Winding String Dynamics in Time-Dependent Beta Deformed Background

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Winding string dynamics in Lorentzian orbifolds has recently been widely studied. Since string theory there is described by a solvable conformal field theory, these time-dependent backgrounds thus provide interesting toy models where both the mini-superspace approximation and the worldsheet theory can be examined.

In this note, we study one exactly solvable non-static string background proposed in hep-th/0509036. It is obtained by applying a Lorentzian analytical continuation to the static β deformed background, which has a metric similar to a flux-brane. The string sigma model there is equivalent to an orbifold after the so-called TsT transformation. The prescription to generate this kind of metric is via applying an $SL(2,R)$ transformation to the parameter $\tau = B_{21} + i\sqrt{g} \;\; \rightarrow \;\; \tau' = \frac{\tau}{1+b\tau}$ of any string background with two $U(1)$ symmetries realized geometrically, i.e. a two torus parameterized by, say, φ_1, φ_2. Note here B_{21} is the B-field along φ_1, φ_2 while $\sqrt{g}$ is the volume of the torus.

The metric we would like to study is $ds^2 = -dt^2 + dr^2 + \frac{t^2}{1+b^2t^2r^2}d\theta^2 + \frac{r^2}{1+b^2t^2r^2}d\varphi^2$ with the B-field $B_{\theta\varphi} = \frac{-bt^2r^2}{1+b^2t^2r^2}$, and the dilaton $e^{2(\Phi-\Phi_0)} = \frac{1}{1+b^2t^2r^2}$ where $-\infty < t, \theta < \infty$, $0 \leq r \leq \infty$ and $\varphi \sim \varphi + 2\pi$. By the mini-superspace approximation where the string worldsheet boils down to a point particle, string dynamics is generally captured by a second-order ordinary differential equation $\ddot{\psi}(t) + \omega(t)^2\psi(t) = 0$ where the dot denotes time derivative. This can be understood as a result from the on-shell constraint for a given string state ψ. Due to the fact that the metric is non-static, any quantum fields propagating within it will give rise to particle production. Therefore, one can solve the ODE to obtain the Bogoliubov coefficient (or S-matrix) which encodes the production rate.

We found that the production rate consists of two parts, and draws a parallel between itself and the case of charged particle pair creation in the Rindler space. That is, one part which is induced by the background B-field corresponds to the usual Schwinger effect, whereas the other which is due to the Milne metric, i.e. $b = 0$ corresponds to the gravitational Unruh effect of Rindler dynamics. Moreover, by means of the D-brane's DBI action, it is also possible to yield the classical one-point correlator by overlapping $\psi(t)$ with the D-brane worldvolume. In addition, from the exact string spectrum on the metric, we are capable of computing the torus and cylinder amplitude which facilitate a non-trivial comparison with the mini-superspace analysis.