

QFT 2003

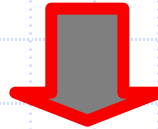
*Review of Dijkgraaf-Vafa
Conjecture and Recent Progress*



大河内 豊 (東工大)

4D $N=1$ Super Yang-Mills

- ◆ Dynamical generation of mass
- ◆ Confinement of Charge
- ◆ Dynamical symmetry breaking



Holomorphy (Seiberg 94')

$N=1, 2$ SQCD

Superpotential, Prepotential

4D N=1 Super Yang-Mills

N=2 Super YM +

$$W_{\text{tree}}(\Phi) = \sum_{k=1}^{n+1} \frac{g_k}{k} \text{Tr} \Phi^k$$

- Matter Content は N=2と同じ
- N=1 Chiral multiplet で deform

$$F_{\text{CI}} = \text{diag}(\underbrace{a_1 \dots a_1}_{N_1}, \underbrace{a_2 \dots a_2}_{N_2}, \dots, \underbrace{a_n \dots a_n}_{N_n})$$

$$W'_{\text{tree}}(a_k) = 0$$

Quantum

$$U(N) \rightarrow \prod_{i=1}^n U(N_i) \xrightarrow{\text{red arrow}} U(1)^n$$

Cachazo-Intriligator-Vafa[0103067]
Cachazo-Fiol-Intriligator-Katz-Vafa
[0110028]

Geometric Transition

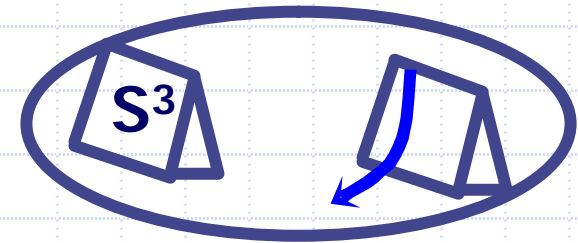
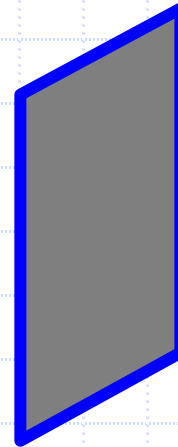
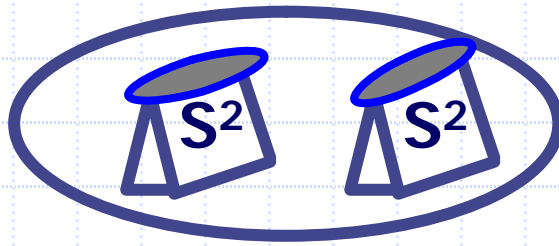
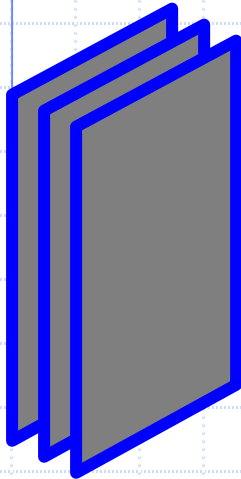
Type IIB String

N=1 SYM
 $W_{\text{tree}}(F)$

Gravity Dual



Classical Gravity



RR Flux

Holomorphic な info を計算する限りにおいては、
内部空間に Geometric Transition を行うだけでよい!!

Classical Gravity

例 $U(N) N=2 \text{ SYM} + W_{\text{tree}}(F)$

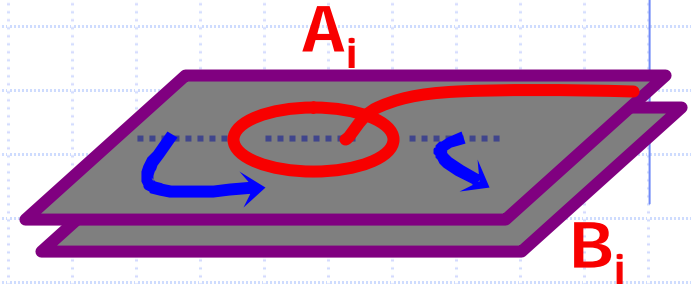


Geometric Transition

$$W'_{\text{tree}}(x^2 + y^2 + z^2 + w^2 + f_{n-1}(x)) = 0$$



$$W'_{\text{tree}}(x^2 + f_{n-1}(x) + y^2) = 0$$



n 個のcut

i -th cut から N_i 本の
RR-Flux

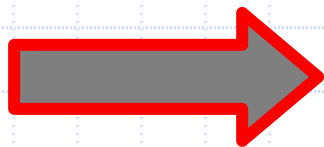
$$S_i = \oint_{A_i} y(x) dx, \quad \Pi_i = \oint_{B_i} y(x) dx = \frac{\partial \mathcal{F}_0}{\partial S_i}$$

$$N_i = \oint_{A_i} h(x) dx, \quad \tau_0 - i b_i = \oint_{B_i} h(x) dx$$

Gauge Theory の情報

Vafa's Identification

$$S_i \rightarrow -\frac{1}{32\pi^2} \text{Tr}_{\text{SU}(N_i)} W^\alpha W_\alpha$$
$$\tau_0 \rightarrow -\frac{8\pi^2}{g_0^2} \quad b_i \rightarrow U(N_i) \text{ の } \theta \text{ term}$$
$$\frac{\partial^2 \mathcal{F}_0}{\partial S_i \partial S_j} \rightarrow U(1)^n \text{ coupling const}$$



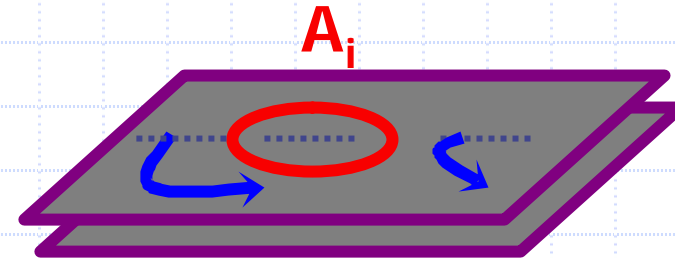
Fluxにより

$$W_{\text{eff}} = \sum_{i=1}^n N_i \frac{\partial \mathcal{F}_0}{\partial S_i} + \tau S_i$$

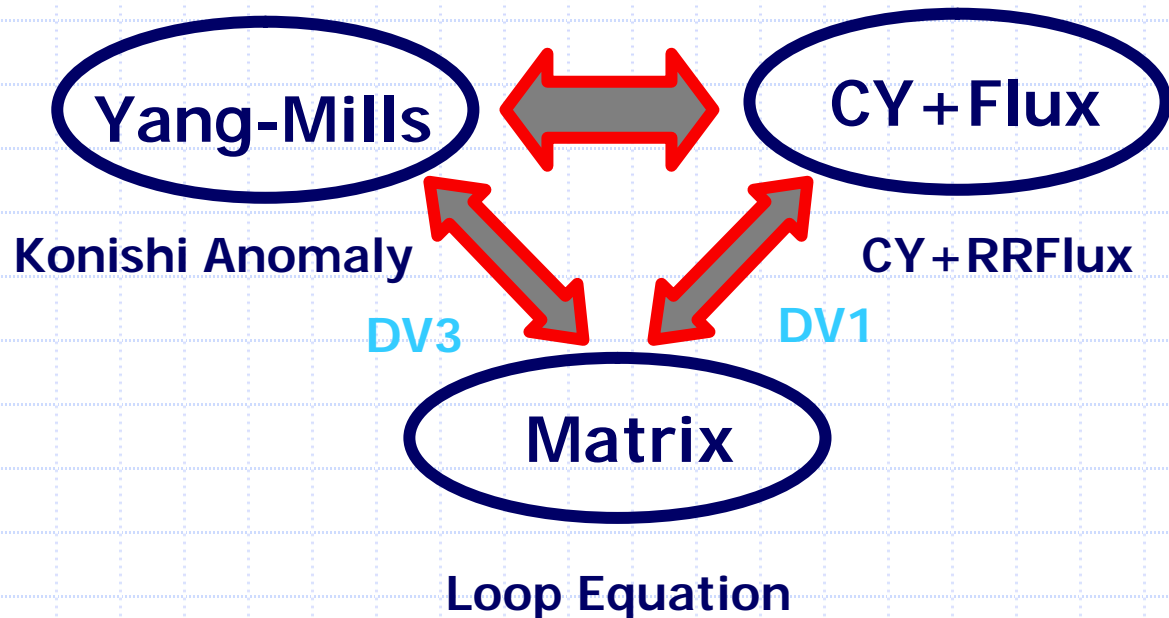
Gukov-Vafa-Witten[9906070]
Gukov[9911011]

Contents

1. Introduction
2. Geometric Transition
3. DV Conjecture
4. Chiral Ring
5. Konishi Anomaly
6. Factrization
7. New Duality
8. Conclusion



Duality Chain



Dijkgraaf-Vafa Conjecture

Dijkgraaf-Vafa[0206255,0207106,0208048]

Dijkgraaf-Gukov-Kazakov-Vafa[0210238]

Dijkgraaf-Grisaru-Lam-Vafa-Zanon[0211017]

Large M Matrix Model

D-V Conjecture

$F : M \times M$ Matrix, $(M \oplus N, g_s \oplus g_{YM}^2)$

$$Z = \int d\Phi \cdot e^{\frac{1}{g_s} \text{Tr} W_{\text{tree}}(\Phi)}$$

$$\sim \exp \left[\sum_g g_s^{2g-2} \mathcal{F}_g^{\text{Mat}}(S_i) \right] \quad \text{Saddle-Point}$$

$$g_s \rightarrow 0, \quad M_i \rightarrow \infty, \quad S_i = g_s M_i : \text{Fixed}$$

E.O.M $R_m \equiv \text{Tr} 1/(z - \Phi)$

$$\underbrace{\left(W'_{\text{tree}} - 2\langle R_m \rangle \right)^2}_y = W'^2_{\text{tree}}(z) + f_m(z) \quad \text{G.T.と同じリーマン面!}$$

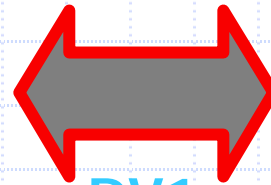
$$\langle R_m(z)^2 \rangle = \langle R_m(z) \rangle^2$$

D-V Conjecture

Perturbative Window

Matrix Model
Free energy

$$\mathcal{F}_0^{\text{Mat}}$$



DV1

リーマン面の
プレポテンシャル

$$\mathcal{F}_0$$

SuperString \wedge



DV3

$$S_i = g_s M_i$$

glueball 場

$$W_{\text{eff}} = \sum_{i=1}^n N_i \frac{\partial \mathcal{F}_0}{\partial S_i} + \tau S_i$$

Matrix Model の
摂動展開より

$$= \sum_{i=1}^n N_i S_i \log \frac{S_i}{\Lambda_0^3} - \tau \sum_{i=1}^n S_i + \sum_k c_{kl\dots}^{ij\dots} S_i^k S_j^l \dots$$

D-V Conjecture

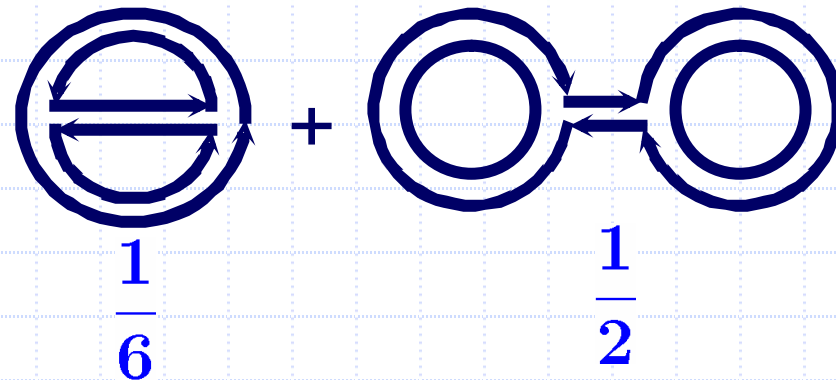
$$W_{\text{tree}} = \frac{m}{2} \text{Tr} \Phi^2 + \frac{\lambda}{3} \text{Tr} \Phi^3$$

U(N) U(N) case : i=1

$$W_{\text{eff}} = \underbrace{NS \log \frac{S}{\Lambda}}_{\text{Veneziano-Yankielowicz}} + \underbrace{3 \frac{2 \lambda^2}{3 m^3} N^3 S^2 + \mathcal{O}(S^3)}_{\text{Veneziano-Yankielowicz}}$$

Veneziano-Yankielowicz

Matrix Model の
2loop diagram



**Gauge Theory の非摂動的な側面が Matrix Model の
摂動展開で理解できる!!**

注意すべき点

- ◆ Matrix のサイズ M_i とゲージ群のランク $U(N_i)$ は別物
- ◆ Matrix Model とYM が等価であるといっているのではない
(Matrix Model だけでは説明できないこともある)

$$W_{\text{eff}} = \sum_{i=1}^n N_i \frac{\partial \mathcal{F}_0}{\partial S_i} + \tau S_i \longleftarrow \text{この形は?}$$

$$W_{\text{eff}} = \sum_{i=1}^n N_i \frac{\partial \mathcal{F}_0}{\partial S_i} + \tau S_i + \overbrace{4\mathcal{G}_{0,1,0}}^{\text{SO/USp}} + \overbrace{\mathcal{F}_{0,0,1}}^{\text{Flavor}}$$

1 boundary

Unoriented

Ookouchi-Watabiki[0301226]

Ookouchi[0211287]

Cachazo-Douglas-Seiberg *Witten's Evolution*

- ◆ 一般の $N=1$ の理論を扱うことができる
- ◆ カイラルな理論も扱える

Chiral Operator

- ◆ CYやMatrix Modelが本質的ではなく Gauge Theoryをよく見直せば 同し構造(リーマン面with Flux)が得られる!
- ◆ さらに拡張した議論ができる(一般のN=1 YM)!

$$\bar{D}_{\dot{\alpha}}\Phi = 0 \text{ のlowest } \longleftrightarrow \{\bar{Q}_{\dot{\alpha}}, \mathcal{O}(x)\} = 0$$

$$\text{SUSY Vacua } \bar{Q}_{\dot{\alpha}}|0\rangle = 0$$

1. Identification

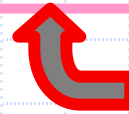
$$\mathcal{O} \simeq \mathcal{O} + \{\bar{Q}_{\dot{\alpha}}, \mathcal{G}^{\dot{\alpha}}\}$$

SUSY Vacua では、 $\langle\{\bar{Q}_{\dot{\alpha}}, \mathcal{G}^{\dot{\alpha}}\}\rangle = 0$ なので自然な同一視

Classification

- ◆ $N=2$ SQCD + W_{tree}

$$QF^kQ, \text{Tr}F^k, \text{Tr}F^kW_a, \text{Tr}F^kW_a W^a \quad k=0,1,2$$



Pure $N=2$ Yang-Mills + W_{tree}

Generating function

$$T(z) = \text{Tr} \frac{1}{z - \Phi} = \sum_{k=0}^{\infty} z^{-1-k} \text{Tr} \Phi^k$$

$$w_{\alpha}(z) = \frac{1}{4\pi} \text{Tr} \frac{W_{\alpha}}{z - \Phi}$$

$$R(z) = \frac{-1}{32\pi^2} \text{Tr} \frac{W_{\alpha} W^{\alpha}}{z - \Phi}$$

2. Cluster decomposition

$$\sigma_{\dot{\alpha}\alpha}^{\mu} \frac{\partial}{\partial x^{\mu}} \mathcal{O}(x) = [\sigma_{\dot{\alpha}\alpha}^{\mu} P_{\mu}, \mathcal{O}(x)] = \{\bar{Q}^{\dot{\alpha}}, [\bar{Q}^{\alpha}, \mathcal{O}(x)]\}$$

Chiral Operator の真空期待値は位置依存性がない!

$$\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle = \langle \mathcal{O}(x_1) \rangle \langle \mathcal{O}(x_2) \rangle = \langle \mathcal{O} \rangle^2$$



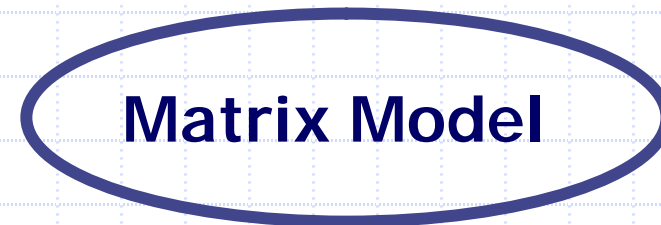
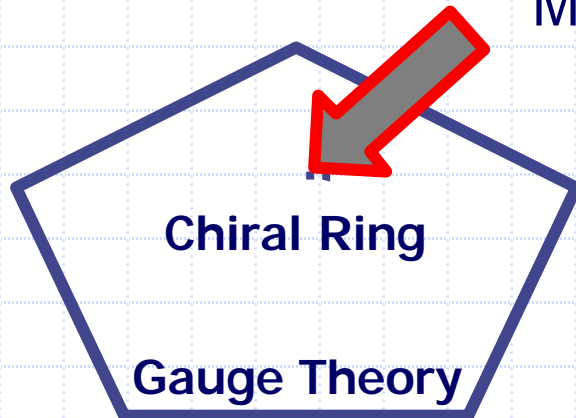
無限に離すことで相関ナシ

Large N Matrix Model が持つ Cluster decomposition と対応

$$\langle \mathbf{R}(x_1) \mathbf{R}(x_2) \rangle = \langle \mathbf{R}(x_1) \rangle \langle \mathbf{R}(x_2) \rangle$$

Chiral Ring

Matrix Model とよ対応する



Schwinger-Dyson

$$R(z) = \frac{-1}{32\pi^2} \text{Tr} \frac{W_\alpha W^\alpha}{z - \Phi}$$

$T(z)$, $w^a(z)$

Schwinger-Dyson

$$R_m(z) = \text{Tr} \frac{1}{z - \Phi}$$

対応物は？

糸山-菅野 [hep-th/0304148]
Super-eigenvalue-model

Konishi-Shizuya[Nuovo Cim A90 (1985)]
 Konishi[Phys Lett B135 (1984)]
 Gorsky[0210281], Tachikawa[0211189]

Konishi Anomaly

普通の小西アノマリー

$$F \longrightarrow F + \varepsilon F$$

$$W_{\text{tree}}(\Phi) = \sum_{k=1}^{n-1} \frac{g_k}{k} \text{Tr} \Phi^k$$

古典的にはSymmetryであるが、
 量子論的にはアノマリーを持つ

チャージkをassign

$$\underbrace{\frac{1}{4} \bar{D}^2 \bar{\Phi} e^V \Phi}_{\{\bar{Q}^{\dot{\alpha}}, \dots\}} = \frac{1}{32\pi^2} \text{Tr} W_\alpha W^\alpha + \text{Tr} \Phi \frac{\partial W_{\text{tree}}(\Phi)}{\partial \Phi}$$

$\{\bar{Q}^{\dot{\alpha}}, \dots\}$

SUSY Vacua + ルジャンドル変換

$$\frac{1}{2} \frac{\partial W_{\text{eff}}(g_k, \Lambda)}{\partial \log \Lambda^{2N}} + \sum_k g_k k \frac{\partial W_{\text{eff}}(g_i, \Lambda)}{\partial g_k} = 0$$

小西アノマリー方程式 ~ W_{eff} の方程式

一般化された小西アノマリー

Chiral Ring Matrix Modelを思いだし、小西アノマリーの変換をChiral Ring全体に格上げすると？

$$\delta\Phi = \epsilon \frac{1}{z - \Phi}, \quad \delta\Phi = \epsilon_\alpha \frac{W^\alpha}{z - \Phi}, \quad \delta\Phi = \epsilon \frac{W_\alpha W^\alpha}{z - \Phi}$$

$$\int \mathcal{D}\Phi' \mathcal{D}\Phi'^\dagger e^{-S(\Phi', \Phi'^\dagger)} = \int \mathcal{D}\Phi \mathcal{D}\Phi^\dagger e^{-S(\Phi, \Phi^\dagger)}$$

Measureからの寄与

$$\mathcal{D}\Phi' = \mathcal{D}\Phi \exp \left[\epsilon \sum_{i,j} \left(\frac{\delta\Phi_{kl}}{\delta\Phi_{ij}} \frac{\partial f_{ij}}{\partial \Phi_{kl}} \right) \right]$$

Regularization

$$\frac{1}{32\pi^2} [W_\alpha, [W^\alpha, T_{kl}]]_{ij}$$

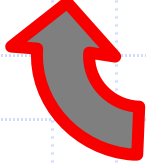
$$T_{lk} = \begin{cases} e_{lk} & \text{for } U(N) \\ e_{lk} - e_{kl} & \text{for } SO(N) \\ e_{lk} + e_{kl} & \text{for } Sp(N) \end{cases}$$

$$(e_{lk})_{ij} = \delta_{il} \delta_{jk}$$

一般化された小西アノマリー方程式

$$\bar{D}^2 J_f = \text{Tr} \left[f(\Phi, W_\alpha) \frac{\partial W_{\text{tree}}(\Phi)}{\partial \Phi} \right] + \frac{1}{32\pi^2} \sum_{ij} \left[W_\alpha, \left[W^\alpha, \frac{\partial f}{\partial \Phi_{ij}} \right] \right]_{ji}$$

それぞれ代入


$$\delta\Phi = \epsilon \frac{1}{z - \Phi}, \quad \delta\Phi = \epsilon_\alpha \frac{W^\alpha}{z - \Phi}, \quad \delta\Phi = \epsilon \frac{W_\alpha W^\alpha}{z - \Phi}$$

$$\text{Tr} \frac{W'_{\text{tree}}(\Phi)}{z - \Phi} = 2R(z)T(z) + w^\alpha(z)w_\alpha(z)$$

$$\frac{1}{4\pi} \text{Tr} \frac{W'_{\text{tree}}(\Phi)}{z - \Phi} = 2R(z)w_\alpha(z)$$

$$-\frac{1}{32\pi^2} \text{Tr} \frac{W'_{\text{tree}}(\Phi) W^\alpha W_\alpha}{z - \Phi} = R(z)^2$$

Konishi Anomaly

Solution $w_\alpha = 0$

$$2R(z) = W'_{\text{tree}}(z) - \sqrt{W'^2 + f_{n-1}(z)}$$

$$y^2 = W'^2_{\text{tree}}(z) + f_{n-1}(z)$$

$T(z) = \dots = y(z)$ の関数で無限遠に留数
 $\pm N$ の simple pole を持つ

$$F_{\mathbf{c}_I} = \text{diag}(a_1 \dots a_1, a_2 \dots a_2, \dots, a_n \dots a_n)$$

$$\oint_{A_i} y(z) dz = \oint_{A_i} \text{Tr} \frac{W_\alpha W^\alpha}{z - \Phi} = S_i$$

$$\oint_{A_i} T(z) dz = \oint_{A_i} \text{Tr} \frac{1}{z - \Phi} = N_i$$

Vafa's
Identification
そのもの

Konishi Anomaly

Generalized Konishi Anomalyを用いる方法は、一般の $N=1$ 理論に適応できる

- ◆ Chiral Gauge Theory

Brandhuber-Ita-Nieder-Oz-Romelsberger[0303006],
Argurio-Ferretti-Heise[0306125], Landsteiner-Lazaroiu-Tatar
[0307182]

- ◆ Sp: antisymmetric F / SO: Symmetric F

Alday-Cirafici[0304119], Kraus-Ryzhov-Shigemori[0304138],
Cachazo[0307063], Landsteiner-Lazaroiu-Tatar[0306236]

一般化された小西アノマリー方程式の解を求めて、YMの詳しい構造を調べればよいのであるが、 $N=2$ YMに $W_{\text{tree}}(F)$ を加えた $N=1$ YMでは、Powerful Tool が存在する!!



Factrization Problem

- ◆ $N=2$ Gauge Theory + $W_{\text{tree}}(F)$
- ◆ Seiberg Witten Curve
- ◆ Vacua with $N-n$ Massless Monop

$$y^2 = P_N(x)^2 - 4\Lambda^{2N} = H_{N-n}(x)^2 F_{2n}(x)$$

$$y_m^2 = F_{2n}(x) = W'_{\text{tree}}(x)^2 + f_{n-1}(x)$$

再び、リーマン面!



Double root と Flux の関係

$$P_N(x) + 2\Lambda^N = H_A(x)^2 F_A(x)$$

$$P_N(x) - 2\Lambda^N = H_B(x)^2 F_B(x)$$

$$H_A H_B = H_{N-2n}, \quad F_A F_B = F_{2n}$$

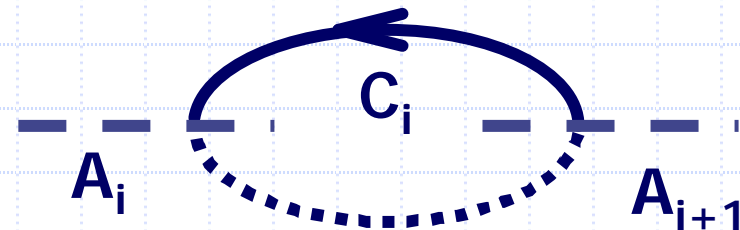
$$T = \left\langle \text{Tr} \frac{1}{x - \Phi} \right\rangle = d \log (P_N + y)$$



Factorization

$U(N_i)$ の q 項

$$\theta_i = \oint_{C_i} T dx$$



U(4) YM: W_{tree} Cubic

$$P_4(x)^2 - 4\Lambda^8 = H_2(x)^2 F_4(x)$$

Solution 1

$$y_m^2 = \left(x^2 + x \frac{\Lambda^4}{a^3} - a^2 \right)^2 - \frac{4\Lambda^4}{a} x - \frac{4\Lambda^8}{a^4}$$

古典極限

- $$\left\{ \begin{array}{l} 1. a \text{ 固定 } L \longrightarrow 0 : U(2) \times U(2) \\ 2. L^4/a^3 \text{ 固定 } L \longrightarrow 0 : U(1) \times U(3) \end{array} \right.$$

Solution 2

$$y_m^2 = (x^2 - ax + b)^2 \pm 4\Lambda^4$$

古典極限

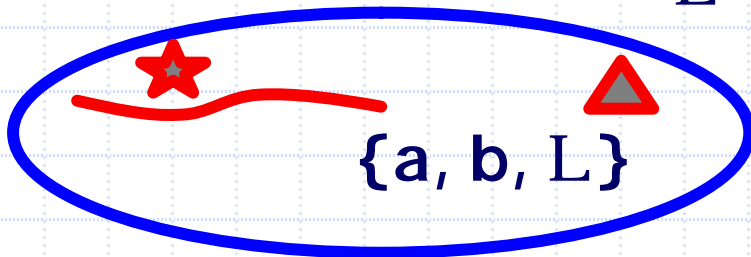
$$L \longrightarrow 0 : U(2) \times U(2)$$

New Duality

$\{g_k, L\}$ を変化させたときに互いに移り合える、異なる半古典理論を互いにDualであると呼ぶ

Solution 2

$L = 0 \quad \mathfrak{U}(2) \times \mathfrak{U}(2)$

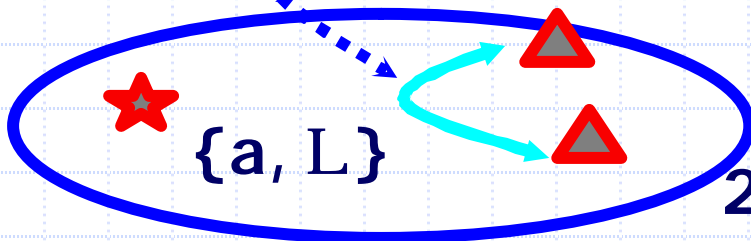


2つの違いは $\{N_i, q_i\}$ によって理解される!

Smooth Interpolation

Solution 1

1. a 固定 $L = 0 \quad \mathfrak{U}(2) \times \mathfrak{U}(2)$

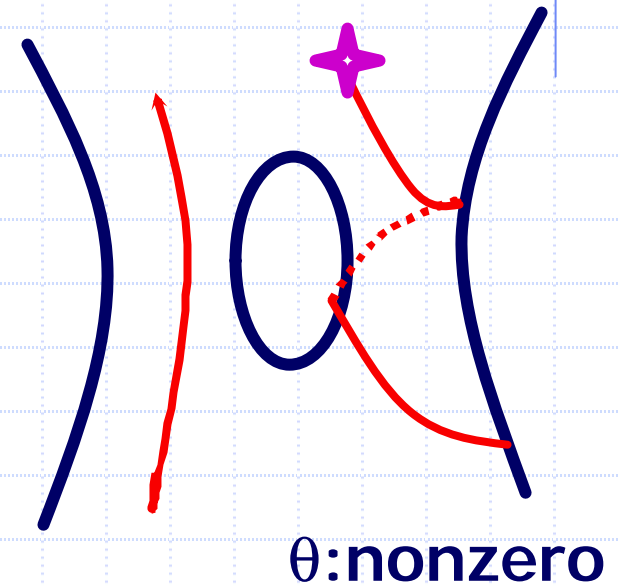
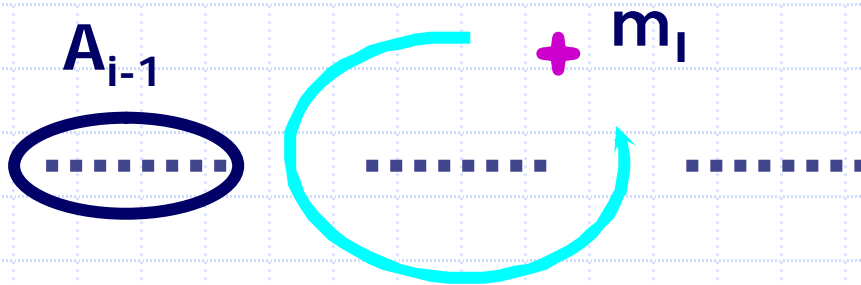


2. L^4/a^3 固定 $L = 0 \quad \mathfrak{U}(1) \times \mathfrak{U}(3)$

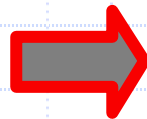
$N=2$ SQCD + W_{tree}

- ◆ Pseudo Confining vacua $Q=0$
- ◆ Higgs vacua $Q \neq 0$

Massive Flavor Singularity



$$q_i \quad q_i + 1$$



$\{N_i, q_i\}$ で Phase を 区別 できない

すべての Phase は スムース に 結ばれている

Massを固定 $W'(-m)=0$

- ◆すべてのmassが等しい
- ◆SO(2N) で W_{tree} が4次の多項式

$$y^2 = P_{2N}(x)^2 - 4x^4 \Lambda^{4N-4-2N_f} (x^2 - m^2)^{N_f}$$

Breaking
pattern

SO(2N)



$$\left\{ \begin{array}{l} U(N) \\ SO(2N_0) \times U(N_1) \\ SO(2N) \end{array} \right.$$

Factrization
From

$$y^2 = \left\{ \begin{array}{l} tF_3(t)[H_{N-2}(t)]^2 \\ F_2(t)[H_{N-1}(t)]^2 \\ tF_1(t)[H_{N-1}(t)]^2 \\ [H_N(t)]^2 \end{array} \right.$$

Over all t^A Factor より $SO(N_0)$ part の相構造が分かる

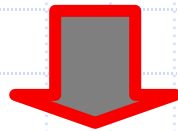
- ◆ A が偶数なら Special branch

Eguchi-Hori-Ito-Yang[9603002]でのクラス1に属し
IRでSupercoformal \wedge

- ◆ A が奇数なら Chebyshev branch

IRでSuperconformal
特性多項式がChebyshev多項式を用いて表せる

Argyres-Douglas Point 江口-菅原[0305050]



N=2 Sup-Conf \longrightarrow New N=1 Sup-Conf

$$P_N(x)^2 - 4\Lambda^{2N} = H_{N-n}(x)^2 [W_{tree}^{1/2}(x) + f_{n-1}(x)]$$

Conclusions

- ◆ Yang-Mills の非摂動的な側面がMatrix Modelの摂動論で理解できる
- ◆ Duality Chain の議論から、 $N=2$ Theory + W_{tree} に対して一般化された小西アノマリーが重要であることが明らかに
- ◆ より一般的な $N=1$ の理論に対して小西アノマリーを用いた議論が現在なされている
- ◆ $N=2 + W_{\text{tree}}$ では相構造を調べるのにSW-CurveのFactrization が有用

紹介できなかったテーマ

- ◆ Super Matrix Model

川合 黒木 森田[0303210]

- ◆ Super Eigenvalue Model

糸山-菅野 [0304148]

- ◆ Gauge群の拡張

Aganagic-Intriligator-Vafa-Warner[0304271], Ahn[0301011, 0301203]

- ◆ Higher ジーナス(重力の効果)

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- ◆ $N=1^*$ Gauge Theory

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