Towards complex Langevin simulation of color superconductivity

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Color superconductivity (CSC)

In extremely dense matter,

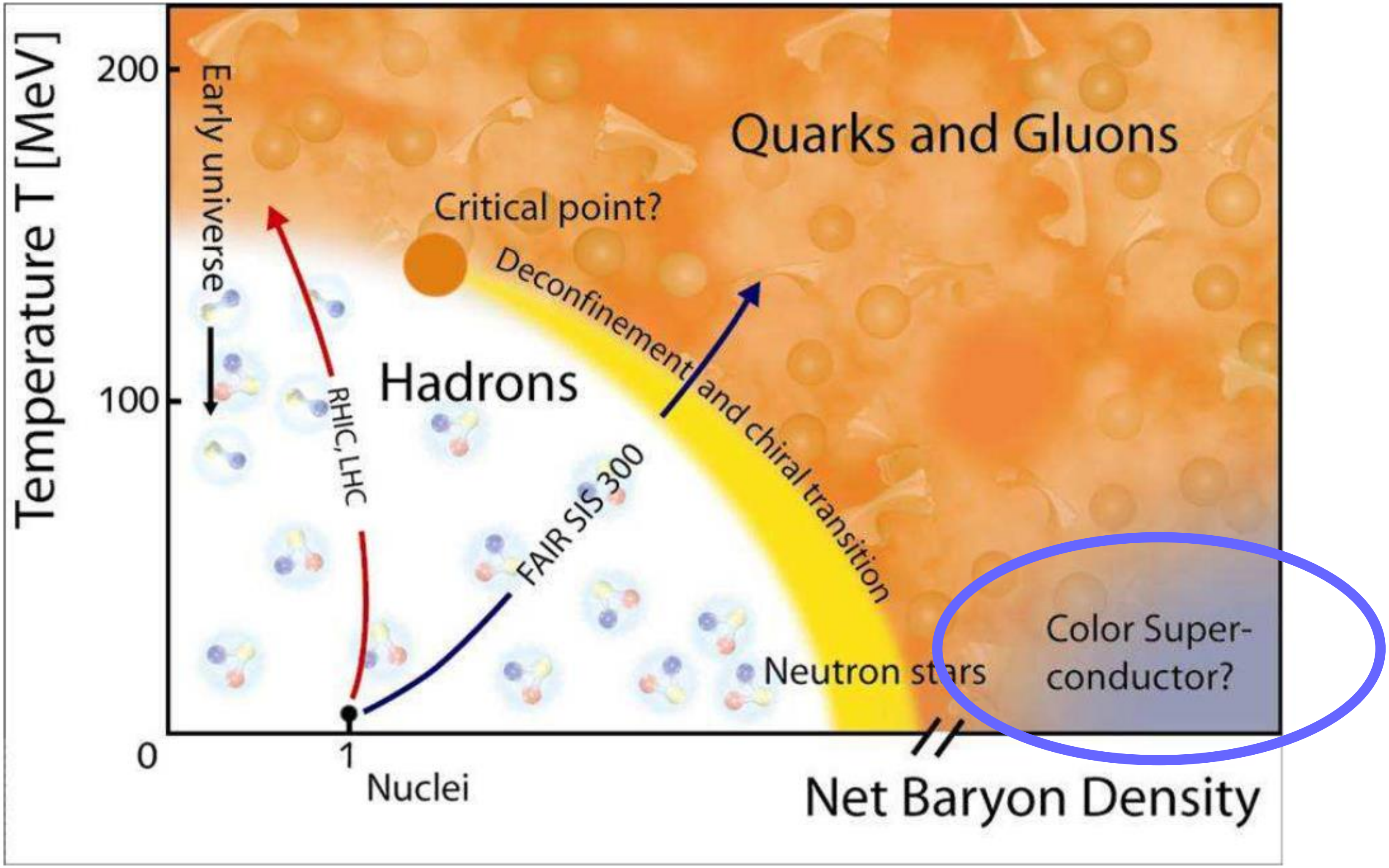
- there are weakly interacting quarks
- highly degenerate Fermi surface is formed at low temperature
- color-antitriplet channel is attractive

↓

Cooper instability

Barrois, NPB (1977), Frautschi (1978), Bailin, Love, Phys. Rept. (1984)
Alford, Rajagopal, Wilczek, PLB (1998), Rapp, Schäfer, Shuryak, Velkovsky, PRL (1998)

QCD at finite density



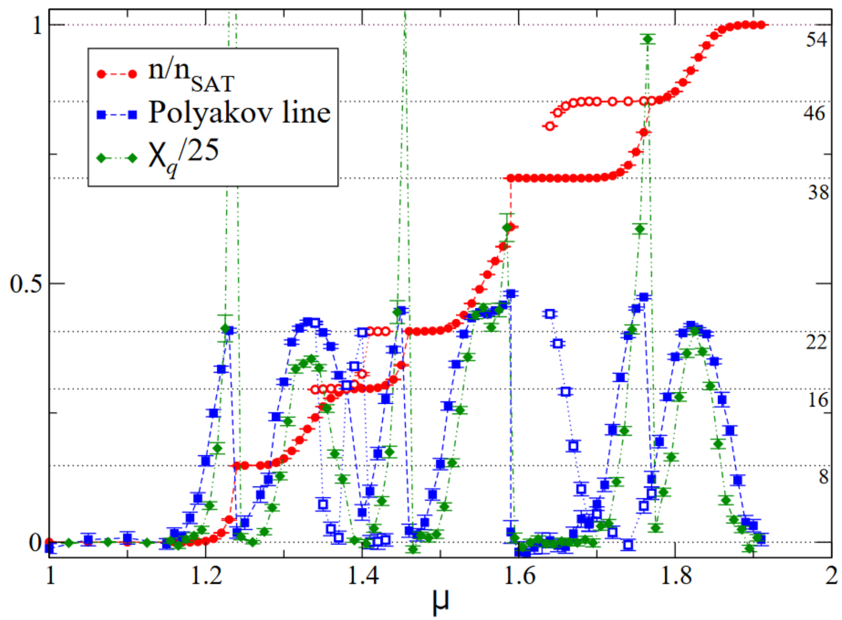
J. Phys.: Conf. Ser. **706** 022004

Dense quark matter in small space

NJL model: Hands, Walters, PLB (2002),
Amore, Birse, McGovern, Walet, PRD (2002)

QCD on $S^1 \times S^3$: Hands, Hollowood, Myers, JHEP (2002),

Two-color QCD: Hands, Walters, JHEP (2002),

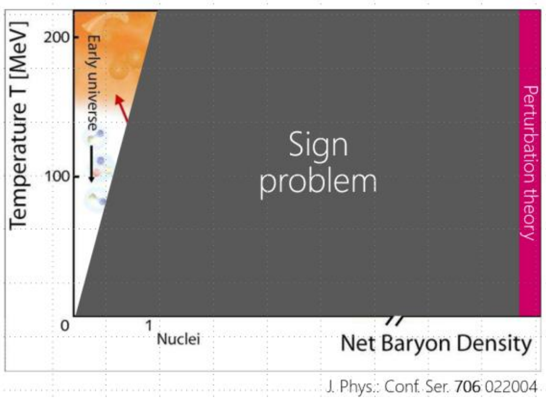


- Stepwise structure of quark number
- Spikes in Polyakov loop

→ What happens in SU(3) QCD?

Dense quark matter in small space

For $SU(3)_c$, first-principle analysis of dense quark matter is hindered by the sign problem.



Our strategy:

- Consider high-β region for the first study
  - high-β = weak coupling = small box
  - Predict phase structure by lattice perturbation theory
- Perform non-perturbative calculation based on complex Langevin method toward low-β region in the large box

Lattice perturbation theory

Our setup

We consider $N_f = 4$ staggered fermions on a lattice.

$$S = S_{\mathbf{fermi}} + S_{\mathbf{gluon}}$$

Fermion bilinear term:

$$S_{\mathrm{fermi}} = \sum_{N, N', \rho, \rho', a, a'} \bar{\chi}^{a}_{\rho}(N) D^{aa'}_{NN', \rho\rho'}(\mu, m) \chi^{a'}_{\rho'}(N')$$

a color

N coordinate

$\rho = (\rho_0, \rho_1, \rho_2, \rho_3), \quad \rho_\mu = 0, 1$

Our setup

We consider N_f = 4 staggered fermions on a lattice.

$$S = S_{\mathrm{fermi}} + S_{\mathrm{gluon}}$$

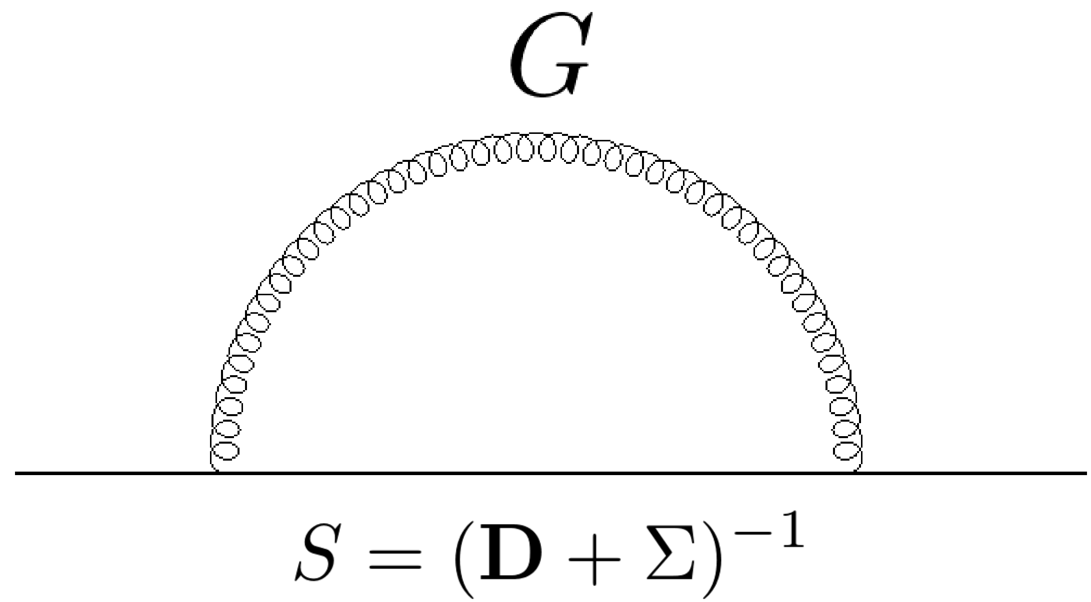
Fermion bilinear term (in Nambu-Gor'kov basis):

$$S_{\mathrm{fermi}} = \frac{1}{2} \sum_{N,N',\rho,\rho',a,a'} \bar{\Psi}^a_\rho(N) \mathbf{D}^{aa'}_{NN',\rho\rho'}(\mu, m) \Psi^{a'}_{\rho'}(N')$$

$$\Psi^a_\rho(N) = \begin{pmatrix} \chi^a_\rho(N) \\ \bar{\chi}^a_\rho(N) \end{pmatrix}$$

Gap equation

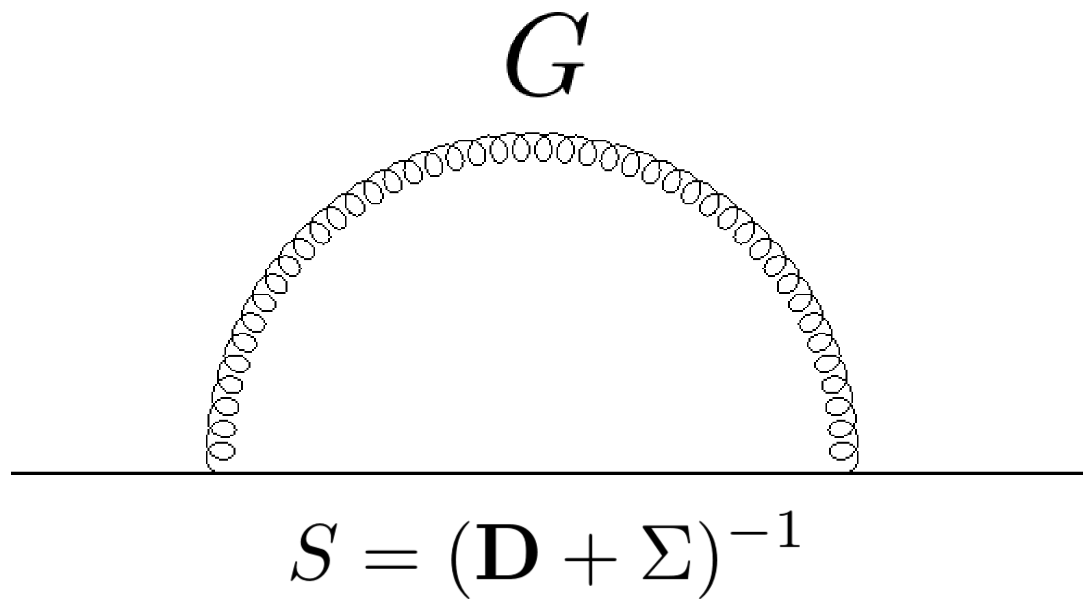
$$\Sigma =$$



Gap equation

Anomalous propagataor

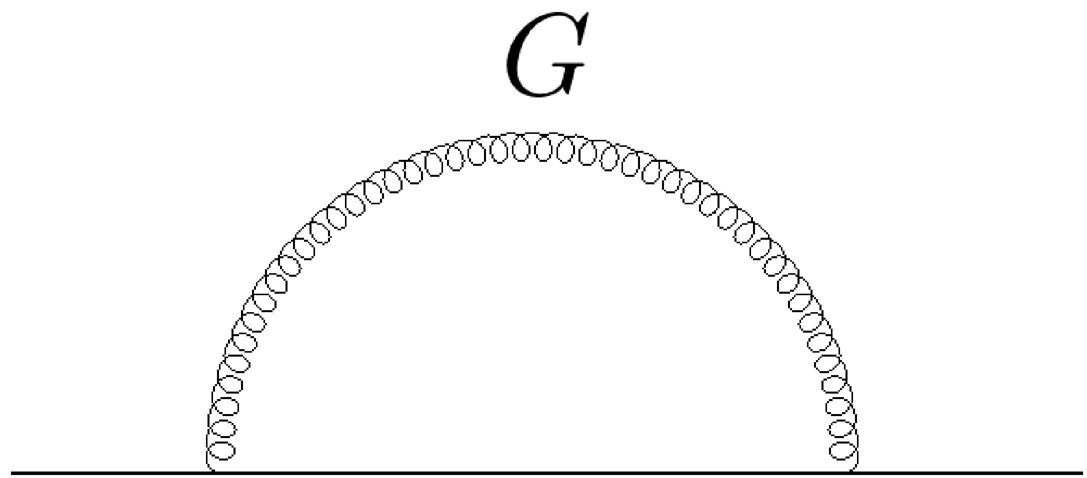
$$\begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} =$$



Criterion for CSC

Anomalous propagataor

$$\begin{pmatrix}\Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22}\end{pmatrix} =$$



$$S = (\mathbf{D} + \Sigma)^{-1}$$

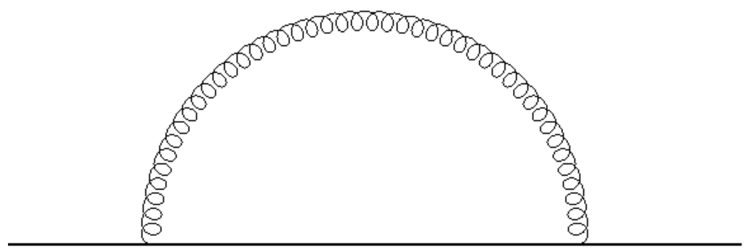
$$\Sigma_{12(21)} \neq 0 \;\Rightarrow\; \text{CSC}$$

Critical β

Calculate critical β at LO in the perturbation theory for a fixed m, μ.

- At LO, $\Sigma \simeq \begin{pmatrix} 0 & \Sigma_{12} \\ \Sigma_{21} & 0 \end{pmatrix}$
- At the transition point, $\Sigma_{12(21)} \ll 1$

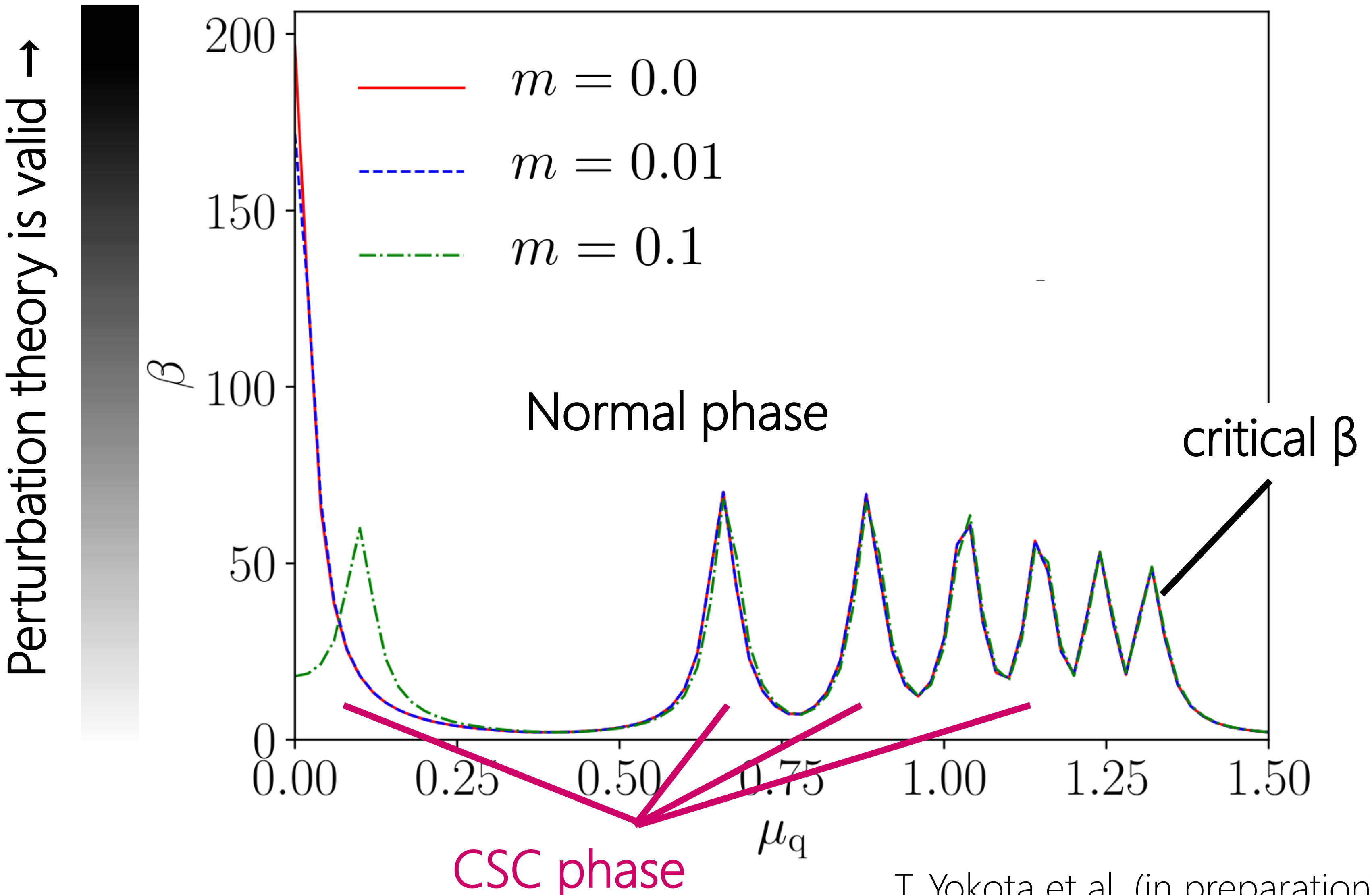
$$\Sigma_{12} \simeq \frac{1}{\beta}$$



$$S_{12} \simeq D_{11}^{-1} \Sigma_{12} D_{22}^{-1}$$

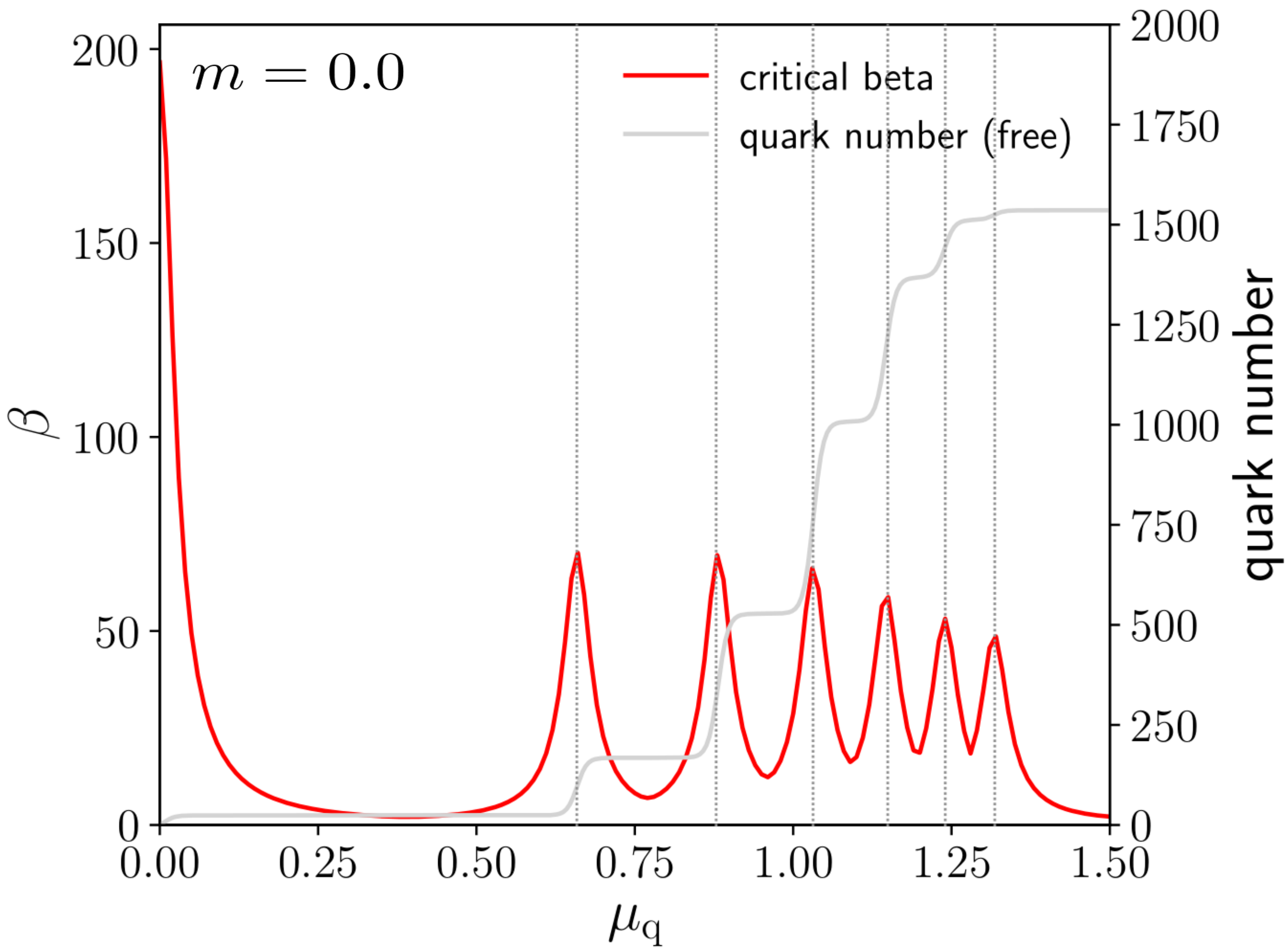
The gap eq. reduces to the linear eq. for Σ_{12}

Critical β on an $8^3 \times 128$ lattice



T. Yokota et al. (in preparation)

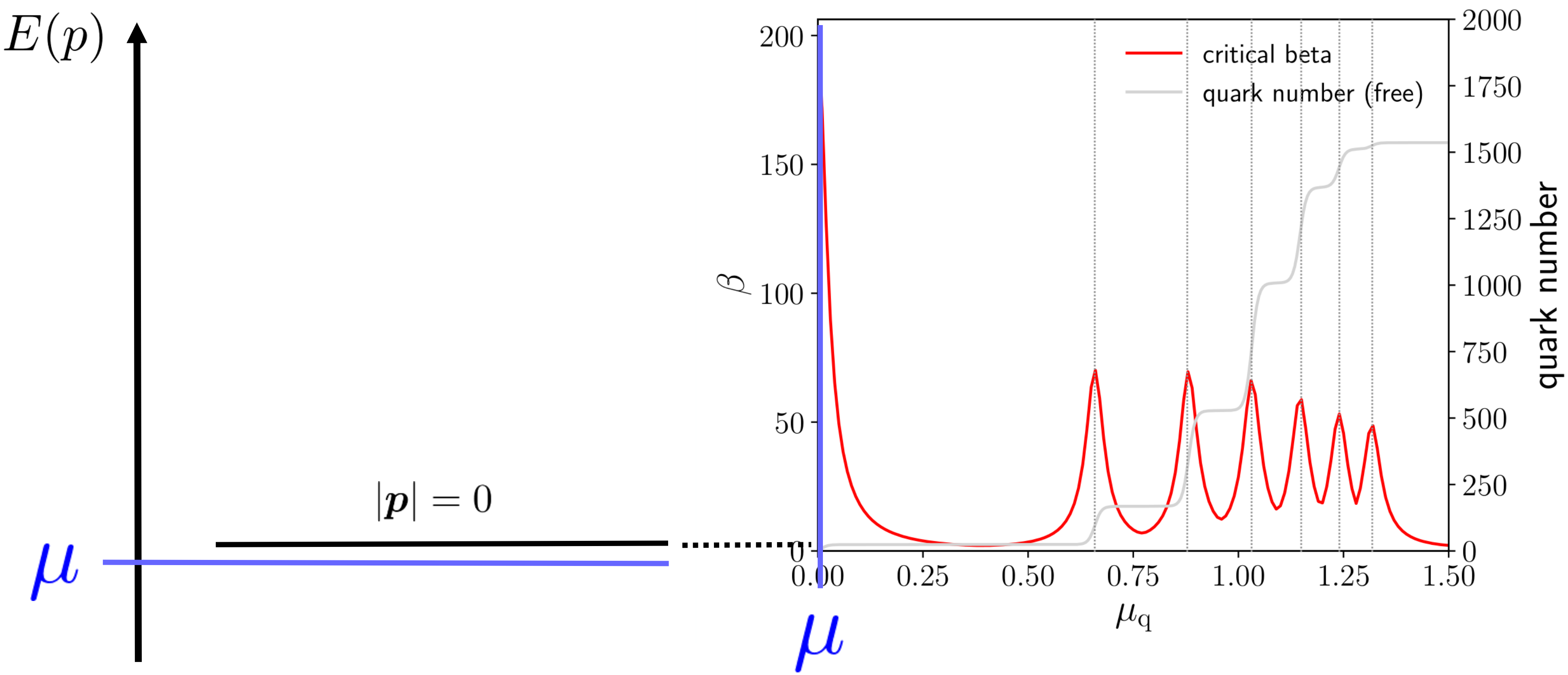
Peak structure



The peak structure corresponds to
the step structure in the number of *free* quarks

Peak structure

Due to the finite volume effect, energy levels of the quarks are discretized as:



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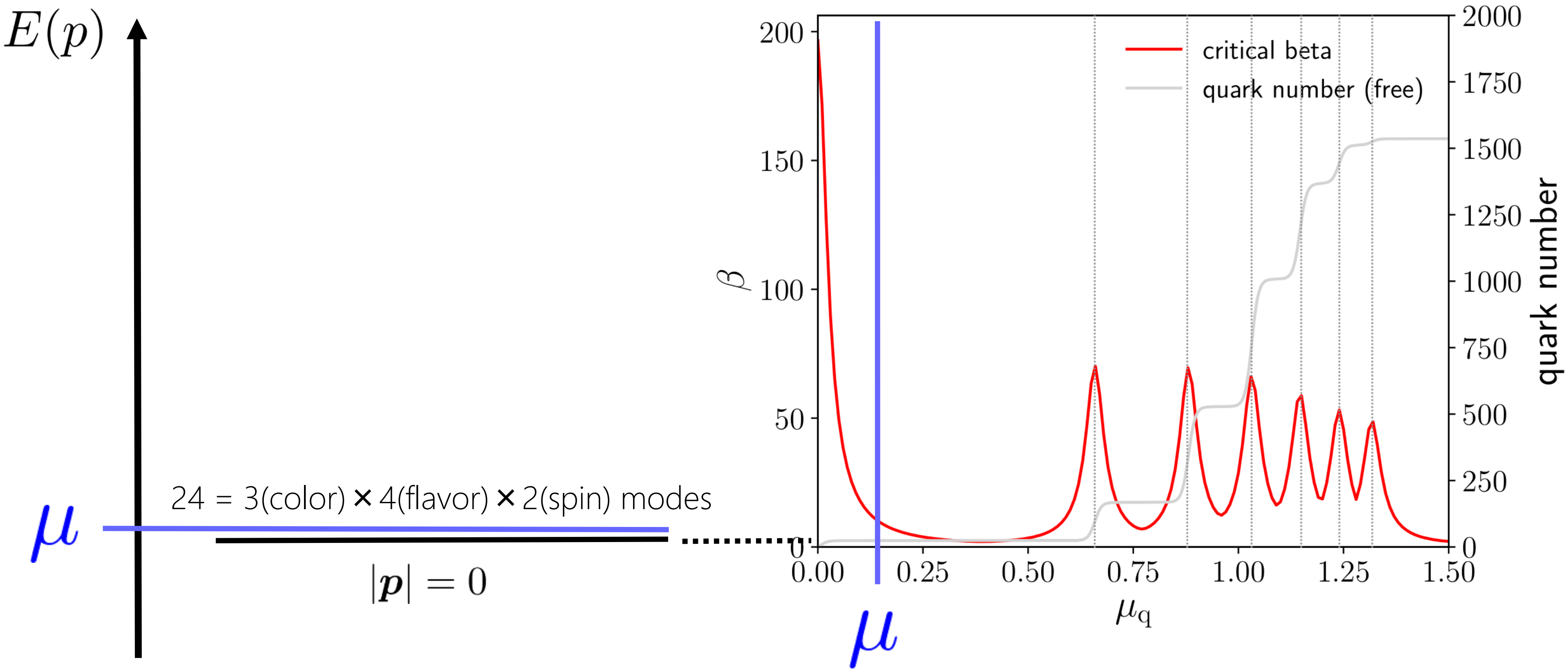

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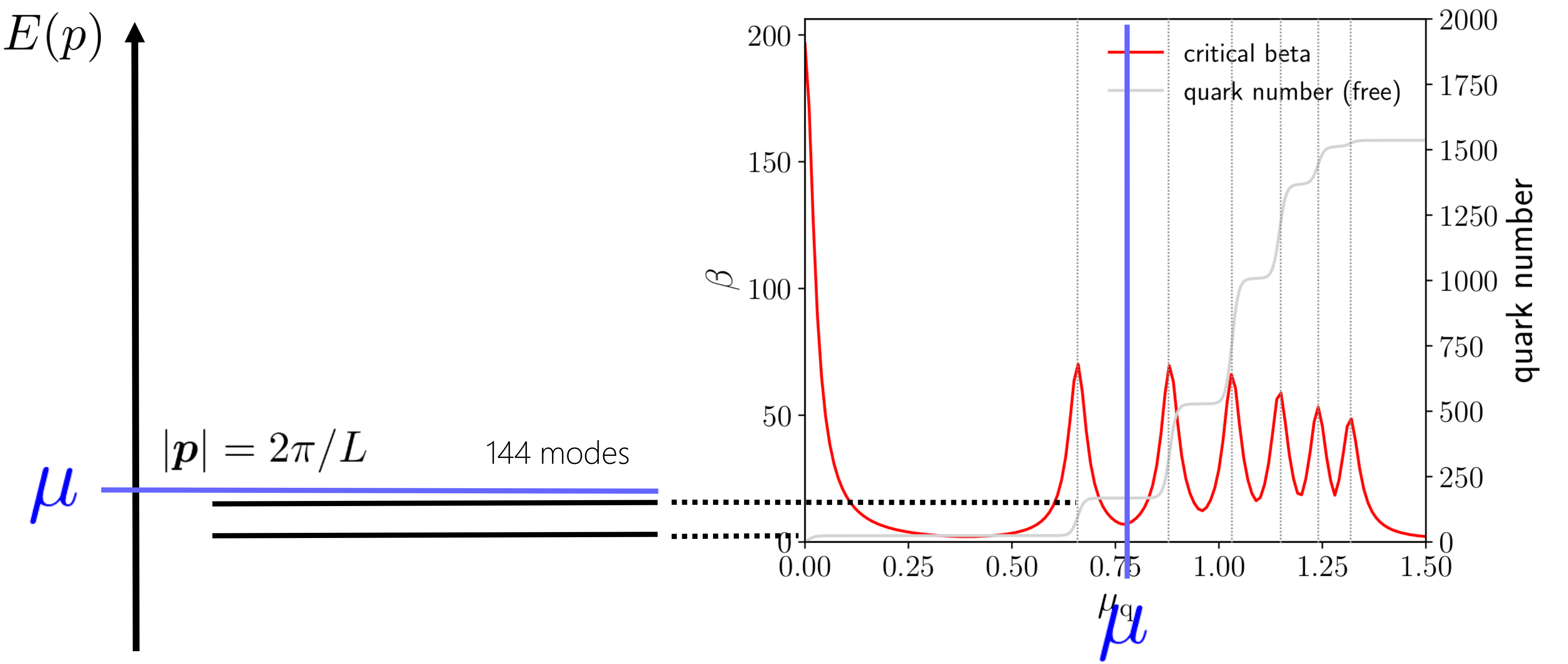

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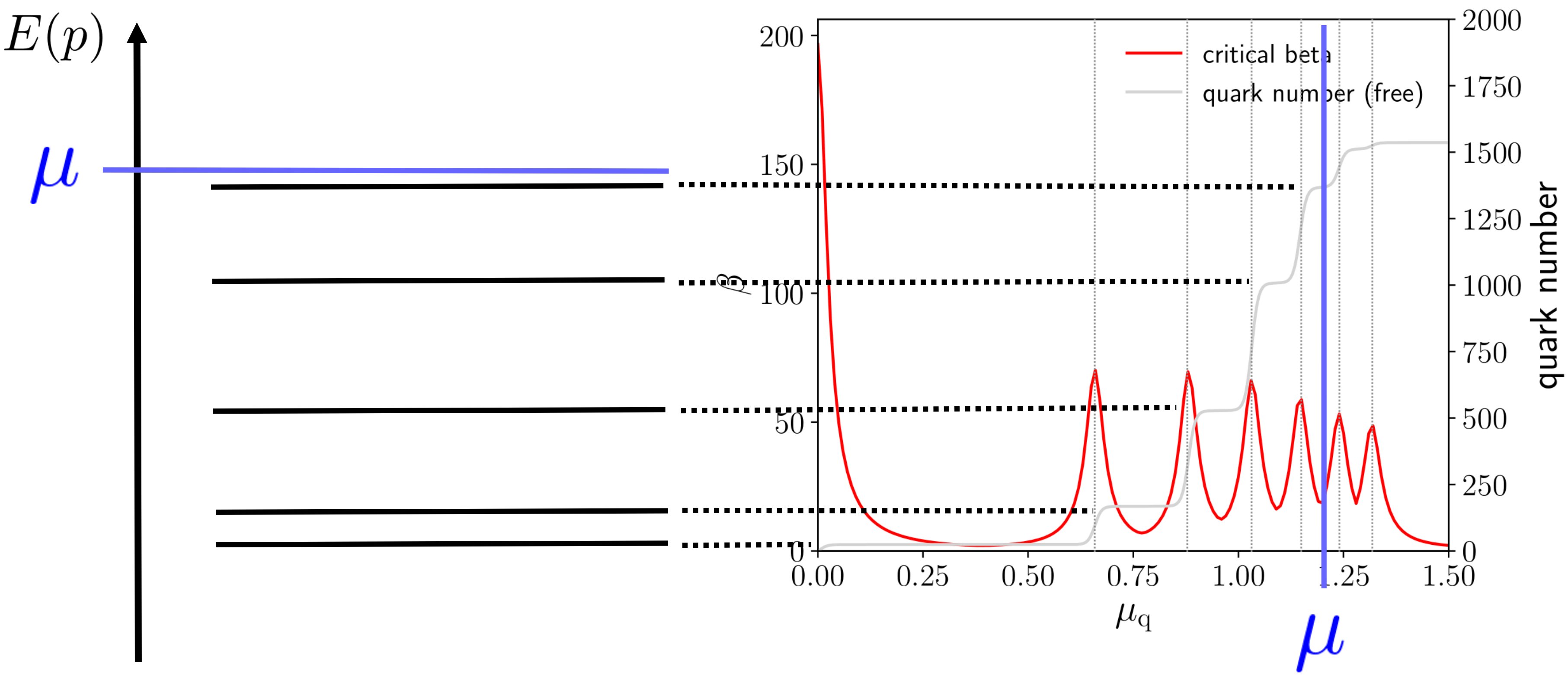
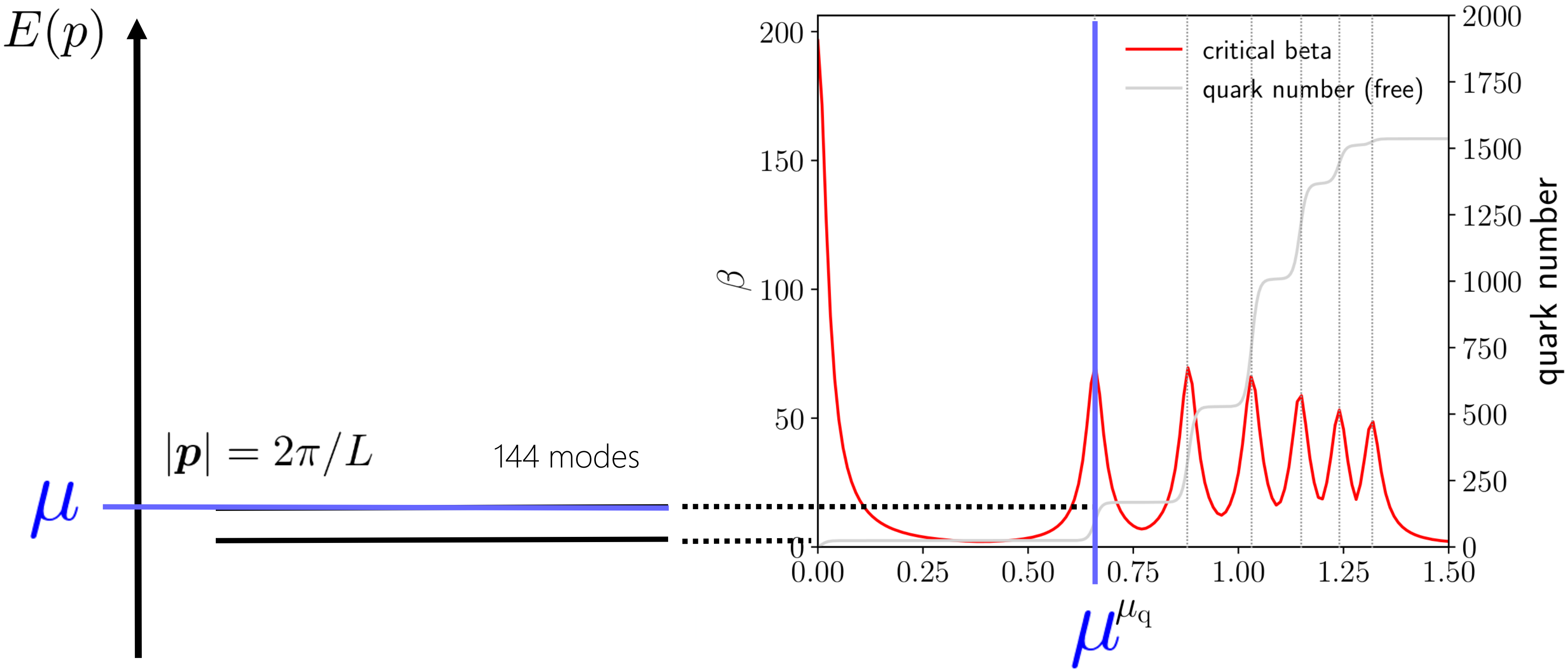


Peak structure

Peak positions correspond to μ at which the quark number changes.

→ At such μ, quarks exist on the Fermi surface.

→ Cooper pairs are formed.



Complex Langevin study

Langevin method (stochastic quantization)

$$\frac{d\phi}{dt} = -\frac{\partial S_{\text{eff}}(\phi)}{\partial \phi} + \eta$$

Reach equilibrium

$$P_{\text{eq}}(\phi) \propto e^{-S_{\text{eff}}(\phi)}$$

Quantum average is obtained by

$$\langle O(\phi) \rangle = \lim_{s \to \infty} \frac{1}{s} \int_{t_0}^{t_0+s} dt \, \langle O(\phi^{(\eta)}(t)) \rangle_{\eta}$$

Complex Langevin

Parisi, Phys. Lett. 131B (1983) 393,
Klauder PRA 29 (1984) 2036a

$$\frac{d\phi}{dt} = -\frac{\partial S_{\text{eff}}(\phi)}{\partial \phi} + \eta$$

⬇

$$P_{\text{eq}}(\phi_{\text{R}}, \phi_{\text{I}})$$

Joint distribution of real and imaginary parts

Criteria for correct convergence:

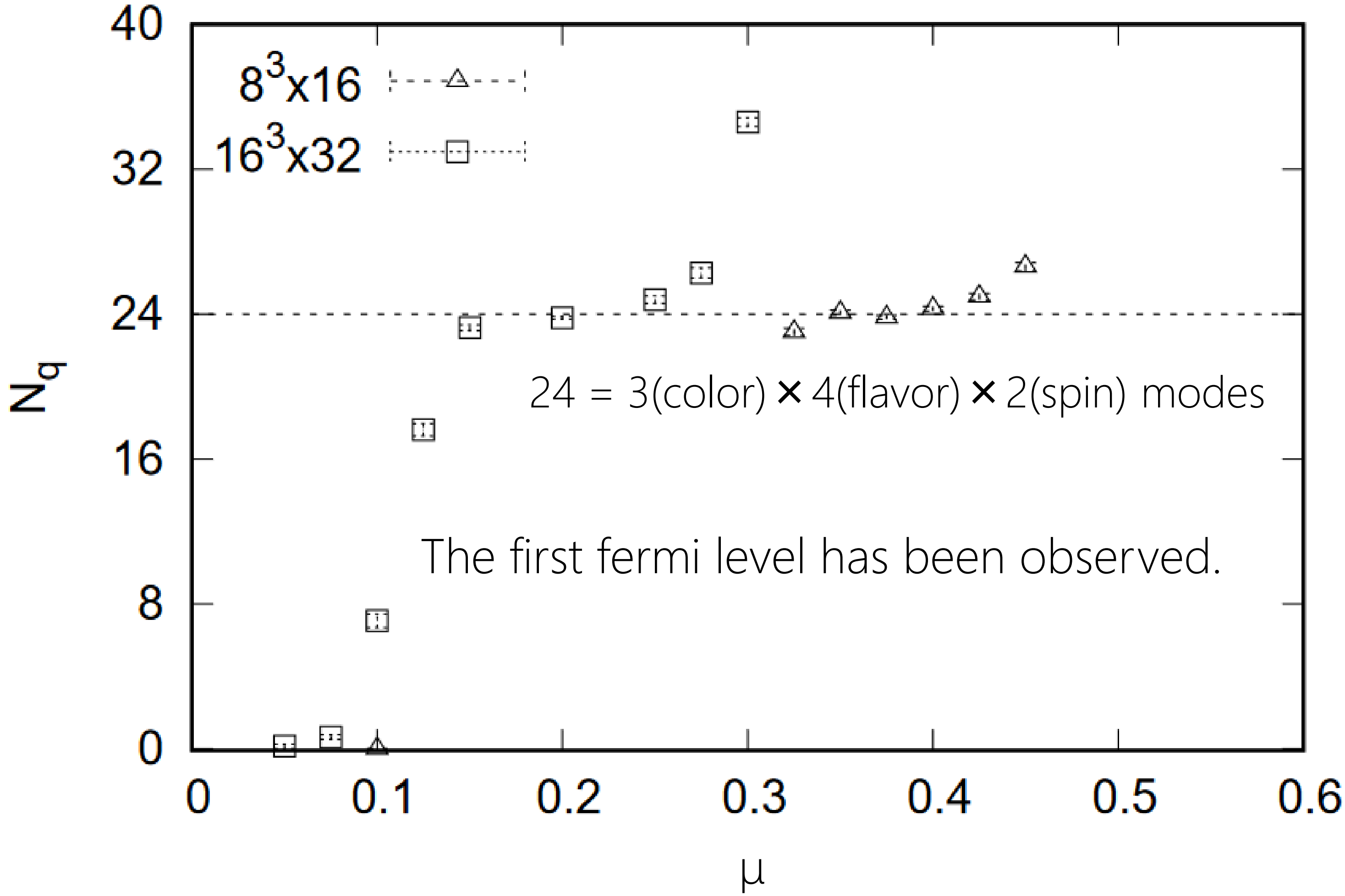
Aarts, Seiler, Stamatescu, PRD 81 (2010) 054608

Aarts, James, Seiler, Stamatescu, EPJ C71 (2011) 1756

Nagata, Nishimura, Shimasaki, PRD 92 (2015) 011501, PTEP 2016 013B01 → Probability distribution of the drift term should have exponential fall-off.

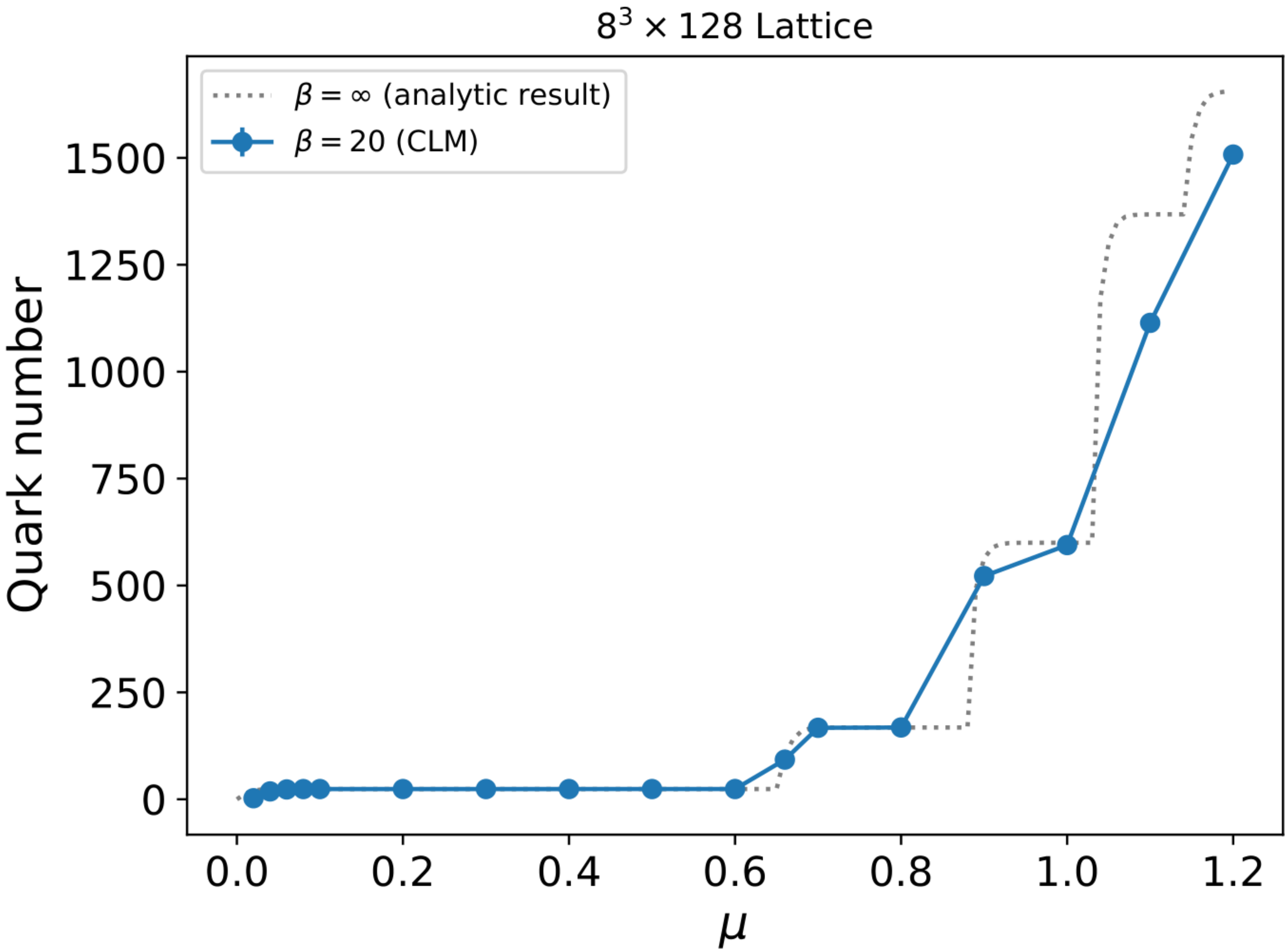
Our previous study

β=5.7, m=0.01

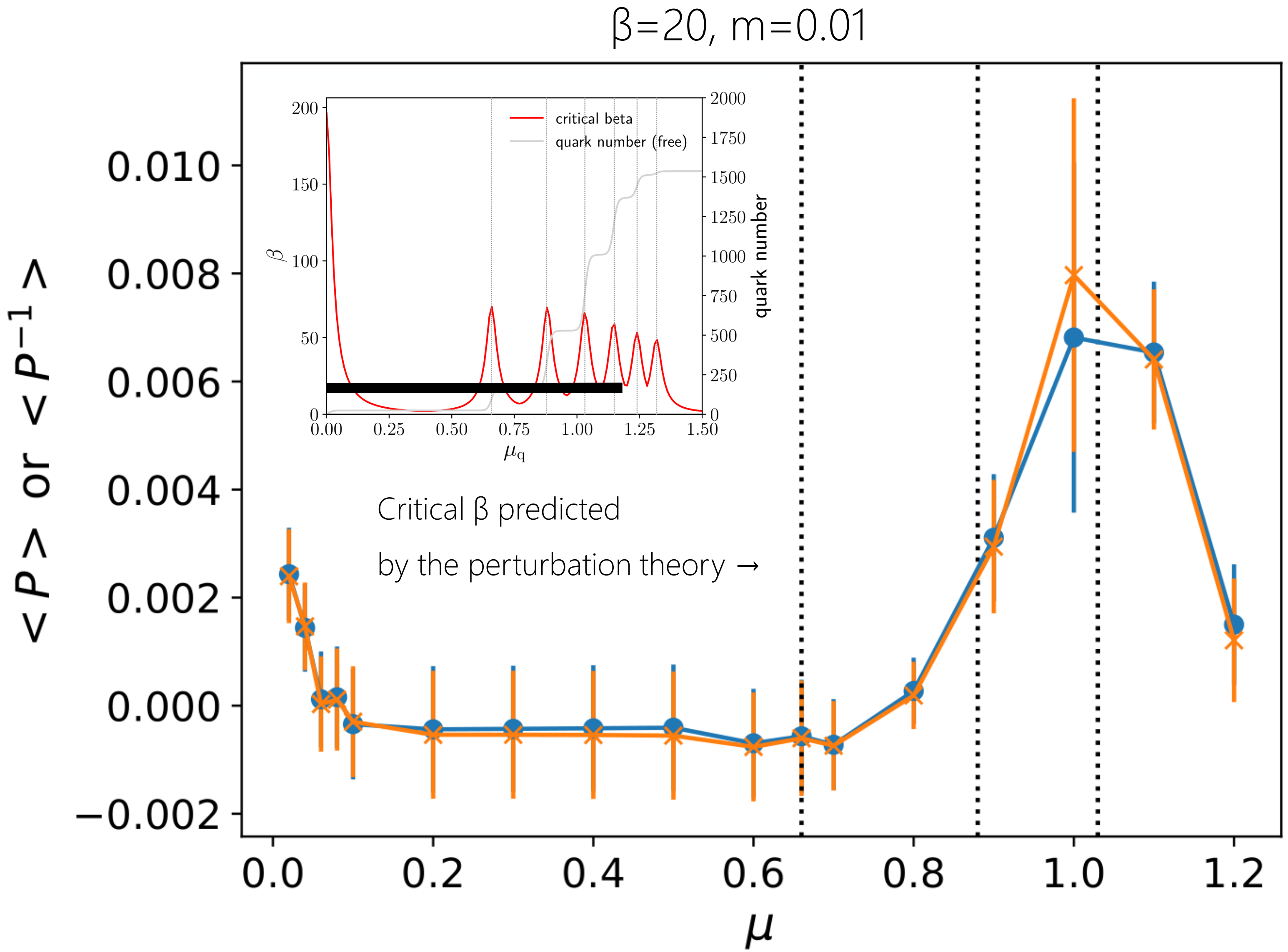


Ito, Matsufuru, Namekawa, Nishimura, Shimasaki, Tsuchiya, ST, JHEP10 (2020) 144

Preliminary results (quark number)

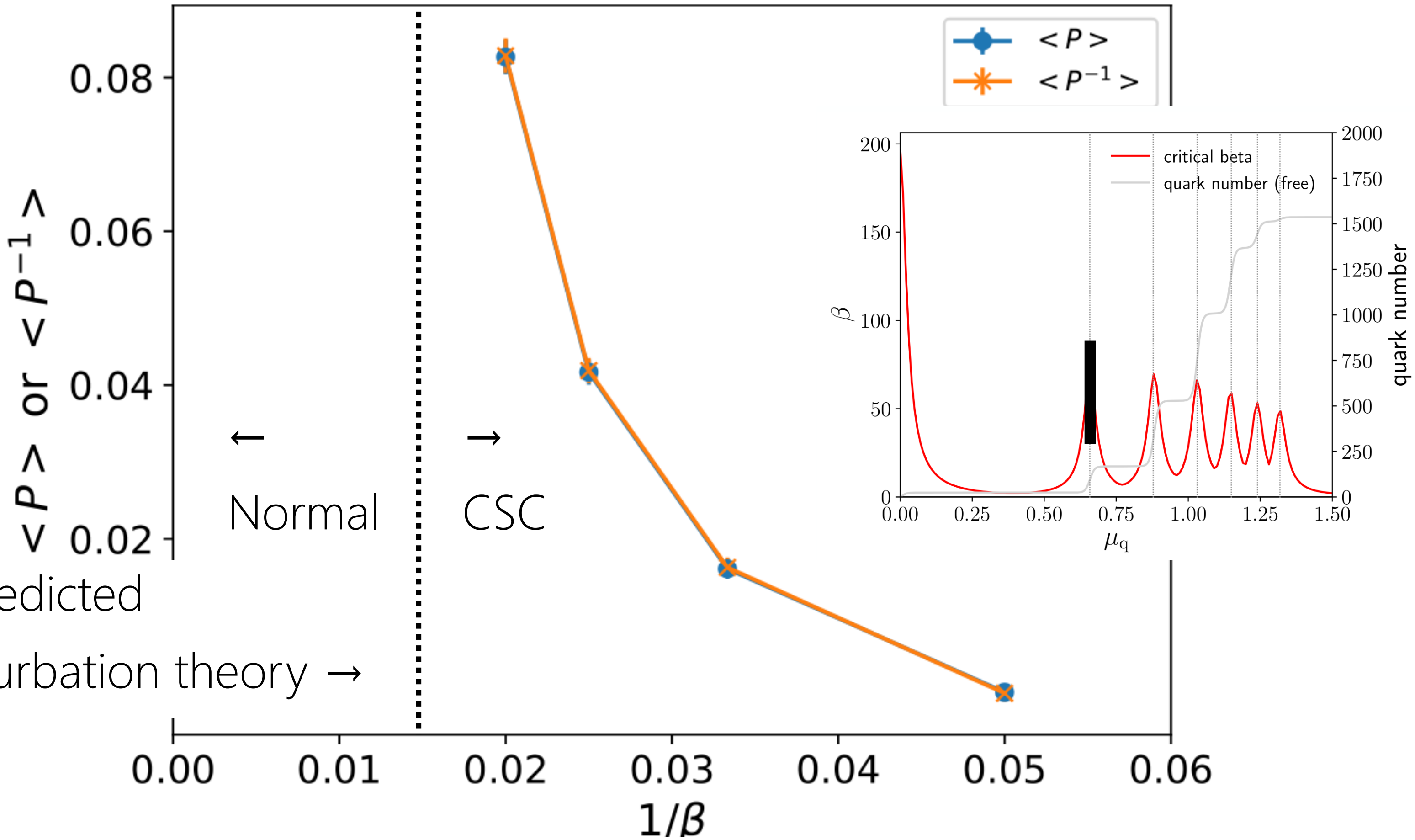


Preliminary results (polyakov loop 1)



Preliminary results (polyakov loop 2)

$\mu=0.66$, $m=0.01$



Critical β predicted

by the perturbation theory →

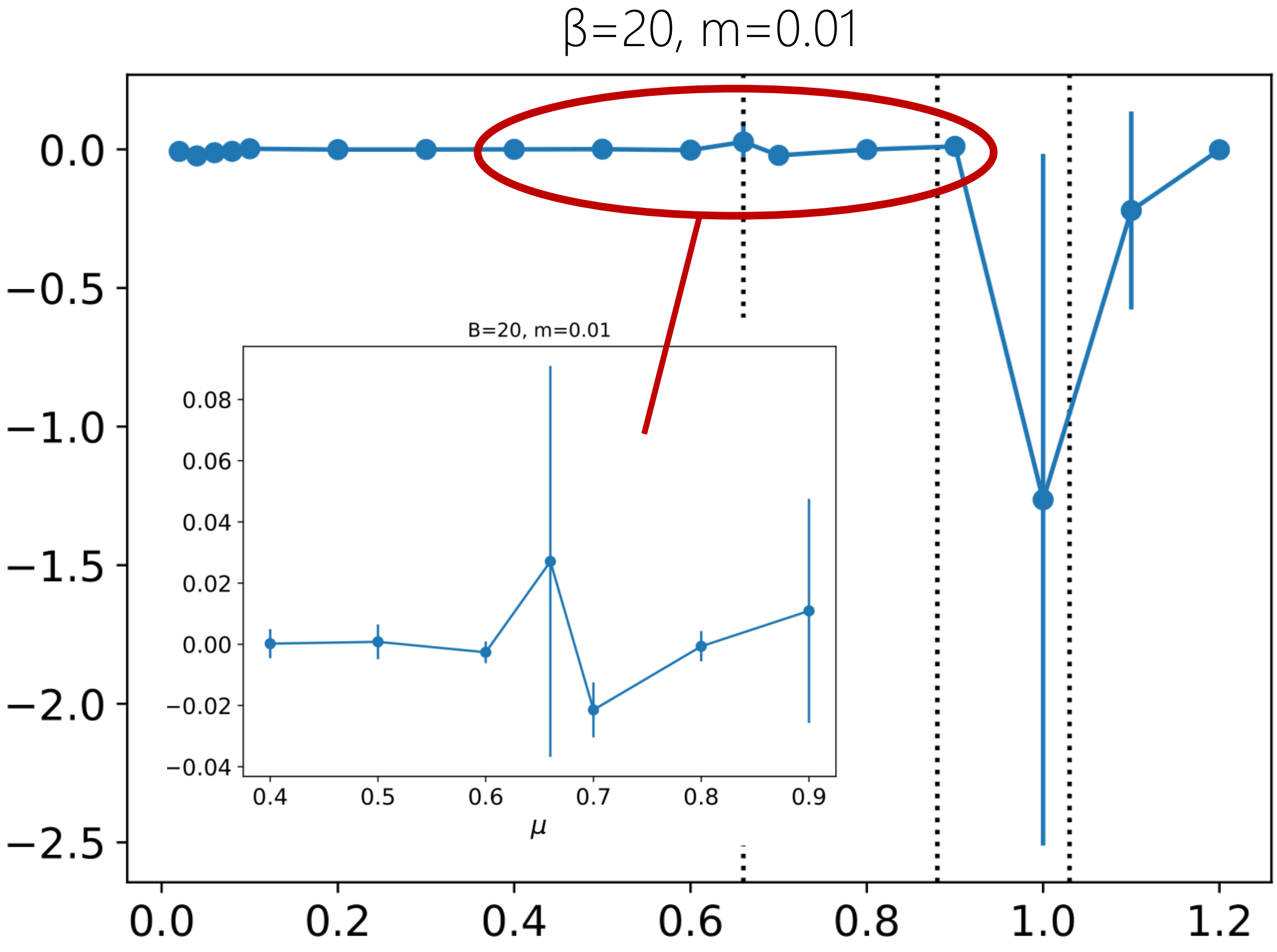
$\langle P\rangle$, $\langle P^{-1}\rangle \neq 0$ for $\beta>\beta_c$

→ Polyakov loop may be enhanced due to gapless excitation from Fermi surface.

(c.f.) perturbative calculations on small S3 Hands, Hollowood, Myers, JHEP (2010)

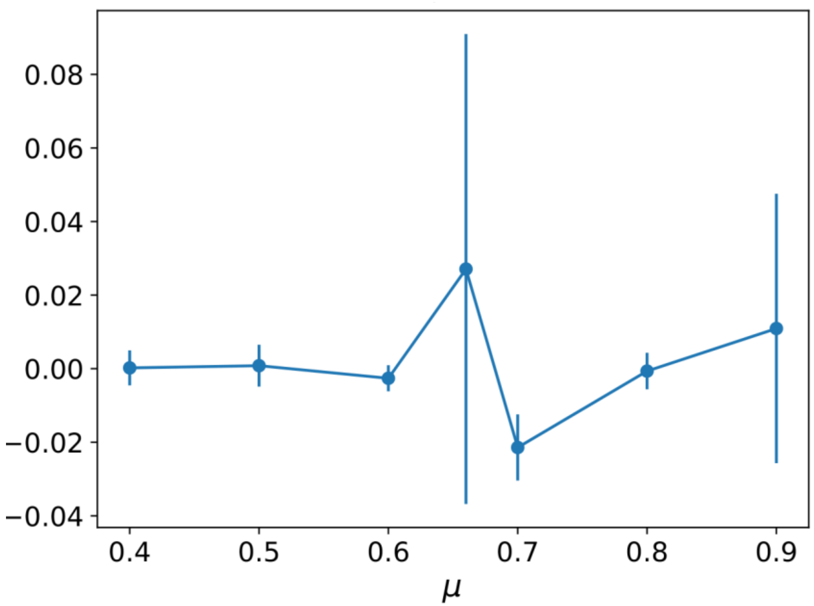
Preliminary results (gauge inv. 4point function)

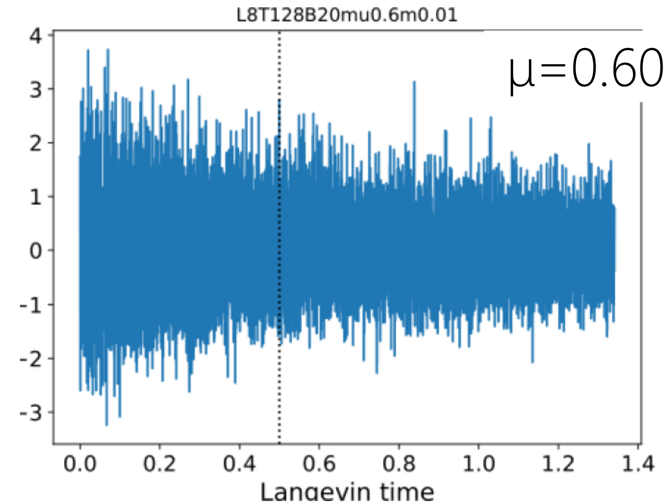
$$O = \varphi_a^\dagger \varphi_a, \quad \varphi_a = \epsilon_{abc} \mathrm{tr} C^{-1} \Psi_b^T C \Psi_c$$

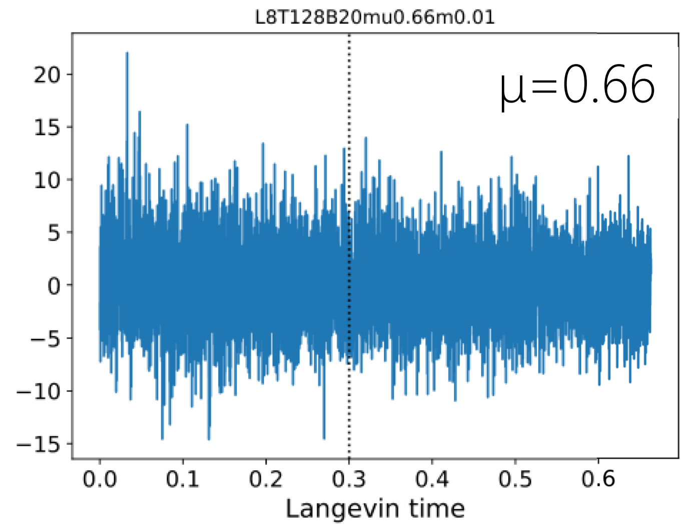


Preliminary results (gauge inv. 4point function)

β=20, m=0.01







At μ=0.66 (position of the first peak),

4-point function shows large fluctuation.

Summary

- We predict the parameter region where CSC appear based on lattice perturbation theory.
- Critical β has a peak structure.
  - Peaks appear when modes of quarks exist on the Fermi surface.
- Complex Langevin shows
  - Occupation of 1^{st}, 2^{nd} , … fermi level is observed.
  - $\langle P \rangle, \langle P^{-1} \rangle \neq 0$ above β_c
  - Large fluctuation of 4-point func. around β_c

Outlook:

- Volume dependence (ongoing)
- Better signal of CSC? (For instance, can we measure gap directly ?)