Deformed Instantons in $\mathcal{N}=1/2$ Super Yang-Mills Theory from The Super ADHM Construction

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Field theory on non(anti)commutative (NAC) superspace arises in string theory as low energy effective theory on D-branes in the presence of a constant graviphoton field strength background [1]. NAC deformation of 4d $\mathcal{N}=1$ SYM theory is called $\mathcal{N}=1/2$ SYM theory, which is realized by the following star product of $\mathcal{N}=1$ superfields : $f*g=f\exp(P)g$, $P=-\frac{1}{2}\overleftarrow{Q_\alpha}C^{\alpha\beta}\overrightarrow{Q_\beta}$, where Q_α is the (chiral) supersymmetry generator and $C^{\alpha\beta}$ is the NAC parameter. Imaanpur has argued [2] that in this theory the anti-self-dual (ASD) instanton equations should be deformed: $v^{\mathrm{SD}}_{\mu\nu}+\frac{i}{2}C_{\mu\nu}\bar\lambda\bar\lambda=0,\quad \lambda=0,\quad \mathcal{D}_\mu\sigma^\mu\bar\lambda=0,\quad D=0$, where $C_{\mu\nu}\equiv C^{\alpha\beta}\sigma_{\mu\nu\alpha}{}^{\gamma}\varepsilon_{\beta\gamma}$.

By extending the super ADHM construction [3] (see also [4]), we formulate [5] a way to construct all the exact solutions to the deformed ASD equations. Given a connection one-form superfield ϕ, the curvature two-form superfield F can be costructed by $F=d\phi+\phi\overset{*}{\wedge}\phi$, where $\overset{*}{\wedge}$ denotes the deformed wedge product [5]. It turns out that the deformed instanton configurations are equivalent to the curvature F satisfying $F_{\mu\dot\alpha}=0$ and $\star F_{\mu\nu}=-F_{\mu\nu}$, as well as the "Yang-Mills constraints" $F_{\alpha\beta}=F_{\dot\alpha\dot\beta}=F_{\alpha\dot\beta}=0$. Our deformed super ADHM construction gives such curvature superfields. We denote the bosonic and fermionic ADHM data (matrix) as a_α and $\mathcal{M}$ respectively. Define the "zero dimensional Dirac operator" $\hat\Delta_\alpha$ as a chiral superfield extension of the one in the purely bosonic ADHM costruction: $\Delta_\alpha(x)=a_\alpha+x_{\alpha\dot\beta}b^{\dot\beta}\longrightarrow\hat\Delta_\alpha\equiv\Delta_\alpha(y)+\theta_\alpha\mathcal{M}$. With the use of the (normalized) zero mode matrix superfield $\hat v$ of $\hat\Delta_\alpha$ such that $\hat\Delta_\alpha*\hat v=0$, the connection one-form for the deformed instanton is given by $\phi=-\hat v^\ddagger*d\hat v$ as long as the condition $\hat\Delta_{(\alpha}*\hat\Delta^\ddagger{}_{\beta)}=0$ is satisfied ($\ddagger$ is explained in [4]). This condition encodes the bosonic and fermionic ADHM constraints and it is found that the bosonic ADHM constraints are modified by terms proportional to $k\times k$ matrices such as $C^{12}\mathcal{M}\mathcal{M}^\ddagger$. To explore the consequences of the deformation terms in the bosonic ADHM constraints would be interesting, because they may be directly contrasted with consequences of noncommutative space by NS-NS B field.

References

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