Classical Simulation of Quantum Fields

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This talk is based on my recent two works [1]. In this talk we study classical field theories in a background where all modes are excited, matching the zero point energy spectrum of quantum field theory. Then we show that even though this is a classical theory, it contains quantum like effects which correspond to loop effects in quantum field theory. When we obtain the solution of equation of motion perturbatically, we have the following form.

$$\begin{aligned}\phi(x) &= \phi_0(x) + \frac{i}{\hbar}\int dy D_P(x-y)\mathcal{L}_i'(\phi_0(y)) \\ &\quad -\frac{1}{\hbar^2}\int dy_1 dy_2 D_P(x-y_1)D_P(y_1-y_2)\mathcal{L}_i''(\phi_0(y_1))\mathcal{L}_i'(\phi_0(y_2)) + \cdots, \end{aligned} \tag{1}$$

where $\phi_0(x)$ is the background configulation, D_P is the Green's function which we take Wheeler-Feynman propagator (the off-shell piece of Feynman propagator), and $\mathcal{L}_i$ is the interaction part of Lagrangian. Since the phase of each mode is an integration constant (each amplitude is fixed to give $\hbar\omega/2$ energy), we take the average over each phase factors. Then we obtain, for example, $\langle\phi_0(x)\phi_0(y)\rangle = D_\delta(x-y)$, where we denote the averaged values by $\langle\cdot\rangle$ and D_δ is the on-shell piece of Feynman propagator. Then two point function $\langle\phi(x)\phi(y)\rangle$ includes loop graphs and we can compare them to Feynman graphs of the corresponding quantum field theory. For the case of ϕ^4 theory, we have recovered the one-loop effect and some pieces of two loop graphs are missing. In order to study the difference caused by missing graphs, we study this classical theory numerically on lattice. We successfully extracted the two loop effects, but the difference from quantum field theory was within the numerical error. We also recovered the almost same critical value in ϕ^4 theory numerically, although we would expect the difference is large in strong coupling. Since the numerical simulation of this classical theory uses only a tiny fraction of CPU time (roughly 10^{-4} compared with Monte Carlo simulation of quantum theory), this classical theory might be quite powerful to compute quantum effects in various theories including QCD.

References

[1] T. Hirayama, B. Holdom, Classical Simulation of Quantum Fields I, hep-th/0507126.
T. Hirayama, B. Holdom, R. Koniuk, T. Yavin, Classical Simulation of Quantum Fields II, hep-lat/0507014.

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