New Formulation of Massive Spin Two Field Theory — Resolution of vDVZ Discontinuity Problem

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We study vDVZ problem of the Fierz-Pauli theory via one parameter (a) extension of its mass term.

$$\mathcal{L}_a = \mathcal{L}_{\rm EH}^{(2)} - \frac{m^2}{2}(h_{\mu\nu}h^{\mu\nu} - ah^2), \tag{1}$$

$a = 1$ case is Fierz-Pauli one. In starting with this Lagrangian, we present a new formulation of the massive spin two model based on two kind of BRS invariance. One is a vector BRS

$$\delta h_{\mu\nu} = \partial_\mu C_\nu + \partial_\nu C_\mu, \quad \delta\theta_\mu = C_\mu, \quad \delta C_\mu = 0, \quad \delta\bar{C}^\mu = iB^\mu, \quad \delta B^\mu = 0, \quad \text{others} = 0, \tag{2}$$

which is corresponding to G.C.T. As mass term breaks G.C.T. invariance, vector BRS invariance is realized by Stueckelberg technic $h_{\mu\nu} \to h_{\mu\nu} - \frac{1}{m}(\partial_\mu\theta_\nu + \partial_\nu\theta_\mu)$ and by using Izawa's BRS procedure for generalized field transformation. Another BRS is a scalar BRS invariance which is corresponding to some kind of Deser-Nepomechie-Waldron-type gauge invariance;

$$\begin{aligned} \delta^S h_{\mu\nu} &= \partial_\mu\partial_\nu C - \eta_{\mu\nu}\left[b\Box + cm^2\right]C, \quad \delta^S\theta_\mu = \frac{m}{2}\partial_\mu C, \\ \delta^S C &= 0, \quad \delta^S\bar{C} = iB, \quad \delta^S B = 0, \quad \text{others} = 0 \end{aligned} \tag{3}$$

In our model, Fierz-Pauli field is defined by $H_{\mu\nu} = h_{\mu\nu} - \frac{1}{m}(\partial_\mu\theta_\nu + \partial_\nu\theta_\mu)$. $H_{\mu\nu}$ is a vector BRS invariant operator. But, its not invariant under scalar BRS. Full BRS invariant operator is given by the trace part $\bar{H}_{\mu\nu} = H_{\mu\nu} - \frac{1}{d}\eta_{\mu\nu}H$. Therefore, trace part H is unphysical, which should be removed by physical state condition.

General BRS invariant Langangian is given by

$$\begin{aligned} \mathcal{L}_{BRS} &= \mathcal{L}_a + m[h_{\mu\nu}(\partial^\mu\theta^\nu + \partial^\nu\theta^\mu) - 2ah\partial^\mu\theta_\mu] - \frac{1}{2}[(\partial_\mu\theta_\nu - \partial_\nu\partial_\mu)^2 - 4(a-1)(\partial^\nu\theta_\nu)^2] \\ &\quad + 2(\partial^\mu h_{\mu\nu} - a'\partial_\nu h + \alpha' m\theta_\nu)B^\nu - \alpha B^\mu B^\nu + 2i\bar{C}^\nu[(\Box + \alpha' m^2)C_\nu + (1-2a')\partial_\nu\partial^\mu C_\mu], \\ &\quad + m^2(h - \frac{2}{m}\partial^\mu\theta_\mu)B + \frac{\alpha'' m^2}{2}B^2 + im^2 d\bar{C}\left[b\Box + cm^2\right]C, \end{aligned} \tag{4}$$

We here adopt the following gauge conditions for vector and scalar gauge invariance;

$$\partial^\mu h_{\mu\nu} - a'\partial_\nu h + \alpha' m\theta_\mu = 0, \quad H = 0,$$

Here, α, α', a' and α'' are gauge parameters. d is a space-time dimension. For the sake of scalar BRS invariance, one can expect that our model contrains correct five degree of freedom that is expected in massive spin two field theory. (In $a' = 1/2$, $\alpha = \alpha' = 1$ case, minxing term between $h_{\mu\nu}$ and h vanishes. In $a' = 1$, scalar BRS invariance is manifest.)

Massless limit of our model in $a \neq 1$ is a graviton theory which is described by the massless dipole propagator of spin two, that dependes on both a' and α. Unfortunately, $a = a' = 1$ case, there still remains some problem.