Generalized symmetries and Anomaly matching

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Outline

1. Anomaly in Quantum Mechanics

   Demonstration of the idea

2. SU(N) Yang-Mills theories

   - What is center sym?
   - Anomaly involving center sym.

3. QCD (-like) theories

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General idea

Solving QFT is a very difficult task.

$\Rightarrow$ We want to get a guidline for possible interesting dynamics.

Strategy

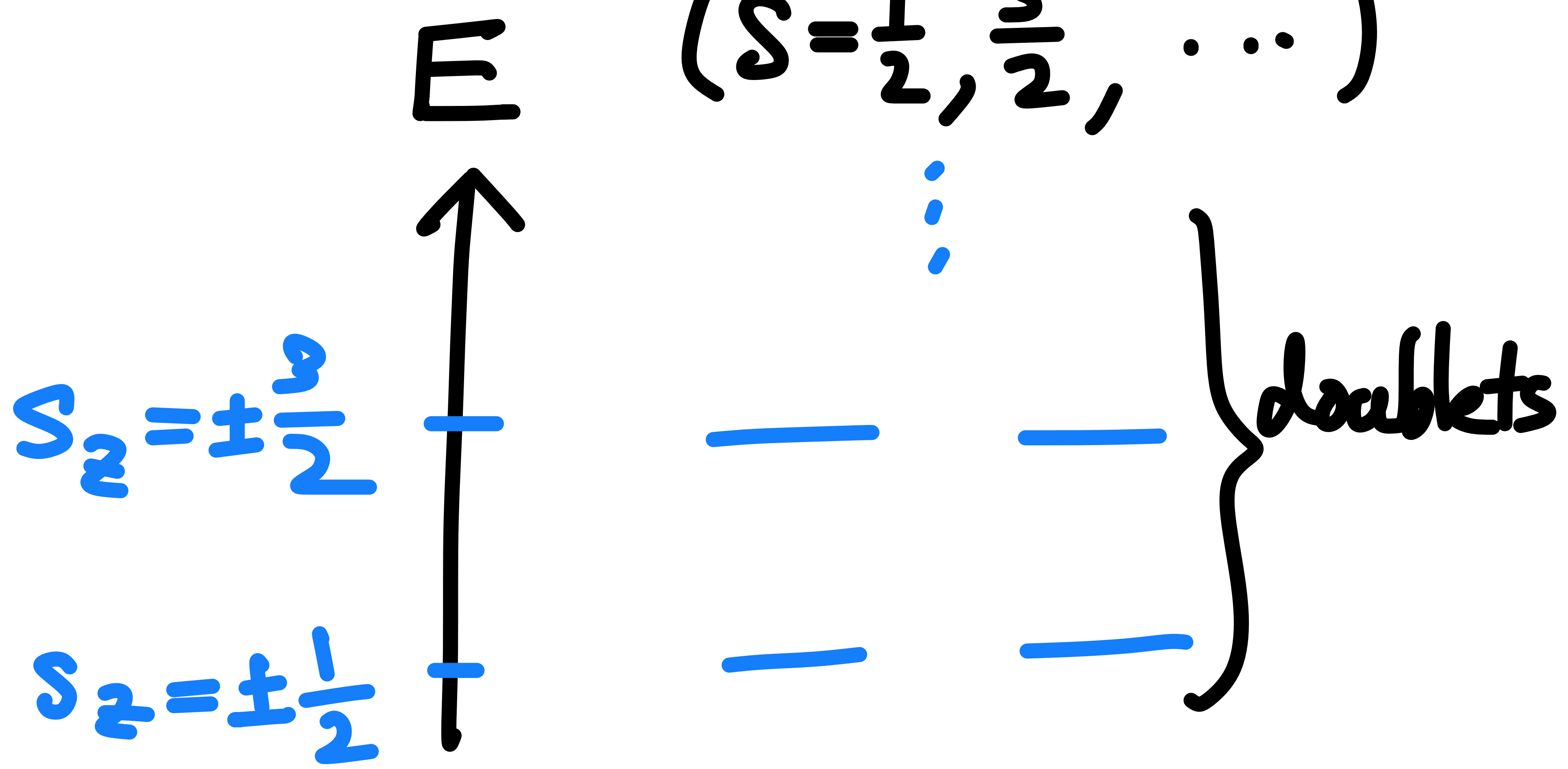
Pay attention to symmetry!

Anomaly of symmetry sometimes tell nontrivial dynamics must occur.

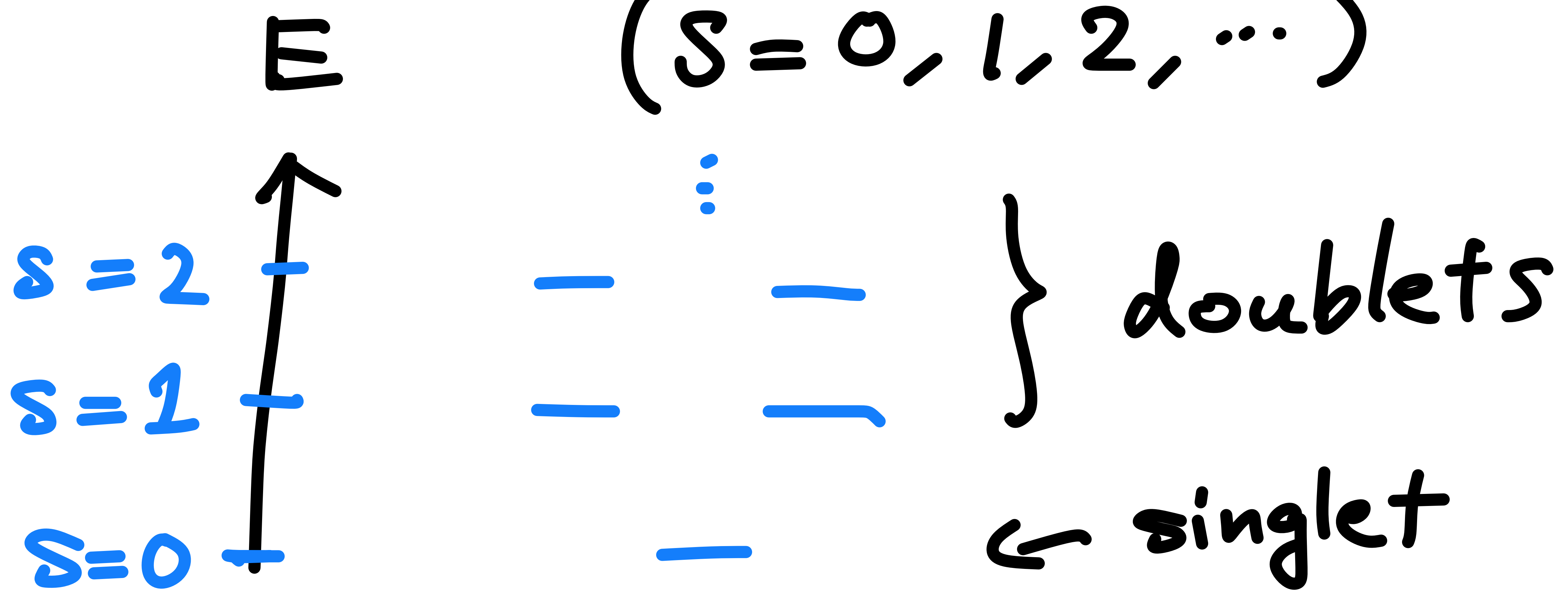
QM of a single spin / QM on a circle S^1

Hamiltonian $\quad \hat{H} = J\,\hat{S}_z^2 \qquad (\hat{S}_z = -S, -S+1, \cdots, S).$

$(S = \tfrac{1}{2}, \tfrac{3}{2}, \cdots)$



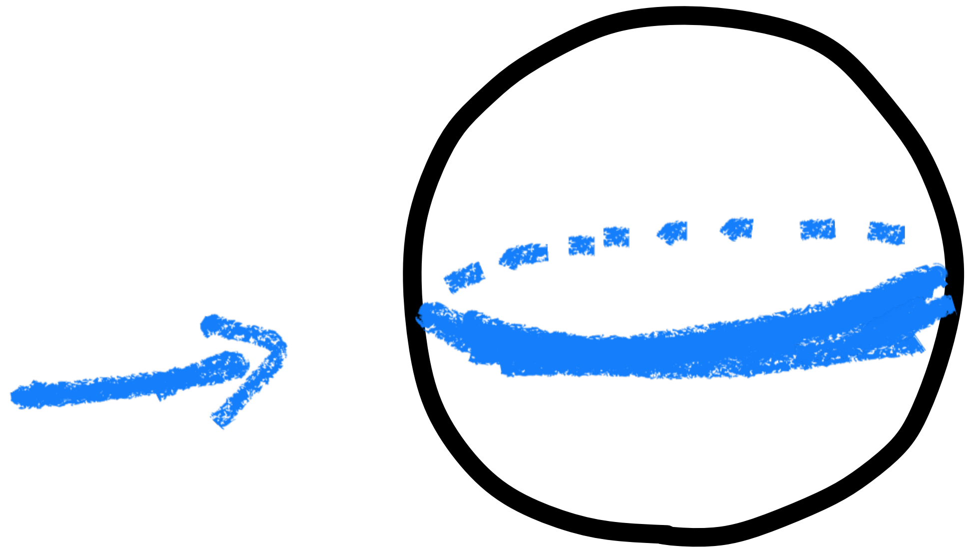
$(S = 0, 1, 2, \cdots)$



In the limit $J \to +\infty$, it becomes QM on a circle.

(Classical) G.S.

$\approx$ Equator



$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\int d\tau \left(\frac{\partial \phi}{\partial \tau}\right)^2 + \cdots .$$

$$(\phi \sim \phi + 2\pi)$$

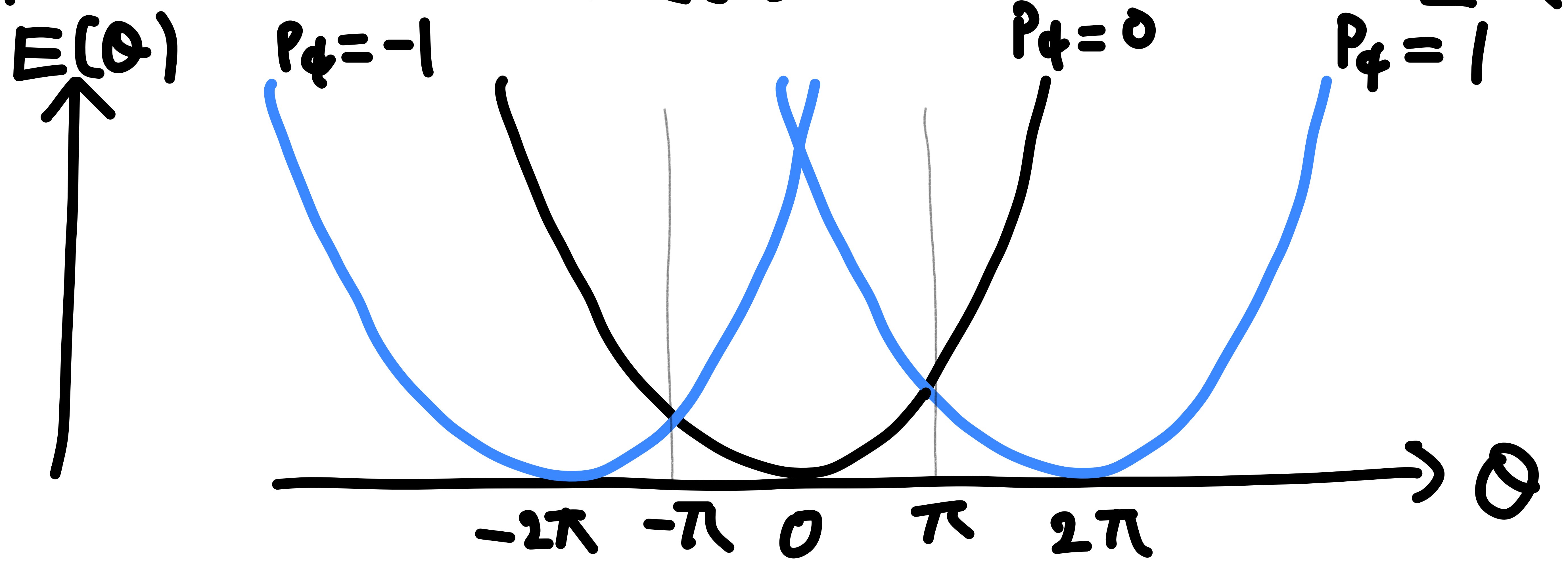
How does low-energy EFT $\mathcal{L}_{eff}$ caputure $S=\frac{1}{2},\frac{3}{2},\cdots$ or $S=1,2,\cdots$?

$\Rightarrow$ θ-angle : $\theta = 2\pi S$

$$\mathcal{L}_{eff} = \int d\tau \, \frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 + \underbrace{\frac{i\theta}{2\pi}\int d\tau \left(\frac{\partial\phi}{\partial\tau}\right)}_{\theta \times \text{winding } \#} .$$

As $\phi \sim \phi + 2\pi$, $\frac{1}{2\pi}\int d\phi \in \mathbb{Z}$. So, $\theta \sim \theta + 2\pi$.

Hamiltonian for $\mathcal{L}_{eff}$: $H_{eff} = \frac{1}{2}\left(\hat{P}_\phi - \frac{\theta}{2\pi}\right)^2$.



Spectral (non)degeneracy from symmetry

- $U(1)$ sym $\hat{U}_\alpha = \exp(i\alpha \hat{p}_\phi) \Rightarrow \hat{p}_\phi = n$ is a good quantum #.

- $\mathbb{Z}_2$ symmetry $\phi \mapsto -\phi$.

$S = 1, 2, \cdots \Leftrightarrow \theta = 0$ i.e. $\hat{H}_{eff} = \frac{1}{2}\hat{p}_\phi^2$.

$\hat{H}_{eff}$ is invariant under $\hat{p}_\phi \mapsto -\hat{p}_\phi$.

$\rightsquigarrow$ $\hat{p}_\phi = 0$ is invariant under both sym.

$S = \frac{1}{2}, \frac{3}{2}, \cdots \Leftrightarrow \theta = \pi$ i.e. $\hat{H}_{eff} = \frac{1}{2}\left(\hat{p}_\phi - \frac{1}{2}\right)^2$.

$\hat{H}_{eff}$ is inv. under $\hat{p}_\phi \mapsto -\hat{p}_\phi + 1$.

$\rightsquigarrow$ No simultaneous eigenstate. Doublet spectrum

Reinterpretation as an Anomaly.

Let us introduce $U(1)$ gauge field A : $d\phi \Rightarrow d\phi + A$.

$$Z_\theta[A] = \int \mathcal{D}\phi \exp\left(-\frac{1}{2}\int |d\phi+A|^2 + i\frac{\theta}{2\pi}\int (d\phi+A)\right).$$

Perform $\mathbb{Z}_2$ transformation : $\phi \mapsto -\phi$, $A \mapsto -A$.

$$\left[\begin{array}{l}\text{At } \theta=0 \text{ (i.e. } S=1,2,\cdots), \\ \quad Z_0[A] \longrightarrow \int \mathcal{D}\phi\, e^{-\frac{1}{2}\int |-d\phi-A|^2} = Z_0[A].\end{array}\right]$$

At $\theta=\pi$ (i.e. $S=\frac{1}{2},\frac{3}{2},\cdots$)

$$\text{“} \frac{\pi}{2\pi}\int (d\phi+A) - \int (\cancel{d\phi}+A)$$

$$Z_\pi[A] = \int \mathcal{D}\phi\, e^{-\frac{1}{2}\int |d\phi+A|^2 + i\boxed{\frac{\pi}{2\pi}\int(-d\phi-A)}} = \underbrace{e^{-i\int A}}_{\text{Anomaly}} \cdot Z_\pi[A]$$

Summary for QM of a spin $\hat{H} = J\hat{S}_z^2$.

- $S = 1, 2, 3, \cdots$

  Symmetry $\underset{A}{\underset{\wr}{U(1)}}$ and $\mathbb{Z}_2$. $Z[A] \xrightarrow{\mathbb{Z}_2} Z[A]$.

  No anomaly for these sym $\Rightarrow$ Singlet state exists.

- $S = \frac{1}{2}, \frac{3}{2}, \cdots$

  Same sym $U(1)$ and $\mathbb{Z}_2$, but $Z[A] \xrightarrow{\mathbb{Z}_2} e^{-i\int A} Z[A]$.

  Gauging $U(1)$ breaks $\mathbb{Z}_2 \Rightarrow$ Anomaly

  $\Rightarrow$ All spectra are doublets.

't Hooft anomaly matching (in a generalized form)

$\boxed{\text{'t Hooft anomaly}}$

Assume d-dim. QFT has a symmetry G.

$$\begin{cases} A : G\text{-gauge field} \\ A \to A + \delta_\lambda A \ : \ G\text{-gauge transformation.} \end{cases}$$

Compute the partition function with this background, $Z[A]$.

$$Z[A + \delta_\lambda A] = \exp\Big(i \int \underline{\mathcal{A}(\lambda, A)}\Big) \cdot Z[A]$$

└ d-dim. local functional of A, λ.

$\mathcal{A}$ is called an 't Hooft anomaly if $\mathcal{A} \neq \delta_\lambda(d\text{-dim. func. of } A)$.

$\boxed{\text{Anomaly matching}}$

't Hooft anomaly is RG-invariant.

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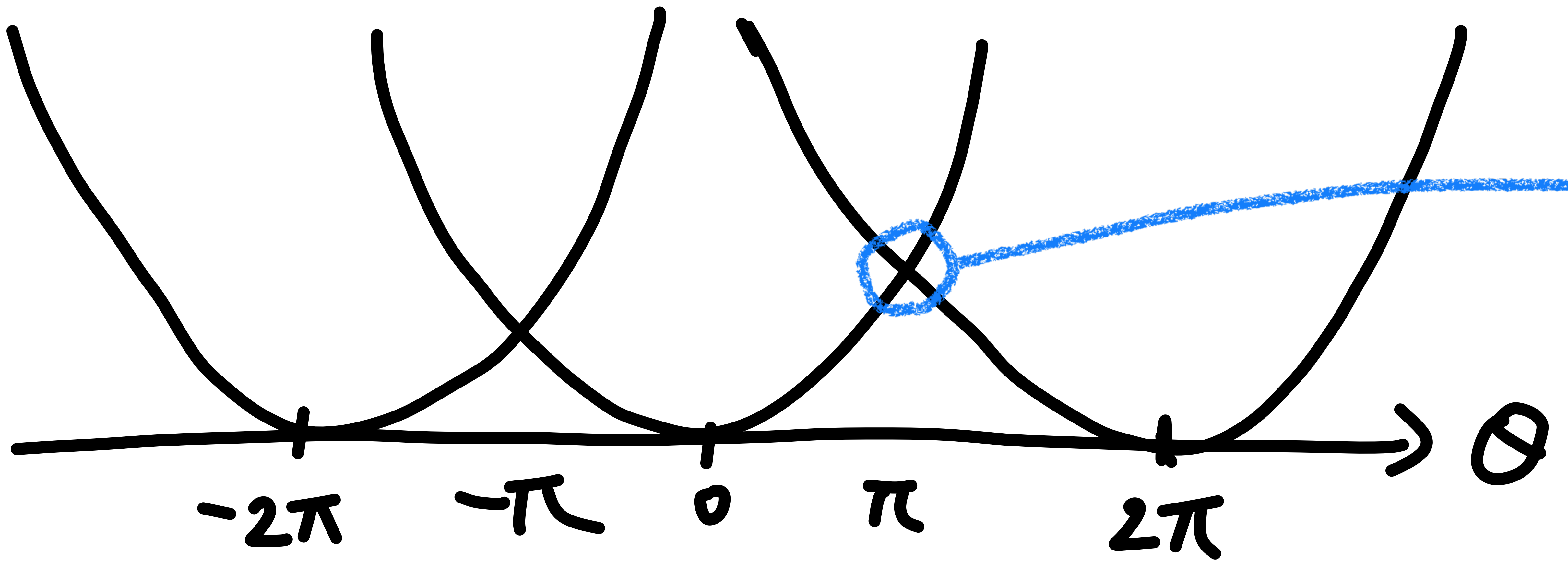
SU(N) Yang-Mills theory

$$S = \frac{1}{g^2}\int |F|^2 + i\underbrace{\frac{\theta}{8\pi^2}\int \mathrm{tr}(F\wedge F)}_{\theta \times \text{instanton \#}} \rightsquigarrow \theta \sim \theta + 2\pi$$

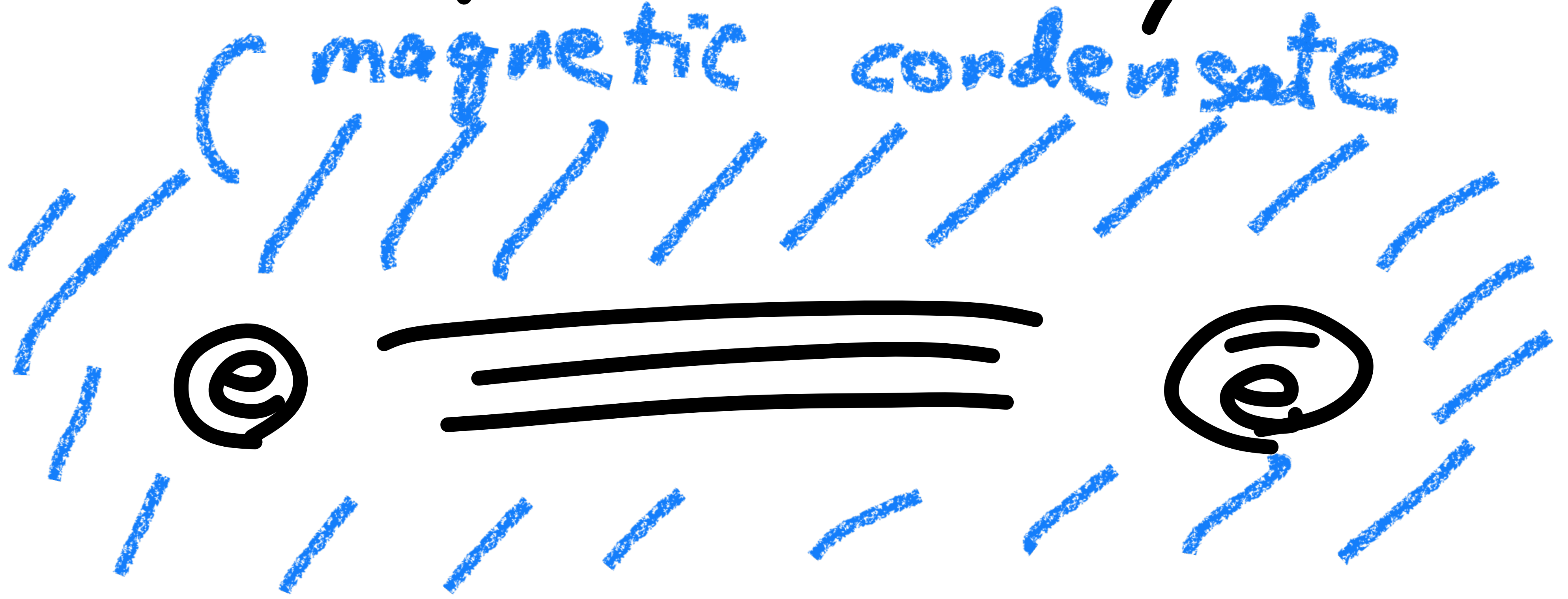
Yang-Mills vacua have interesting response for θ



spontaneous CP breaking

(Large-N : Witten '80, '98
Chiral model: Dashen '71, Di Vecchia, Veneziano '80
...)

Intuitive explanation via dual superconductivity.



(At least in Abelianized regime)
there are monopole/dyon
in YM theory

$$(\vec{e}, \vec{m}) = (n\vec{\alpha}_i, \vec{\alpha}_i).$$

$$\text{Coulomb energy} \sim g^2 e^2 + \frac{1}{g^2} m^2.$$

With the θ-angle, Witten effect tells

$$\text{Coulomb energy} \sim g^2\left(e + \frac{\theta}{2\pi} m\right)^2 + \frac{1}{g^2} m^2$$
$$\sim g^2\left(n + \frac{\theta}{2\pi}\right)^2$$

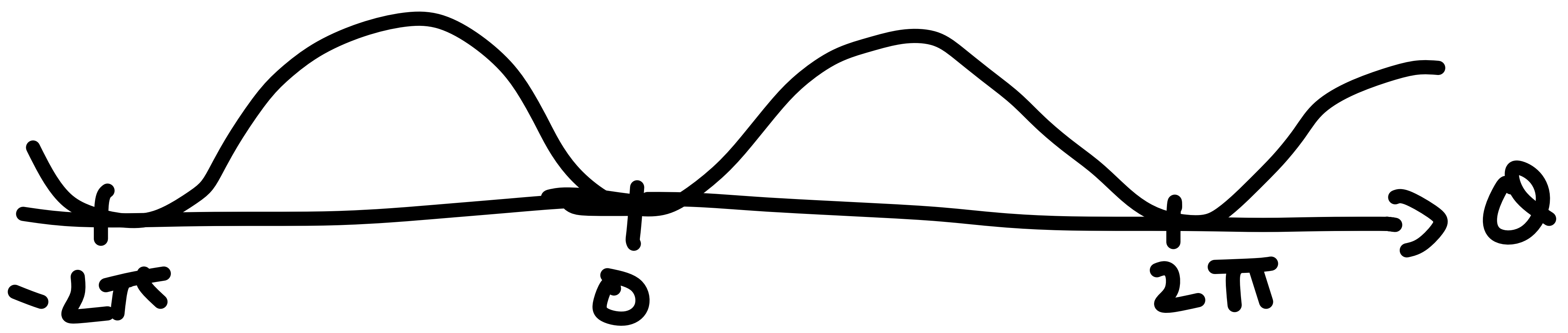
$-\pi < \theta < \pi \Rightarrow$ Monopole (i.e. $n=0$) is preferred as condensate.
$\pi < \theta < 3\pi \Rightarrow$ Dyon ($n=1$) is preferred. ('t Hooft '81, Cardy, Rabinovici '82 ...)

I like this intuition, but it is also puzzling:

No order parameter distinguishes monopole/dyon condensates.

$\rightsquigarrow$ Landau's classification does not require CP-breaking at $\theta = \pi$.

what's wrong with



?

- This is inconsistent with 't Hooft anomaly involving the center symmetry. (Gaiotto, Kapustin, Komargodski, Seiberg '17)

Center symmetry

Entering the graduate school, learning non-Abelian YM theories, we are told about a mysterious sym., center sym.

Standard story :

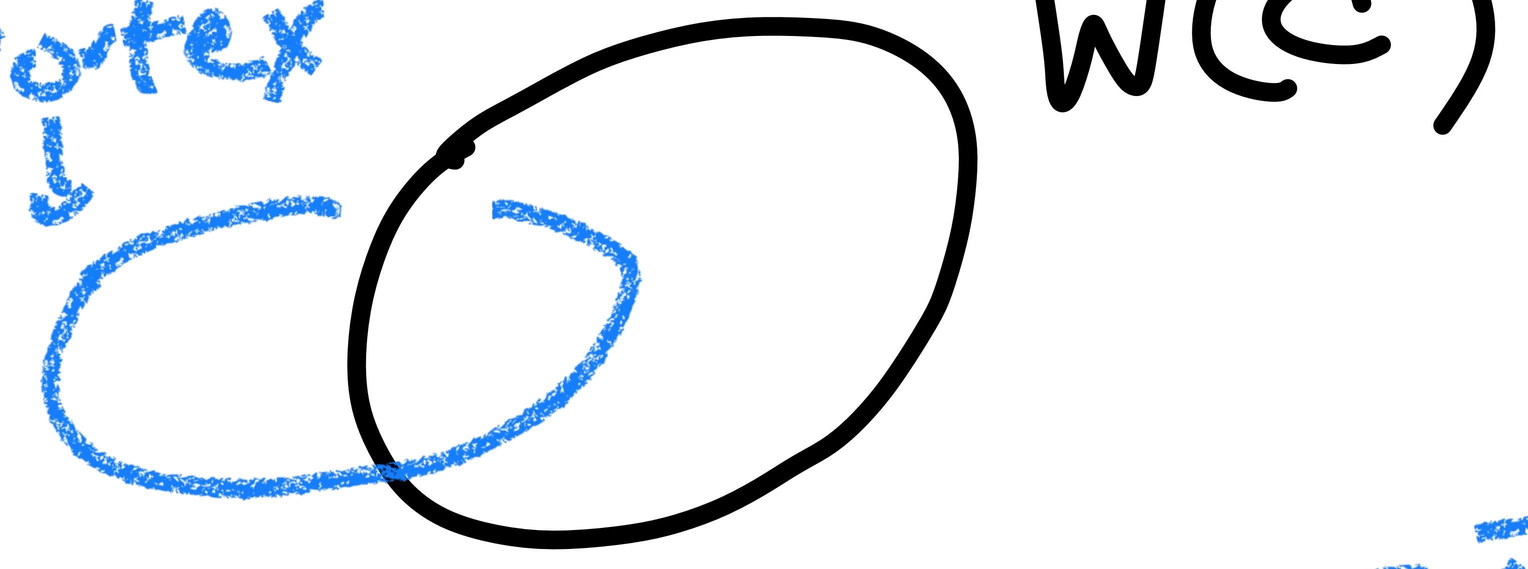
- YM does not have global symmetry. (except Poincaré & charge-conj.)
- But, Confinement / Higgs phases are separated.
- Once you compactify on a torus, $\mathbb{Z}_N$ sym appears.
  Polyakov loop $P \to e^{\frac{2\pi i}{N}} P$.

$\mathbb{Z}_N$ 1-form symmetry

1-form symmetry provides a systematic tool to formulate center symmetry on a general 4-dim. spacetime.

(Gaiotto, Kapustin, Seiberg, Willet '14)

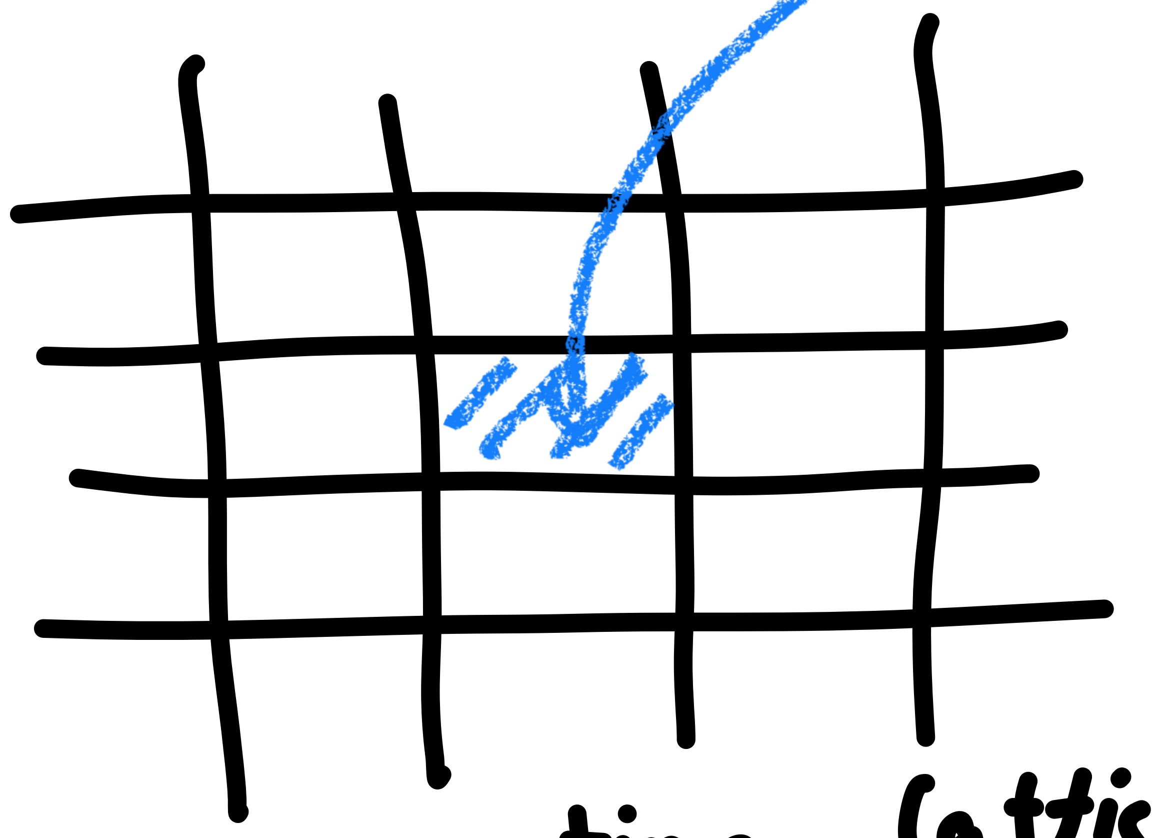
(Roughly) $\mathbb{Z}_N$ center vortex



$$W(C) = e^{\frac{2\pi i}{N}} W(C).$$

$\mathbb{Z}_N$-twist on a plaquette. (= Gukov-Witten surface operator)

$\rightsquigarrow$

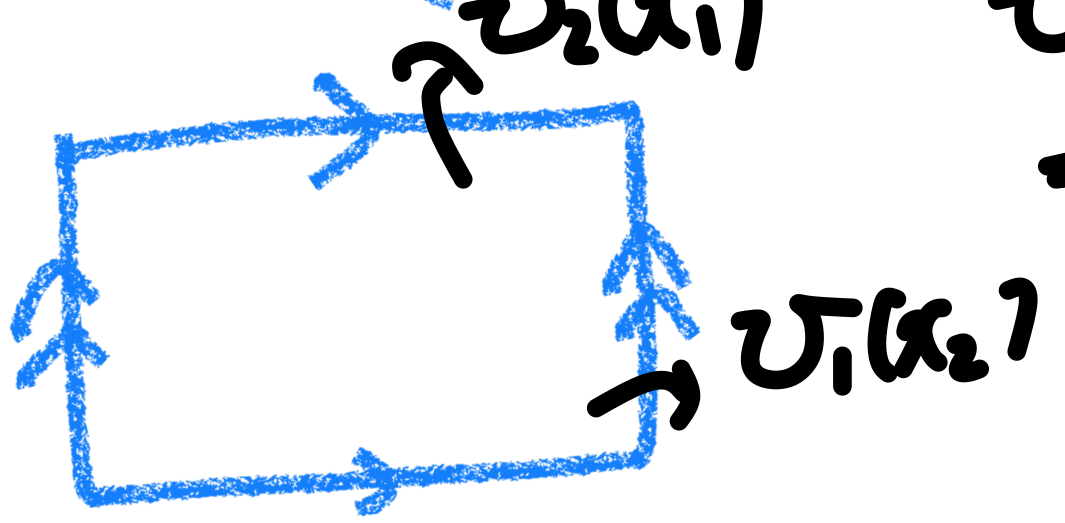


spacetime lattice

- Location of the twist can be changed freely $\Leftrightarrow$ Conservation law.
- If the move crosses $W(C)$, $\mathbb{Z}_N$ phase appears.

Using this (abstract/fancy) terminologies,

we can make the precise meaning of 't Hooft twisted b.c.



$$U_1(L)\,U_2(0) = U_2(L)\,U_1(0) \times e^{\frac{2\pi i}{N} n_{12}}.$$

Gauging of $\mathbb{Z}_N$ 1-form sym

$\Rightarrow$ $\mathbb{Z}_N$ 2-form gauge field B

$$\int_{(T^2)_{1,2}} B = 2\pi\, n_{12} \quad \text{is the 't Hooft twist.}$$

Anomaly involving $\mathbb{Z}_N$ 1-form sym. at $\theta=\pi$.

Introducing 't Hooft twist.

$$Q_{top} = \underbrace{-\frac{1}{N}\times\frac{n_{ij}\tilde{n}_{ij}}{4}}_{-\frac{N}{4\pi}\int B\wedge B \text{ in the current terminology.}} + \text{integer.} \qquad \text{(van Baal '82)}$$

Using this,

$$Z_{\theta=0}[B] \xrightarrow{CP} Z_{\theta=0}[B]$$

but

$$Z_{\theta=\pi}[B] \xrightarrow{CP} \underbrace{e^{-i\frac{N}{4\pi}\int B^2}}_{\text{Anomaly.}} Z_{\theta=\pi}[B].$$

What do we get?

Monopole/dyon condensing vacua cannot be distinguished by the Landau's order parameter.

However

monopole vacuum

$$Z_{\theta=0}[B] = 1\times |Z_{\theta=0}[B]|$$

different phase factors with B-field.

$$Z_{\theta=2\pi}[B] = e^{i\frac{N}{4\pi}\int B\wedge B}\,|Z_{\theta=0}[B]|.$$

dyon vacuum

These two states are different as Symmetry-Protected Topological phase with $\mathbb{Z}_N$ 1-form symmetry.

(Gaiotto, Kapustin, Komargodski, Seiberg '17, Tanizaki, Kikuchi, '17 ...)

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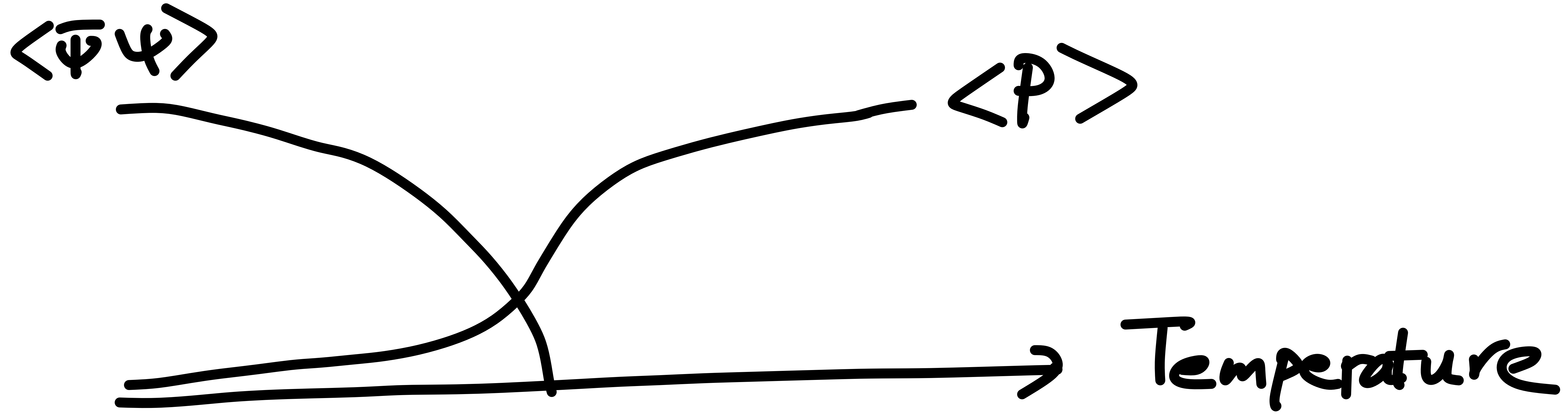
↳ I'm skipping many details and present results.

Please see the next talk by Takuya Furusawa.

Chiral symmetry vs. Confinement in QCD

QCD with fundamental quarks :



Chiral restoration occurs at the temperature, around which $\langle P \rangle$ rises up.

But, fund. quarks explicitly break center symmetry.

$\rightsquigarrow$ Can we make the precise connection ?

Two interesting setups

$\mathbb{Z}_N$-twisted QCD (Kouno et al. '12, Poppitz, Sulejmanpasic '13)

Take $N_c = N_f (= N)$ and the $SU(N)_F$-twisted b.c. for quarks:

$$q_f(\boldsymbol{x}, x_4 + L) = e^{\frac{2\pi i}{N} f} q_f(\boldsymbol{x}, x_4) \quad (f = 1, \cdots, N).$$

⤳ There is a center-like sym:

$$P \to e^{\frac{2\pi i}{N}} P, \qquad q_f \to q_{f+1}.$$

Large-N_c QCD (N_f: fixed)

String-breaking by dynamical quarks is $\frac{1}{N_c}$-suppressed.

⤳ $\mathbb{Z}_{N_c}$ center sym. is approximately good.

In these setups, we can show

unbroken center(-like) sym $\Rightarrow$ chiral symmetry breaking

(Tanizaki, (Kikuchi), Misumi, Sakai '17
Shimizu, Yonekura '17 ...)

Faithful symmetry of QCD

$$\frac{SU(N_f)_L \times SU(N_f)_R \times U(1)_V}{\mathbb{Z}_{N_f} \times \mathbb{Z}_{N_c}}$$

nontrivial quotient exists.

$\Rightarrow \left\{ \begin{matrix} B_f \\ B_c \end{matrix} \right\}$ can be introduced

$$Z[B_c, B_f] \xrightarrow[\text{axial rotation}]{\text{discrete}} \underbrace{e^{i\frac{N}{2\pi}\int B_c \wedge B_f}}_{\text{Anomaly}} Z[B_c, B_f].$$

Summary

- Symmetry and Anomaly provide us a useful guidline toward interesting nontrivial dynamics
- Of course, this is an "antique" technology, but many new results are still found there.