# ゲージ理論をめぐって

## — Yang-Mills 50年 —

2004. 7. 14

「場の量子論 2004」

九後 太一

# The Dawning of Gauge Theory    O'Raifeartaigh

**1918** Weyl  Gravitation and Electricity

$$g_{\mu\nu}(x) \rightarrow \lambda(x)\, g_{\mu\nu}(x)$$

Schrödinger '22 '26  $\partial_\mu \rightarrow \partial_\mu - \frac{ie}{\hbar} A_\mu$

London '26  $\psi \rightarrow e^{\frac{ie}{\hbar}\int A}\, \psi$

**1929** Weyl  Electron and Gravitation  Zeit f. Physik (1929)

ゲージ不変性  $e^{ie\lambda(x)}\psi(x)$

$$\Rightarrow \begin{cases} \partial_\mu \rightarrow D_\mu = \partial_\mu - ieA_\mu \\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \end{cases}$$

ゲージ場の存在 minimal int.

kinetic term

35 湯川

**1939** Klein  Conf. report (Kazimierz, Poland 1939)
SU(2) YM

49 朝永-Schwinger-Feynman-Dyson

50 Gupta Bleuler

**1953** Pais  局所 isospin
Pauli  Letters to Pais  SU(2) YM

**1954** Yang-Mills  Phys. Rev. 96 (1954) 191

1955 Shaw (Ph.D Thesis)

1956 内山  Phys. Rev. 101 (56) 1597

1960 南部 Gauge Inv. in Superconductivity

J. J. Sakurai } Massive YM ρ
61 Glashow } W, Z

61 Goldstone

63 Feynman

64
66 Higgs

66 中西-Lautrup
共変ゲージ
不定計量正準理論

67 Kibble

Weinberg-Salam

67 DeWitt
Faddeev-Popov

69 Adler, Bell-Jackiw

1971 't Hooft YMのくりこみ

71 修士入学 T.K.

73 Gross-Wilczek, Politzer
漸近自由性

73 Nakanishi
N-L Zの Higgs 模型

74 't Hooft, Polyakov monopole

75 Nielsen-Olsen vortex
BPZ

75 Becchi-Rouet-Stora BRS変換
Tyutin

78 Kugo-Ojima
$Q_B|phys\rangle = 0$

79 Fujikawa
path-int measure → anomaly

1984 Green-Schwarz

Annomaly cancellation

d=10 SYM SO(32)

$E_8 \times E_8$

1995 Seiberg-Witten $\mathcal{N}=2$ SYM

exact solution

Seiberg $\mathcal{N}=1$ ele-mag duality

Polchinski D-brane

S, T, U duality

M theory

'97 Maldacena AdS/CFT

Gauge Symmetry is not a Symmetry

# Gauge Symmetry とは何?

Is it so powerful?

| Symmetry | Global | Local |
|---|---|---|
| sp. unbroken | • Multiplet str. spectrum<br>• symmetry relations in amplitudes<br>Wigner 相 | |
| sp. broken | • ∃ NG bosons<br>• Low energy theorems<br>NG phase | |

| | Local symmetry |
| --- | --- |
| unbroken | Massless vector 有　Coulomb 相　　colored states Wigner 相と同じ |
| | Massless vector 無　Confinement 相　　color singlets のみ |
| broken | Massless vector 無　Higgs phase　　no charges<br>(if $g$: 小 ⇒ ∃ Massive gauge boson) |

Massless vector 粒子が存在する場合は

Global sym の Wigner phase と同じ（それ以上）が言える.

注意）上の sym は, cov. gauge の際に持る global symmetry

## i) Fradkin-Shenker

No invariant distinction between

Confinement phase $\longleftrightarrow$ Higgs phase

連続的につながる

However, in cov. gauge

unbroken $\longleftrightarrow$ broken

## ii) Higgs 逆定理

Massless vector が無い in $A_\mu^a$

$\Rightarrow$ Color is sp. broken for $1+u \neq 0$ (Higgs phase)

Confinement occurs for $1+u=0$ (Confinement)

$$\langle D_\mu c^a \ \partial_\nu \bar{c}^b \rangle = -\frac{p_\mu p_\nu}{p^2}\delta_{ab}$$

$$\langle D_\mu c^a \ g(A_\nu \times \bar{c})^b \rangle = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) u_{ab}(p^2)$$

Maxwell-YM eq. $\partial^\nu F_{\mu\nu}^a = g J_\mu^a - \{Q_B, D_\mu \bar{c}^a\}$

対偶をとれば

Color is unbroken with $1+u \neq 0$ $\Rightarrow$ Massless vector 存在

i.e. U(1) なら photon 存在
unbroken

Non-Abelian なら

Color unbroken $\exists$ two choice

$$\begin{cases} 1+u \neq 0 & \text{Coulomb} \\ 1+u = 0 & \text{Confinement} \end{cases}$$

$\Leftrightarrow$ 物理的には

Asymptotic non-free $\leftrightarrow$ InfraRed Free $g \to 0$ Coulomb

Asymptotic Free $\leftrightarrow$ InfraRed slavery $g \to \infty$ Confinement

# Is broken gauge symmetry meaningful ?

⇒ An Answer

( Weak coupling and 理論の cutoff scale が大 $\Lambda$ ) の時には意味あり (のみ)

Massive gauge boson が無いと何も言えない

有っても

例えば coupling universality

Pauli term $\bar{\Psi}\sigma^{\mu\nu}\psi F_{\mu\nu}$

[massless だと $p \to 0$ (onshell) 消える]

massive だと on-shell coupling に効く!

→ non-universal

しかし weak coupling なら

$M \sim gv \ll \Lambda$ $\qquad$ $\frac{1}{\Lambda}\underbrace{\bar{\Psi}\sigma^{\mu\nu}\psi F_{\mu\nu}}_{\text{canonical dimension}}$

(くりこみ可能性の理由)

Coupling が大きいと意味がない!?

**系** Massless vector bosons の場合は
universal coupling.
( $p_\mu \to 0$ で on-shell )

逆に,

$p_\mu \to 0$ で non-zero coupling を持つ
spin $j$ ($\geq 1$) massless particle の存在

$\Rightarrow$ ${}^\exists$ conserved charge of rank $j-1$ tensor に couple

spin 1 $\longrightarrow$ scalar charge $Q$
〃 3/2 $\longrightarrow$ spinor charge $Q_\alpha$ (SUSY)
〃 2 $\longrightarrow$ vector charge $P_\mu$

Weinberg, Kugo-Uehara

# Is Confined Gauge Symm Meaningful ?

⇨ Answer よくわからない

我々の $SU(3)_{color}$

$N_c = 3$ is confirmed in

- Drell ratio $R$
- $\pi^0 \to 2\gamma$
- Baryon $qqq$ 3体

etc

whereas in Seiberg's duality

$$SU(N_c) \text{ with } N_f \leftrightarrow SU(\tilde{N}_c) \text{ with } N_f$$

$$N_c + \tilde{N}_c = N_f$$

初命 $g$ 大 ↔ $\tilde{g}$ 小 duality あり

<u>Weak coupling なら意味がある</u> ?

→ しかしこれは無意味

massless particle は特殊

— これに対しては色々な事が言える.

- **Weinberg-Witten の定理**
- **(Dynamical) gauge boson の存在定理**

1) Spin 1 case

$A_\mu$ : auxiliary field  e.g. $\sim \phi^\dagger \overleftrightarrow{\partial_\mu} \phi$ in $CP^{N-1}$

> $\delta A_\mu = \partial_\mu \Lambda(x)$  U(1) gauge sym.
>
> if U(1) : sp. unbroken
>
> $\Rightarrow$ $^\exists$ massless vector in $A_\mu$

(∵) gauge 固定  $\mathcal{L}_{GF} = B\partial A + \frac{1}{2}\alpha B^2$

$\Lambda(x) = a_\mu x^\mu + b$  still sym.

$$\delta A_\nu(x) = [i(a_\mu Q^\mu + bQ), A_\nu] = \partial_\nu \Lambda = a_\nu$$

$$\hookrightarrow \begin{cases} [iQ^\mu, A_\nu] = \delta^\mu_\nu & \to \text{always sp. broken!} \\ [iQ, A_\nu] = 0 & \to \text{U(1) neutral} \end{cases}$$

$$\begin{cases} Q & \text{——} \quad j_\rho \\ Q^\mu & \text{——} \quad j^\mu_\rho = x^\mu j_\rho + \cdots \end{cases}$$ higher moment current

if $\underset{Q}{U(1)}$ is sp. broken, i.e. $j_\rho = v\,\partial_\rho \tilde{\phi} + \cdots$

then
$$\downarrow$$
$$j^\mu_\rho = v'(x^\mu \partial_\rho - \delta^\mu_\rho)\tilde{\phi} + \cdots$$

$\exists\phi \qquad [\phi, \tilde{\phi}] = D$

$$\langle [Q, \phi] \rangle = v$$

$$\langle [Q^\mu, \partial_\rho \phi] \rangle = v' \delta^\mu_\rho$$

$$\hookrightarrow \quad A_\mu = v'^{-1} \partial_\mu \phi$$

一般に lower mom charge が sp. broken

→ higher mom charge も 一般に broken

∴ もし $Q$ is unbroken

$\Rightarrow \quad A_\mu = a_\mu \quad$ : massless vector

no massless scalar ∃ so that

Ferrari - Picasso (1971)

exact masslessness + photon

also proves dynamical generation

## exact masslessness of graviton

GC 不変性を持つ系

$g_{\mu\nu}$ : auxiliary field (でも良い)

$$\mathcal{L}_{GF} = -\sqrt{-g}\, g^{\mu\nu} \partial_\mu B_\nu \quad \text{de Donder gauge}$$

↓ GC のうち

$$\Lambda^\mu(x) = \underset{GL(4)}{\underline{a^\mu{}_\nu}\, x^\nu} + \underset{T_4}{\underline{b^\mu}}$$

$$\delta g_{\rho\sigma} = [i(a^\mu{}_\nu M_\mu{}^\nu + b^\mu P_\mu),\, g_{\rho\sigma}]$$

↳

$$[P_\mu,\, g_{\rho\sigma}] = i\partial_\mu g_{\rho\sigma}$$

$$[M_\mu{}^\nu,\, g_{\rho\sigma}] = i(\delta_\rho^\nu g_{\mu\sigma} + \delta_\sigma^\nu g_{\rho\mu}) + ix^\nu\partial_\mu g_{\rho\sigma}$$

**transl. inv.**
**non-degenerate** $\langle g_{\mu\nu}(x) \rangle \longrightarrow \eta_{\mu\nu}$ by GL(4)

$$\langle [M_\mu{}^\nu,\, g_{\rho\sigma}] \rangle = i(\delta_\rho^\nu \eta_{\mu\sigma} + \delta_\sigma^\nu \eta_{\mu\rho})$$

10 $\quad M_{\{\mu\nu\}} \equiv \frac{1}{2}(M_\nu{}^\rho \eta_{\mu\rho} + M_\mu{}^\rho \eta_{\nu\rho})$

10 out of 16 GL(4) is sp. broken

$P_\mu$ : unbroken $\Rightarrow$ 10 NG sym. tensor

$\exists$ graviton !

massless

$\theta$-vacuum exists ?