMK^{-1} = \mathcal{M}^*$ with $K \equiv C\gamma_5\tau_2$

$m = \mu = 0$:
global $SU(2N_f)$ symmetry $\longrightarrow$ $Sp(2N_f)$ by $\langle\overline{\psi}\psi\rangle \neq 0$.

$$\Rightarrow N_f(2N_f - 1) - 1 \text{Goldstone modes}$$

$N_f = 2$: 5 modes

$\overline{\psi}\vec{\sigma}\gamma_5\psi$ pion $\psi^T\epsilon\tau_2 C\gamma_5\psi,\quad \overline{\psi}\epsilon\tau_2 C\gamma_5\overline{\psi}^T$ scalar diquark

Diquark condensation

Diquarks are colour singlets in QC_2D

$\rightarrow$ superfluidity rather than colour superconductivity

$\rightarrow$ exact Goldstone mode from breaking of $U(1)_B$ symmetry

Bose–Einstein Condensation:

Condensation of tightly bound diquarks (Goldstone baryons)

$\leftrightarrow$ Chiral perturbation theory

$$\langle\psi\psi\rangle \propto \sqrt{1-(\mu/\mu_o)^4}$$

Bardeen–Cooper–Schrieffer:

Pairing of quarks near the Fermi surface

$$\langle\psi\psi\rangle \propto \Delta\mu^2$$

Bulk thermodynamics expectations

Chiral perturbation theory

- Pseudo-Goldstone bosons (diquarks)
- Separation of scales $m_\pi \ll m_\rho$
- Corresponds to BEC

$$n_q^{\chi PT} = 8\mu N_f F^2 \left[1 - \left(\frac{\mu_o}{\mu} \right)^2 \right]$$

Free fermions

- Fermi sphere of weakly interacting quarks
- Corresponds to BCS

$$n_q^{SB} = \frac{N_f N_c}{3} \left(\mu T^2 + \frac{\mu^3}{\pi^2} \right)$$

Two-colour quarks and gluons

Gluodynamics — SU(2) and SU(3) very similar?

- Effects of deconfinement on gluon propagation?
- Gap equation with effective or one-gluon interaction used to determine superconducting gap $\rightarrow$ more realistic input?

Quark propagator

- Details of phase diagram depend critically on the effective quark mass in the medium.
- Dynamical quark masses $\rightarrow$ effective strange quark mass?
- Location of Fermi surface?
- Direct determination of diquark gap, size of Cooper pairs?

Lattice formulation

We use Wilson fermions:

- Correct symmetry breaking pattern, Goldstone spectrum
- $N_f < 4$ needed to guarantee continuum limit
- No problems with locality, fourth root trick
- Chiral symmetry buried at bottom of Fermi sea

$$S = \overline{\psi}_1 M(\mu)\psi_1 + \overline{\psi}_2 M(\mu)\psi_2 - J\overline{\psi}_1(C\gamma_5)\tau_2\overline{\psi}_2^T + \overline{J}\psi_2^T(C\gamma_5)\tau_2\psi_1$$

$$\gamma_5 M(\mu)\gamma_5 = M^\dagger(-\mu), \quad C\gamma_5\tau_2 M(\mu) C\gamma_5\tau_2 = -M^*(\mu)$$

Diquark source $J \equiv \kappa j$ introduced to

- lift low-lying eigenmodes in the superfluid phase
- study diquark condensation without uncontrolled approximations

Simulation parameters

Name	β	κ	a	am_π	m_π/m_ρ
Coarse	**1.9**	**0.1680**	**0.18fm**	**0.65**	**0.80**
Light	**1.7**	**0.1810**	**0.19fm**	**0.44**	**0.61**
Fine	**2.1**	**0.1577**	**0.14fm**	**0.45**	**0.81**

Ensemble	N_s	N_τ	T (MeV)	μa	ja
Coarse	12	24	47	0.25–1.10	0.02, 0.04 (0.03)
	16	24	47	0.30–0.90	0.04
	12	16	70	0.30–0.90	0.04
	16	12	94	0.20–0.90	0.02, 0.04
	16	8	141	0.10–0.90	0.02, 0.04
Fine	16	32	45	0.15–0.80	0.02, 0.03 (0.01)
	16	20	71	0.20–0.60	0.02, 0.03
	16	16	89	0.20–0.60	0.02, 0.03
	16	12	119	0.20–0.60	0.02, 0.03
Light	12	24	43	0.10–0.80	0.02, 0.04 (0.03)
	16	12	87	0.10–0.80	0.02, 0.04
	16	8	130	0.10–0.70	0.02, 0.04

Simulation parameters

T-scans, fixed μ (coarse lattice)

All simulations done on $16^3 \times N_\tau$ lattices

μa	ja	N_τ
0.0	0.0	4–10
0.35	0.02	4–13, 16
	0.04	4–12, 14, 16
0.40	0.02	5–13, 16
	0.04	4–13
0.50	0.02	6–12, 16
	0.04	4–16, 18, 20
0.60	0.02	6–12, 14, 16
	0.04	6–16, 20

In addition, 300 trajectories were generated at $ja = 0.03, 0.05$ for $N_\tau = 9, 10, 11$ at all $\mu > 0$.

Simulation parameters

T-scans, fixed μ (fine lattice)

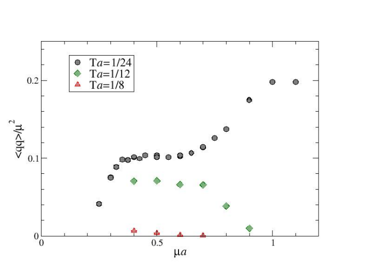
All simulations done on $16^3 \times N_\tau$ lattices

μa	ja	N_τ
0.0	0.0	4–10, 12, 16
0.2	0.03	4–14, 16, 18
0.3	0.02	4, 6, 8, 10, 12–14, 16
	0.03	4–14, 16, 18
0.4	0.02	4, 6, 8, 10, 12–14, 16
	0.03	4–14, 16, 18, 20
0.5	0.02	4, 6, 10, 12–14, 16
	0.03	4, 6–14, 16, 18

Diquark condensate — μ-scan

Coarse ensemble

Results shown are for linear extrapolation
Power law $\langle qq \rangle = Aj^{\alpha}$ works for $\mu a \lesssim 0.4$, with $\alpha = 0.85 - 0.5$.
[Effective field theory predicts $\alpha = \frac{1}{3}$ near onset]



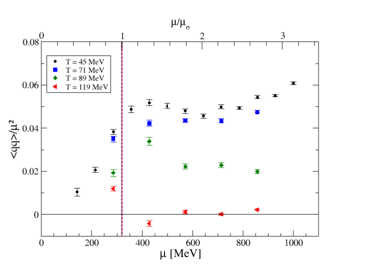
- BCS scaling $\langle qq \rangle \sim \mu^2$ for $0.35 \lesssim \mu a \lesssim 0.7$
- Melted at $T = 141$MeV ($N_\tau = 8$)
- New transition for $\mu a \gtrsim 0.7$?
- Melting for $N_\tau = 12, \mu a \gtrsim 0.7$?

$N_\tau = 16$ results are very close to $N_\tau = 24$ results.

Diquark condensate — μ-scan

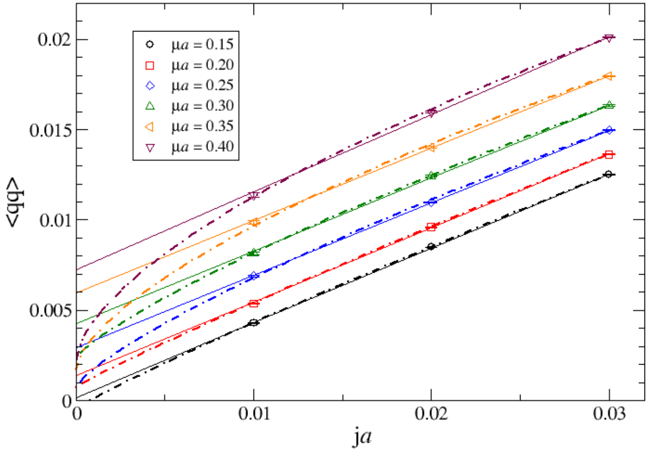
Fine ensemble

Results shown are for linear extrapolation
$\langle qq \rangle = \Delta + Aj^{1/3}$ does not give satisfactory results

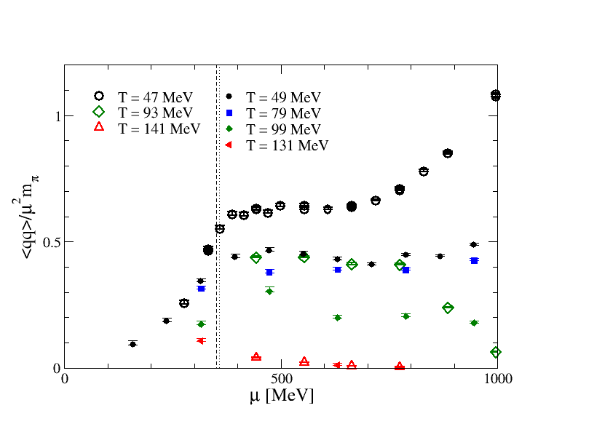


- BCS scaling $\langle qq \rangle \sim \mu^2$ confirmed
- Melted at $T \approx 130$MeV ($N_\tau = 12$)
- No sign of second transition

Diquark source extrapolation



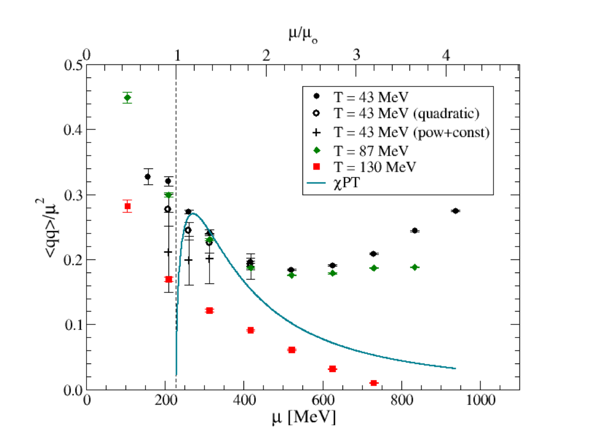
Coarse and fine ensemble



Smaller magnitude on fine lattice: renormalisation?

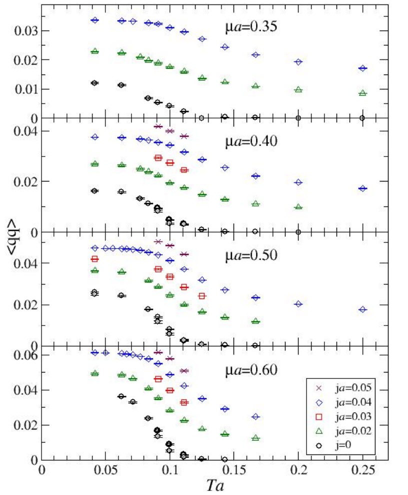
Diquark condensate — μ-scan

Light ensemble



Signs of a BEC window?

Superfluid to normal transition — coarse ensemble



$j \to 0$ extrapolation not fully under control

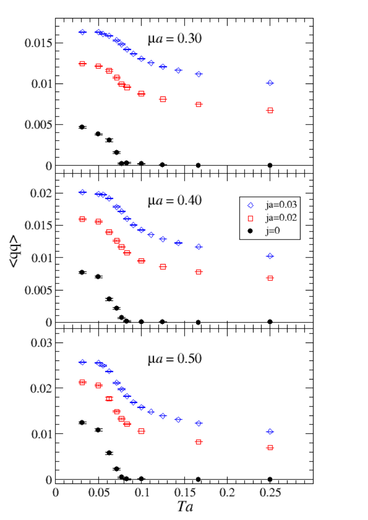
Linear form used here

Transition temperatures from inflection points (and $j \to 0$)

$a\mu$	T_s (MeV)
0.35	82(27)
0.40	94(9)
0.50	93(6)
0.60	93(7)

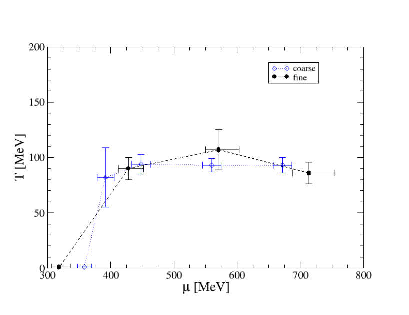
Remarkably constant!

Fine ensemble



Transition temperatures

$a\mu$	T_s (MeV)
0.30	90(10)
0.40	107(18)
0.50	86(10)



Agreement coarse–fine

Deconfinement transition

Polyakov loop L requires renormalisation,

$$L_R = e^{-F_q/T} = e^{-(F_0+\Delta F)/T} = Z_L^{N_\tau} L_0$$

We use two schemes to determine $Z_L = \exp(-a\Delta F)$,

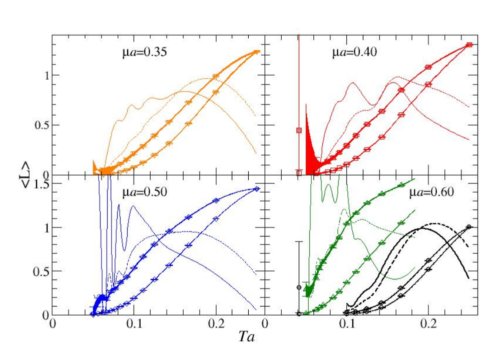
Scheme A $\quad L_R(T = \frac{1}{4a}, \mu = 0) = 1\,,$

Scheme B $\quad L_R(T = \frac{1}{4a}, \mu = 0) = 0.5\,.$

We determine the deconfinement temperature (crossover region) from the inflection point (linear region) of L_R

Deconfinement transition

Coarse ensemble

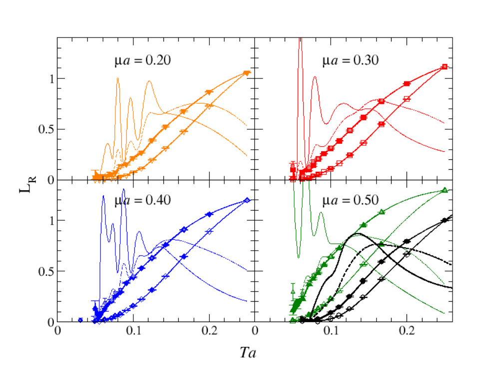


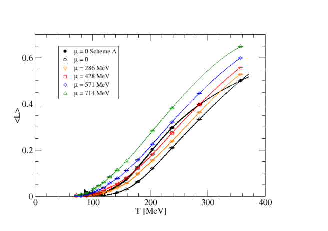
Scheme dependence ⟷ broad crossover?

Estimates from Scheme B, encompassing Scheme A

μa	$T_d a$	T_d (MeV)
0.0	0.193(20)	217(23)
0.35	0.140–0.220	157–247
0.40	0.108–0.200	121–225
0.50	0.080–0.200	90–225
0.60	0.060–0.135	67–152

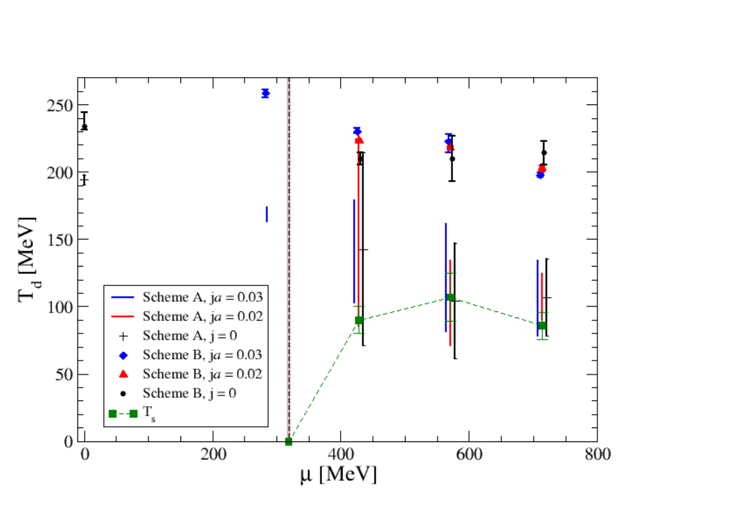
Fine ensemble



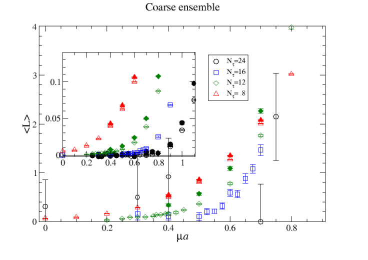


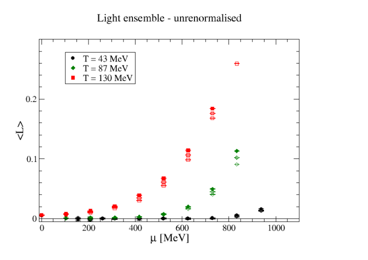
Qualitative features as for coarse ensemble

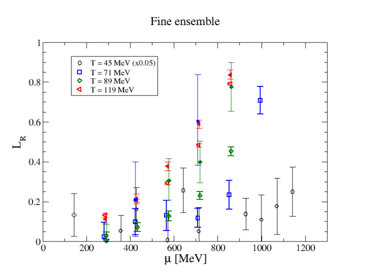
Fine ensemble: transitions



Polyakov loop — μ-scan

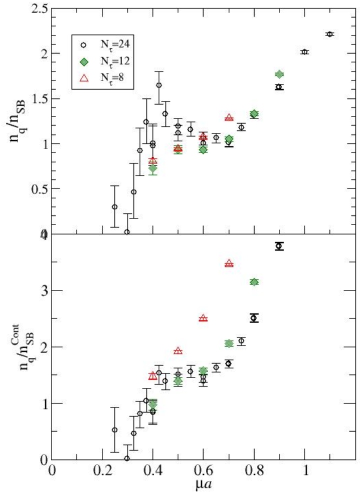






- Clear sign of deconfinement at high T and μ
- $\mu_d(T)$ decreasing with increasing T
- Deconfinement at low T: lattice artefact?

Number density: Coarse ensemble

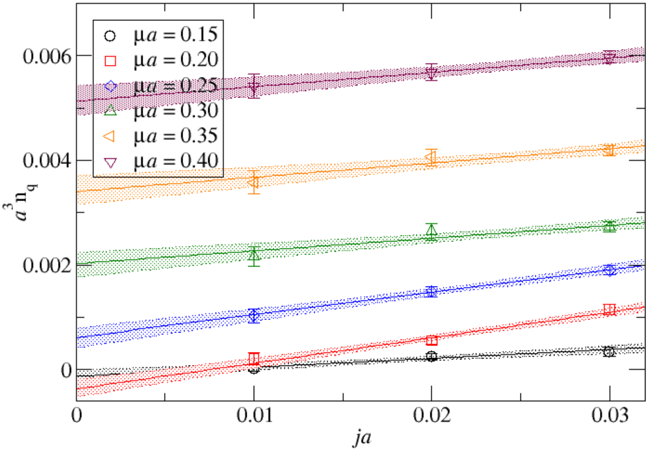


- Normalised to lattice SB or

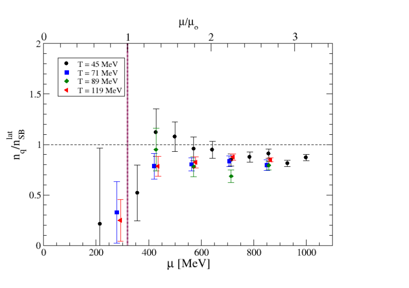
$$n_{SB}^{\text{Cont}} = \frac{4\mu T^2}{3} + \frac{4\mu^3}{3\pi^2}$$

- Close to SB scaling for $0.4 \lesssim \mu a \lesssim 0.7,\ N_\tau = 24, 12$.
- Little T-dependence for $T \lesssim 100\text{MeV}$.
- Strong thermal effects for $N_\tau = 8$
- $N_\tau = 16, 24$ almost identical

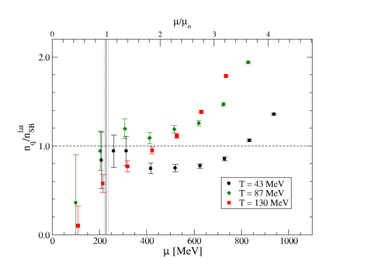
Diquark source extrapolation: fine ensemble

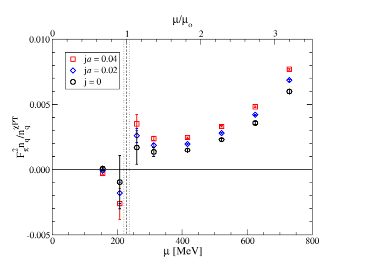


Number density on fine and light ensemble

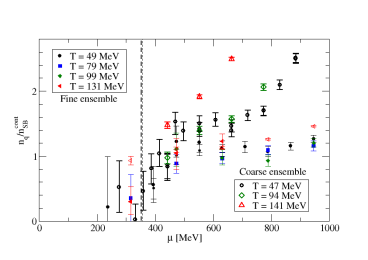


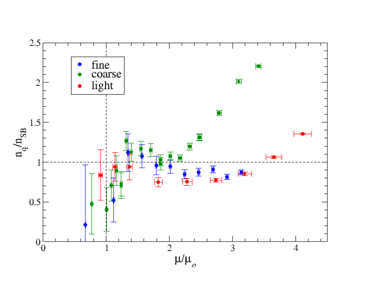
Fine ensemble:
Same qualitative features as coarse!

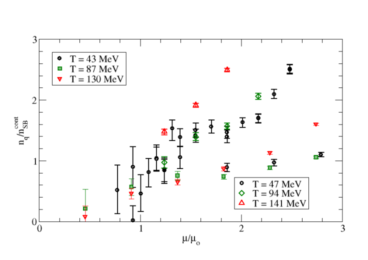


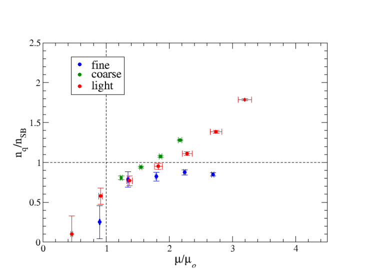


Number density: comparison

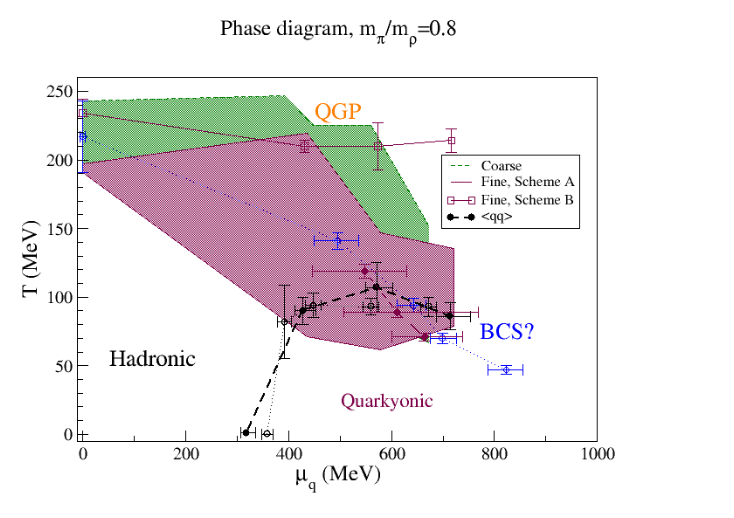








Summary


Phase diagram, m_π/m_ρ=0.8
QGP
Hadronic
Quarkyonic
BCS?
Coarse
Fine, Scheme A
Fine, Scheme B
<qq>
T (MeV)
μ_q (MeV)
0
50
100
150
200
250
0
200
400
600
800
1000

Gluon propagator

- Essential ingredient in gap equation

$$S^{-1}(p) = S_0^{-1}(p) + Z_2 \int d^4q \Gamma_\mu(p,q) D_{\mu\nu}(q) S(p-q)\gamma_\nu$$

  used to determine dynamical fermion mass and superfluid/superconducting gap
- Link to functional methods (DSE, FRG)
- Electric gluon may signal deconfinement transition

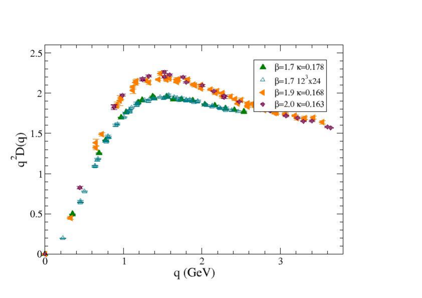
Tensor structure in medium

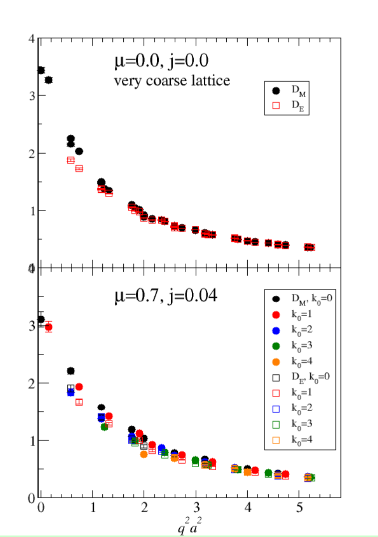
$$D_{\mu\nu}(\vec{q}, q_0) = P^T_{\mu\nu} D_M(\vec{q}^{\,2}, q_0^2) + P^E_{\mu\nu} D_E(\vec{q}^{\,2}, q_0^2) + \xi \frac{q_\mu q_\nu}{(q^2)^2}$$

$$P^M_{\mu\nu}(\vec{q}, q_0) = (1-\delta_{0\mu})(1-\delta_{0\nu})(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{\vec{q}^{\,2}}) \,,$$

$$P^E_{\mu\nu}(q_0, \vec{q}\,) = (\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) - P^M_{\mu\nu}(q_0, \vec{q}\,) \,.$$

Gluon propagator results

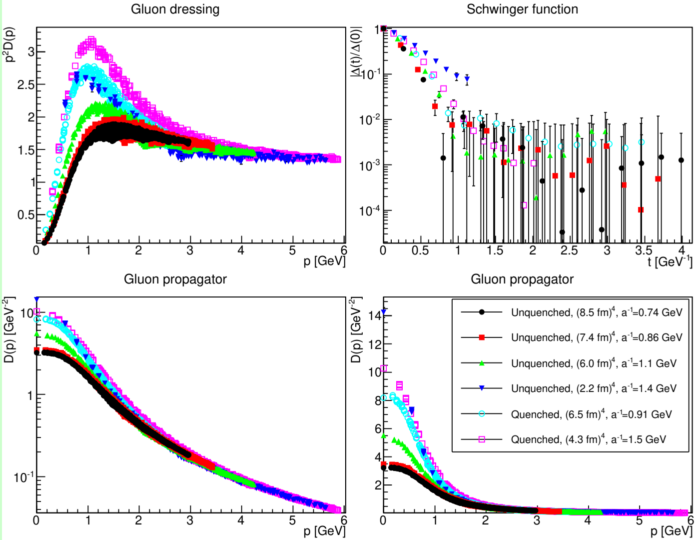




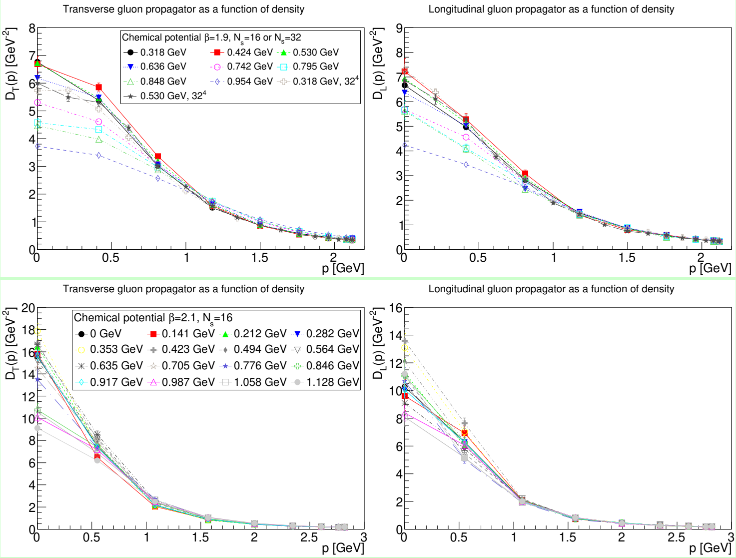
Some finite volume and lattice spacing effects at $\mu = 0$

In-medium modifications, incl. violations of Lorentz symmetry, visible in magnetic gluon at high density

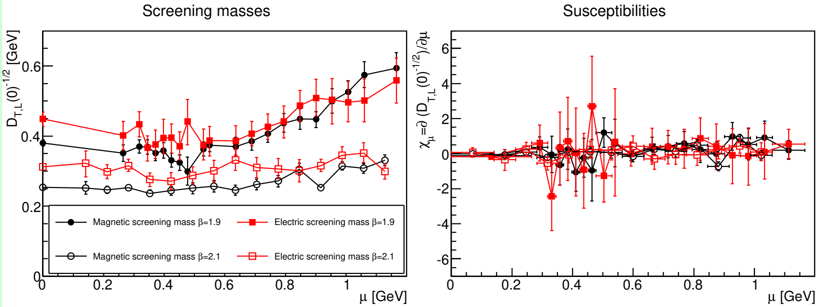
Gluon propagator in vacuum



Gluon propagator: varying μ

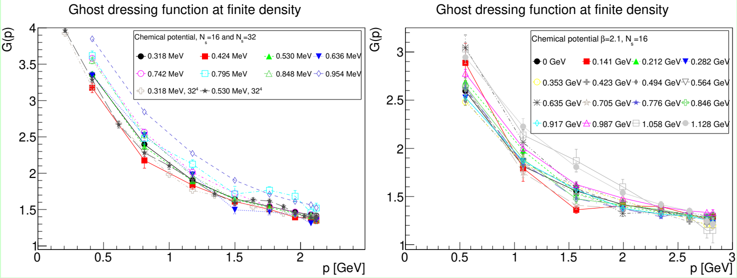


Screening masses



- Increasing with μ on coarse ensemble
  $\leftrightarrow$ infrared suppression
- No discernible effect on fine ensemble
- No significant difference electric/magnetic

Ghost propagator



- Very little μ dependence
- Changes at large μ: lattice artefact?

Quark propagator — tensor structure

$$S^{-1}(\vec{p},\tilde{\omega}) = i\vec{p}\!\!\!/ A(\vec{p}^2,\tilde{\omega}^2) + i\gamma_4\tilde{\omega}C(\vec{p}^2,\tilde{\omega}^2) + B(\vec{p}^2,\tilde{\omega}^2) + i\gamma_4\vec{p}\!\!\!/ D(\vec{p}^2,\tilde{\omega}^2)$$

$$S(\vec{p},\tilde{\omega}) = i\vec{p}\!\!\!/ S_a + i\gamma_4\tilde{\omega}S_c + S_b + i\gamma_4\vec{p}\!\!\!/ S_d$$

where $\tilde{\omega} \equiv p_4 - i\mu$.

In general the form factors are complex!

The D form factors can be shown to vanish [Rusnak&Furnstahl 1995]

Gor'kov formalism

Quarks and antiquarks are in the same representation.
Construct Gor'kov spinor

$$\Psi = \begin{pmatrix} \psi \\ \overline{\psi}^T \end{pmatrix} \quad \Longrightarrow \quad \langle \Psi(x)\overline{\Psi}(y)\rangle \equiv \mathcal{G}(x,y) = \begin{pmatrix} S_N & -S_A \\ \overline{S}_A & \overline{S}_N \end{pmatrix}$$

S_A contains information about anomalous propagation
Self-energies are diquark gaps Δ (superfluid/superconducting)
General tensor structure is the same as for normal components, scalar diquark gap is Δ_b.

From isospin and charge conjugation symmetry it follows that

$$\overline{S}_N(x,y) = -S_N(y,x)^T\,, \qquad S_A(x,y) = S_A(y,x)^T$$

Fermi surface and Cooper pairs

Fermi surface

In a Fermi liquid the Fermi surface is given by

$$\det S^{-1}(\vec{p_F}, p_4 = 0) = 0 \quad \Longleftrightarrow \quad \vec{p}^2 A^2 + \tilde{\omega}^2 C^2 + B^2 = 0$$

Pole in propagator

In gapped phase: zero crossing!

Size of Cooper pair

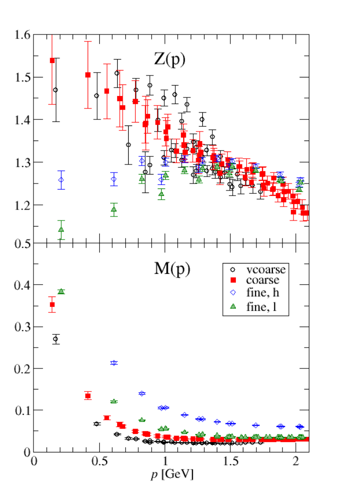
If we know the anomalous propagator $S_A(x)$ we can compute the size of the Cooper pairs:

$$\xi^2 = \frac{\int d^3x \, \vec{x}^2 |\frac{1}{2} \operatorname{Tr}(S_A(x)\Lambda^+)|^2}{\int d^3x \, |\frac{1}{2} \operatorname{Tr}(S_A(x)\Lambda^+)|^2}$$

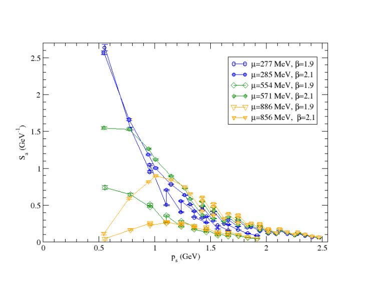
Quark propagator results

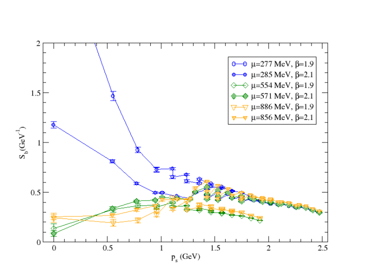
Quark propagator in vacuum

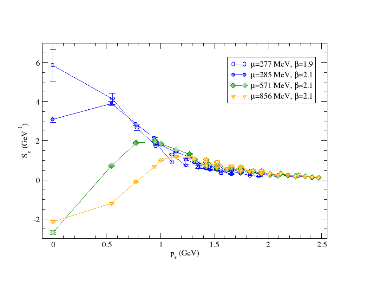
- Large lattice spacing dependence
- Substantial quark mass dependence for $Z(p)$
- Unusual p-dependence in $Z(p)$
- infrared suppression recovered in low-mass and continuum limit?
- M(p) not yet properly corrected!



Quark propagator: normal components

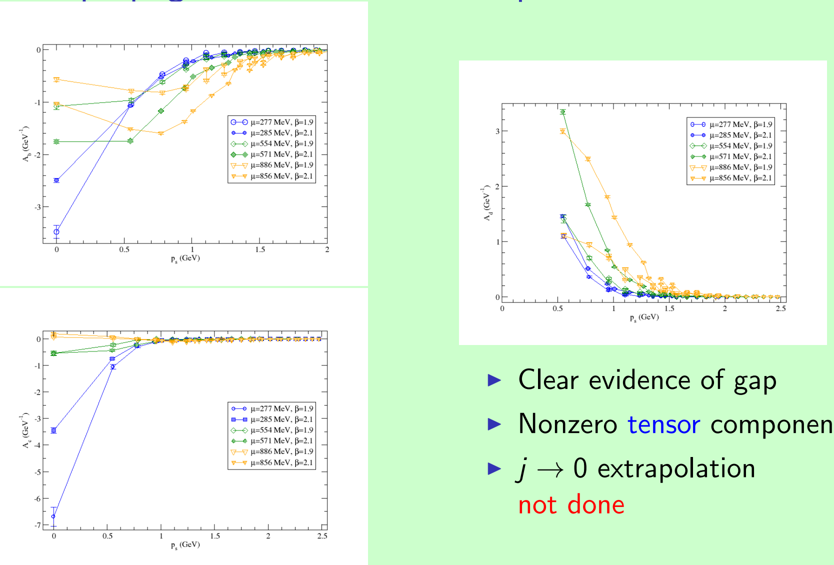






- Significant scaling violations
- Large medium modifications — anomalous propagation?
- Evidence of gap in S_c

Quark propagator: anomalous components



- Clear evidence of gap
- Nonzero tensor component
- $j \to 0$ extrapolation not done

Summary

Evidence for three phases/regions

- Vacuum/hadronic phase below $\mu_o = m_\pi/2$, low T
- BCS/quarkyonic for intermediate μ, low T
- Deconfined/QGP matter at high T
- Superfluid to normal $T_s(\mu)$ remarkably flat above μ_o
- Deconfinement crossover $T_d(\mu)$ decreasing with μ
- Consistent picture from coarse and fine ensembles
- Second transition at large μ may be lattice artefact
- BEC window opening with lighter quarks?

Outlook

- Attempt O(2) scaling fit for superfluid to normal transition?
  - $\rightarrow$ Requires several larger lattice volumes
- Full analysis of quark propagator in progress
- Quark–gluon vertex?