

# Relation between linear and nonlinear supersymmetry in curved spacetime

Saitama Institute of Technology, Motomu Tsuda  
E-mail: [tsuda@sit.ac.jp](mailto:tsuda@sit.ac.jp)

Nonlinear (NL) realization of supersymmetry (SUSY) and NL SUSY action given by Volkov-Akulov (VA) are described in terms of a Nambu-Goldstone (NG) fermion indicating the spontaneous SUSY breaking (SSB). The equivalence of the VA model of NL SUSY to various (renormalizable) linear (L) supermultiplets was shown by many authors, and in the relation between the NL and the L SUSY basic fields in the L supermultiplets are expressed as composites of the NG fermion in SUSY invariant way: Indeed, for  $N = 1$  SUSY it is well known that the VA model is related to the scalar supermultiplet of L SUSY of Wess and Zumino. Also it has been shown that the VA model is related to the gauge supermultiplet of L SUSY with a Fayet-Iliopoulos (FI)  $D$  term indicating SSB, and a  $U(1)$  gauge field is constructed as an axial vector in terms of the NG fermion field. For  $N = 2$  SUSY, we have also shown that a  $U(1)$  gauge field can be constructed as a vector in terms of the NG fermion fields in the linearization of the  $N = 2$  NL SUSY, i.e., we have proved that the  $N = 2$  VA model is equivalent to the  $N = 2$  gauge supermultiplet with  $SU(2) \times U(1)$  invariance, in which L SUSY action possesses FI terms indicating SSB.

Recently, towards a unified theory from the viewpoint of compositeness of matter based on  $SO(10)$  super-Poincaré (SP) symmetry, a new Einstein-Hilbert (EH) type action with a small cosmological constant which describes the (globally) NL SUSY invariant gravitational interaction of NG fermions has been proposed by Shima as a fundamental action of the superon-graviton model (SGM). The EH type action of SGM is constructed by extending the geometrical arguments of general relativity (GR) on Riemann spacetime to new spacetime where the coset space coordinates of  $superGL(4,R)/GL(4,R)$  turning to the NG fermion degrees of freedom are attached at every Riemann spacetime point. Contrary to the simple EH type expression of the action, its expansion in terms of graviton and the NG fermions possesses a very complicated (highly nonlinear) and rich structure, which consists of the EH action of GR, the VA action and their interactions. The SGM action has at least a  $[global\ NL\ SUSY] \otimes [local\ GL(4R)] \otimes [local\ Lorentz] \otimes [global\ SO(10)]$  invariance as a whole which is isomorphic to  $SO(10)$  SP symmetry. In order to obtain the low energy physical contents from such a highly NL theory described by the SGM action, it is inevitable to linearize the (global) NL SUSY in SGM and derive a corresponding L supermultiplet.

In the linearization of the NL SUSY of VA by systematic arguments, it is important to provide a superspace transformation law with a parameter of the NG fermion given by Ivanov-Kapustnikov (IK). It gives superfields which transform homogeneously under supertranslations, and as a result the superfields lead to SUSY invariant relations which connect the L supermultiplets with the VA model of NL SUSY. In SGM, we have recently shown that a new definition of supertranslations of superspace coordinates in curved spacetime gives superfields in terms of the IK superspace transformation law, which transform homogeneously under the new supertranslations. A corresponding L supermultiplet of SGM under the new definition of supertranslations is now under investigations.