String cosmology and the index of the Dirac operator

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Strings and Fields

YITP, August 20 2005

Outline

- String Cosmology, Flux Compactification, Stabilization of Moduli, Metastable de Sitter Space, KKLT construction

- When stabilizing instanton corrections are possible?
  How fluxes affect the standard condition ?

$$W_{inst} = Ae^{-(\text{Vol}+i\alpha)}$$

- Index of a Dirac operator on Euclidean M5 brane and D3 brane with background fluxes:

  general condition for existence of instanton corrections

- An example of fixing of all moduli: M-theory on $K3 \times K3$ and IIB on $K3 \times T^2/Z_2$: D3/D7 cosmological model

Work with **Aspinwall, Bergshoeff, Kashani-Poor, Sorokin,Tomasiello**

hep-th/0501081, hep-th/0503138, hep-th/0506014, hep-th/0507069

Our Universe is an Ultimate Test of Fundamental Physics

- High-energy accelerators will probe the scale of energies way below GUT scale
- Cosmology and astrophysics are sources of data in the gravitational sector of the fundamental physics (above GUT, near Planck scale)

Impact of the discovery of the current acceleration of the universe

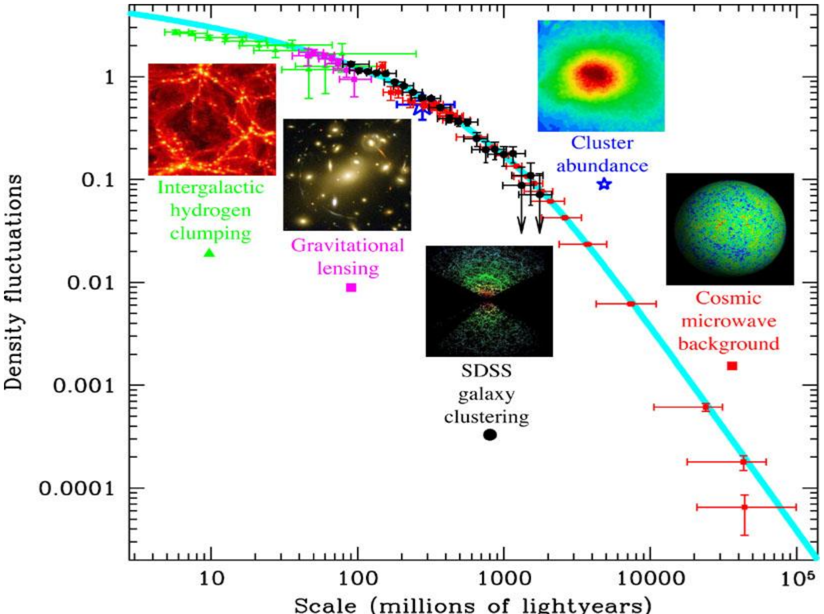
Until recently, string theory could not describe **acceleration of the early universe** (inflation)

The discovery of **current acceleration** made the problem even more severe, but also helped to identify the root of the problem

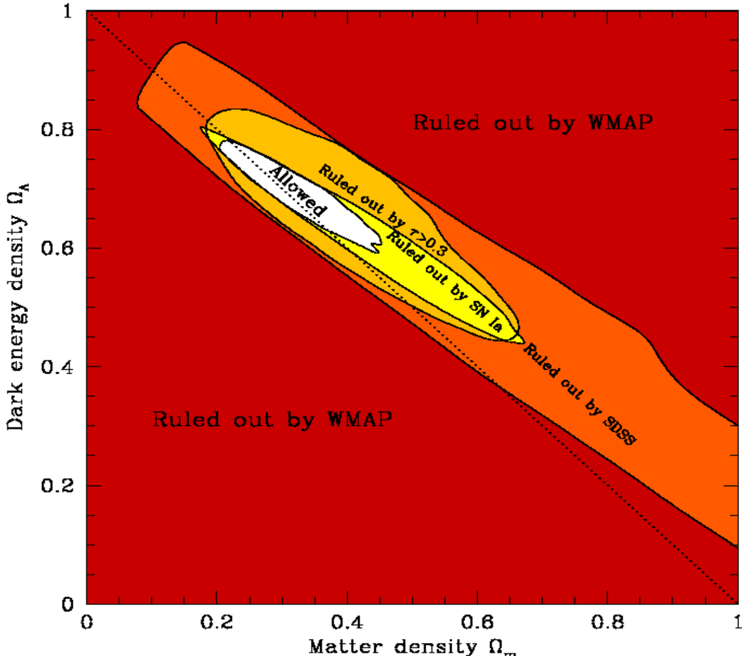
String Theory and Cosmology

How to get the 4d near de Sitter and/or de Sitter space from the compactified 10d string theory or 11d M-theory?

$$H_{infl} \leq 10^{-5} M_p$$



$$H_{accel} \sim 10^{-60} M_P$$



No-Go Theorems for 4d de Sitter Space
from 10/11d string/M theory

- Gibbons **1985**
- de Wit, Smit, Hari Dass, **1987**
- Maldacena, Nunez, **2001**

How to go around the conditions for de Sitter no-go theorems?

How to perform a compactification from 10/11 dimensions to 4 dimensions and stabilize the moduli?

First proposal

Towards cosmology in type IIB string theory

Dilaton and complex structure stabilization **Giddings, Kachru and Polchinski**

Volume stabilization, <u>KKLT construction of de Sitter space</u>

Kachru, R.K., Linde, Trivedi

2003

Maloney, Silverstein, Strominger,

in non-critical string theory

Kachru, R. K., Maldacena, McAllister, Linde, Trivedi

INFLATION

The KKLMMT model

FLUX COMPACTIFICATION and MODULI STABILIZATION

FLUXES $\Rightarrow$ small numbers in string theory for cosmology

- **Best understood example: resolved conifold**

$$w_1^2 + w_2^2 + w_3^2 + w_4^2 = z$$

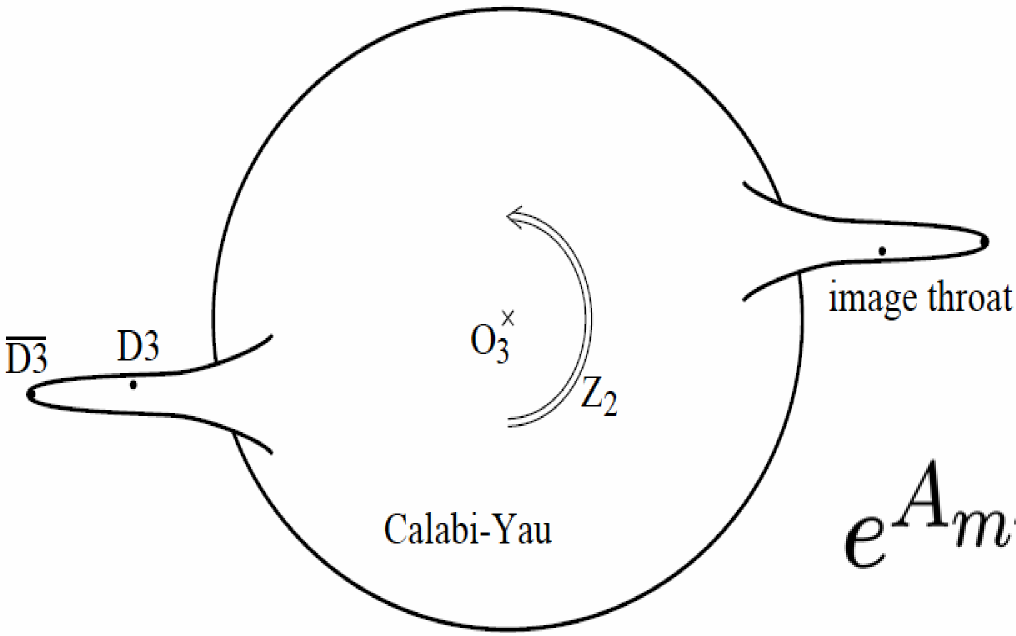
K and M are integer fluxes

$$\int F_{(3)} = 2\pi M$$

$$\int H_{(3)} = -2\pi K$$

$$z \sim e^{-\frac{2K}{Mgs}}$$

The throat geometry has a highly warped region



$$e^{A_{min}} \sim z^{1/3} \sim e^{-\frac{2\pi K}{3Mg_s}}$$

$$ds^2 = e^{2A(y)} ds_4^2 + ds_y^2 \qquad e^{2A} \ll 1$$

Redshift in the throat

Flux compactification and moduli stabilization in IIB string theory (supergravity + local sources)

The potential with respect to dilaton and volume is very steep, moduli run down and V vanishes, the space tends to decompactify to 10d and string coupling tends to vanish, unless both are stabilized at some finite values.

Dilaton stabilization

$$G_3 = F_3 - \tau H_3$$

$$\tau = C_0 + ie^{-\phi}$$

$$\tilde{F}_5 = F_5$$

Warping fixed by local sources (tadpole condition) and non-vanishing ISD fluxes

$$*G_3 = iG_3$$

This equation fixes the shape of CY and the dilaton-axion

4d description

* SUSY at scale 1/R(CY) $\longrightarrow$ N=1 effective action
* Specify the Kahler potential, superpotential and gauge couplings

$$K = -3\ln[-i(\rho-\bar\rho)] - \ln[-i(\tau-\bar\tau)]$$
$$-\ln[-i\int_M \Omega\wedge\bar\Omega]$$

- **Add the superpotential due to fluxes**

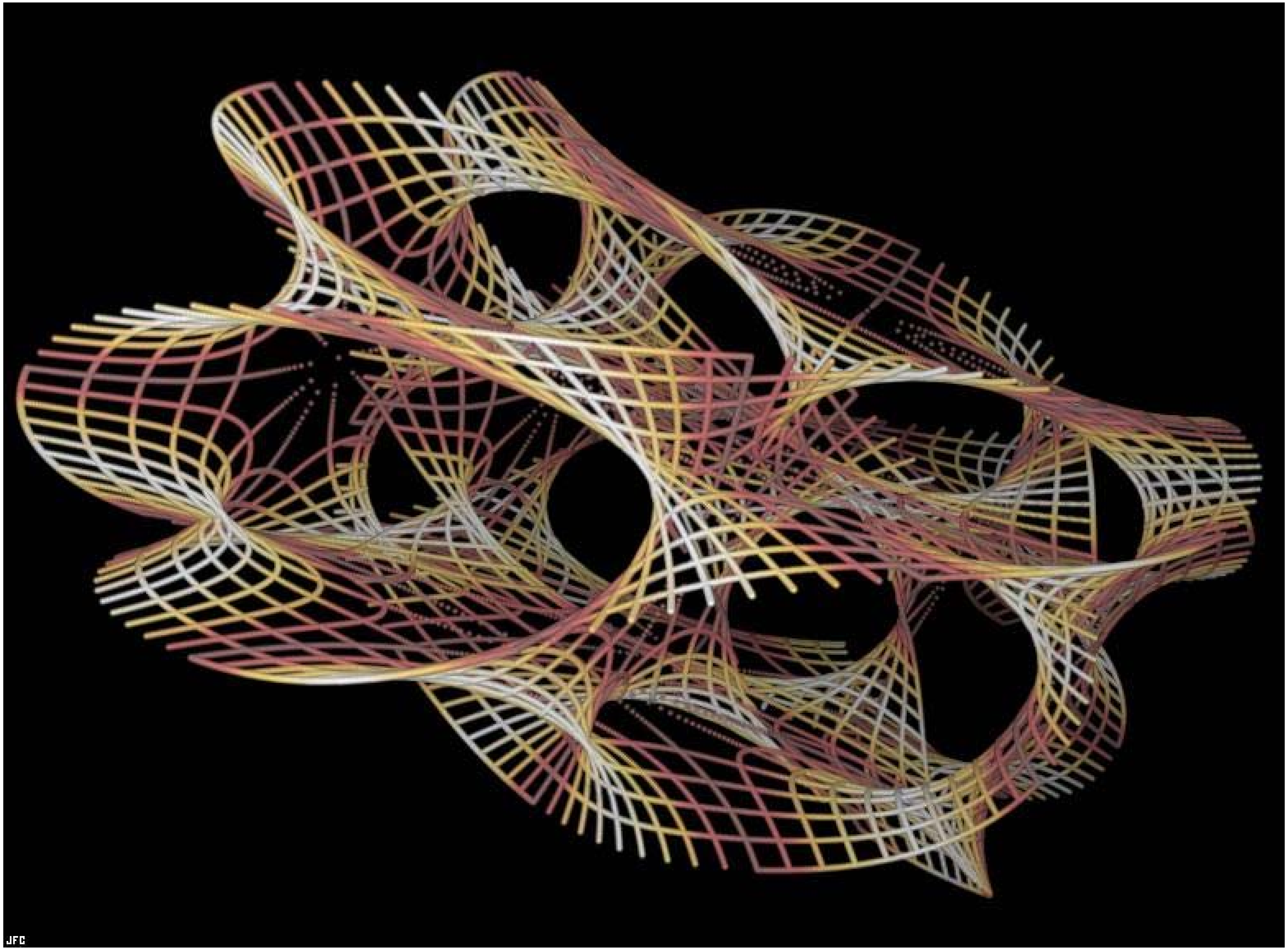
$$W[z_\alpha,\tau] = \int G_3(\tau)\wedge\Omega(z)$$

* Solve equations $D_{z_\alpha}W = D_\tau W = 0$

The complex structure fields and the axion-dilaton are fixed.

The overall volume still has a runaway potential

z_α **Shape moduli fixed by fluxes**



$\rho = \alpha + i\sigma$ **Total volume σ not fixed by fluxes**

Potential

No-scale supergravity

$$V = e^K (\sum_{\rho, z_\alpha, \tau} g^{k\bar{l}} D_k W \overline{D_l W} - 3|W|^2)$$

No potential for the volume moduli. Dilaton and shape

$$V = e^K \sum_{z_\alpha, \tau} g^{a\bar{b}} D_a W \overline{D_b W} \geq 0$$

moduli are generically fixed in Minkowski space!

***** Kahler moduli problem (in particular, overall volume)**

*****KKLT proposal**

i) non-perturbative superpotential from Euclidean D3-branes wrapped on special 4-cycles

ii) non-perturbative superpotential from pure SYM on a stack of D7's on Σ_4

Effective theory for the volume moduli

$$W = W_0(z_{cr}, \tau_{cr}) + Ae^{-ia\rho} + \ldots \qquad \rho = \alpha + i\sigma$$

Solve $D_\rho W = 0 \Longrightarrow \sigma_{cr} \sim \frac{1}{a} \ln W_0$

$C_2(G) > 1 \;, \qquad W_0 \ll 1$ **the volume is stabilized in**

AdS critical point in the regime of validity of calculations!

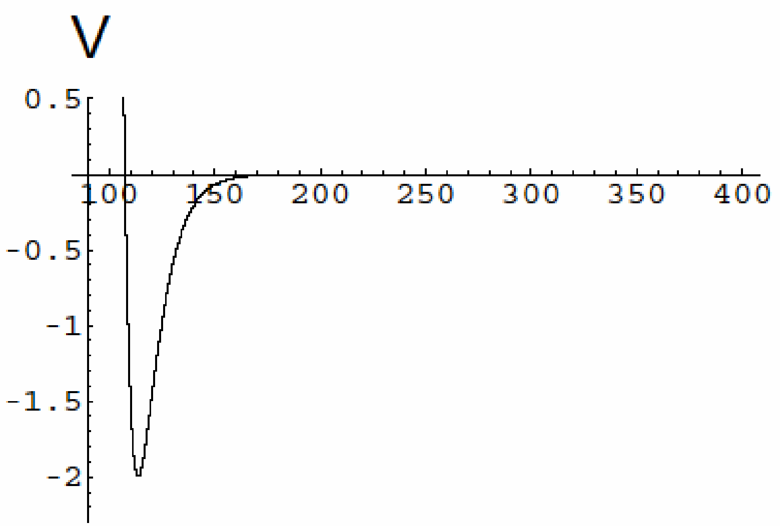
$$V_{cr} = -3e^K |W_{cr}|^2 \;, \qquad D_l W = 0$$

Volume stabilization

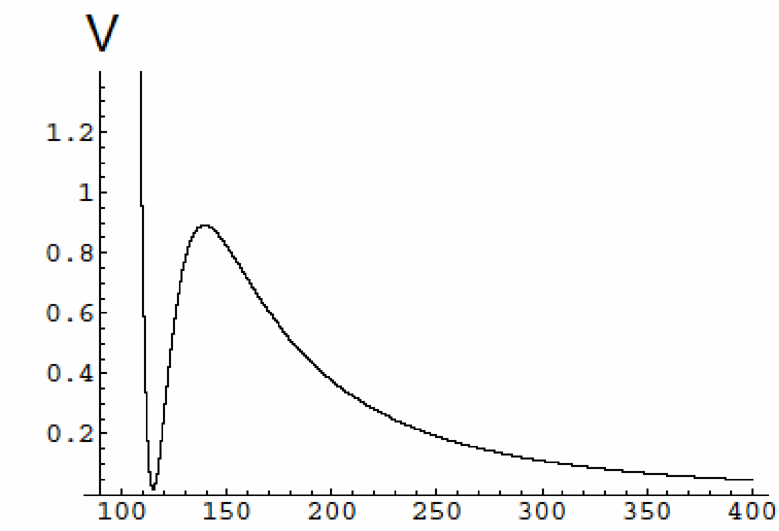
Basic steps:

Warped geometry of the compactified space and nonperturbative effects (gaugino condensation, instantons) lead to AdS space negative vacuum energy) with unbroken SUSY and stabilized volume

Uplifting AdS space to a metastable dS space (positive vacuum energy) by adding anti-D3 brane at the tip of the conifold (or D7 brane with fluxes)



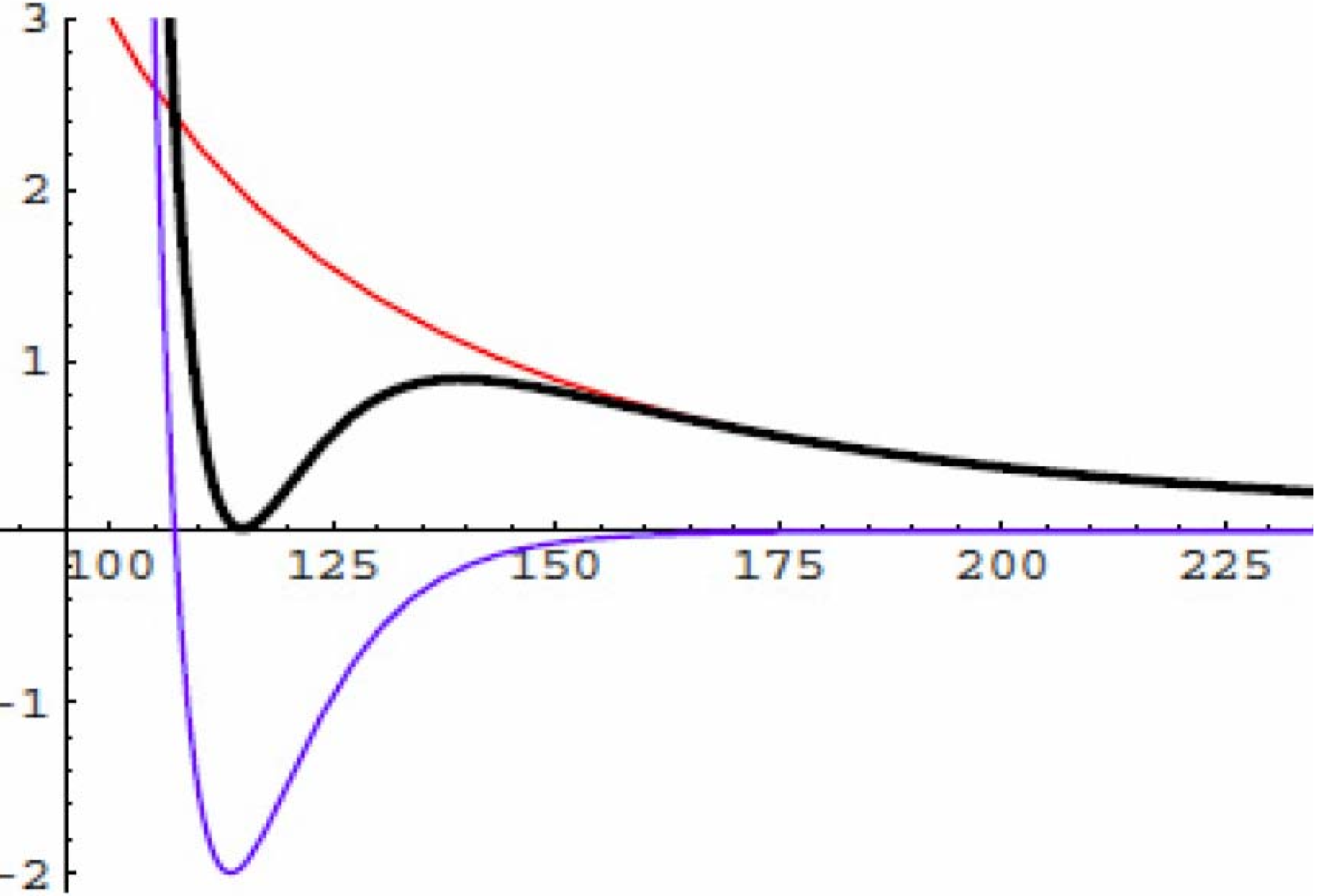
AdS minimum



Metastable dS minimum

$V(\overline{D3}) = \sqrt{g}_4 \sim 1/\sigma^2$ in KS warped geometry

V(AdS) + V(anti-D3)= V(dS)



Metastable dS vacua

RECENT DRAMATIC PROGRESS

in moduli stabilization in string theory

- KKLT scenario, 2003
- Building a better racetrack**,** Denef, Douglas, Florea, 2004
- Fixing all moduli in an F-theory compactification, Denef, Douglas, Florea, Grassi, Kachru, 2005
- Fluxes and gaugings, Derendinger, Kounnas, Petropoulos, Zwirner, 2005
- Type IIA moduli stabilization, DeWolfe, Giryavets, Kachru, Taylor, 2005
- **Fixing all moduli in M-theory on K3xK3, Aspinwall, R.K. 2005**
- On de Sitter vacua in type IIA, Saueressig, Theis, Vandoren, 2005

IIB nonsusy AdS vacua with exponentially large volume

In [Balasubramanian-Berglund, Balasubramanian-Berglund-Conlon-Quevedo, Conlon-Quevedo-Suruliz] it was shown that, when taking into account α' corrections to the Kähler potential, a new branch of vacua can appear as nonsusy AdS minima of the potential.

Rough idea: keep some divisor volumes $\rho_i \sim O(1)$ while sending overall vol to infinity, and balance nonperturbative $e^{-\rho_i}$ off against perturbative α' corrections.

$\Rightarrow$ Volume stabilized at exponentially large value:

$$\mathrm{Vol} \sim W_0 \, e^{c/g_s}$$

where W_0 and g_s are fixed by the fluxes.

IIB string compactified on $\mathrm{K3x}\frac{T^2}{\mathbf{Z}_2}$

A natural space for D3/D7 cosmological model

Dasgupta, Herdeiro, Hirano, R.K; Koyama, Tachikawa and Watari

- Flux vacua in this model were studied by **Trivedi, Tripathy** in string theory, and by **Angelantonj, D'Auria, Ferrara, Trigiante** in string theory and 4d gauged supergravity. **Kähler moduli remained non-fixed.**

- The **minimal** remaining moduli space is

Vectors

Hypers

$$\frac{U(1,1+n_3)}{U(1)\times U(1+n_3)} \times \frac{SO(2,18)}{SO(2)\times SO(18)}$$

KKLT stabilization is possible for the volume of K3!

Is KKLT stabilization possible for the volume of $\frac{T^2}{\mathbf{Z}_2}$?

KKLT 1: gaugino condensation. Works only for vector multiplets, does not work for hypers.

KKLT 2: instantons from Euclidean 3-branes wrapped on 4-cycles. May work for vector multiplets and hypers.

- On K3 x $\frac{T^2}{Z_2}$ there are 4-cycles which may or may not lead to non-vanishing instantons

Status in 2004

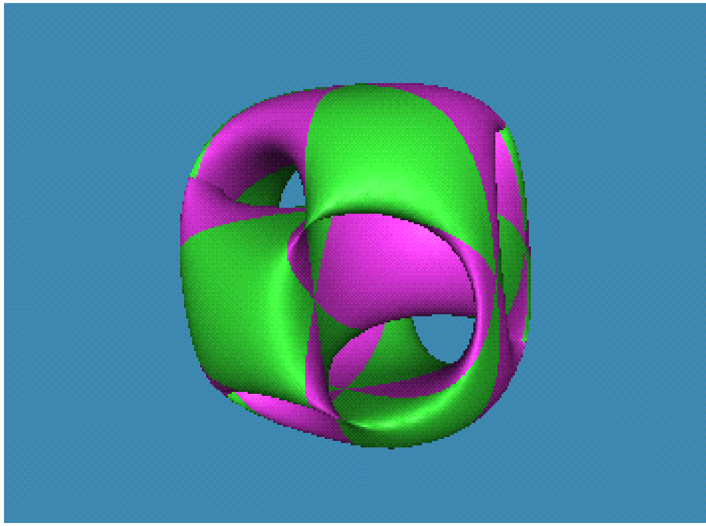
According to the old rules (**established before fluxes were introduced**), the relevant M-theory divisors in our model have an **arithmetic genus 2**. Therefore **one could incorrectly conclude that there are no stabilizing instantons for our favorite cosmological model**.

Witten 1996: in type IIB compactifications under certain conditions there can be corrections to the superpotential coming from Euclidean D3 branes. The argument is based on the M-theory counting of the fermion zero modes in the Dirac operator on the M5 brane wrapped on a 6-cycle of a Calabi-Yau four-fold. He found that **such corrections are possible only in case that the four-fold admits divisors with holomorphic characteristic equal to one,**

$$\chi_D = \frac{1}{2}(N_+ - N_-) \equiv \sum(-1)^n h^{(0,n)} = 1$$

In presence of such instantons, there is a correction to the superpotential which at large volume yields the term required in the KKLT construction

$$W_{\mathrm{inst}} = A e^{-(\mathrm{Vol}+i\alpha)} \qquad A \neq 0$$



The equations made explicit in terms of forms

Requiring the preservation of $N = 1$ supersymmetry imposes the 4-flux is (2,2) and primitive, Becker[2].

$$D\epsilon_+ = 0\,, \qquad D\epsilon_- = F\epsilon_+$$

The M5 brane is wrapped on a 6-cycle inside the compactifying 4-fold. Following Witten, we identify the bundle S in which the spinors take values as

$$S = S_+ \otimes (N^{1/2} \oplus N^{-1/2}) = K^{1/2} \oplus (K^{1/2} \otimes \Omega^{0,2}) \otimes (K^{1/2} \oplus K^{-1/2})$$

This is equal to

$$\mathcal{O} \oplus \Omega^{0,2} \oplus K \oplus (\Omega^{0,2} \otimes K)\,.$$

The index thus results to be

$$\chi_D = (-)^p h^{(0,p)}$$

which is called the holomorphic characteristic, or arithmetic genus of the divisor D.

In presence of fluxes there were indications that the rule $\chi_D = 1$ may not be necessary

Robbins, Sethi (2004); Gorlich, Kachru, Tripathy and Trivedi (2004); Tripathy and Trivedi (2005); Saulina (2005); Berglund, Mayr (2005); Gomis, Marchesano and Mateos (2005)

We established a <u>new rule</u>, replacing the rule $\chi_D = 1$ in presence of fluxes

- Constructed the Dirac operator on M5 with background fluxes
- Performed the counting of fermionic zero modes and found the generalized index
- Studied interesting examples, like stabilization of all moduli in M-theory on K3xK3 and its F-theory limit
- Constructed the Dirac operator on D3 brane with background flux and defined its index
- Studied interesting examples in type IIB: K3 x T^2/Z_2, general Fano manifolds, orientifold T^6/Z_2

M5 brane

- Dirac action on M5 with background fluxes

$$\Gamma^a \hat{\nabla}_a \theta = 0$$

- Here $\hat{\nabla}_a$ is a super-covariant derivative including **torsion when fluxes in the background M theory are present**

$$\Gamma^a (\nabla_a + T_a{}^{\underline{abcd}} F_{\underline{abcd}}) \theta_- = 0$$

R.K. and Sorokin

New Dirac Equation on the Brane

$$(\tilde{\gamma}^a \nabla_a - \frac{1}{24} \gamma^i \tilde{\gamma}^{abc} F_{abci})\theta = 0$$

INDEX OF THE DIRAC OPERATOR:

Can flux affect it?

Solving spinor equations and counting zero modes

$$D\epsilon_+ = 0\,, \qquad D\epsilon_- = F\epsilon_+$$

Ansatz

$$\epsilon_+ = \phi|\Omega\rangle + \phi_{\bar a\bar b}\Gamma^{\bar a\bar b}|\Omega\rangle$$

$$\epsilon_- = \phi_{\bar z}\Gamma^{\bar z}|\Omega\rangle + \phi_{\bar z\bar a\bar b}\Gamma^{\bar z\bar a\bar b}|\Omega\rangle$$

One of the equations depends on flux

$$[\partial^A_{\bar a}\phi_{\bar z} + 4g^{\bar b c}\partial^A_c\phi_{\bar b\bar a\bar z} + 8F_{\bar a\bar z bc}\phi^{bc}]\Gamma^{\bar a\bar z}|\Omega\rangle = 0$$

$$\mathcal{H}(F_{\bar a\bar z bc}\phi^{bc}dz^{\bar a}) = 0$$

Here $\mathcal{H}$ is the projector into harmonic forms, such that it gives zero on any exact or co-exact form

$$[\partial_{\overline{a}}\phi + 4g^{\overline{b}c}\partial_c\phi_{\overline{b}\overline{a}}]\Gamma^{\overline{a}}|\Omega\rangle = 0$$

$$[\partial^A_{\overline{a}}\phi_{\overline{z}} + 4g^{\overline{b}c}\partial^A_c\phi_{\overline{b}\overline{a}\overline{z}} + 8F_{\overline{a}\overline{z}bc}\phi^{bc}]\Gamma^{\overline{a}\overline{z}}|\Omega\rangle = 0$$

$$[\partial_{\overline{a}}\phi_{\overline{b}\overline{c}}\Gamma^{\overline{a}\overline{b}\overline{c}}]|\Omega\rangle = 0$$

$$[\partial^A_{\overline{a}}\phi_{\overline{b}\overline{c}\overline{z}}]\Gamma^{\overline{a}\overline{b}\overline{c}\overline{z}}|\Omega\rangle\ .$$

$F = 0$ $\chi = \frac{1}{2}(N_+ - N_-) = h^{0,0} - h^{0,1} + h^{0,2} - h^{0,3}$

New constraint on zero modes

$$\mathcal{H}(F_{\overline{a}\overline{z}bc}\phi^{bc}dz^{\overline{a}}) = 0$$

- We can interpret this equation as a linear operator $\mathcal{H}F$ annihilating ϕ^{bc}

➢ For a generic choice of flux the system is of maximal rank, and hence admits no solutions. This kills all of the $(0,2)$ forms.

Counting fermionic zero modes on M5 with fluxes

R.K., Kashani-Poor, Tomasiello

- New computation of the normal bundle U(1) anomaly

$$\chi_D(F) = \chi_D - (h^{(0,2)} - n)$$

- Here n is the dimension of solutions of the constraint equation which depends on fluxes.

$$0 \leq n \leq h^{(0,2)}$$

- To have instantons we need

$$\chi_D(F) = 1 \quad \Longrightarrow \quad \begin{cases} \chi_D = 1 \\ \chi_D \neq 1 \end{cases}$$

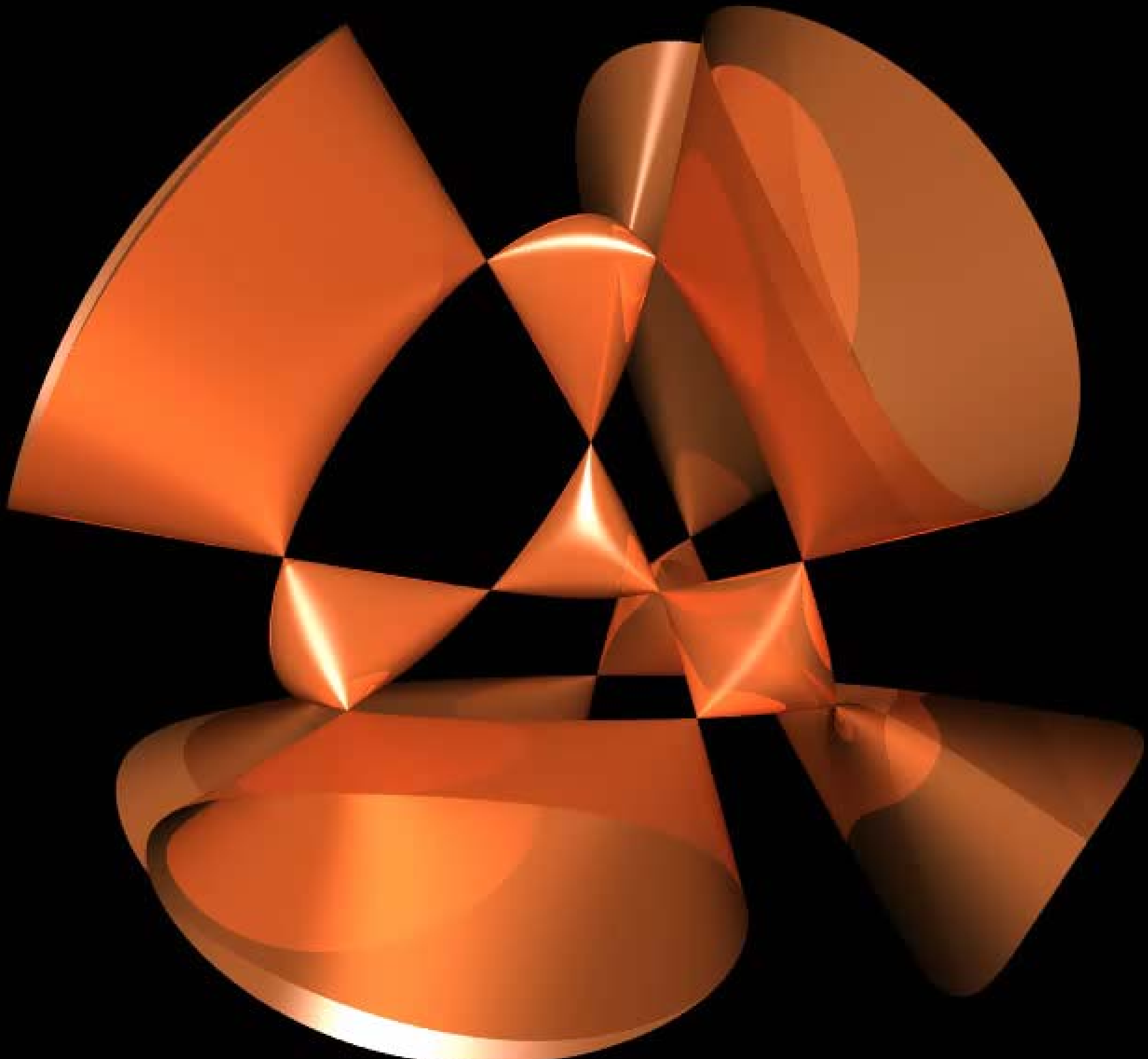
Fixing All Moduli for M-Theory on K3 x K3

Aspinwall, R. K.

Paul Aspinwall's K3 movie: K3 surface

(a **non-singular** quartic surface in projective space of three dimensions)

with **variation of the deformation parameter**



M-theory compactified on K3xK3:

incredibly simple and elegant

- Without fluxes in the compactified 3d theory there are two 80-dimensional quaternionic **Kähler** spaces, one for each K3.
- With non-vanishing primitive (2,2) **flux**, (2,0) and (0,2), each K3 becomes an **attractive K3**: one-half of all moduli are fixed
- 40 in each K3 still remain moduli and need to be fixed by instantons.

There are **20 proper 4-cycles in each K3. They provide instanton corrections from M5-branes wrapped on these cycles:**

moduli space is no more

ATTRACTIVE K3 SURFACES

- G. Moore, in lectures on Attractors and Arithmetic
- **In M-theory on K3xK3** Aspinwall, R. K.
- **The complex structures are uniquely determined by a choice of flux G**

The K3 surface is attractive if the rank of the Picard lattice has the maximal value, 20, and the rank of the transcedental lattice (orthogonal complement of the Picard lattice) is 2.

- They are in one-to-one correspondence with $Sl(2,Z)$ equivalence classes of positive-definite even integral binary quadratic forms, which can be written in terms of a matrix

$$Q_j = \begin{pmatrix} p_j^2 & p_j.q_j \\ p_j.q_j & q_j^2 \end{pmatrix} = \begin{pmatrix} 2a & b \\ b & 2c \end{pmatrix} \qquad j = 1,2$$

- **Both K3 surfaces whose complex structures are fixed by G are forced to be attractive**

$$\Omega_j = p_j + \tau_j q_j \qquad \tau_j = \frac{-p_j.q_j + i\sqrt{\det Q_j}}{q_j^2}$$

Obstructed instantons

- For general choice of fluxes we find conditions when the instantons can be generated.

- When these conditions are not satisfied, fluxes will obstruct the existence of the stabilizing instanton corrections.

The orientifold on K3 x $\frac{T^2}{\mathbf{Z}_2}$

- **In F-theory compactifications on K3 x K3 one of the attractive K3 must be a Kummer surface to describe an orientifold in IIB**

$$Q = 2R$$

- **Attractive K3 surface is a Kummer surface if, and only if, the associated even binary quadratic form is twice another even binary quadratic form.**

Instanton corrections

- With account of the new index theorem we find that instantons are generated for M5 branes wrapping $K3_1 \times P^1$ and $P^1 \times K3_2$
- For a flux of the form $G = Re(\gamma\Omega_1 \wedge \bar{\Omega}_2)$ each K3 surface is attractive and, as such, has Picard number equal to 20. This leaves each K3 with 20 complexified Kahler moduli.
- **We proved that there are 40 independent functions on 40 variables. All moduli unfixed by fluxes are fixed by instantons.**

Orientifold limit

- **Take an F-theory limit of M-theory on K3xK3. We find an equivalent statement about instanton corrections for IIB on K3x** $T^2/\mathbf{Z}_2$

- The M5 instanton must wrap either an elliptic fibre or a "bad fibre" (fibre which is not an elliptic curve), classified by Kodaira. With account of these two possibilities we find

- Instantons from D3 branes wrapping $P^1 \times \frac{T^2}{Z_2}$ from M5 on $P^1 \times K3_2$
  Instanton from D3 wrapped on K3x pt from M5 which was wrapped on P^1 where $K3_1 \times P^1$ is a "bad fibre".

- We find the right number of cycles to fix all moduli which were not fixed by fluxes (**one should be careful about obstructed instantons**).

The index of the Dirac operator on D3 brane with background fluxes

Bergshoeff, R. K., Kashani-Poor, Sorokin, Tomasiello

- We study the instanton generated superpotentials in Calabi -Yau orientifold compactifications **directly in IIB**.

$$\chi_D(F) = \frac{1}{2}(N_+ - N_-)$$

- We derive the Dirac equation on a Euclidean D3 brane in the presence of background flux, which governs the generation of a superpotential in the effective 4d theory by D3 brane instantons.

  A classical Dirac action is

$$L_f^{D3} = \frac{1}{2} e^{-\phi} \sqrt{-\det g}\, \bar{\theta}(1 - \Gamma_{D3})[\Gamma^\alpha \delta\psi_\alpha - \delta\lambda]\theta$$

Marolf, Martucci, Silva

Duality covariant gauge-fixing kappa-symmetry, compatible with orientifolding

- For D3 $(1-\Gamma_{D3})\,\theta = 0$ $\Longrightarrow$ $\theta_2 = i\gamma_5\theta_1$

- Gauge-fixed action

$$L_f^{D3} = 2\sqrt{\det g}\,\theta_1\{e^{-\phi}\Gamma^\alpha\nabla_\alpha + \tfrac{1}{8}\tilde{G}_{\alpha\beta i}\Gamma^{\alpha\beta i}\}\theta_1$$

$\tilde{G}_{(3)} = iG_{(3)}$ on states of positive chirality

$\tilde{G}_{(3)} = -i\bar{G}_{(3)}$ on states of negative chirality

$G_{(3)}$ is the standard primitive (2,1) 3-form of type IIB string theory

Count fermionic zero modes using the ansatz analogous to M5

- Fluxes lead to new constraints on fermionic zero modes

$$\mathcal{H}(\bar{G}_{\bar{a}\bar{z}b}\phi^{b}) = 0$$

$$\mathcal{H}(G_{ab\bar{z}}\phi^{ab}) = 0$$

- Orientifold projection may cut some zero modes when the divisor hits the O-plane

EXAMPLES

- Applying the formalism to the K3x$\frac{T^2}{Z_2}$ orientifold we show that our results are consistent with conclusions attainable via duality from an M-theory analysis.
- In case of T^6/Z_2 we find that $\chi_D(F) = 1$ and **instanton corrections are possible** when the divisor hits the O-plane. We also find that $\chi_D(F) = -3$ and **instanton corrections are not possible** when the divisor is off the O-plane, in agreement with Trivedi,Tripathy
- D3 branes on **general Fano manifolds** without fluxes:

Holomorphic characteristics of the orientifold locus in perfect agreement with M-theory

$$\chi_{D3} = h^{0,0} - h^{0,1} + h^{0,2}$$

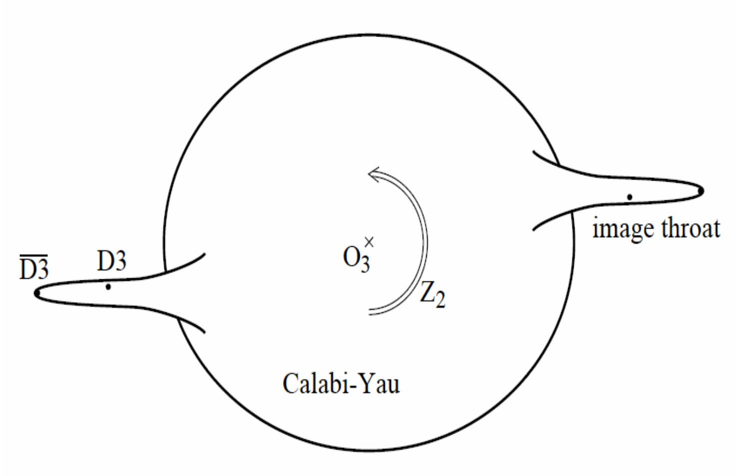
$$\chi_{M5} = \chi(P^1) \times \chi_{D3} = \chi_{D3}$$

Back to String Cosmology

The goal is to stabilize all moduli, but the inflaton field should correspond to a flat direction of the potential

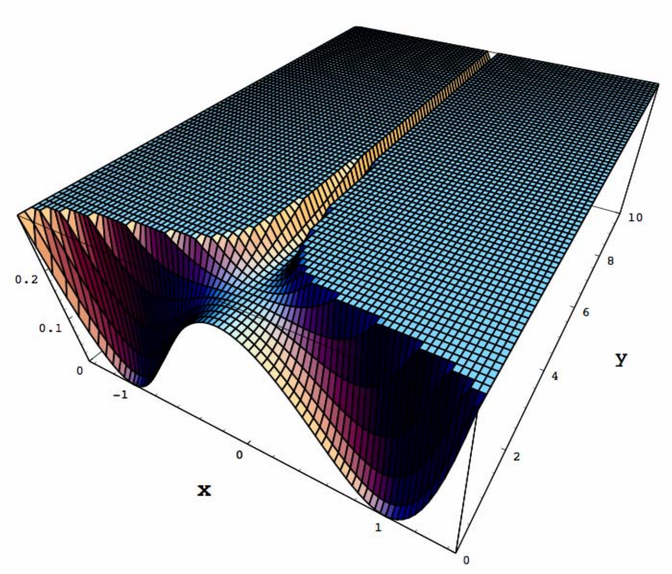
In several versions of string inflation scenario, the inflaton field corresponds to the position of the D3 brane. Thus we would like to keep D3 brane mobile

Inflationary models using mobile D3 branes



KKLMMT brane-anti-brane inflation

Fine-tuning



D3/D7 brane inflation

with volume stabilization and

shift symmetry, slightly

broken by quantum corrections

$$n_s = 0.98$$

No fine-tuning?

D-term
inflation

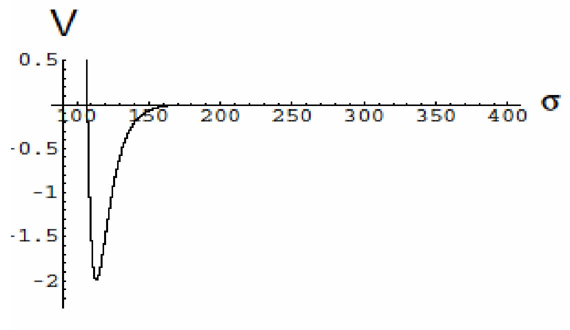
Moduli Stabilization and D3/D7 Inflation

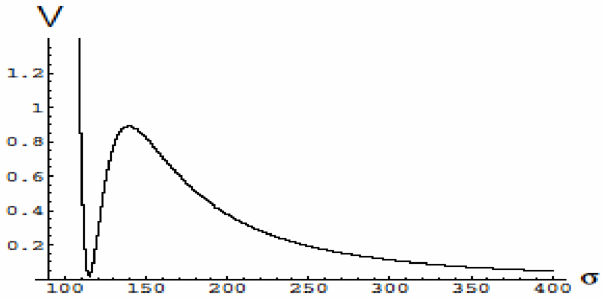
- The purpose of our studies of instanton corrections was, in particular, to clarify the case of moduli stabilization in D3/D7 inflationary model
- Our new results show that the goal of fixing all moduli (except the positions of D3 branes) in this model is now accomplished (either by duality from M-theory or directly in IIB)
- The classical shift symmetry of this model, which implies flatness of the inflaton direction associated with the position of the D3 brane, may survive under certain conditions

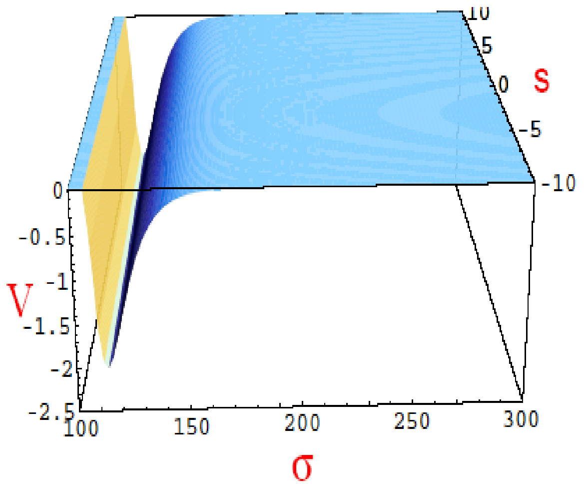
Hsu, R.K., Prokushkin; Firouzjahi, Tye

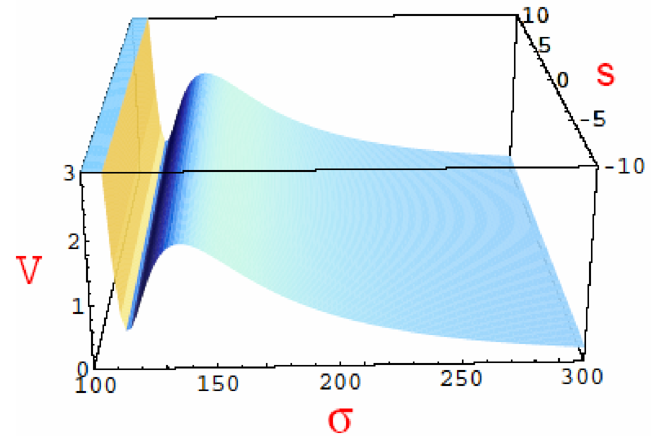
Inflaton Trench

Hsu, R.K., Prokushkin









SHIFT SYMMETRY

Mobile D3 brane?

Is the inflaton direction flat?

Previous investigations suggesting that D3 may be fixed:

Ori Ganor??? **1996, no fluxes**

Berg, Haack, Kors ???

1. Calculations valid only in absence of flux
2. All 16 D7 on top of each other (different from D3/D7 scenario)
3. Unlike in their work, we have no gaugino condensation

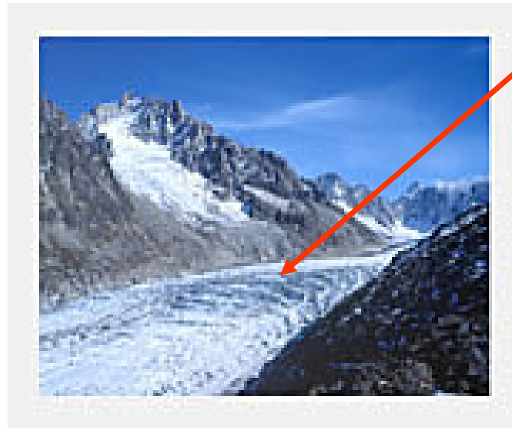
Berglund, Mayr ???

Assumption about the use of the worldsheet instantons and duality in presence of RR fluxes with N=1 susy ???

In our direct approach, the positions of D3 branes are not fixed by either fluxes or known instantons from wrapped branes, i.e. the inflaton direction is flat

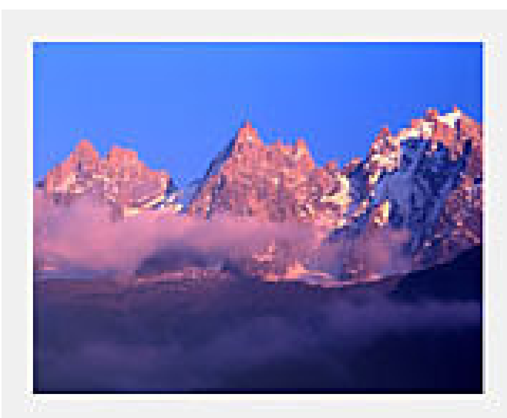
SLOW-ROLL

Aiguille du Dru



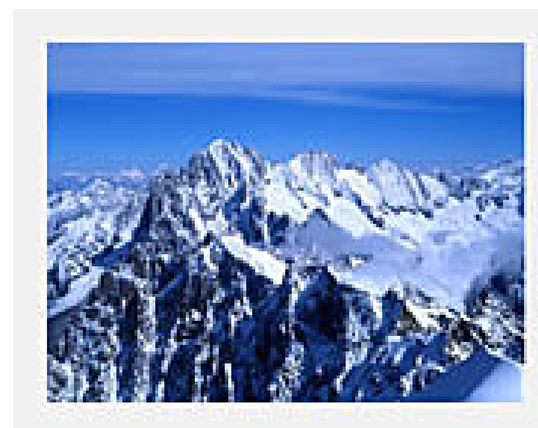
Mt. Dolent, Argentiere Glacier

Aiguille Verte, Afternoon Clouds



Alpenglow, Chamonix Aiguilles

Aiguille du Dru



Aiguille Verte