Auxiliary Field Method in a Quantum Mechanical Four-Fermi Model

Tomohiko Sakaguchi (Kyushu Univ.) and Taro Kashiwa (Ehime Univ.)
E-mail: tomohiko@higgs.phys.kyushu-u.ac.jp

In our previous studies [1, 2], we have found that we can improve its results by carrying out the loop expansion in the auxiliary field to higher orders even if the loop expansion parameter $(1/N, \hbar)$ is not small. Being based on these results, we investigate validity of the auxiliary field method in a Quantum Mechanical Four-Fermi Model, whose Hamiltonian is given by

$$H(\hat{\boldsymbol{a}}, \hat{\boldsymbol{a}}^\dagger) = -\omega \hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{a}}^\dagger + \frac{\lambda^2}{2M} \sum_{i,j=1}^{M} \hat{a}_i \hat{a}_j \hat{a}_j^\dagger \hat{a}_i^\dagger \ , \quad \hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{a}}^\dagger = \sum_{i=1}^{M} \hat{a}_i \hat{a}_i^\dagger \ ,$$

$$\{\hat{a}_i, \hat{a}_j^\dagger\} = \delta_{ij}, \ \ \{\hat{a}_i, \hat{a}_j\} = \{\hat{a}_i^\dagger, \hat{a}_j^\dagger\} = 0 \ \ (i, j = 1, 2, \cdots, M) \ .$$

In this talk, especially, we take notice of the difference of obtained results between the discrete time and continuum time formalism (Fig. 1). As a result, in the discrete case, as the degree of the loop expansion increases the ground state energy is improved (see the left figure). Whereas, in the continuum case, the ground state energy is not improved because the higher order terms do not contribute at all (see the right figure).

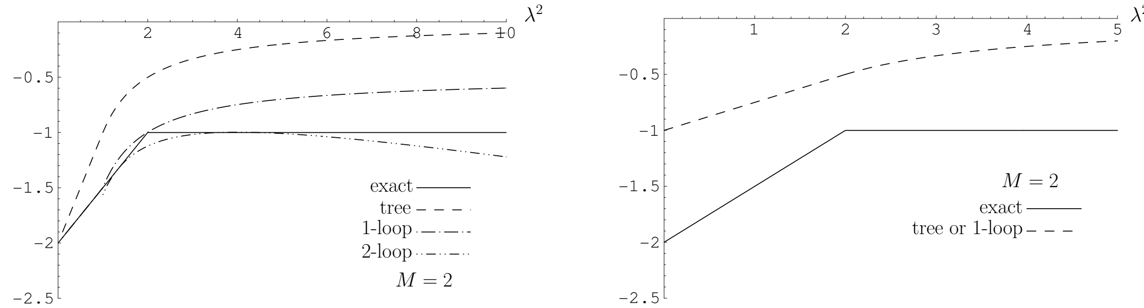

Figure 1: Four-Fermi coupling dependence of the ground state energy at $\omega = 1$ and $M = 2$: the left and right figures represent the result of the discrete and continuum case, respectively. The solid curve stands for the exact ground state energy and the other curves stand for the approximations of the ground state energy.

References

[1] T. Kashiwa, Phys. Rev. **D59**, 085002 (1999).

[2] T. Kashiwa and T. Sakaguchi, Prog. Theor. Phys. Vol.110, No.3, 589 (2003). T. Kashiwa and T. Sakaguchi, Phys. Rev. **D68**, 065002 (2003).