Worldwide Wall Webs

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Domain walls are the simplest topological defects and have repeatedly been studied in many area of the physics such as particle physics, cosmology and condensed matter physics. Espceially, it is well known that there exists an intimate connection between supersymmetry and solitons since BPS solitons preserves a part of supersymmetry. While 1/2 BPS instantons, 1/2 BPS monopoles and 1/2 BPS vortices have intensively been studied since many years ago, the history of the 1/2 BPS domain walls is not so old. Recently the 1/2 BPS domain walls were investigated in $\mathcal{N} = 2$ supersymmetric Abelian gauge thoery with $N_{\rm F}$ hypermultiplet in the fundamental representation [1]. In that model there exist $N_{\rm F}$ descrete vacua, then parallel $N_{\rm F} - 1$ domain walls interpolating these vacua coexist as the 1/2 BPS states. The total moduli space of the 1/2 BPS domain walls was found to be isomorphic to the complex projective space $\mathbf{C}P^{N_{\rm F}-1}$. While the parallel walls preserve a half of the supersymmetry, the walls with angles preserves a quater of the supersymmetry and make junctions of the walls. When there exist lots of non-parallel walls, they develop the 1/4 BPS webs of the domain wall junctions. The BPS equations are [2]

$$F_{12} = 0,\ \partial_1\Sigma_2 - \partial_2\Sigma_1 = 0,\ \mathcal{D}_\alpha H = HM_\alpha - \Sigma_\alpha H,\ \partial_1\Sigma_1 + \partial_2\Sigma_2 = (g^2/2)\left(c - HH^\dagger\right),$$

where Σ_α ($\alpha = 1, 2$) are complex scalars in the vector multiplet and H is $N_{\rm F}$ component hypermultiplet complex scalar with the FI parameter c, the gauge coupling g and M_α is the diagonal mass matrix. The first three equations are solved as

$$H = S^{-1}(x^1, x^2)H_0 e^{M_1x^1+M_2x^2}, \quad W_\alpha - i\Sigma_\alpha = -iS^{-1}(x^1, x^2)\partial_\alpha S(x^1, x^2).$$

Here H_0 is a $N_{\rm F}$ component constant matrix which we call moduli matrix since all the constant parameters in this matrix are moduli paramters. For a given moduli matrix $S(x^1, x^2)$ is determined by the last of the above BPS equations. Clearly there exist an equivalence relation $(H_0, S) \sim V(H_0, S)$ ($V \in \mathbf{C}$) which does not affect the physical configurations by definition. Thus we find the total moduli space of the 1/4 BPS webs of walls is $H_0 \sim VH_0$, namely this corresponds to the complex projective space $\mathbf{C}P^{N_{\rm F}-1}$ which agree with that of the 1/2 BPS parallel walls. We study the 1/4 BPS webs of walls by using the moduli matrix H_0 and find very complicated configurations of domain walls which can include any number of the external legs and internal loops. We also find that the property of the 1/4 BPS webs of walls is quite similar to that of the (p, q)-string/5brane webs in the superstring theory.

[1] Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, Phys. Rev. Lett. **93** (2004) 161601
[2] M. Eto, Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, "Webs of walls," arXiv:hep-th/0506135.