タキオン凝縮と弦の場の理論

髙橋 智彦 (奈女理)

§ Introduction

§ Bosonic D-brane

- A. Sen. "Descent Relations Among Bosonic D-branes", hep-th/9902105

§ String Field Theory

§ Tachyon Condensation in SFT

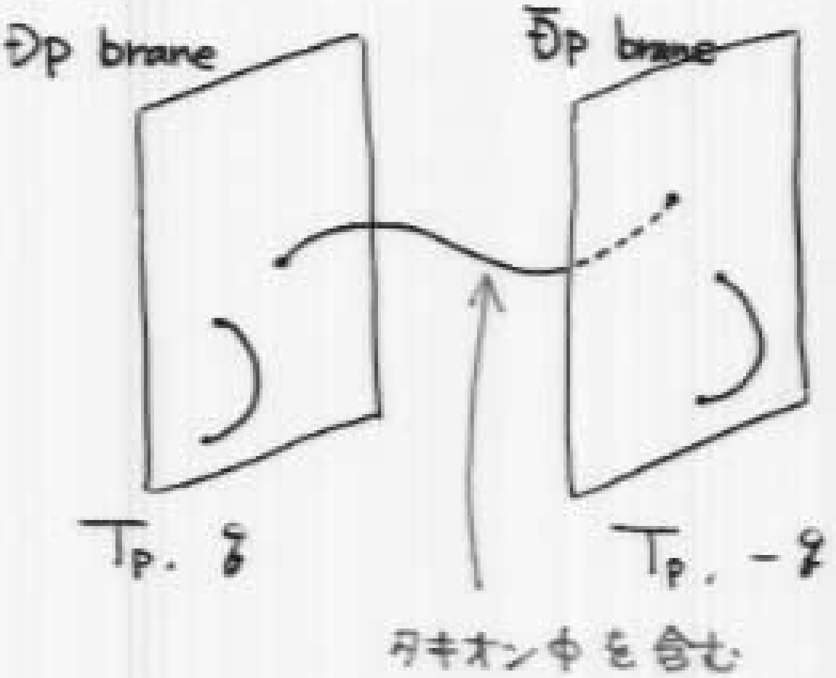
- A. Sen. "Universality of the Tachyon Potential", hep-th/9911116
- A. Sen, B. Zwiebach, "Tachyon Condensation in String Field Theory", hep-th/9912249

§ D-branes as Non-commutative Solitons

- R. Gopakumar, S. Minwalla, A. Strominger, "Noncommutative Solitons", hep-th/0003160
- J. Harvey, P. Kraus, F. Larsen, E. Martinec, "D-branes and Strings as Noncommutative Solitons", hep-th/0005031
- E. Witten, "Noncommutative Tachyons And String Field Theory", hep-th/0006071

§ Introduction

Type II



$Dp-\bar{D}p$ の RR charge $= g + (-g) = 0$

$Dp-\bar{D}p$ の 質量 $= T_p + T_p = 2T_p$

$\Downarrow$

Dp と $\bar{D}p$ は 対消滅をして

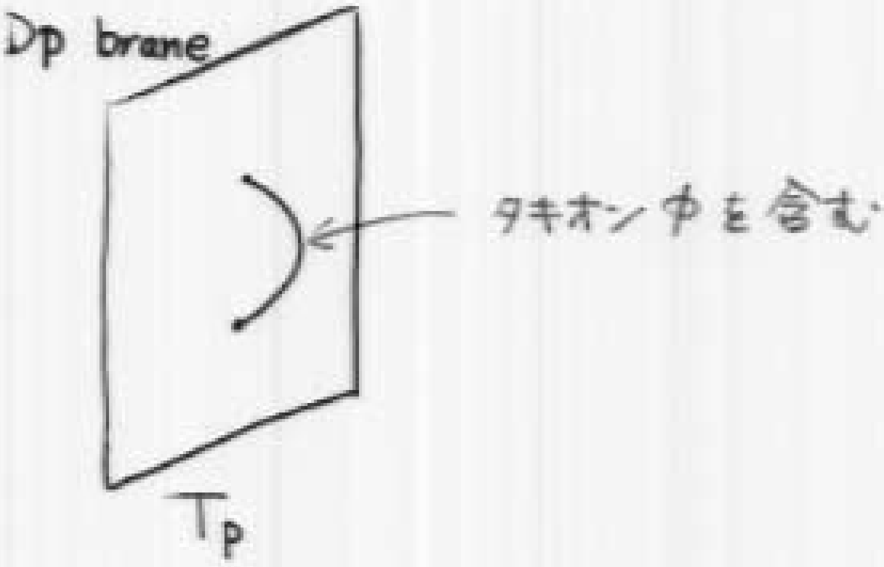
Brane がない真空（閉弦だけの真空）になる.

Sen's conjecture

$$\boxed{2T_p + V(\phi_0) = 0}$$

$V(\phi)$: タキオンのポテンシャル.

Bosonic string



$\Downarrow$

タキオンが凝縮して、Dp braneがない真空になる

Sen's conjecture

$$\boxed{T_p + V(\phi_0) = 0}$$

Sen's conjecture

$$\boxed{T_p + V(\phi_0) = 0}$$

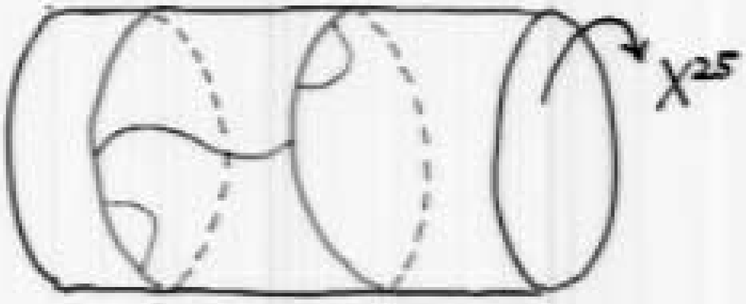
「弦の場の理論(SFT)を使った定量的テストから
このconjectureが非常に支持できる.」

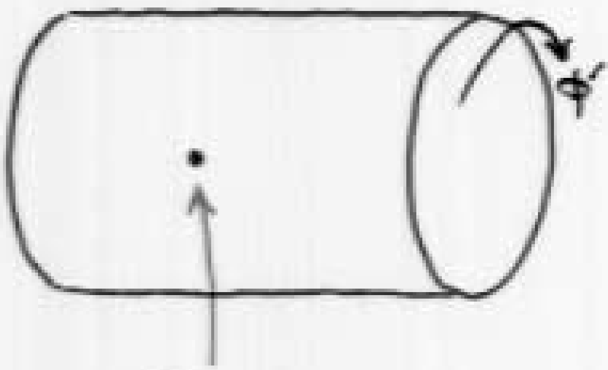
↑
前半の話

Off-shell 解析

§2 Bosonic D-brane

S^1 $R=\frac{1}{2}$ $\qquad\qquad$ S^1 $R=\frac{1}{2}$



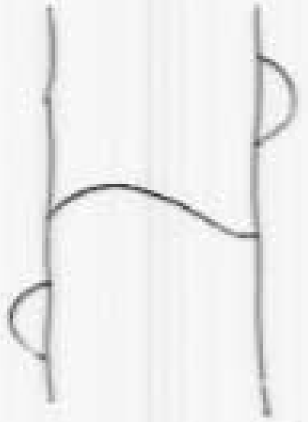


Dstring $\qquad$ Dstring + Wilson line $\frac{1}{2}$ $\qquad$ = $\qquad$ D particle

タキオンが凝縮 ($\alpha=1$)

CFTとして二つの理論は等価

$\downarrow$ $R\to\infty$ ← [同じ marginal operator が対応] → $\downarrow$ $R\to\infty$



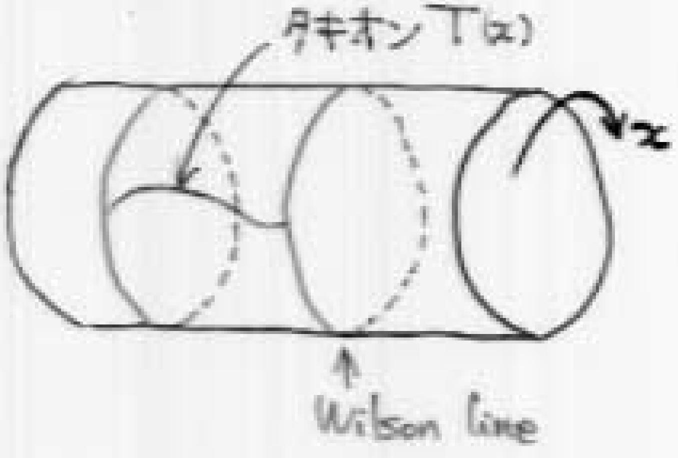
= $\qquad$ 

D particle

Dstring $\qquad$ Dstring

タキオンが凝縮.

• D string, D string + Wilson line $\frac{1}{2}$



T(x) のモード展開

$$T(x) = \sum_{n\in\mathbb{Z}} T_{n+\frac{1}{2}}\, e^{i(n+\frac{1}{2})\frac{x}{R_c}}$$

$$\left(R_c = \frac{1}{2}\right)$$

$T_{n+\frac{1}{2}}$ の質量 $m_{n+\frac{1}{2}}$

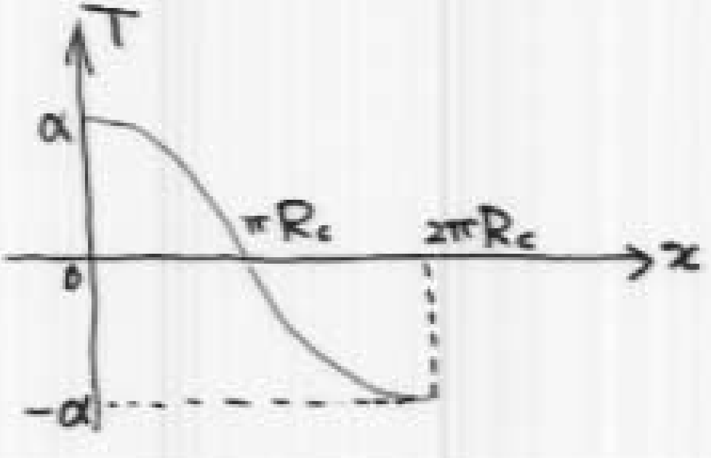
$$m^2_{n+\frac{1}{2}} = \frac{(n+\frac{1}{2})^2}{R_c^2} - 1 \qquad (\alpha' = 1)$$

$$\boxed{T_{\pm\frac{1}{2}} \text{ は massless} \longleftrightarrow \text{marginal operator}}$$

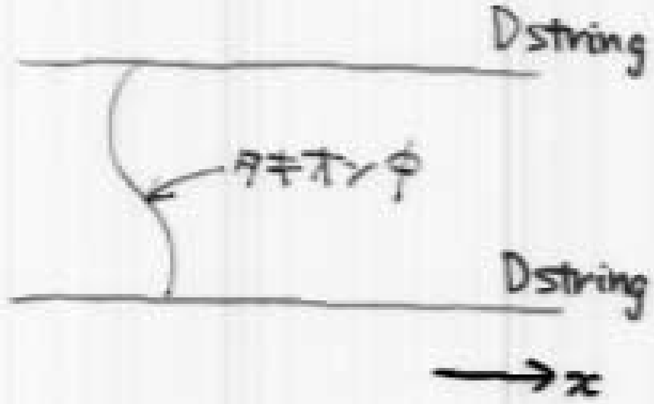
$T_{\frac{1}{2}} + T_{-\frac{1}{2}} = \alpha,\quad T_{\frac{1}{2}} - T_{-\frac{1}{2}} = 0,\quad T_r = 0 \quad (|r| > \frac{1}{2})$

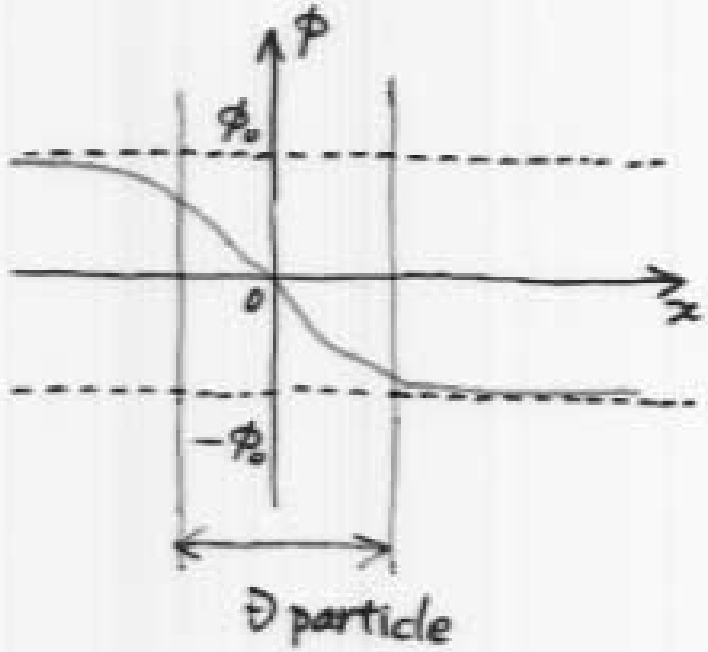
とすると、

$$T(x) = \alpha \cos\left(\frac{x}{2R_c}\right) = \alpha \cos x.$$



S^1 上のタキオン Kink.





$$\int_{-\infty}^{\infty} dx \left(2T_1 + V(\phi(x)) \right) \sim T_0 \quad (\text{有限})$$

$\therefore$

$$\boxed{2T_1 + V(\phi_0) = 0}$$

であるべき.

§ String Field Theory

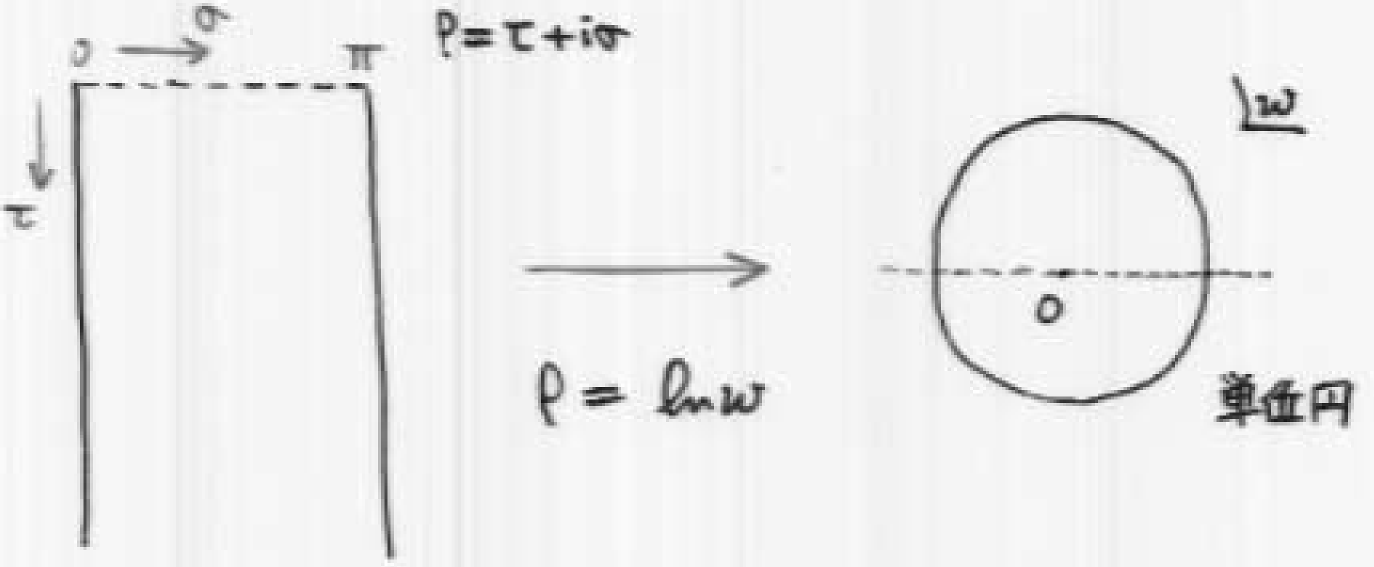
String Field

$$|\Psi\rangle = \phi(x)\, c_1 |0\rangle + A_\mu(x)\, \alpha^\mu_{-1}\, c_1 \underbrace{|0\rangle}_{\text{SL(2,R) vacuum}} + \cdots$$

$$\boxed{\text{String field } \Psi \longleftrightarrow \text{CFT の状態空間 } \mathcal{H} \text{ のある状態 } (\#\text{ghost} = 1)}$$

$$|\Psi\rangle = \Psi(w=0)|0\rangle$$

$\rho = \tau + i\sigma$



Action

$$S = -\frac{1}{g^2}\left(\frac{1}{2}\langle \Psi, Q_B \Psi\rangle + \frac{1}{3}\langle \Psi, \Psi, \Psi\rangle\right)$$

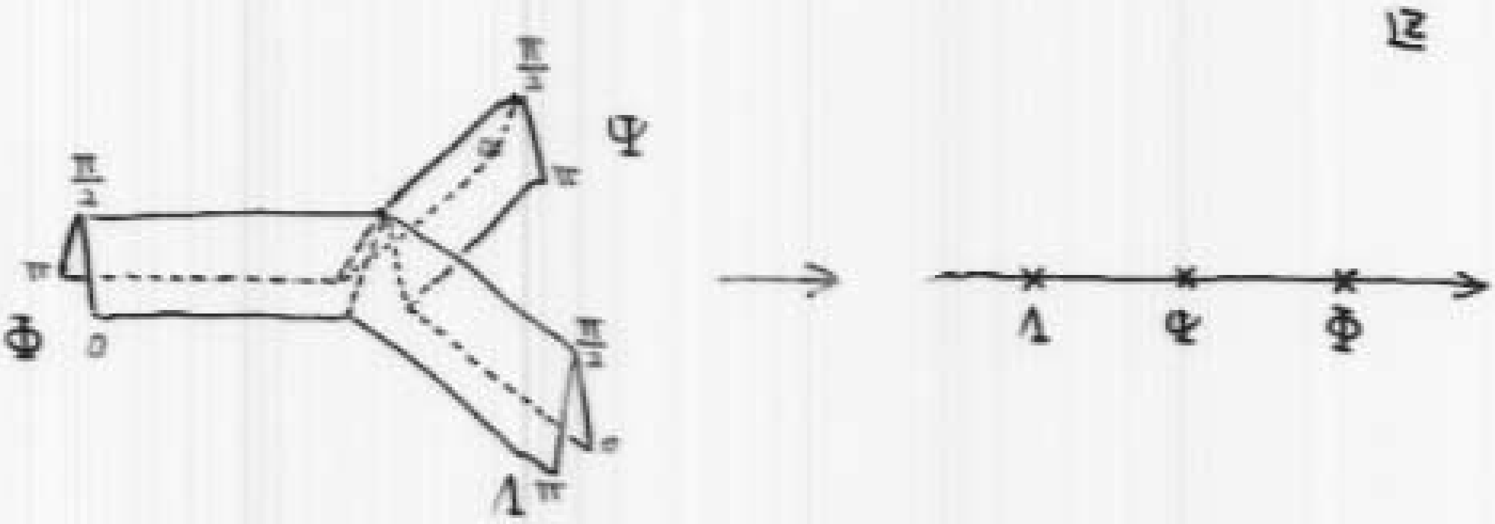
$\langle \Phi, \Psi \rangle$

$$\langle \Phi, \Psi \rangle = \langle I \circ \Phi(0)\ \Psi(0) \rangle$$

↖ z平面での CFTの相関関数

$$I(z) = -1/z$$

$\langle \Phi, \Psi, \Lambda \rangle$



$$\langle \Phi, \Psi, \Lambda \rangle = \langle h_1 \circ \Phi(0)\ h_2 \circ \Psi(0)\ h_3 \circ \Lambda(0) \rangle$$

$$\langle \Psi, \Psi, \Psi \rangle = \langle V_3 | |\Psi\rangle_1 |\Psi\rangle_2 |\Psi\rangle_3$$

$$\langle V_3 | \in \mathcal{A}_1^* \otimes \mathcal{A}_2^* \otimes \mathcal{A}_3^*$$

$$\langle V_3 | = {}_3\langle 0|\, {}_2\langle 0|\, {}_1\langle 0|\, e^{\frac{1}{2}\sum_{\substack{m,n\geq 0 \\ r,s=1,2,3}} \bar{N}^{rs}_{mn} \alpha^{(r)}_{-m} \alpha^{(s)}_{-n}} \times (\text{ghost})$$

$\bar{N}^{rs}_{mn}$: Neumann 係数.

$$\bar{N}^{rr}_{00} = -\frac{1}{2} \ln\left(\frac{3^3}{4^2}\right)$$

$$\bar{N}^{11}_{11} = -\frac{5}{27}, \quad \bar{N}^{12}_{11} = \frac{16}{27}, \quad \bar{N}^{13}_{11} = \frac{16}{27}$$

$\vdots$

tachyon potential $V(T)$

$$|T\rangle = T(0)|0\rangle \in \mathcal{A}$$

$$V(T) = -S(T)\Big/\int d^{26}x = T_{25}\, f(T)$$

$\therefore$

$$\boxed{f(T) = 2\pi^2\left(\frac{1}{2}\langle T, Q_B T\rangle + \frac{1}{3}\langle T, T, T\rangle\right)}$$

Sen's conjecture

$$\boxed{1 + f(T_0) = 0}$$

universality (background indepedence)

$$\mathcal{A} = \mathcal{A}_1 \oplus \mathcal{A}_2$$

$$\mathcal{A}_1 : \{ L_n^{(m)},\ c_n,\ b_n \} \times |0\rangle$$

$$\mathcal{A}_2 : \{ L_n^{(m)},\ c_n,\ b_n \} \times \underbrace{\phi(0)|0\rangle}_{h>0 \text{ primary}}$$

$|\phi_1\rangle \in \mathcal{A}_1$, $|\phi_2\rangle \in \mathcal{A}_2$ とすると

- $\langle \phi_1, Q_B \phi_2 \rangle = 0$
- $\langle \phi_1, \phi_1, \phi_2 \rangle = 0$

$\therefore$ $f(T)$ の中に $\mathcal{A}_2$ 成分は２次以上の項でしか現れない.

$$\boxed{\begin{array}{l} f(T) \sim \phi_2\phi_2 + \phi\phi_2\phi_2 \\ \dfrac{\partial f}{\partial \phi_2} \sim \phi_2 + \phi\phi_2 = 0 \implies \phi_2 = 0 \end{array}}$$

$\Downarrow$

$$\boxed{\begin{array}{c} f(T) \text{ は } \mathcal{A}_1 \text{ のみに依存する.} \\ \text{(up to factor)} \\ \parallel \\ \text{universality} \end{array}}$$

$f(T_0)$ の評価

$$|T\rangle = a\, c_1|0\rangle + b\, L_{-1} c_1|0\rangle$$
$$+ c\, c_{-1}|0\rangle + d\, L_{-2} c_1|0\rangle + \cdots$$
$$(a, b, c, d, \cdots = \text{const.})$$

$f(T)$ は無限個の変数の potential.

そこで

level truncation

$|T\rangle$ を level N までの状態で近似する

$\langle T, T, T\rangle$ の level の和 $\leq 2N$.

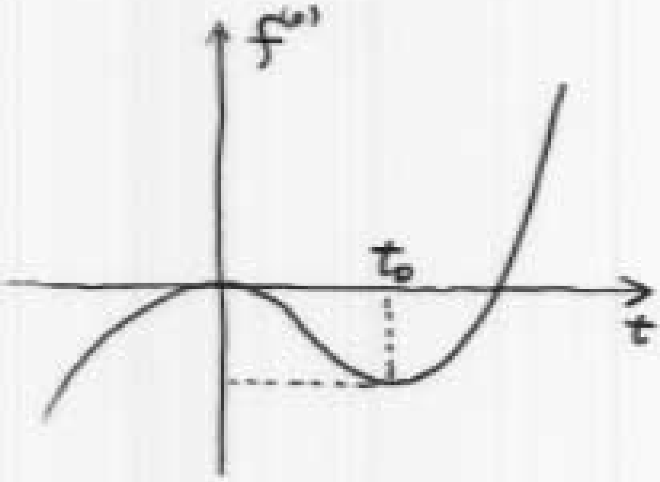
level 0 近似

$$|T\rangle = t\, c_1 |0\rangle$$

$$f^{(0)}(t) = 2\pi^2 \left(-\frac{1}{2} t^2 + \frac{1}{3} \left(\frac{4}{3\sqrt{3}} \right)^{-3} t^3 \right)$$

$$\therefore \quad t_0 = \left(\frac{4}{3\sqrt{3}} \right)^3$$

$$f^{(0)}(t_0) = \underline{-0.684}$$



"Sen's conjecture" の値の 約70% を与える.

level 2 近似

$$|T\rangle = t\, c_1|0\rangle + u\, c_{-1}|0\rangle + v \frac{1}{\sqrt{13}} L_{-2} c_1 |0\rangle$$

$$\boxed{f^{(2)}(T_c) = \underset{\sim\sim\sim}{-0.949}}$$

level 4 近似

$$|T\rangle = t\, c_1|0\rangle + u\, c_{-1}|0\rangle + v \frac{1}{\sqrt{13}} L_{-2} c_1 |0\rangle$$
$$+ A L_{-4} c_1 |0\rangle + B L_{-2} L_{-2} c_1 |0\rangle + C c_{-3} |0\rangle$$
$$+ D b_{-3} c_{-1} c_1 |0\rangle + E b_{-2} c_{-2} c_1 |0\rangle + F L_{-2} c_{-1} |0\rangle$$

$$\boxed{f^{(4)}(T_c) = \underset{\sim\sim\sim}{-0.9864}}$$

> "Sen's conjecture" は 99 % 正しい.

Moeller & Taylor

<u>level 10 近似</u>

$|T\rangle = \cdots$ 252項

$\langle T.T.T\rangle = \cdots$ 138,202項!

$$\boxed{f^{(20)}(T_0) = -0.99912}$$

$$\boxed{99.91\% \text{ 正しい}}$$

N. Berkovits, A. Sen, B. Zwiebach

"Tachyon Condensation in Superstring Field Theory"

hep-th/0002211

Super SFT. level 3 85% ok

non-poly open. Tachyon Kink 95% ok

⋮

J. Harvey, P. Kraus

"D-branes as unstable lumps in bosonic open string field theory"

hep-th/0002117

§ D branes as Noncommutative Solitons

$$S = \frac{1}{g^2}\int d^2z \left(\partial_z\phi\, \partial_{\bar z}\phi + V(\phi) \right)$$

$$\phi * \phi = e^{\frac{\theta}{2}(\partial_z \partial_{\bar z'} - \partial_{z'} \partial_{\bar z})} \phi(z,\bar z)\phi(z',\bar z')\Big|_{z'\to z}$$

$z \to \sqrt{\theta}\, z$ と変数変換して、$\theta \to \infty$ の極限をとると

$$S = \frac{\theta}{g^2}\int d^2z\, V(\phi)$$

$$\phi * \phi = e^{\frac{1}{2}(\partial_z \partial_{\bar z'} - \partial_{z'} \partial_{\bar z})} \phi(z,\bar z)\phi(z',\bar z')\Big|_{z'\to z}$$

$$\begin{cases} \phi_0 : & \phi_0 * \phi_0 = \phi_0 \\ \lambda : & \dfrac{dV(\lambda)}{d\lambda} = 0. \end{cases}$$ とすると、

$$V(\lambda\phi_0) = V(\lambda)\cdot\phi_0$$

$\therefore$ $\boxed{\phi = \lambda\phi_0 \text{ は 古典解.}}$

$$\phi(z,\bar{z}) = \frac{1}{(2\pi)^2}\int d^2k\, \tilde{\phi}(k)\, e^{-i(k_q x + k_p y)}$$

$(z = x + iy)$

$\phi(z,\bar{z})$ に $\mathcal{O}_\phi(\hat{p},\hat{q})$ を対応させる

$$\mathcal{O}_\phi(\hat{p},\hat{q}) = \frac{1}{(2\pi)^2}\int d^2k\, \tilde{\phi}(k)\, e^{-i(k_q \hat{q} + k_p \hat{p})}$$

$$\left([\hat{q},\hat{p}] = i \right)$$

このとき

- $\frac{1}{2\pi}\int d^2z\, \phi(z,\bar{z}) = \mathrm{Tr}_{\mathcal{H}}\, \mathcal{O}_\phi(\hat{p},\hat{q})$

- $\mathcal{O}_f \cdot \mathcal{O}_g = \mathcal{O}_{f*g}$

$$\phi_0 * \phi_0 = \phi_0 \longleftrightarrow \mathcal{O}_{\phi_0}^2 = \mathcal{O}_{\phi_0}$$

$\boxed{\phi_0 \text{ は } \mathcal{H} \text{ における projection operator に対応}}$

$$a = \frac{1}{\sqrt{2}}(\hat{q} + i\hat{p}), \quad a^\dagger = \frac{1}{\sqrt{2}}(\hat{q} - i\hat{p})$$

$$[a, a^\dagger] = 1$$

$$|n\rangle = \frac{1}{\sqrt{n!}}(a^\dagger)^n |0\rangle$$

<u>Projection operator</u>

$$P = \sum_n c_n |n\rangle\langle n|$$

$U^\dagger P U$ も projection operator. だから

Noncommutative Soliton は <u>$U(\infty)$ の対称性</u>をもつ.

Tachyon の有効理論

$$S = \frac{C}{g_s}\int d^{26}x \sqrt{g}\left(f(t)\, g_{\mu\nu}\partial^\mu t\, \partial^\nu t - V(t) + \cdots\right)$$

$(C = g_s T_{25})$

$\downarrow\ \theta \to \infty$

$$S = -\frac{C}{G_s}\int d^{26}x \sqrt{G}\, V(t)$$

◎ $t = t_* \phi_0(z)$ を代入　（$|0\rangle\langle 0|$ に対応する解）

$$S = -\frac{C\,V(t_*)}{G_s}\int d^{24}x \int d^2z \sqrt{G}\, \phi_0(z)$$

$$= -\frac{2\pi\theta\, C\, V(t_*)}{G_s}\int d^{24}x \sqrt{G}$$

◎ $V(t_*) = 1$ (Sen's conjecture) と $G_s = \dfrac{g_s\sqrt{G}}{2\pi\alpha' B\sqrt{g}}$ を代入

$$S = -(2\pi)^2\alpha' \frac{C}{g_s}\int d^{24}x \sqrt{g}$$

$$= -T_{23}\int d^{24}x \sqrt{g} \qquad (T_{23} = (2\pi)^2\alpha' T_{25})$$

$\boxed{t = t_* \phi_0 \text{ は D23 brane に対応している.}}$

$\theta \to \infty$ の極限で
~~$(B \to \infty)$~~

$$|\Psi\rangle = |\Psi_0\rangle \otimes e^{ip\underset{\sim}{X}}(w=0)\,|0\rangle$$

非可換な方向（2p次元）

$$\underset{\mathcal{A}}{\Uparrow} = \underset{\mathcal{A}_0}{\Uparrow} \otimes \underset{\mathcal{A}_1}{\Uparrow}$$

$$\left(|\Phi * \Psi\rangle = |\Phi_0 * \Psi_0\rangle \otimes |\Phi_1 * \Psi_1\rangle \right)$$

$B = tB_0$, $X^i = Y^i/\sqrt{t}$, $t \to \infty$

$$\underset{\mathcal{A}_1}{\underset{\Uparrow}{e^{ipY}(\tau)}} \cdot \underset{\mathcal{A}_1}{\underset{\Uparrow}{e^{iqY}(\tau')}} \sim e^{-\frac{i}{2}\theta^{ij}p_i q_j}\, \underset{\mathcal{A}_1}{\underset{\Uparrow}{e^{i(p+q)Y}(\tau')}}$$

$$\underset{\mathcal{A}_0}{\underset{\Uparrow}{\partial^n Y(\tau)}} \cdot \underset{\mathcal{A}_1}{\underset{\Uparrow}{e^{iqY}(\tau')}} \sim \frac{1}{t}\frac{1}{(\tau-\tau')^n} e^{iqY} \underset{\sim\sim}{\to 0}$$

◎ $\mathcal{A}_1$ を $\mathcal{H}$ 上の operator で表現する.

$\{|0\rangle, |1\rangle, |2\rangle, \cdots\}$

($\mathcal{A}_1$ を $\infty \times \infty$ 行列 ($U(\infty)$ 行列) とみなす.)

◎ $\mathcal{H} = V \oplus W$

W : $\mathcal{H} - V$

V : N次元部分空間 $\{|0\rangle, |1\rangle, \cdots, |N-1\rangle\}$

<u>projection operator</u>

$$P_N = |0\rangle\langle 0| + |1\rangle\langle 1| + \cdots + |N-1\rangle\langle N-1|$$

$$\left(\begin{array}{l} P_N V = V, \quad P_N W = 0 \\ P_N^2 = P_N \end{array} \right)$$

$$\boxed{|\Psi_0\rangle = \underbrace{|T_0\rangle}_{\text{D25 brane が消滅する解}} \otimes (1-P_N)}$$

は、String Field の運動方程式

$$Q_B\Psi + \Psi * \Psi = 0$$

の解である。

- $Q_B T_0 + T_0 * T_0 = 0$
- $[Q_B, P_N] \to 0 \quad (\theta \to \infty)$
- $(1-P_N)^2 = 1-P_N$

$\Psi = \Psi_0 + \Psi'$ と展開する

Ψ' が VV open string のとき

$$S = \int \left(\Psi' * Q_B \Psi' + \frac{2}{3} \Psi' * \Psi' * \Psi' \right)$$

◎ $|\Psi'\rangle \in \mathcal{A}_0 \otimes M_N$

↑ complex $N \times N$ 行列

≅ CP factor

◎また $|\Psi'\rangle$ は可換な方向だけ運動量をもつ.

$\Rightarrow$ Ψ' : N枚の D(25−2p) brane 上の open string.

(古典解で brane U(N) sym. が見えている)

Ψ' が WW open string のとき

$$S = \int \left(\Psi' * Q_B' \Psi' + \frac{2}{3} \Psi' * \Psi' * \Psi' \right)$$

$$Q_B' \phi = Q_B \phi + T_0 * \phi + (-)^{|\phi|} \phi * T_0$$

Sen's conjecture より Ψ' に物理的モードはない.

VW open string についても同様

§ まとめ

◎ Sen's conjecture

$$T_p + V(\Phi_0) = 0$$

◎ SFTの解析より、Sen's conjecture は 99.9% 正しい。 (Off-shell)

◎ $\theta \to \infty$ の極限で

D25 $\longrightarrow$ N枚 D(25-2p)

が解析できる。

このとき SFT が役立つ (?)

課題

◎ $1 + f(T_0) = 0$ は 100% 正しいのか?

◎ さらに非摂動論的解析が SFT に可能か?

Closed String

True Vacuum

⋮