N.C.Supersymmetric Gauge Theories and their Unified perspective

Department of Mathematics, Faculty of Science and Technology, Keio University

3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522, Japan.

Akifumi.Sako

E-mail: sako@math.keio.ac.jp

We investigated cohomological gauge theories in noncommutative $\mathbb{R}^{2D}$ (N.C.$\mathbb{R}^{2D}$). We showed that vacuum expectation values of the theories do not depend on noncommutative parameters, and the large noncommutative parameter limit is equivalent to the dimensional reduction. From these facts, following nature of the partition functions are derived.

Claim

Let Z_{2D} and $\langle O \rangle_{2D}$ be a partition function and vacuum expectation value of O of a cohomological field theory in N.C.$\mathbb{R}^{2D}$ with $D \geq 1$ such that $\delta_\theta Z_{2D} = 0$ and $\delta_\theta \langle O \rangle_{2D} = 0$. Here, zero mode integrals and contributions from BPS solutions that become singular at large noncommutative parameter limit are removed from the path integral of Z_{2D} and $\langle O \rangle_{2D}$. Let Z_{2D-2} and $\langle O \rangle_{2D-2}$ be the partition function and vacuum expectation value of O of a noncommutative cohomological field theory in N.C. $\mathbb{R}^{2D-2}$, where they are given by dimensional reduction of Z_{2D} and $\langle O \rangle_{2D}$. Then,

$$Z_{2D} = Z_{2D-2} \quad , \quad \langle O \rangle_{2D} = \langle O \rangle_{2D-2} \ , \tag{1}$$

i.e. the partition functions of such theories do not change under dimensional reduction from $2D$ to $2D - 2$.

Therefore, we find several partition functions of supersymmetric gauge theories in various dimensions are equivalent. Using this technique, we determine the partition function of the $\mathcal{N} = 4$ U(1) gauge theory in noncommutative $\mathbb{R}^4$, where its action does not include a topological term.

$$Z_{\mathcal{N}=4}^{4dim} = \frac{\pi^2}{6}. \tag{2}$$

The result is common among (8-dim , $\mathcal{N} = 2$), (6-dim , $\mathcal{N} = 2$), (2-dim , $\mathcal{N} = 8$) and the IKKT matrix model given by their dimensional reduction to 0-dim.

References

[1] A.Sako, *Noncommutative Cohomological Field Theories and Topological Aspects of Matrix models*, `hep-th/0312120` .

[2] A.Sako, T.Suzuki , *Partition functions of Supersymmetric Gauge Theories in Noncommutative $\mathbb{R}^{2D}$ and their Unified Perspective*, `hep-th/0503214`.