Supersymmetry in gauge theories with extra dimensions

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Much attention has been paid recently to gauge theories with extra dimensions to explore new possibilities for gauge symmetry breaking and solving the hierarchy problem without introducing additional Higgs fields.

In [1], we have investigated gauge invariant theories with extra dimensions and observed the quantum-mechanical $N = 2$ supersymmetric structure between 4d and extra-space components of gauge fields. However, the quantum-mechanical supersymmetry which the whole action possesses is not mentioned in the paper.

In this talk, we show that the higher dimensional gauge theory possesses the quantum-mechanical supersymmetry and the supersymmetry is a part of the higher dimensional gauge symmetry. Let us consider a $4+1$-dimensional Abelian gauge theory with a curved extra dimension. The metric is assumed to be of the form $ds^2 = \Delta(y)^2 \left(\eta_{\mu\nu}dx^\mu dx^\nu + dy^2\right)$. The action we consider is

$$S = \int d^4x \int dy \sqrt{-g} \left(-\frac{1}{4} g^{MM'} g^{NN'} F_{M'N'} F_{MN}\right) = \int d^4x \int dy \ \mathcal{L} \tag{1}$$

where $F_{MN}(x,y) = \partial_M A_N(x,y) - \partial_N A_M(x,y)$. We adopt the gauge fixing as $\partial^\mu A_\mu + \frac{1}{\Delta}\partial^y \Delta A_y = 0$ and decompose the 4d component of gauge field into the longitudinal mode and transverse one such as $A_\mu = A^L_\mu + A^T_\mu$ with $\partial^\mu A^T_\mu = 0$, so that the lagrangian can be written as

$$\mathcal{L} = \frac{1}{2}\Delta(A^y \ \rho) \begin{pmatrix} \partial_\mu\partial^\mu & 0 \\ 0 & \partial_\mu\partial^\mu \end{pmatrix} \begin{pmatrix} A_y \\ \rho \end{pmatrix} - \frac{1}{2}\Delta(A^y \ \rho) H \begin{pmatrix} A_y \\ \rho \end{pmatrix} + \frac{1}{2} A^{\mu T}(\partial_\nu\partial^\nu + \Delta^{-1}\partial_y \Delta \partial_y) A^T_\mu \tag{2}$$

where

$$H = \begin{pmatrix} -\partial_y \Delta^{-1} \partial_y \Delta & 0 \\ 0 & -\Delta^{-1}\partial_y \Delta \partial_y \end{pmatrix}, \quad A^L_\mu \equiv \frac{1}{\sqrt{\partial_\nu \partial^\nu}} \partial_\mu \rho. \tag{3}$$

Here, we assumed that the surface terms vanish. The lagrangian (2) is invariant under the supersymmetry transformation

$$\begin{pmatrix} A_y \\ \rho \end{pmatrix} \to \begin{pmatrix} A'_y \\ \rho' \end{pmatrix} = e^{i\theta Q} \begin{pmatrix} A_y \\ \rho \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & i\partial_y \\ i\Delta^{-1}\partial_y \Delta & 0 \end{pmatrix} \text{ with } Q^2 = H, \tag{4}$$

because of $[Q, H] = 0$. Here, θ is a bosonic parameter and the gauge fixing condition is also invariant under the supersymmetry transformation using the equation of motion.

The transformation can be regarded as a part of the 5d gauge transformation, since the infinitesimal transformation of (4) is written by $\boldsymbol{\delta A_M = -\theta \partial_M \rho}$. Thus, we can define the supersymmetry transformation of matter fields in any gauge invariant theories with matter fields.

Reference

[1] C.S. Lim, T. Nagasawa, M. Sakamoto and H. Sonoda, Phys. Rev. **D72**(2005)064006, hep-th/0502022.

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