Proof of all-order conformal invariance of β-deformed $\mathcal{N} = 4$ YM in the planar limit

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Quantum field theory with conformal symmetry are important since they are fixed points of renormalisation group flow, essential to dynamics of quantum field theory. In particular, it is of basic interests to study the case where these fixed points constitue smooth manifold. In four dimension, the prime example is the $\mathcal{N} = 4$ supersymmetric Yang-Mills model, which forms a fixed line. The so-called β-deformed $\mathcal{N} = 4$ YM is a deformation of this theory, which is conjectured to be conformal, extending the fixed line to a fixed surface. Also, the interest in the deformed theory is recently enhanced, in the context of AdS/CFT correspondence, by the proposal of the string dual to the deformed theory.

Recently, we have proved the all-order conformal invariance of β-deformed YM in the planar limit. [1] Here we shall give brief accounts of the proof.

Our proof closely follows that for the $\mathcal{N} = 4$ theory by Mandelstam and Brink et al., using lightcone $\mathcal{N} = 4$ superspace. Although the deformed theory has only $\mathcal{N} = 1$ SUSY, we find that it can be formulated using the original $\mathcal{N} = 4$ superspace: the deformation amounts to replacing every product in the superspace action by a newly defined Moyal-like $\star$-product acting on the Fermionic part of the superspace.

With this formulation, our argument goes as follows. The conformal invariance follows from the finiteness of all Green functions in this gauge, which in turn is shown by Weinberg's theorem. The theorem guarantees finiteness of a Feynman graph when the so-called superficial degrees of divergence D are negative. The bulk of our proof is devoted to estimating D.

The estimate consists of two steps. In the first step, we make a preliminary estimate, without distinguishing internal and external momenta and using the explicit form of the vertices. The resulting estimate, by power counting rules for supergraphs [2], of D is equal to zero. In the second step, we refine this estimate by using explicit form of the vertices to show that the D is strictly negative. The manipulation in this step consists of partial integrating derivatives from internal to external lines, and explicit cancellation between various contractions. This estimate of D is sufficient to prove that all diagrams are finite, except for a few exceptions. The proof is completed by explicitly showing that these exceptions are also finite.

To conclude, we mention that there are various possible generalisations of our argument. In particular, by modifying the $\star$-products it is possible to break the SUSY completely, and our proof works exactly the same. To analyse non-planar graphs is also clearly important.

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[1] Ananth, Kovacs, H. Shimada, e-print hep-th/0609149.

[2] This rule is modified by the deformation for non-planar graphs, hence our proof is only for the planar limit.