String phenomenology:
heterotic vs intersecting D-brane models

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Here we discuss phenomenological aspects of heterotic string models and intersecting D-brane models. In particular, we consider the structure of Yukawa matrices in heterotic orbifold models and intersecting D-brane models.

1 Introduction

Superstring theory is a promising candidate for unified theory including gravity. Thus, it is very important to derive the standard model from superstring theory, including values of gauge couplings, fermion masses and other phenomenological aspects.

For such purpose, several types of 4D string models have been constructed so far. Recently, much attention has been paid in intersecting D-brane models [1, 2, 3, 4]. Actually, many intersecting D-brane models have been constructed and their phenomenological aspects have been studied.

On the other hand, before discovery of D-branes, heterotic string models have been almost a unique target of phenomenological studies. Indeed, a number of 4D heterotic models have been constructed and several phenomenological studies have been done. As one of recent topics, heterotic orbifold models [5] have been studied to realize interesting field-theoretical orbifold models [6, 7]. In heterotic orbifold models, some modes live in the 10D bulk, and other modes live in the 6D and 4D space-time. Thus, field-theoretical brane-world scenario can be realized within the framework of heterotic orbifold models as well as D-brane models. Hence heterotic orbifold models and intersecting D-brane models have the similarity that string modes corresponding to matter fields are localized in the compact space.

In this talk, we discuss phenomenological aspects of heterotic orbifold models and intersecting D-brane models, comparing each other. In particular, we consider the structure of Yukawa coupling matrices in heterotic orbifold models and intersecting D-brane models. Those are very important to understand fermion masses and mixing angles. Here we concentrate mainly to $D6$-branes on the $T^6/(\mathrm{Z}_2 \times \mathrm{Z}_2)$ with $T^6 = T^2 \times T^2 \times T^2$, although some comments hold true for other types of intersecting D-brane models.

2 4D effective field theory and gauge couplings

In general, massless spectra of 4D string models include gravitons, gauge bosons, matter fermions, Higgs scalars and moduli fields. Their action, i.e. gauge couplings, Yukawa couplings, Kähler potential and so on, can be obtained by the dimensional reduction and CFT calculations, and it is also constrained by (stringy) symmetries. The above couplings as well as kinetic terms depend on vacuum expectation values of dilaton and moduli fields.

Here, we briefly give comments on gauge kinetic functions in heterotic orbifold models and intersecting D-brane models. The experimental values of three gauge couplings of the standard gauge group meet around 2×10^{16} GeV, when the field content above the weak scale is the same as one in the minimal supersymmetric standard model. In $E_8 \times E_8$ heterotic models, gauge bosons are originated from the 10D bulk modes. Thus, gauge couplings as well as gauge kinetic functions are the same in 4D effective theory, even if the standard gauge group, but not a unified gauge group is obtained directly at the compactification scale. That is consistent with the experimental values at the first level. At the next precise level, we must consider the reason of the difference between the string scale and 2×10^{16} GeV.

On the other hand, in intersecting D-brane models, the $SU(3)$ and $SU(2)$ of the standard gauge group can correspond to different D-branes, while $U(1)_Y$ is a linear combination of $U(1)$'s corresponding to several D-branes. In this case, the gauge couplings are obtained as $g_a^2 = g_{D6}^2/V_a$, where V_a is the volume of $D6$-brane in the compact space. Thus, there is no prediction on gauge couplings in generic model. To realize the gauge coupling unification, we have to require $V_{SU(3)} = V_{SU(2)}$.

Whether the gauge kinetic functions are the same or different between $SU(3)$, $SU(2)$ and $U(1)_Y$ is also significant for low-energy supersymmetric models. Different gauge kinetic functions, in general, means non-universal gaugino masses including their CP phases, when moduli fields appearing in gauge kinetic functions contribute to supersymmetry breaking. When gaugino masses are of $O(100)$ GeV, such non-universal CP phases affect significantly experiments concerned about CP violations like electric dipole moments of electron and neutron.

3 Yukawa matrices in heterotic orbifold models

What is the origin of fermion masses and mixing angles is one of important issues in particle physics. They are determined by Yukawa couplings within the framework of the standard model as well as its extension. In a sense, $O(1)$ of Yukawa couplings seem natural. From this viewpoint, how to derive hierarchically suppressed Yukawa couplings is a key-point in understanding the hierarchy of fermion masses and mixing angles.

Yukawa couplings have been studied in several types of 4D string models, that is, selection rules have been investigated and $O(1)$ of Yukawa couplings have been calculated explicitly in many 4D string models. Among them, heterotic orbifold models as well as intersecting D-brane models are interesting, because they lead to suppressed Yukawa couplings depending on moduli. Calculations of such moduli-dependent Yukawa couplings are possible in orbifold models [8, 9], since string theory can be solved on orbifolds. Calculations of Yukawa couplings in intersecting D-brane models are similar to those in heterotic orbifold models [10, 11, 12]. Furthermore, the selection rule due to space group invariance in orbifold models seems unique [8, 13, 14], e.g. compared with Z_N discrete symmetries. It allows non-trivially off-diagonal couplings. Hence, orbifold models have a possibility for leading to realistic mixing angles as well as fermion masses. Therefore, it is important to study systematically the possibility for leading to realistic fermion masses and mixing angles in heterotic orbifold models.

In this talk we study the possibility for predicting a realistic mixing angle as well as mass ratios. We concentrate ourselves mainly to (2×2) sub-matrices of the second and third quark families. We study systematically the possibility for obtaining realistic values of V_{cb} and mass ratios m_c/m_t and m_s/m_b. Then, we will show examples to lead to them. To our knowledge, our result is the first examples, which show explicitly the possibility for predicting realistic values of mixing angles by use of only renormalizable couplings in string models, when we consider a pair of up and down Higgs fields, although already there are proposals to introduce more Higgs fields to lead to realistic Yukawa matrices.

A 6D orbifold is defined as a division of a 6D torus by a discrete twist θ. The 6D $Z_6 - I$ orbifold is a direct product of two 2D Z_6 orbifolds and a 2D Z_3 orbifold. On an orbifold, there are twisted strings, which satisfy the following condition,

$$X^i(\sigma = 2\pi) = (\theta^k X)^i(\sigma = 0) + n_\alpha e^i_\alpha, \tag{1}$$

where e^i_α is the lattice vector defining the 6D torus and n_α are integers. This twisted string belongs to the $\hat{T}_k$ sector. Its center of mass corresponds to the fixed point f on the orbifold, satisfying $f^i = (\theta^k f)^i + n_\alpha e^i_\alpha$. This fixed point f is presented by the corresponding space group element $(\theta^k, n_\alpha e^i_\alpha)$. The fixed points are defined up to the conjugacy class, that is, two fixed points $(\theta^k, n_\alpha e^i_\alpha)$ and $(\theta^k, n'_\alpha e^i_\alpha)$ are equivalent when $n_\alpha e^i_\alpha - n'_\alpha e^i_\alpha = (1-\theta^k)\Lambda$, where Λ denotes the lattice spanned by e^i_α. The number of fixed points, that is, the number of twisted ground states, is determined when we fix an orbifold. For example, the $\hat{T}_1$ of the 2D Z_6 orbifold has a single fixed point, while the $\hat{T}_2$ and $\hat{T}_3$ sectors have three and four fixed points, respectively. All of them are not fixed points under θ, and we have to take linear combinations of the corresponding states.

Next we consider the selection rule for allowed Yukawa couplings. Three states corresponding to three fixed points $(\theta^{k_i}, (1-\theta^{k_i})f_i)$ for $i = 1, 2, 3$ can couple if the product of their space group elements $\prod_i(\theta^{k_i}, (1-\theta^{k_i})f_i)$ is equivalent to identity, up to conjugacy class. For example, the selection rule in Z_3 orbifold models allow only diagonal couplings. On the other hand, non-prime order orbifold models allow off-diagonal Yukawa couplings.

The strength of Yukawa couplings has been calculated by use of 2D conformal field theory. It depends on locations of fixed points. The Yukawa coupling strength of the $\hat{T}_1\hat{T}_2\hat{T}_3$ coupling in Z_6-I orbifold models is obtained for the $G_2 \times G_2$ part as [8, 9, 14]

$$Y = \sum_{f_{23}=f_2-f_3+\Lambda} \exp[-\frac{\sqrt{3}}{4\pi} f_{23}^T M f_{23}], \tag{2}$$

up to an overall normalization factor, where

$$M = \begin{pmatrix} R_1^2 & -\frac{3}{2}R_1^2 & 0 & 0 \\ -\frac{3}{2}R_1^2 & 3R_1^2 & 0 & 0 \\ 0 & 0 & R_2^2 & -\frac{3}{2}R_2^2 \\ 0 & 0 & -\frac{3}{2}R_2^2 & 3R_2^2 \end{pmatrix}, \tag{3}$$

in the $G_2 \times G_2$ root basis. Here, f_2 and f_3 denote fixed points of $\hat{T}_2$ and $\hat{T}_3$ sectors, respectively, and R_i corresponds to the radius of the i-th torus, which can be written as a real part of the i-th Kähler moduli T_i up to a constant factor. The states with fixed points in the same conjugacy class contribute to the Yukawa coupling. Thus, we take summation of those contributions in eq. (2). However, the states corresponding to the nearest fixed points (f_2, f_3) contribute dominantly to the Yukawa coupling for a large value of R_i. Hence, we calculate Yukawa couplings for the nearest fixed points (f_2, f_3). Similarly, the strength of $\hat{T}_2\hat{T}_2\hat{T}_2$ Yukawa couplings is obtained as

$$Y = \sum_{f_{23}=f_2-f_3+\Lambda} \exp[-\frac{\sqrt{3}}{16\pi} f_{23}^T M f_{23}], \tag{4}$$

Class	Q	u	d	H_u	H_d
Assignment 1	$\hat{T}_2$	$\hat{T}_3$	$\hat{T}_3$	$\hat{T}_1$	$\hat{T}_1$
Assignment 2	$\hat{T}_3$	$\hat{T}_2$	$\hat{T}_2$	$\hat{T}_1$	$\hat{T}_1$
Assignment 3	$\hat{T}_2$	$\hat{T}_3$	$\hat{T}_2$	$\hat{T}_1$	$\hat{T}_2$
Assignment 4	$\hat{T}_2$	$\hat{T}_2$	$\hat{T}_3$	$\hat{T}_2$	$\hat{T}_1$
Assignment 5	$\hat{T}_2$	$\hat{T}_2$	$\hat{T}_2$	$\hat{T}_2$	$\hat{T}_2$

表 1: Five classes of Assignments

Class	Q_2, Q_3	u_2, u_3	d_2, d_3	H_u	H_d	$(R_1)^2$	$(R_2)^2$	$\frac{m_c}{m_t}$	$\frac{m_s}{m_b}$	V_{cb}
1	$\hat{T}_2^{(2)}, \hat{T}_2^{(4)}$	$\hat{T}_3^{(3)}, \hat{T}_3^{(2)}$	$\hat{T}_3^{(1)}, \hat{T}_3^{(3)}$	$\hat{T}_1$	$\hat{T}_1$	27.8	107	0.0038	0.029	0.041
2	$\hat{T}_3^{(2)}, \hat{T}_3^{(4)}$	$\hat{T}_2^{(3)}, \hat{T}_2^{(2)}$	$\hat{T}_2^{(1)}, \hat{T}_2^{(3)}$	$\hat{T}_1$	$\hat{T}_1$	24.0	150	0.0038	0.032	0.041
3	$\hat{T}_2^{(1)}, \hat{T}_2^{(4)}$	$\hat{T}_3^{(2)}, \hat{T}_3^{(4)}$	$\hat{T}_2^{(2)}, \hat{T}_2^{(4)}$	$\hat{T}_1$	$\hat{T}_2^{(4)}$	196	316	0.0038	0.019	0.042
4	$\hat{T}_2^{(2)}, \hat{T}_2^{(4)}$	$\hat{T}_2^{(2)}, \hat{T}_2^{(3)}$	$\hat{T}_3^{(1)}, \hat{T}_3^{(4)}$	$\hat{T}_2^{(4)}$	$\hat{T}_1$	416	226	0.0040	0.035	0.035
5	$\hat{T}_2^{(2)}, \hat{T}_2^{(4)}$	$\hat{T}_2^{(2)}, \hat{T}_2^{(4)}$	$\hat{T}_2^{(3)}, \hat{T}_2^{(2)}$	$\hat{T}_2^{(4)}$	$\hat{T}_2^{(4)}$	368	400	0.0038	0.029	0.041
Central values from experiments								0.0038	0.025	0.041

表 2: Realistic examples

where f_2 and f_3 denote two of three fixed points in $\hat{T}_2$ sectors.

Here we study systematically the possibilities for leading to realistic quark masses and the mixing angle for the second and third families in Z_6-I orbifold models by use of the structure of fixed points and the strength of Yukawa couplings explained above. We assume the minimal number of up and down Higgs fields. We concentrate to the mass ratios, m_c/m_t and m_s/m_b, and the mixing angle V_{cb}.

The $\hat{T}_1$ sector has a single relevant state for the $G_2 \times G_2$ part, that is, there is no variety for families. That implies that we have to assign matter fields with $\hat{T}_2$ and $\hat{T}_3$. Hence, we have five classes of assignments, which are shown in Table 1.

We investigate systematically all of these possibilities,varying two independent parameters R_1 and R_2. We find many configurations leading to realistic values of m_c/m_t, m_s/m_b and V_{cb}, which are consistent with experimental values up to $O(1)$ factor. In particular, the numbers of realistic examples in Assignments 1 and 2 are larger than those in other assignments. Here we show one of the best fitting examples in each class of Assignment. Table 2 shows examples leading to realistic values of m_c/m_t, m_s/m_b and V_{cb}. The first column shows the class of Assignments. The second column shows assignments of quarks and Higgs fields with twisted states. The third and fourth columns show the values of R_1^2 and R_2^2 corresponding to the best fit with the experimental values. The last three columns show predicted values of m_c/m_t, m_s/m_b and V_{cb}.

As results, we have found many examples of assignments leading to realistic values of the mass ratios m_c/m_t and m_s/m_b and the mixing angle V_{cb}. (See for detail Ref. [16].)

4 Yukawa matrices in intersecting D-brane models

In intersecting D-brane models, bi-fundamental matter fields appear as open strings stretching D-branes, which intersect each other. Thus, such massless modes are localized at intersecting points of D-branes in the compact space. For example, the quark doublets are localized at intersecting points between $SU(3)$ D-brane and $SU(2)$ D-brane. Furthermore, the family number is obtained as the intersecting number.

Calculation of yukawa couplings in intersecting D-brane models is quite similar to one in heterotic orbifold models. However, the structure of Yukawa matrices depends on D-brane configurations, that is, it is model-dependent. Most of explicit models, which have been constructed so far, seem to lead to the factorizable form, $Y_{ij} = a_i b_j$. That is not realistic, because that is a rank-one matrix, that is, it can derive only non-vanishing mass for the third family, but not non-vanishing values of lighter family masses and mixing angles. We may need alternative flavor structure other than the intersecting number.

A new type of origin for the flavor structure has been proposed in Ref. [17]. In the model, the dynamically generated flavor structure has been studied. For example, its gauge group includes $SU(3) \times SU(2) \times USp(2)^6$, the matter content includes C_α and D_α, where C_α and D_α have $(\mathbf{3},\mathbf{1},\mathbf{2})$ and $(\mathbf{1},\mathbf{2},\mathbf{2})$ representations under $SU(3) \times SU(2) \times USp(2)_\alpha$, respectively, and those are singlets under $USp(2)_\beta$ for $\beta \neq \alpha$. The $USp(2)_\alpha$ gauge coupling is stronger, because the corresponding D-branes have smaller volume than $SU(3)$ and $SU(2)$, and the dynamical scale of $USp(2)_\alpha$ is expected just below the string scale. Then the matter fields C_α and D_α confine, so as to give six generations of quark doublets, $Q_\alpha \sim C_\alpha D_\alpha$ $(\alpha = 1, \cdots, 6)$. The model has two anti-generations, and they have mass terms with Q_α, such that two of six Q_α become quite heavy and four generations remain massless at this level.

Yukawa matrices of the model has been analyzed in Ref. [18]. The detailed analysis of the full Yukawa matrices is rather complicated. For simplicity, we show the (2×2) sub-matrices corresponding to two families of heavier quarks. Those Yukawa matrices are obtained as

$$Y^u \quad \rightarrow \quad \begin{pmatrix} 1 & 0 \\ \varepsilon_1^2 \varepsilon_3 & \varepsilon_1 \varepsilon_3 \end{pmatrix}, \tag{5}$$

for the up sector,

$$Y^d \quad \rightarrow \quad \begin{pmatrix} \varepsilon_1 & 0 \\ \varepsilon_1 \varepsilon_3 & \varepsilon_3 \end{pmatrix}, \tag{6}$$

for the down sector. Here, ε_i denotes the suppression factor estimated as e^{-A_i}, where A_i is the area which string sweep on the i-th torus.

Now we can calculate mass eigenvalues of two heavy modes among four generations, i.e. $m_{u,3}$ and $m_{u,4}$ for the up sector and $m_{d,3}$ and $m_{d,4}$ for the down sector, and their mixing angle V_{34}. The mass ratios and the mixing angle is obtained as

$$\frac{m_{u,3}}{m_{u,4}} \approx \varepsilon_1 \varepsilon_3, \qquad \frac{m_{d,3}}{m_{d,4}} \approx \frac{\varepsilon_3}{\varepsilon_1}, \qquad V_{34} \approx \varepsilon_3, \tag{7}$$

that is, we have the following relation,

$$\sqrt{\frac{m_{u,3}}{m_{u,4}} \frac{m_{d,3}}{m_{d,4}}} \approx V_{34}, \tag{8}$$

at the composite scale.

It is interesting to compare these results with the experimental values of quark mass ratios, $\frac{m_c}{m_t}$ and $\frac{m_s}{m_b}$, and the mixing angle V_{cb}. At the weak scale, the experimental values of mass ratios, $\frac{m_c}{m_t} = 0.0038$ and $\frac{m_s}{m_b} = 0.025$, lead to

$$\sqrt{\frac{m_c}{m_t}\frac{m_s}{m_b}} = 0.01, \tag{9}$$

and the mixing angle is

$$V_{cb} = 0.04. \tag{10}$$

We find that the values of parameters $\varepsilon_1 \sim 0.5$ and $\varepsilon_3 \sim 0.01$ lead to almost realistic structure of quark Yukawa coupling matrices.

We have shown non-vanishing mixing angles are obtained, and in particular we can derive realistic values of mixing angles and mass ratios when we take suitable values of radius parameters. Further detailed study including the lepton sector is interesting.

5 Conclusion

We have discussed some phenomenological aspects in heterotic models and intersecting D-brane models.

Concerned with Yukawa couplings in heterotic orbifold models, we have systematically studied the possibility for leading to realistic values of m_c/m_t, m_s/m_b and V_{cb} in Z_6-I orbifold models. We have found realistic examples of Yukawa matrices. In particular, the classes of Assignments 1 and 2 have many realistic Yukawa matrices. Our result is the first examples to show the possibility for deriving the realistic mixing angle by renormalizable couplings in string models with one pair of H_u and H_d.

To realize our results, the moduli R_1 and R_2 must be stabilized at proper values. How to stabilize these moduli is an important issue to study further.

One can extend our analysis to other non-prime order Z_N orbifold models. Similarly we can discuss $Z_N \times Z_M$ orbifold models. Another important extension is to study the lepton sector. The situation would change for realizing large mixing angles. It is interesting to investigate systematically whether one can obtain realistic lepton masses and mixing angles by renormalizable couplings derived from string models. Such systematical analysis will also be done elsewhere.

Concerned about Yukawa matrices in intersecting D-brane models, there is a serious difficulty to have a realistic Yukawa coupling matrices in the models in which the generation structure of quarks and leptons is originated from the multiple intersection of D-branes. On the other hand, it has been shown that the structure of Yukawa coupling matrices in the models with dynamical generation of Yukawa coupling matrices can be realistic. Indeed, realistic values of the mixing angles V_{cb} and mass ratios m_c/m_t and m_s/m_b can be realized. The most relevant fact is the different origin of the generation. The origin of the generation is not the multiple intersection of D-branes, but many different D-branes with the same multiplicity and the same winding numbers.

Recently, many intersecting D-brane models have been constructed. It is the time to study their phenomenological aspects, e.g. gauge couplings, Yukawa couplings and supersymmetry breaking.

Another recent topic on moduli stabilization and supersymmetry breaking is concerned about string models with flux. That would open a new possibility in string phenomenology.

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