Scalar Laplacian on Sasaki-Einstein Manifolds $Y^{p,q}$ [1]

Osaka City University Advanced Mathematical Institute: Hironobu Kihara
E-mail: kihara@sci.osaka-cu.ac.jp

We study the spectrum of the scalar Laplacian on the five-dimensional toric Sasaki-Einstein manifolds $Y^{p,q}$, which is topologically $S^2 \times S^3$. The eigenvalue equation reduces to Heun's equation, which is a Fuchsian equation with four regular singularities. We show that the ground states, which are given by constant solutions of Heun's equation, are identified with BPS states corresponding to the chiral primary operators in the dual quiver gauge theories. The excited states correspond to non-trivial solutions of Heun's equation. It is shown that these reduce to polynomial solutions in the near BPS limit.

The metric tensor of $Y^{p,q}$ parameterized by two positive integers p, q $(p > q)$ is written as [2]

$$\begin{aligned} ds^2 &= \frac{1-y}{6}\left(d\theta^2 + \sin^2\theta d\phi^2\right) + \frac{1}{w(y)q(y)}dy^2 + \frac{q(y)}{9}(d\psi - \cos\theta d\phi)^2 \\ &\quad + w(y)\left[d\alpha + f(y)(d\psi - \cos\theta d\phi)\right]^2 . \end{aligned} \tag{1}$$

There are three Killing vectors, ∂_α, ∂_ψ and ∂_ϕ, which commute each other. The eigenvalue equation, $\Box\Psi = -4\lambda(\lambda+2)\Psi$, for the scalar Laplacian, $\Box = \frac{1}{\sqrt{g}}\partial_\mu\sqrt{g}g^{\mu\nu}\partial_\nu$, reduces to two linear ordinary differential equations owing to the isometry. The eigenvalues of $Q_R = 2\partial_\psi - 1/3\partial_\alpha$, ∂_ψ and ∂_α are identified with the $\mathcal{R}$-charge, spin and baryon number of the mesons, $\mathcal{S}$, $\mathcal{L}_\pm$, in the dual quiver gauge theory, [3].

The two equations are Gauss's hypergeometric differential equation for θ and Heun's differential equation for y. The first equation has regular solutions described by Jacobi polynomials. Heun's equation is a Fuchsian type equation with four regular singularities.

For Heun's type equation for y, we find two families of solutions with eigenvalues $\lambda = \frac{3}{4}Q_R, \frac{3}{4}Q_R + 1$. For $\lambda = \frac{3}{4}Q_R$ the eigen functions are constant and for $\lambda = \frac{3}{4}Q_R + 1$ those are polynomials with degree one. Let $S, L_\pm$ be $\mathcal{R}$-charges for $\mathcal{S}, \mathcal{L}_\pm$ respectively, then the charges for multi meson, Q_R are $\mathcal{A}S + \mathcal{B}L_+ + \mathcal{C}L_-$, where $\mathcal{A}, \mathcal{B}, \mathcal{C}$ are non-negative integers.

REFERENCES

[1] H. Kihara, M. Sakaguchi and Y. Yasui, Phys. Lett. B **621**, 288 (2005) [arXiv:hep-th/0505259].
[2] J. P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, arXiv:hep-th/0403002.
[3] S. Benvenuti, S. Franco, A. Hanany, D. Martelli and J. Sparks, arXiv:hep-th/0411264;

[1] joint work with Makoto Sakaguchi(Okayama Institute for Quantum Physics) and Yukinori Yasui(Osaka City University)[1]