Localization on D-brane and Gauge theory/Matrix model

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In this talk, I talked about the relation between the instanton counting of four-dimensional $\mathcal{N}=2$ supersymmetric gauge theories [1] and two-dimensional (bosonic) gauge theories based on the work [2].

Let us consider Type IIB superstring theory on $\mathbb{R}^{1,3}\times\mathbb{C}\times\mathcal{M}_4$, where $\mathcal{M}_4$ is a four-dimensional ALE space. If N_c D5-branes are wrapped on a 2-cycle Σ in $\mathcal{M}_4$, 4D $SU(N_c)$ $\mathcal{N}=2$ SYM theory appears on the extra $\mathbb{R}^{1,3}$ space on the D5-branes except for the compactified 2-cycle. We also introduce a noncommutativity to the $\mathbb{R}^{1,3}$ space-time, which does not affect the instanton contribution to the prepotential of the $\mathcal{N}=2$ SYM theory. According to the concept of the large N reduction and the noncommutativity, the gauge theory on the D5-branes reduces to a two-dimensional large N topological field theory on the internal 2-cycle.

By evaluating the vacuum expectation value of an operator which couples to D-instanton on the D-strings and taking into account the moduli parameters of the 4D SYM, we found that we obtain the partition function of the two-dimensional generalized Yang-Mills theory [2],

$$Z=\sum_{\{n_i\}}\prod_{1\le i<j\le N}\left(g_s n_i-g_s n_j\right)^2 e^{-\frac{A}{g_s}\sum_i W(g_s n_i)}. \tag{1}$$

If we regard $\{n_i\}$ as "positions" of free fermions, any configuration of $\{n_i\}$ can be thought to be an excitation state from the "vacuum state" that is defined by the densest state around the critical points of the potential $W(x)$. As a result, there appear two fermi surfaces in this system. In addition, we take a large N limit by fixing the combination, $\left(\frac{g_sN}{2N_c}\right)^{2N_c}e^{-A\mu\left(\frac{g_sN}{2N_c}\right)^{N_c}}\equiv\Lambda^{2N_c}$. As a result, we can rewrite (1) as $Z_{gYM_2}=Z_{\rm Nek}\left(\boldsymbol{a};\Lambda\right)^2$, where $Z_{\rm Nek}\left(\boldsymbol{a};\Lambda\right)$ is so-called Nekrasov's partition function for 4D $\mathcal{N}=2$ SYM [1].

We also showed that the similar partition functions for $cN=2$ theory with hypermultiplets can be obtained from suitably deformed two-dimensional theories. As an important by-product, we derived the instanton partition function for $\mathcal{N}=2$ A_2-type quiver gauge theory.

References

[1] N. Nekrasov and A. Okounkov, arXiv:hep-th/0306238.

[2] S. Matsuura and K. Ohta, arXiv:hep-th/0504176.