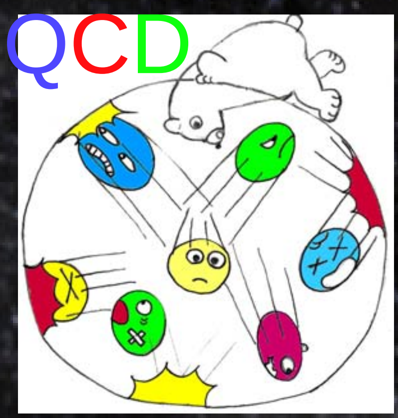
クォーク･グルーオン･プラズマ
– 高温･高密度で実現された物質の新しい形態–

弦理論と場の量子論における新たな進展

2006 年9 月12 日～16 日　京都

広島大学･情報メディア教育研究センター
中村純



Linear Response Theory

今日の話を要約すると

- 超高エネルギーで重い原子核同士を衝突させたら、温度が上がり閉じ込め/非閉じ込め相転移温度をおそらく超えた

実験の概要

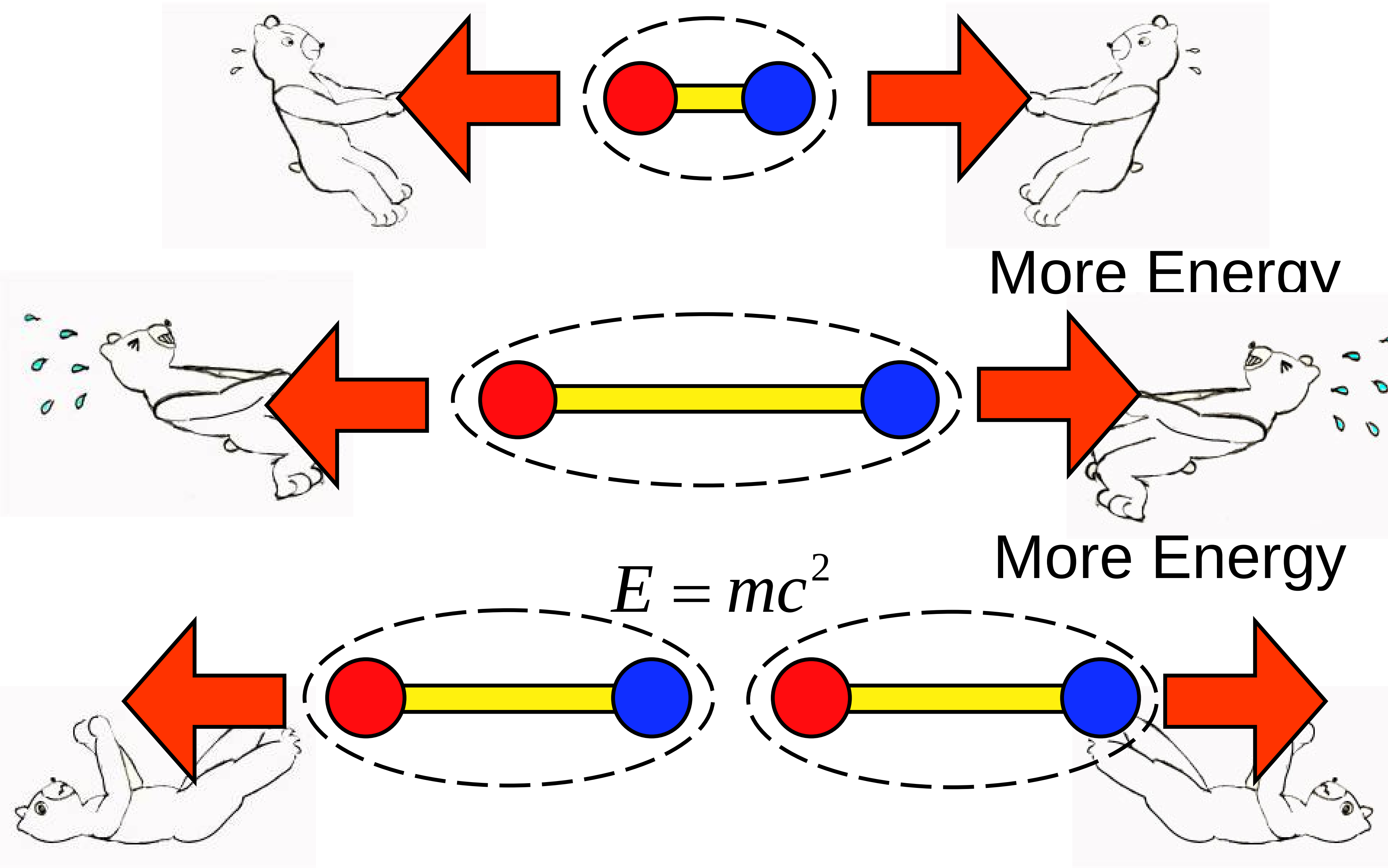
- そこで作られたQCD物質は、クォークとグルーオンが自由に飛んでいるガス状のものではなかった
- 完全流体に近いもの？

どうしてそう思われるの？

- 格子QCDで輸送係数の計算をいろいろ苦労しながら試みている

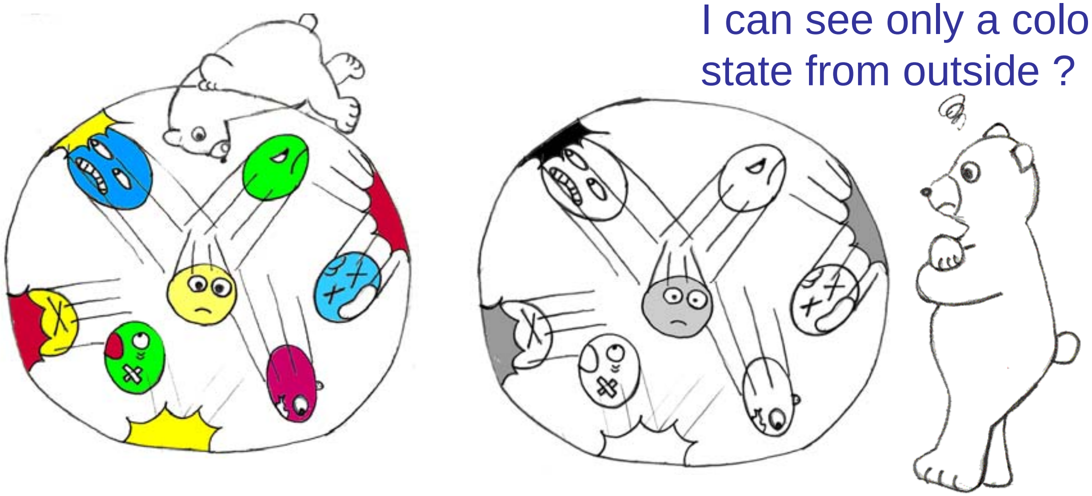
何が面白いの？

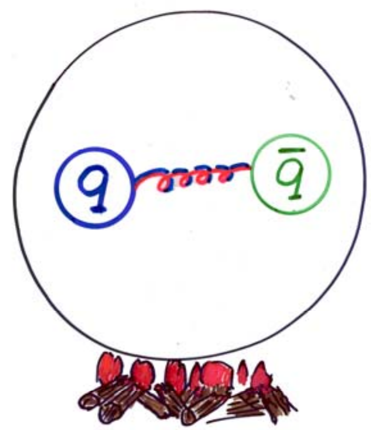
Confinement

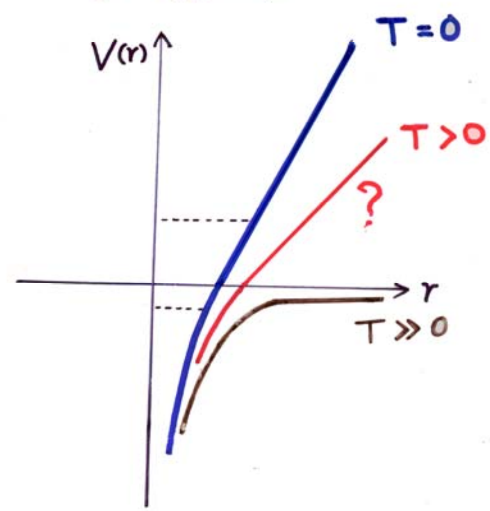


Confinement (2)

I can see only a colorless state from outside ?





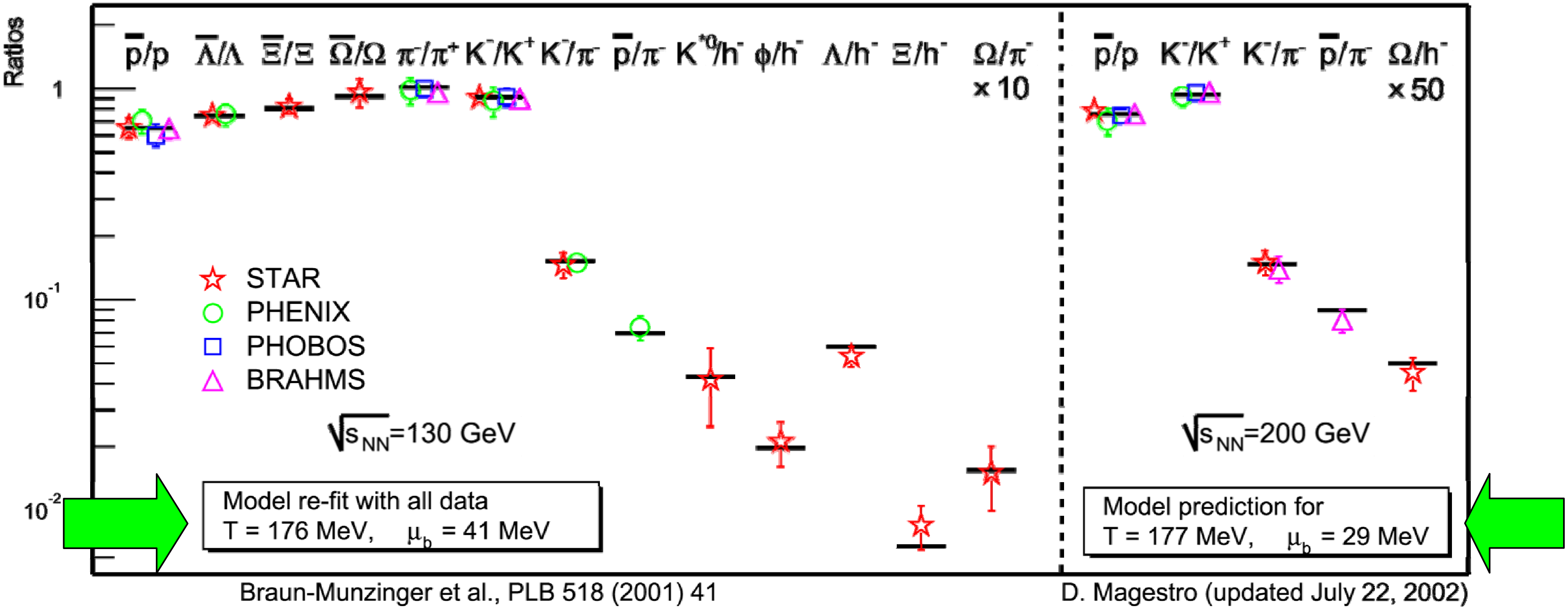


Confinement Potential is “screened” at finite temperature.

→ Deconfinement

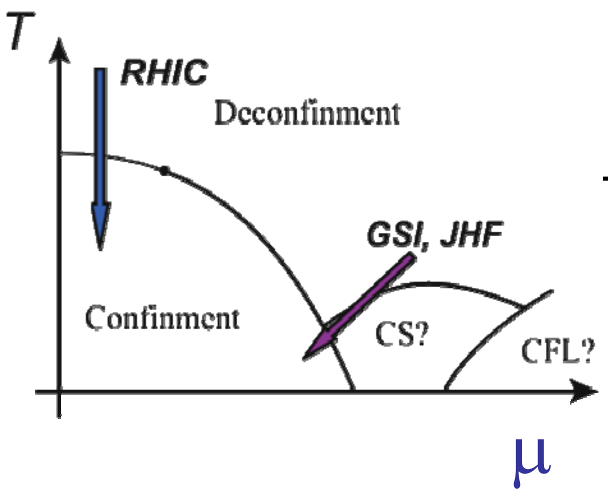
統計モデルでデータをフィット

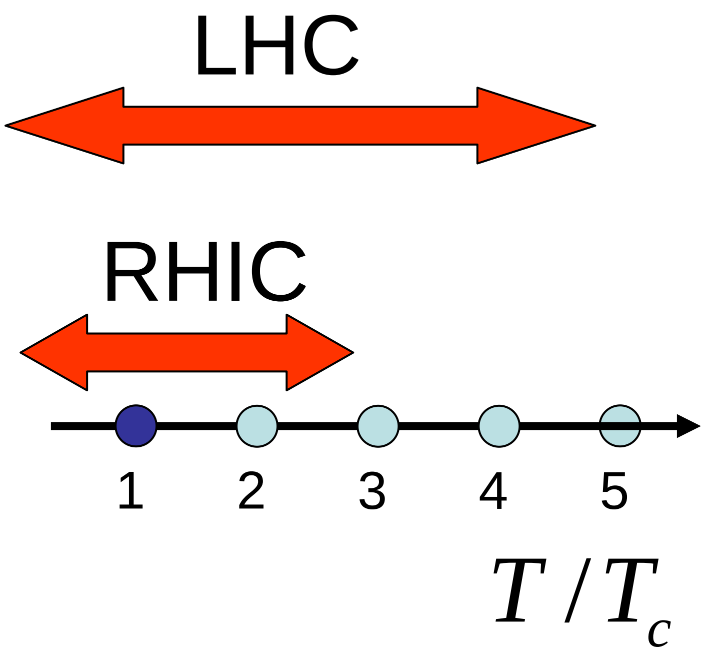
$$n_i = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_b B_i - \mu_s S_i)/T] \pm 1}$$

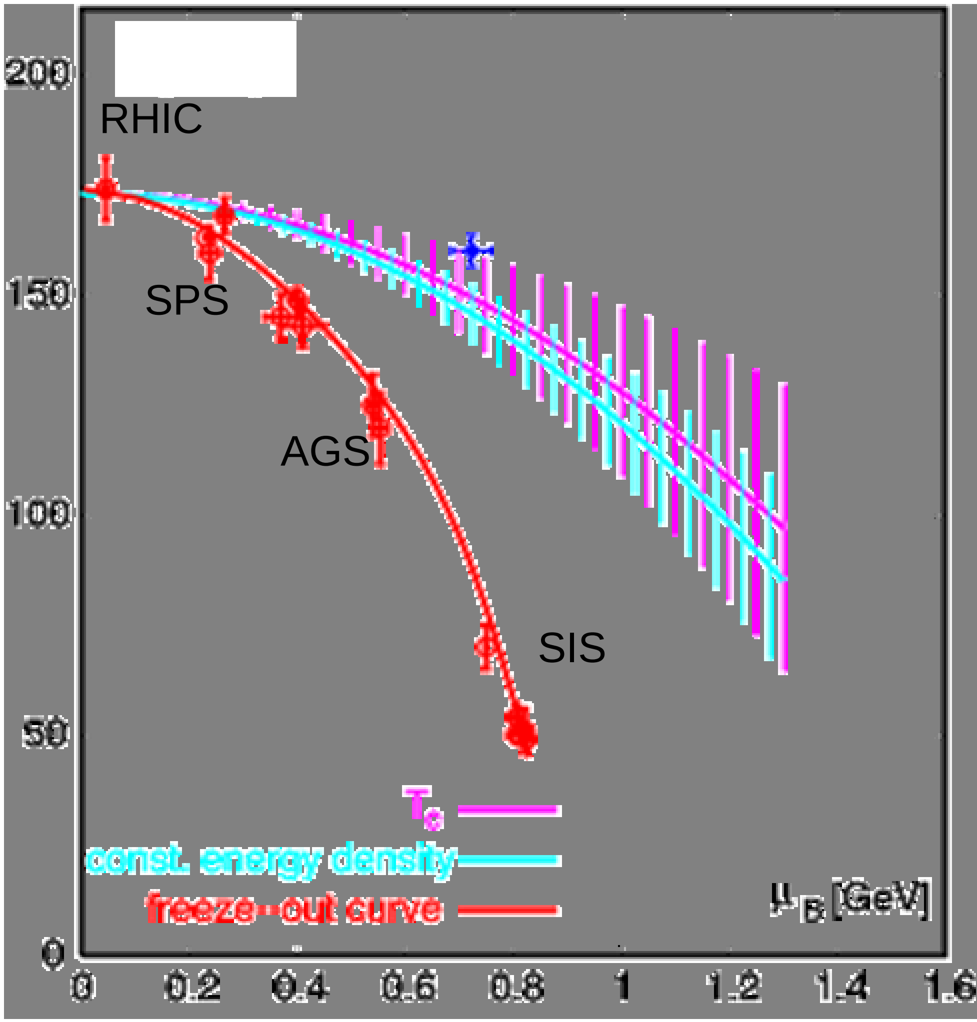


最近Photonで測られた温度は300 - 400MeV

A Comparison with Lattice Results
P. Braun-Munzinger, K. Redlich and J. Stachel







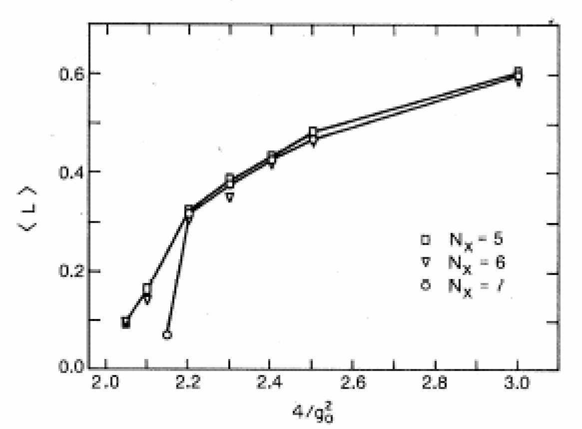
Observation of a Phase Transition at Finite Temperature on the Lattice

1981, McLerran and Svetitsky, Kuti, Polonyi and Szlachanyi, Engels et al.

$$Z = e^{-\beta F} = \mathrm{Tr} e^{-\beta(H-\mu N)} = \sum_{\phi} \left\langle \phi \middle| e^{-\beta(H-\mu N)} \middle| \phi \right\rangle$$

$$e^{-\beta \Delta F} = \frac{Z(\text{Gluons+A Static Quark})}{Z(\text{Gluons})} = \left\langle L(\vec{x}) \right\rangle$$

Excess Energy when a quark exists.



$$e^{-\beta \Delta F} = \frac{Z(\text{Gluons+Static Quark+Anti-Quark})}{Z(\text{Gluons})}$$

$$= \left\langle L(\vec{x}) L^{\dagger}(\vec{y}) \right\rangle$$

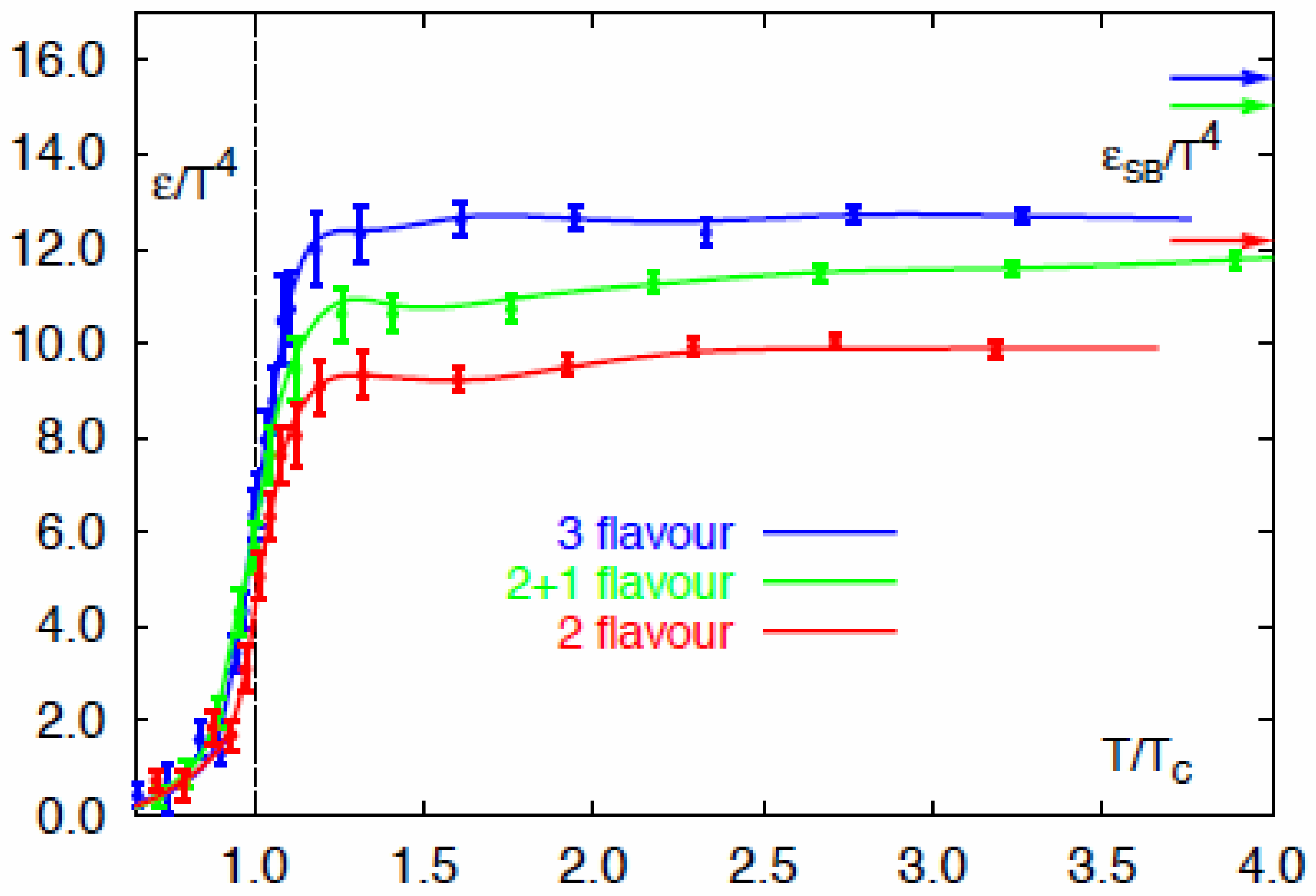
Excess Energy when a quark and an anti-quark exist.

⇨ Heavy Quark Potential

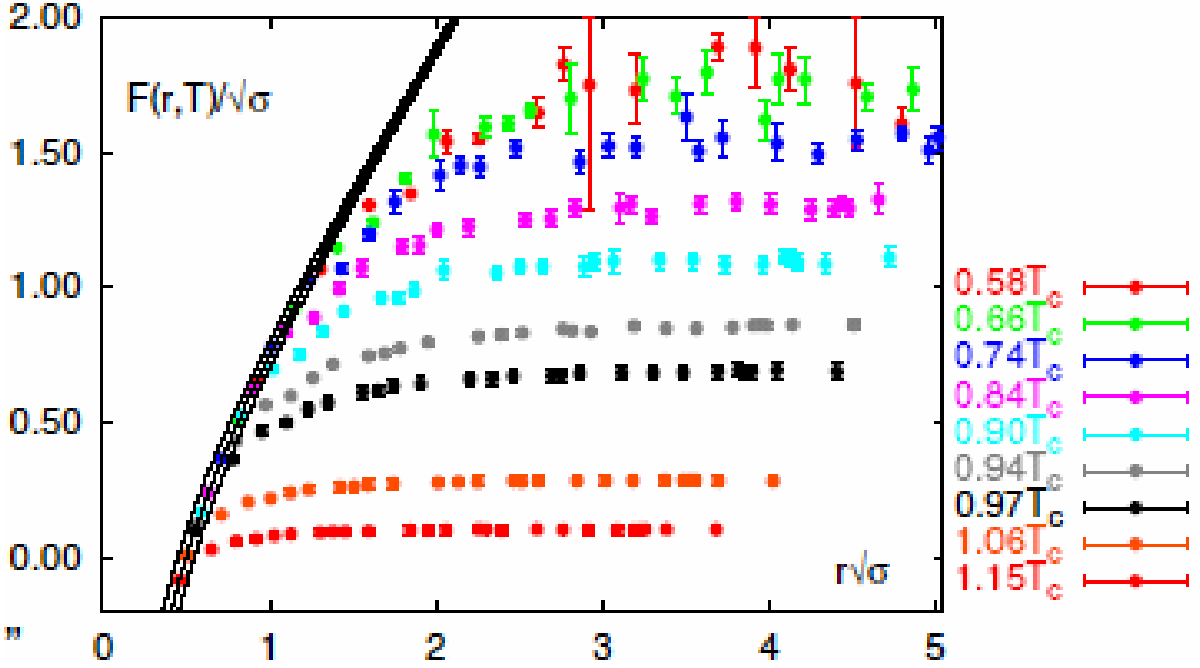
McLerran and Svetitsky, PRD24,（1981）

Lattice QCD Calculations

F. Karsch, Lect. Notes Phys. 583 (2002) 209.



Heavy Quark Potential with Dynamical Quarks

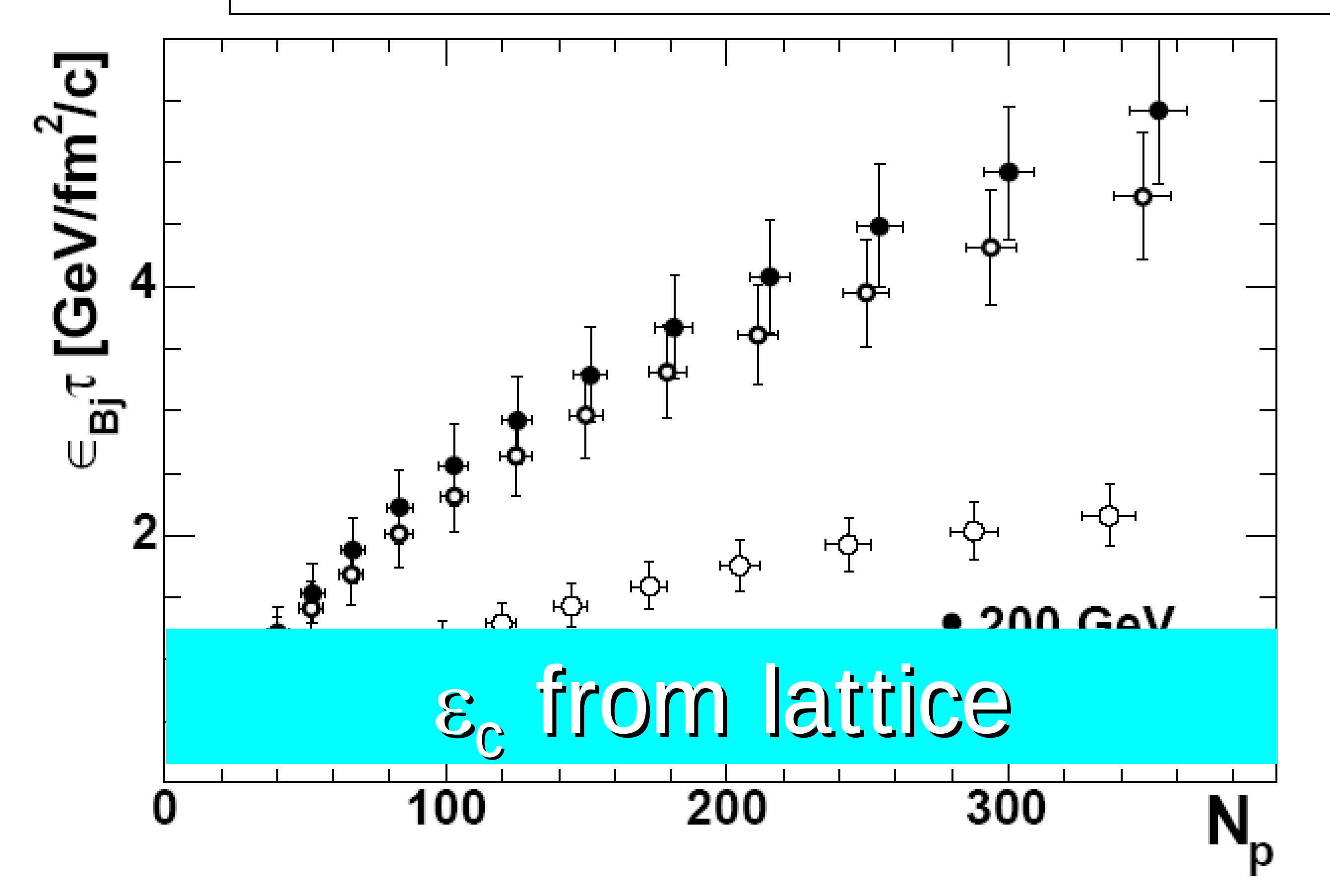


Bielefeld

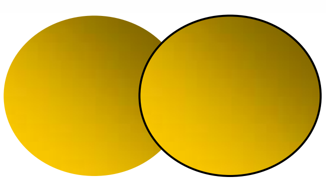
$$\varepsilon_{Bj}(\tau) = \frac{\langle m_T \rangle}{\tau \pi R^2} \frac{dN}{dy}$$

Bjorken('83)

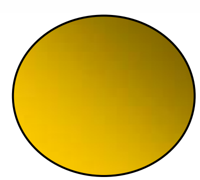
τ: proper time
y: rapidity
R: effective transverse radius
m_T: transverse mass



τ=1fm/c 程度に取れば、中心衝突では格子QCDからのε_Cをはるかに超えている

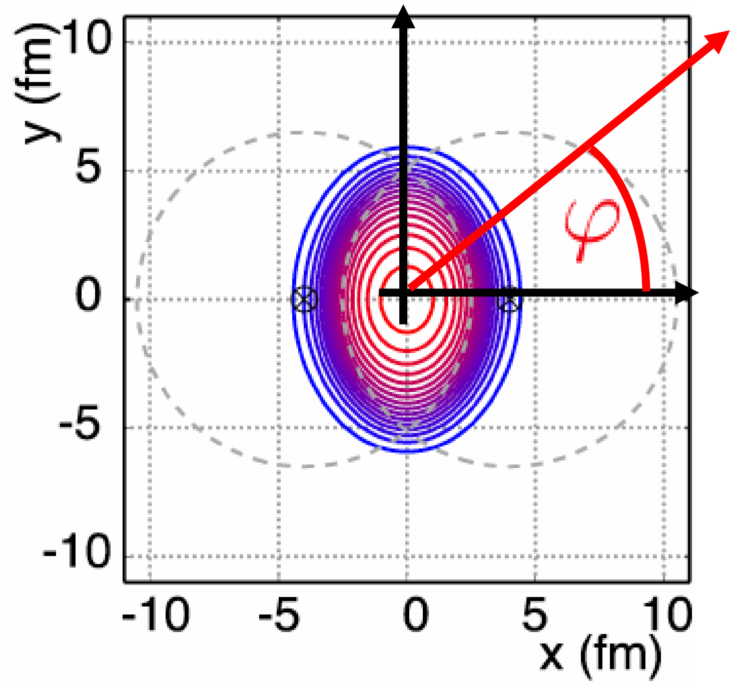


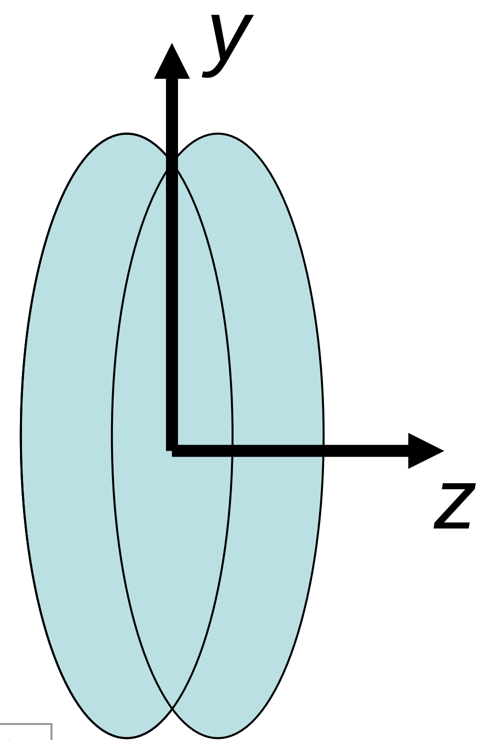
PHENIX('05)

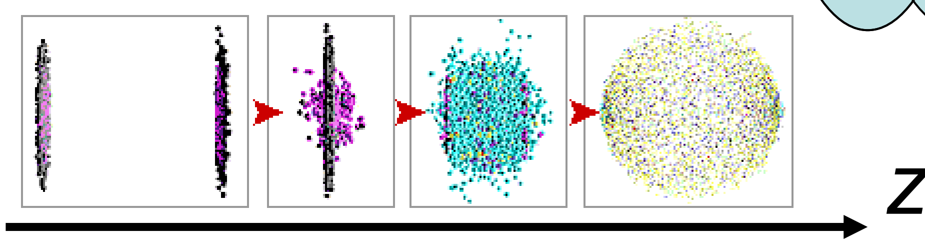


RHIC加速器で閉じ込め/非閉じ込め相転移温度Tcを超えたのはほぼ確実

Collective Flow （ガスだったら無い）

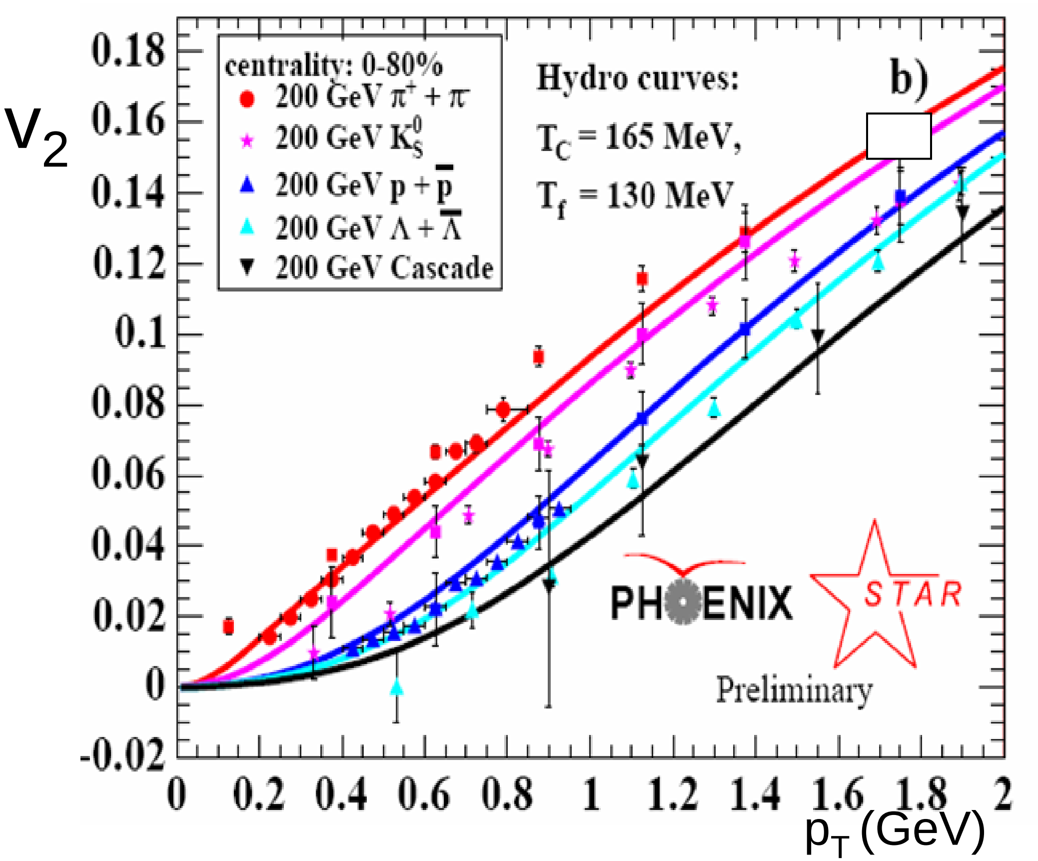






$$\frac{dN}{p_T dp_T dy d\varphi}(p_T, \varphi; b) = \frac{dN}{2\pi p_T dp_T dy}\left(1 + 2v_2(p_T; b)\cos(2\varphi) + ...\right)$$

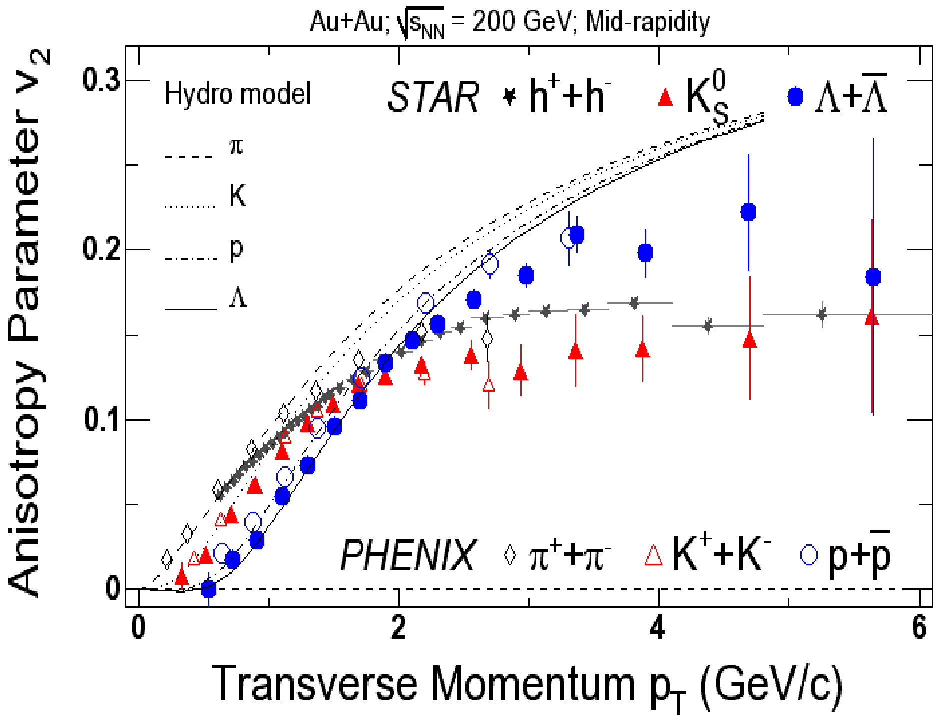
Like a Fluid?



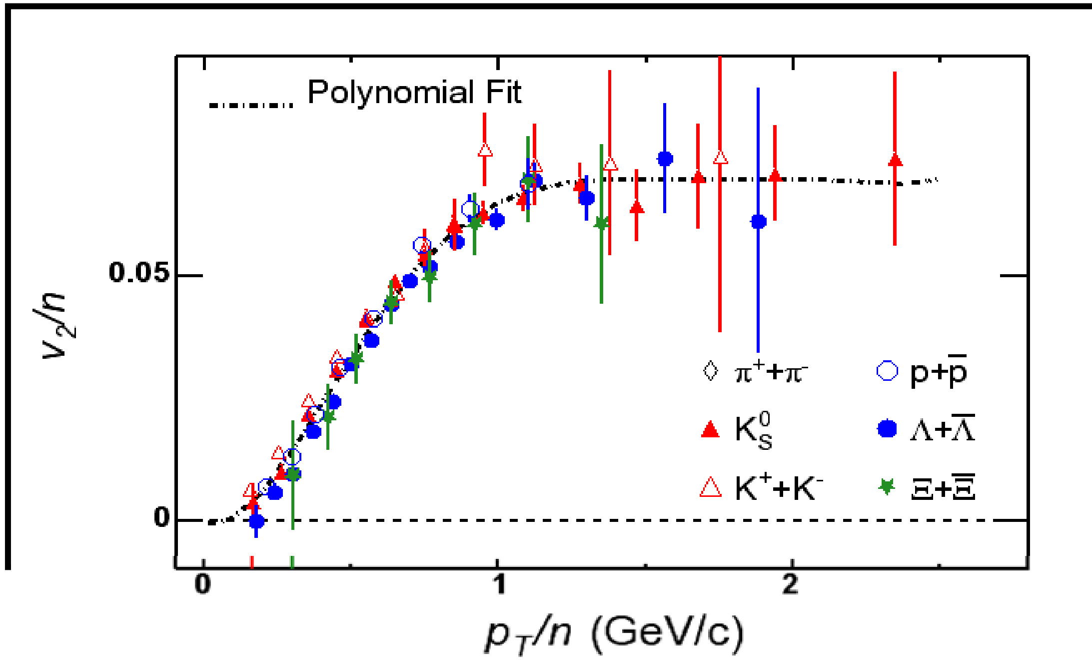
Lines:
Hydrodynamics calc.
with QGP type EoS.

*viscosity = resistance of liquid to shear forces (and hence to flow)

Thermalization time <u>t=0.6 fm/c</u> and <u>ε=20 GeV/fm^3</u>
Required QGP Type EoS in Hydro model



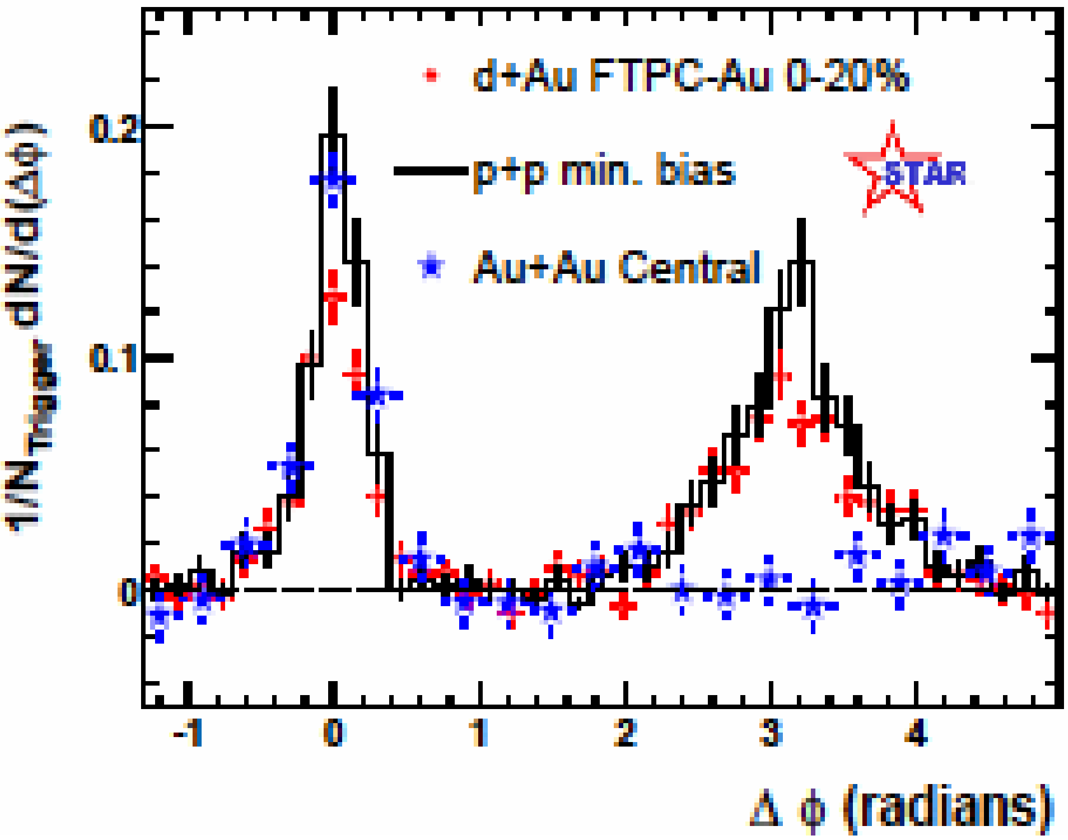
流れているのは
クォーク？！

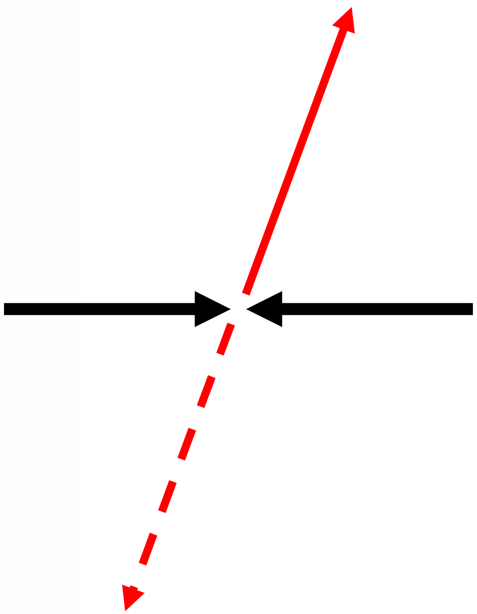


n=2 for mesons

n=3 for baryons !

Jet Quenching





いない！

Partonと非常に強く相互作用する物質が作られている？

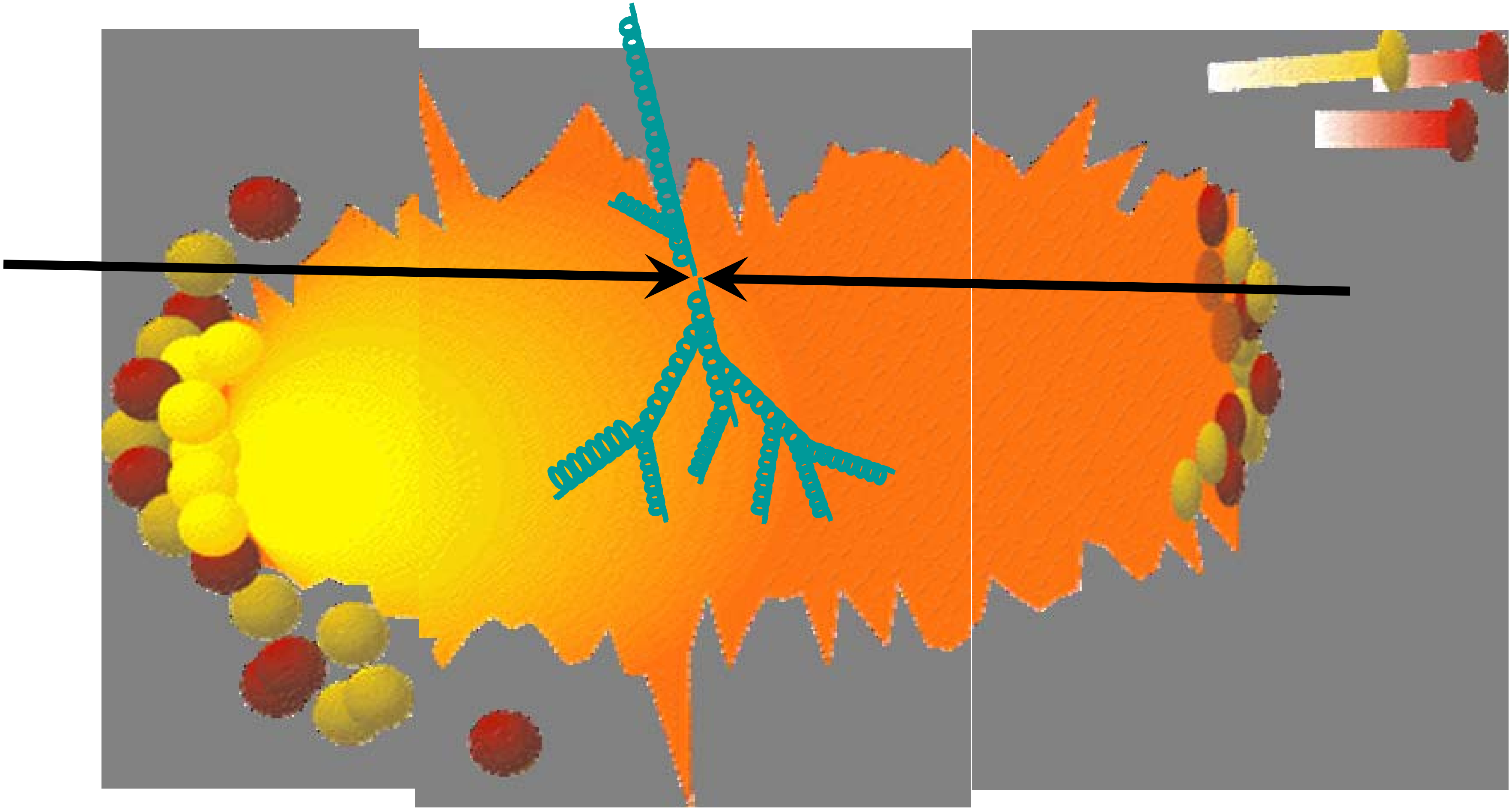
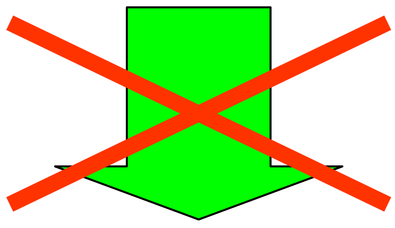


Figure stolen from Chujo's talk

Deconfinement

(Disappearing of the confinement potential)

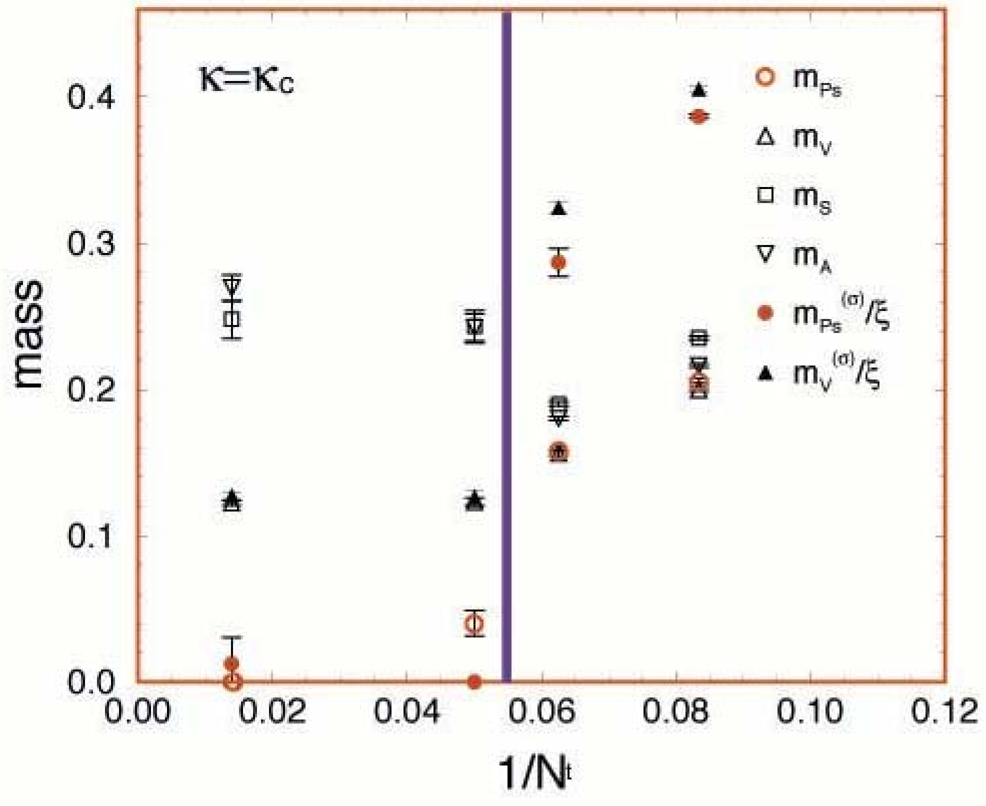


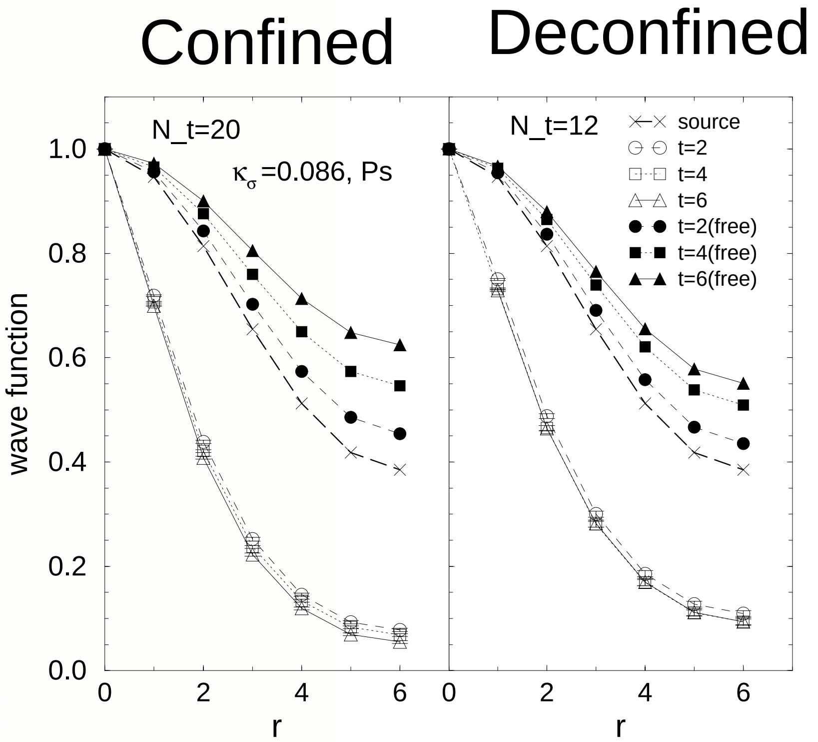
No Bound State

- QED is a Deconfinement theory, but there are Positroniums.
- Mass and Width may change.

Progress of Lattice Technology (2) - Hadrons at finite Temperature -

QCD-Taro Collaboration, Phys.Rev. D63 (2001) 054501,hep-lat/0008005

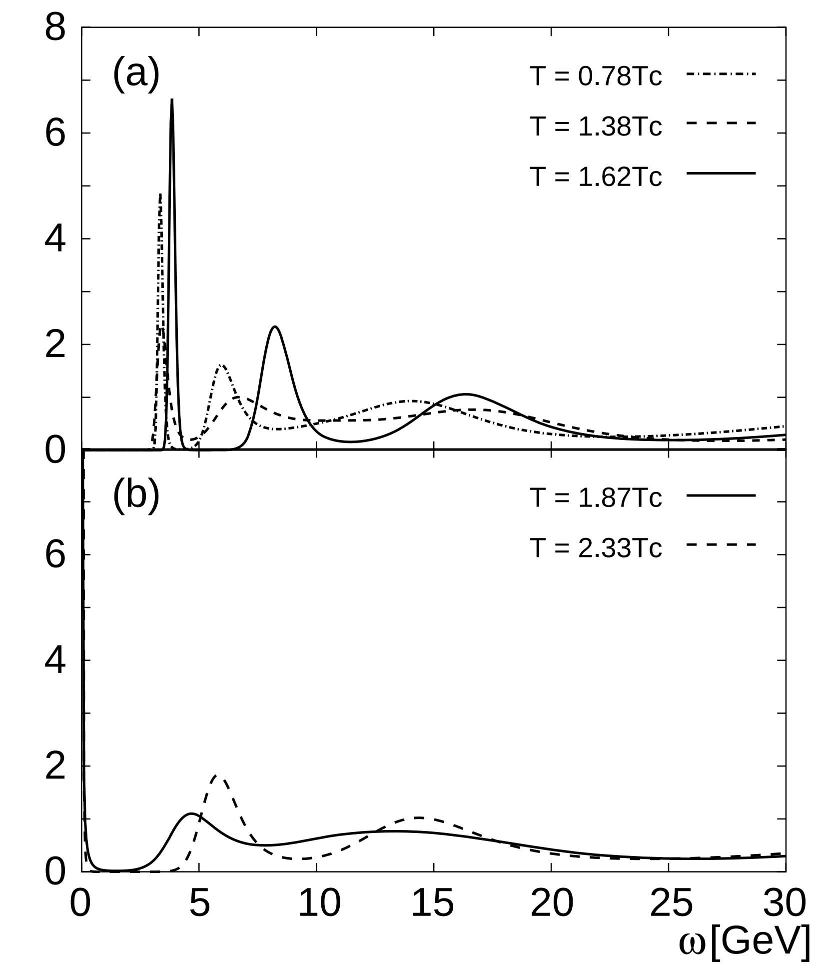


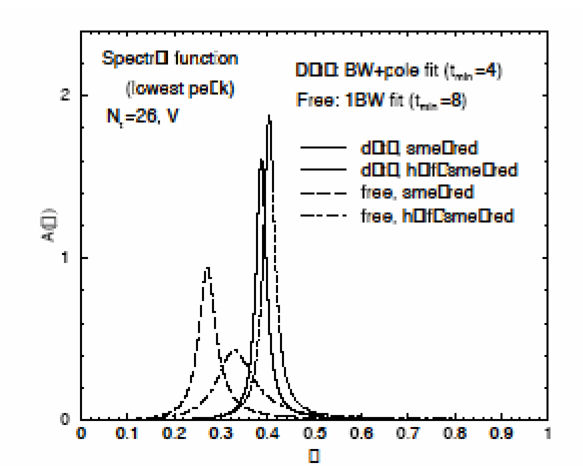


Spectral Functions at finite *T*

- Asakawa-Hatsuda
  - Phys.Rev.Lett. 92 (2004) 012001
- Umeda et al.
  - Nucl.Phys. A721 (2003) 922
- Datta et al.
  - Phys.Rev. D69 (2004) 094507

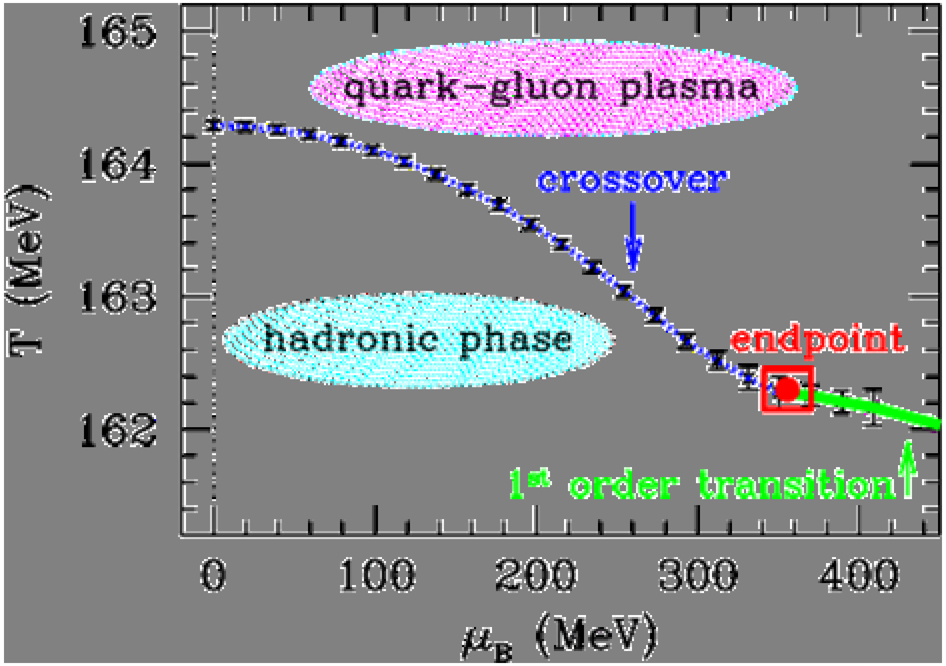
Asakawa-Hatsuda

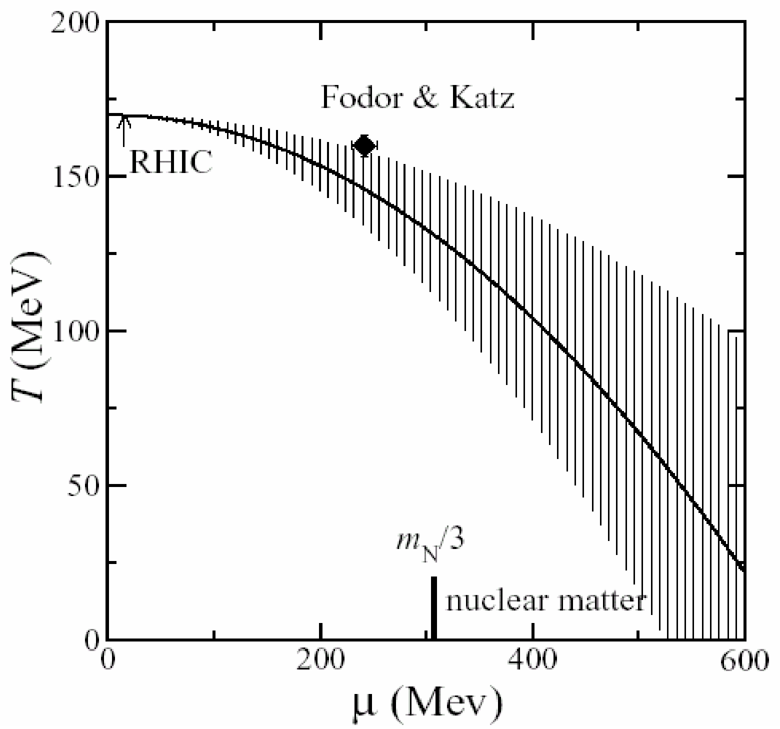


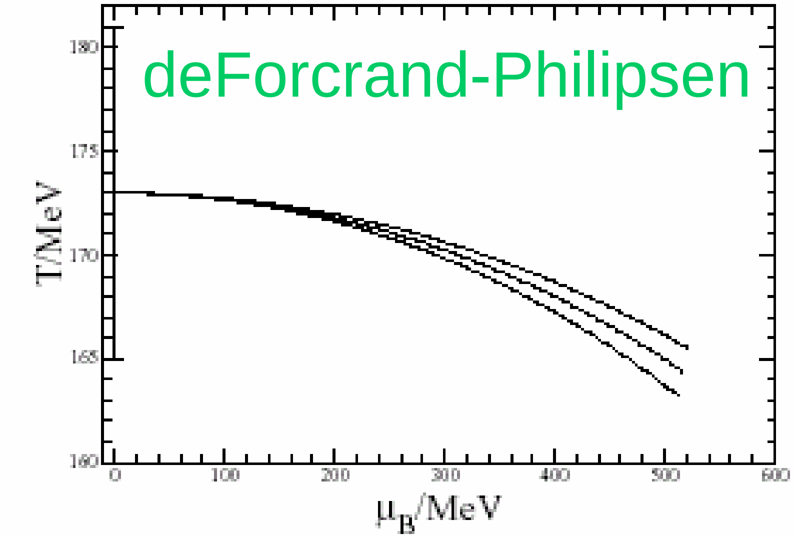


Umeda et al.

Progress of Lattice Technology (3)
- QCD Simulations at Finite Density -



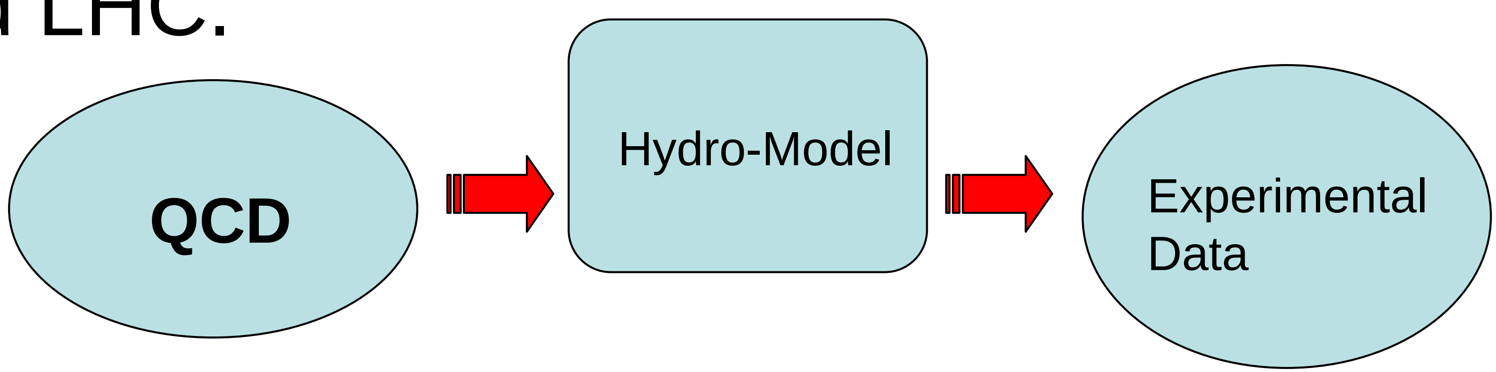




Transport Coefficients

A. Nakamura S.Sakai

- A Step towards Gluon Dynamical Behavior.
- They can be (in principle) calculated by a well established formula (Linear Response Theory).
- They are important to understand QGP which is realized in RHIC (and CERN-SPS) and LHC.



RHIC-data ⇨ Big Surprise !

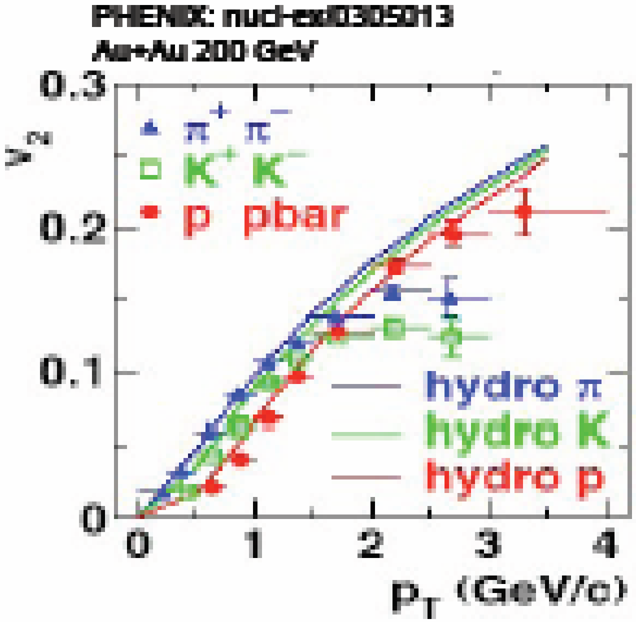
Hydro-dynamical
Model describes
RHIC data well !

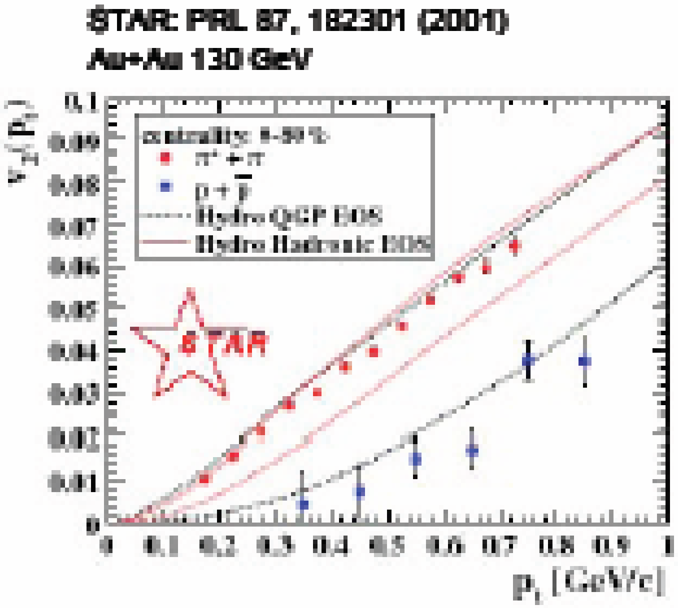
At SPS, the Hydro describes well one-particle distributions,

HBT etc., but fails for the elliptic flow.

Oh,
really ?

Hydro describes well v2





Hydrodynamical calculations are based on Ideal Fluid, i.e., zero shear viscosity.

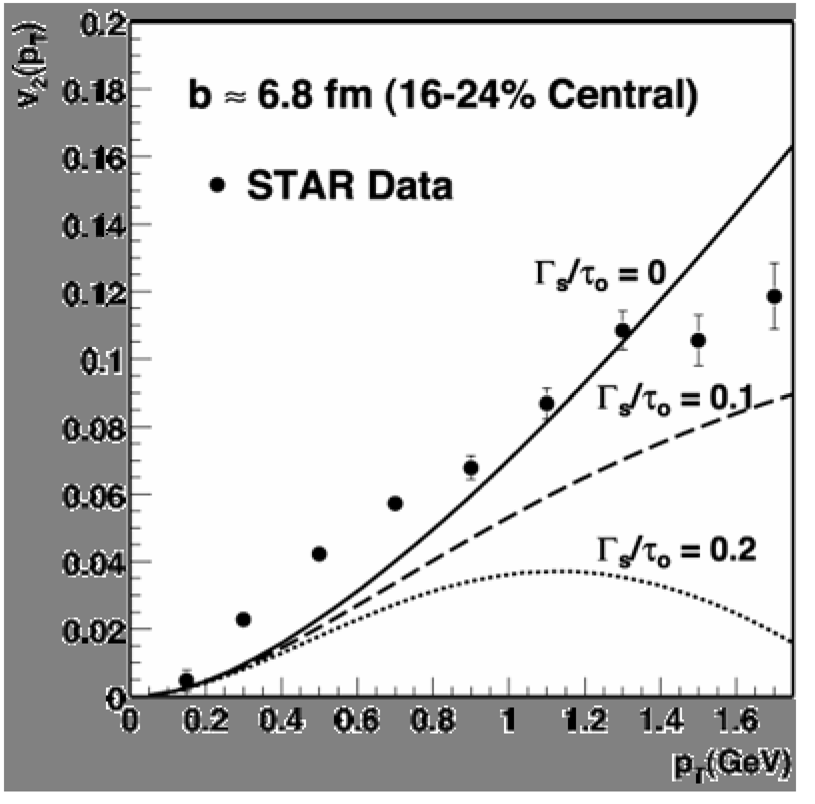
Or not so surprise …

- E. Fermi, Prog. Theor. Phys. 5 (1950) 570
  - Statistical Model
- S.Z.Belen’skji and L.D.Landau, Nuovo.Cimento Suppl. 3 (1956) 15
  - Criticism of Fermi Model
    “Owing to high density of the particles and to strong interaction between them, one cannot really speak of their number.”

Hagedorn, Suppl. Nuovo Cim. 3 (1956) 147. Limiting Temperature

Teaney, Phys.Rev. C68 (2003) 034913
(nucl-th/0301099)

$$\Gamma_s \equiv \frac{\frac{4}{3}\eta}{sT}$$

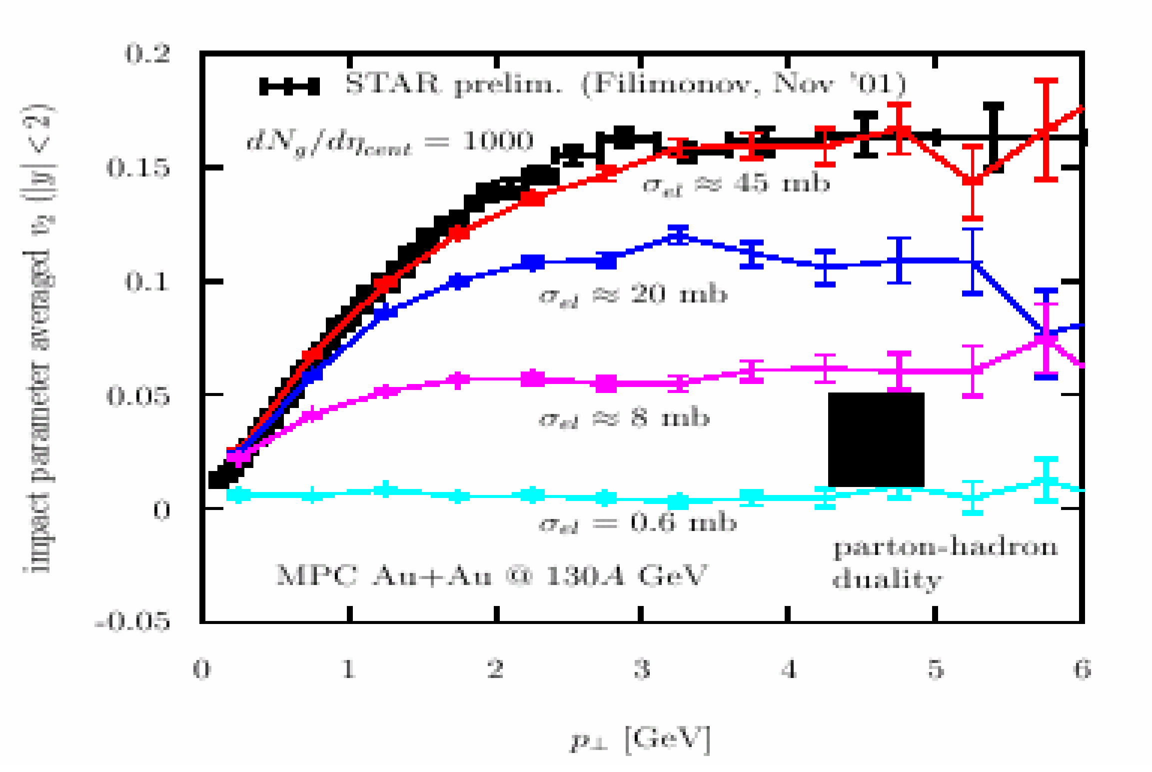


η : shear viscosity

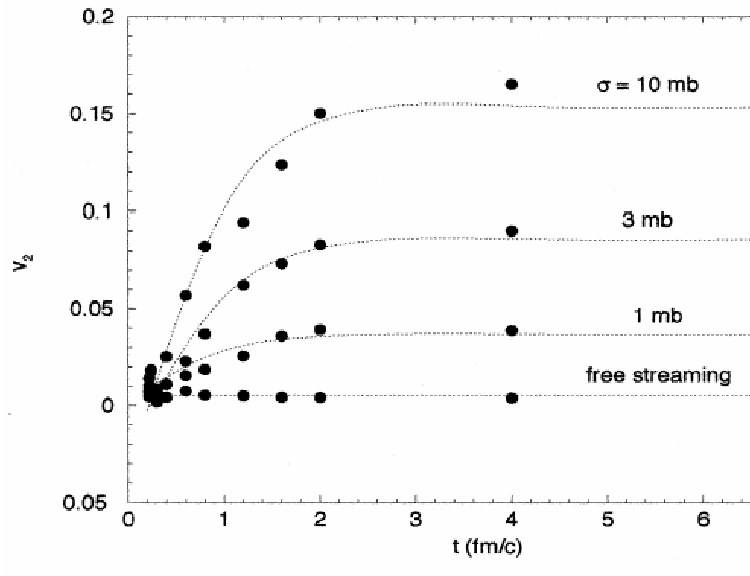
$\tau = \sqrt{t^2 - z^2}$: Time scale of the expansion

D. Molnar and M. Gyulassy, Nucl. Phys. A697 (2002) 495. (Erratum ibid. A703 (2002) 893);

“15 times above the perturbative estimate” (Molnar Quark Matter 2005)



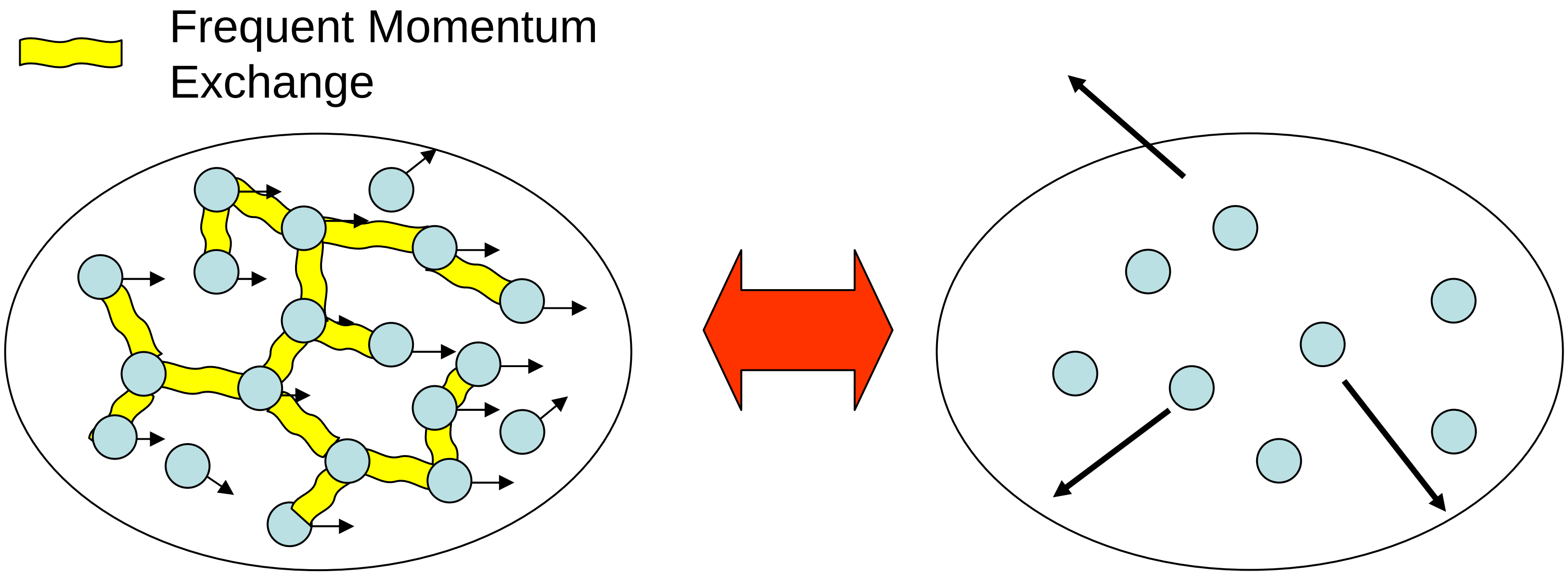
Zhang et al.(’99)



Another Big Surprise !

- The Hydrodynamical model assumes zero viscosity,
  i.e., Perfect Fluid.
- Phenomenological Analyses suggest also small viscosity.

Liquid or Gas ?

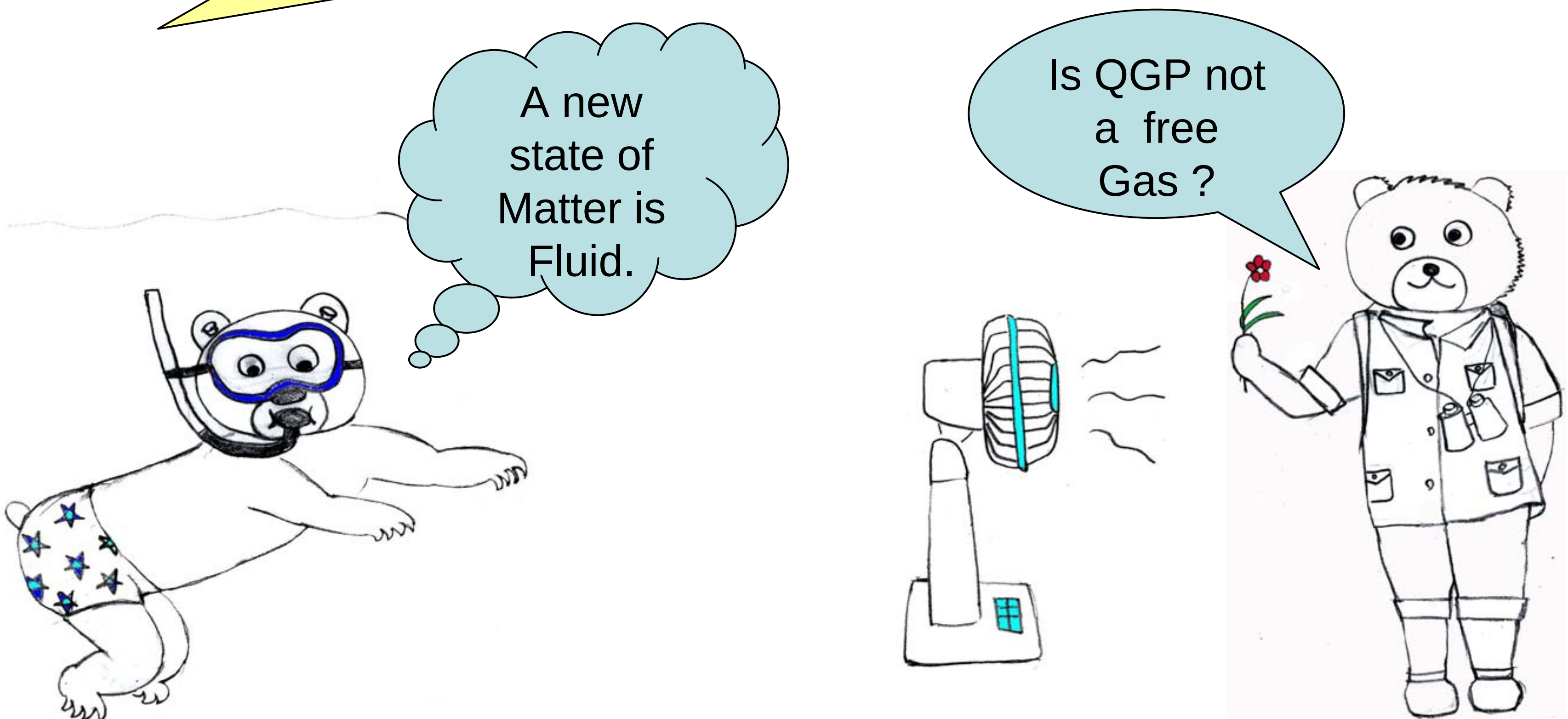


Perfect fluid

Opposite
Situation

Ideal Gas

If produced matter at RHIC is (perfect) Fluid, not Free Gas what does it mean ?



Lowest Perturbation
(Illustration purpose only)

Pressure

$$P = \frac{\pi^2}{90} T^4 \left(1 - \frac{15}{8} \left(\frac{g}{\pi}\right)^2 + \cdots \right)$$

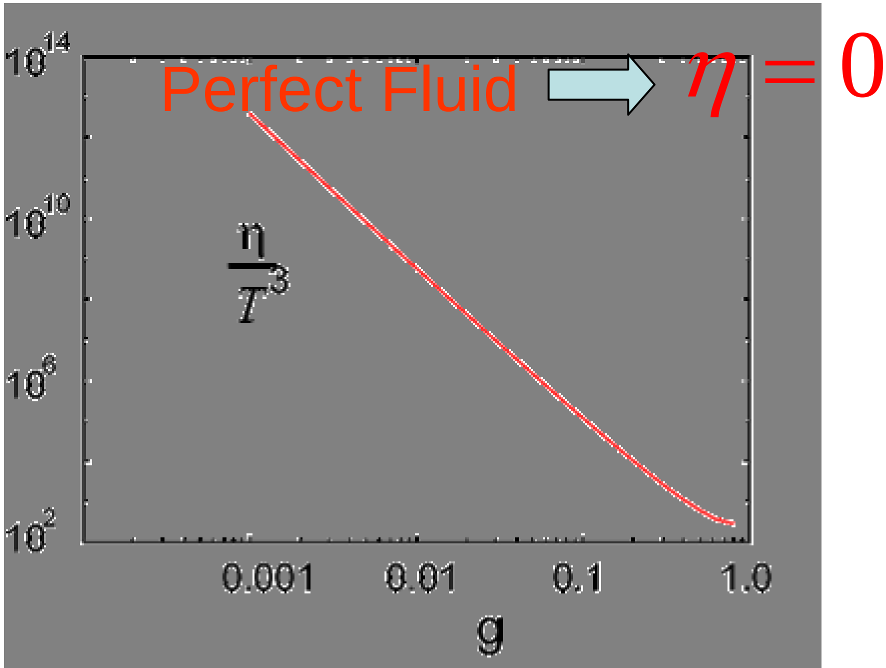
Ideal Free Gas

Viscosity

$$\eta = \kappa \frac{T^3}{g^4 \ln g^{-1}}$$

$$\kappa = 27.126 (N_f = 0), \quad 86.473 (N_f = 2)$$

- At weak coupling, it increases.



Literature (1) - 流体模型

- ランダウ･リフシッツ理論物理学教程
  - 流体力学（2） 絶版！
    - 第15章：相対論的流体力学
- J. D. Bjorken
  - Phys. Rev. D 27, 140–151 (1983)
    - Highly relativistic nucleus-nucleus collisions: The central rapidity region

Literature (1) –　流体模型（続き）

- Iso, Mori and Namiki, Prog. Theor. Phys. 22 (1959) pp.403-429
  - The first paper to analyze the Hydrodyanamical Model from Field Theory.
  - Applicability Conditions were derived:
    - Correlation Length << System Size
    - Relaxation time << Macroscopic Characteristic Time
    - Transport Coefficients must be small

Literature (2) – 輸送係数（摂動）

- G. Baym, H. Monien, C. J. Pethick and D. G. Ravenhall,
  - Phys. Rev. Lett. 16 (1990) 1867.
- P. Arnold, G. D. Moore and L. G. Yaffe
  - JHEP 0011 (2000) 001, (hep-ph/0010177).
  - Leading-log results"
- P. Arnold, G. D. Moore and L. G. Yaffe
  - JHEP 0305 (2003) 051, (hep-ph/0302165).
  - Beyond leading log"

Literature (3) – 輸送係数

- Hosoya, Sakagami and Takao, Ann. Phys. 154 (1984) 228.
  - Transport Coefficients Formulation
- Hosoya and Kayantie, Nucl. Phys. B250 (1985) 666.
- Horsley and Shoenmaker, Phys. Rev. Lett. 57 (1986) 2894; Nucl. Phys. B280 (1987) 716.
- Karsch and Wyld, Phys. Rev. D35 (1987) 2518.
  - The first Lattice QCD Calculation
- Aarts and Martinez-Resco, JHEP0204 (2002)053
  - Criticism against the Spectrum Function Ansatz.
- Petreczky and Teaney, hep-ph/0507318
  - Impossible to determine Heavy Quark Transport coefficient

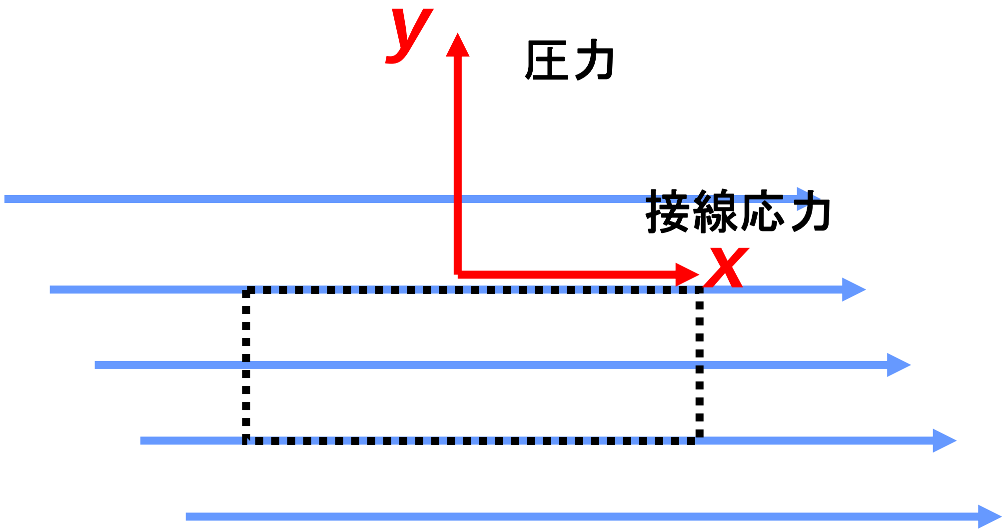
Literature (4) – 輸送係数（格子QCD）

- Masuda, A.N.,Sakai and Shoji
  Nucl.Phys. B(Proc.Suppl.)42, (1995),526
- A.N., Sakai and Amemiya
  Nucl.Phys. B(Proc.Suppl.)53, (1997), 432
- A.N, Saito and Sakai
  Nucl.Phys. B(Proc.Suppl.)63, (1998), 424
- Sakai, A.N. and Saito
  Nucl.Phys. A638, (1998), 535c
- A.N, Sakai
  Phys.Rev.Lett. 94 (2005) 072305
  hep-lat/0406009

Lietature(5) - Linear Response Theory

- ✓ Zubarev
  "Non-Equilibrium Statistical Thermo-dynamics"
- ✓ Kubo, Toda and Saito
  - Statistical Mechanics (Springer-Verlag, 1983)
  - 岩波講座　現代物理学の基礎（第2版）第5巻　統計物理学
- ✓ 木村他「中野藤生先生インタビュー ～線形応答理論から半世紀を経て～」
  - 物性研究　2005年 5月号 1000円
- ✓ 中嶋貞雄　線形応答理論の成立
  - 日本物理学会誌　第51巻10号
  - http://wwwsoc.nii.ac.jp/jps/jps/butsuri/50th/noframe/50(10)/50th-p699.html

粘性係数



$$\tau_{xy} = \eta \frac{\partial u_x}{\partial y}$$

接線応力が流れの勾配に比例
⇒　ニュートン流体

τ_{xy}：接線応力（ずれ応力）

$$\tau_{xy} = \tau_{yx} = 2\eta \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right)$$

（中村純　「物理とテンソル」（共立出版）　は別に参考にはなりません。
買ってくれてもいいけど）

ランダウの相対論的流体モデル

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu}$$

$$\partial_\mu j_i^\mu = 0$$ i : Baryon Number, Entropy

（+ 状態方程式）

$$T^{\mu\nu} \Rightarrow T^{\mu\nu} + 2\eta X^{\mu\nu} + \cdots$$

$$X_{\mu\nu} \equiv \frac{1}{2}\left(\Delta_{\mu\rho}\Delta_{\nu\sigma} + \Delta_{\mu\sigma}\Delta_{\nu\rho} - \frac{2}{3}\Delta_{\mu\nu}\Delta_{\rho\sigma}\right)\nabla^\rho u^\sigma$$

$$\Delta_{\mu\nu} \equiv g_{\mu\nu} - u_\mu u_\nu$$

$$\nabla^\mu \equiv \Delta^{\mu\nu}\frac{\partial}{\partial\sigma^\nu}$$

Tsumura, Kunihiro and Ohnishi hep-ph/0609056

$$\rho : \; e^{-A+B} : \text{ non-equilibrium statistical operator}$$

$$A = \int d^3x \beta(x,t) u^\nu T_{0\nu}(x,t)$$

$$B = \int d^3x \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} T_{\mu\nu}(x,t) \partial^\mu (\beta(x,t) u^\nu)$$

Using: $e^{-A+B} = e^{-A} + \int_0^1 d\tau e^{A\tau} B e^{-A\tau} e^{-A} + \cdots$

$$\rho \approx \rho_{eq} + \int_0^1 d\tau (e^{A\tau} B e^{-A\tau} e^{-A} - \langle B \rangle_{eq}) \rho_{eq}$$

$$\rho_{eq} \equiv e^{-A} / \mathrm{Tr} e^{-A} \rightarrow \exp(-\beta H) / \mathrm{Tr} e^{-A}$$

in the co-moving frame, $u^\mu = (1 \quad 0 \quad 0 \quad 0)$

$$\left\langle T_{\mu\nu} \right\rangle = \left\langle T_{\mu\nu} \right\rangle_{eq} +$$

$$+\int d^3x' \int_{-\infty}^{t} dt' e^{\varepsilon(t'-t)} (T_{\mu\nu}(x,t), T_{\rho\sigma}(x',t'))_{eq} \partial^{\rho}(\beta u^{\sigma})$$

where $(T_{\mu\nu}(x,t), T_{\rho\sigma}(x',t'))_{eq}$

$$\equiv \int_0^1 d\tau \left\langle T_{\mu\nu}(x,t) \left(e^{-A\tau} T_{\rho\sigma}(x',t') e^{A\tau} - \left\langle T_{\rho\sigma}(x',t') \right\rangle_{eq} \right) \right\rangle_{eq}$$

$$\left\langle T^{ij} \right\rangle = \eta (\partial^i u^j + \partial^j u^i)/2$$

$$\left\langle T^{0i} \right\rangle = -\chi (\beta^{-1}(x,t) \partial^i \beta + \partial_{\alpha} u^{\alpha})$$

$$\left\langle p \right\rangle - \left\langle p \right\rangle_{eq} = -\varsigma \partial_{\alpha} u^{\alpha}$$

$$p \equiv -\frac{1}{3} T^{i}{}_{i}$$

- One can show

$$(T_{\mu\nu}(x,t),T_{\rho\sigma}(x',t'))_{eq} = -\beta^{-1}\int_{-\infty}^{t'} dt''\left\langle T_{\mu\nu}(x,t),T_{\rho\sigma}(x',t'')\right\rangle_{ret}$$

Transport Coefficients are expressed by Quantities at Equilibrium

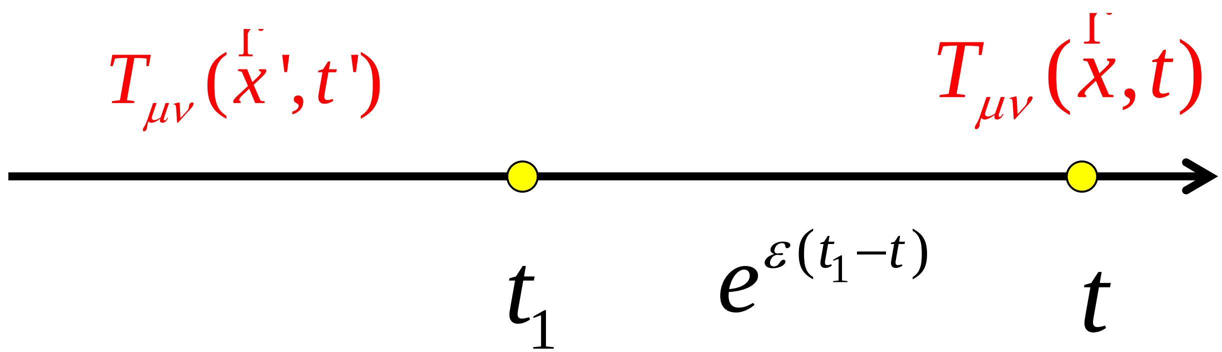
$$\eta = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1-t)} \int_{-\infty}^{t_1} dt' < T_{12}(\vec{x},t) T_{12}(\vec{x}',t') >_{ret}$$

$$\frac{4}{3}\eta + \varsigma = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1-t)} \int_{-\infty}^{t_1} dt' < T_{11}(\vec{x},t) T_{11}(\vec{x}',t') >$$

$$\chi = -\frac{1}{T}\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1-t)} \int_{-\infty}^{t_1} dt' < T_{01}(\vec{x},t) T_{01}(\vec{x}',t') >_{ret}$$

η : Shear Viscosity **ς : Bulk Viscosity**

χ : Heat Conductivity ⇒ we do not consider in Quench simulations.



$$-\infty < t' < t_1 < t$$

Energy Momentum Tensors

$$T_{\mu\nu} = 2Tr(F_{\mu\sigma}F_{\nu\sigma} - \frac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma})$$

$$(T_{\mu\mu} = 0)$$

$$U_{\mu\nu}(x) = \exp(ia^2 g F_{\mu\nu}(x))$$

$$F_{\mu\nu} = \log U_{\mu\nu} / ia^2 g$$

or

$$F_{\mu\nu} = \left(U_{\mu\nu} - U_{\mu\nu}{}^{\dagger}\right) / 2ia^2 g$$

Real Time Green function vs. *Temperature* Green function

Hashimoto, A.N. and Stamatescu, Nucl.Phys.B400(1993)267

$$<<\frac{1}{i}[\phi(t,\vec{x}),\phi(t',\vec{x}')]>>\equiv\frac{1}{Z}\mathrm{Tr}(\frac{1}{i}[\phi(t,\vec{x}),\phi(t',\vec{x}')]e^{-\beta H})$$

$$=F\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\Lambda(\omega,\vec{p})$$

$$\phi(t,\vec{x})=e^{itH}\phi(0,\vec{x})e^{-itH}$$

$$G_{\beta}^{ret/adv}(t,\vec{x};t',\vec{x}')=\pm\theta(t-t'/t'-t)<<\ldots>>$$

$$=F\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}K_{\beta}^{ret/adv}(\omega,\vec{p})$$

$$K_{\beta}^{ret/adv}(\omega,\vec{p})=\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\Lambda(\omega')}{\omega-\omega'\pm\varepsilon}$$

Temperature Green function

$$G_\beta(\tau,\vec{x};\tau',\vec{x}') = \ll T_\tau \phi(\tau,\vec{x})\phi(\tau',\vec{x}') \gg$$

$$\phi(t,\vec{x}) = e^{\tau H}\phi(0,\vec{x})e^{-\tau H}$$

$$G_\beta(\tau,\vec{x};0,0) = G_\beta(\tau+\beta,\vec{x};0,0)$$

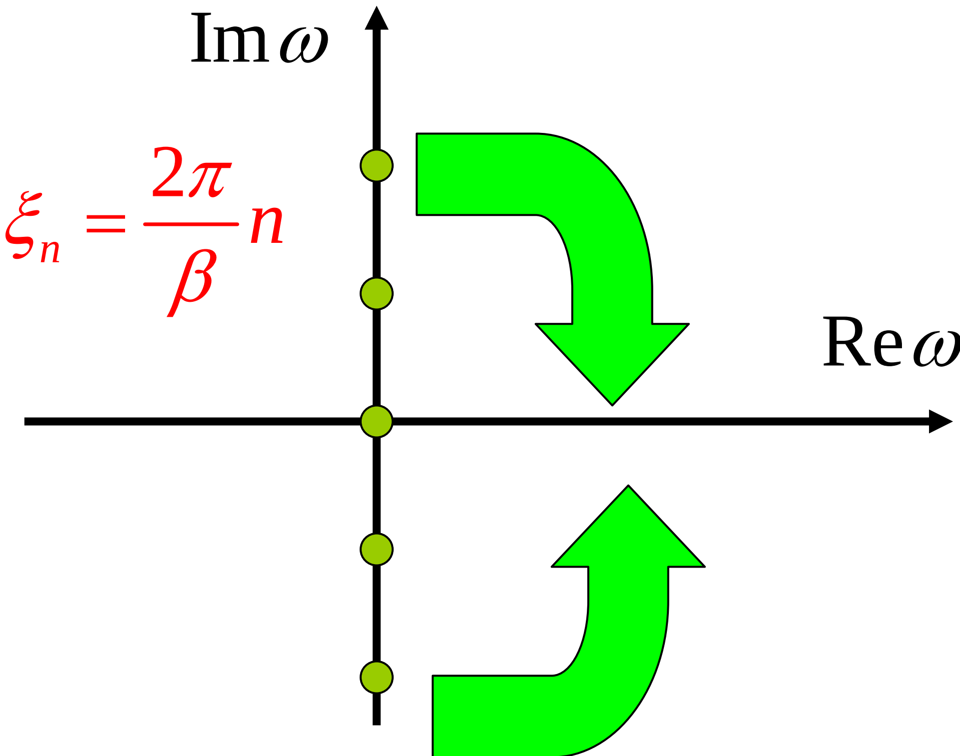
$$\hat{K}_\beta(\xi_n,\vec{p}) = F^{-1}\int_0^\beta d\tau e^{-i\xi_n(\tau-\tau')}G_\beta(\tau,\vec{x};\tau',\vec{x}')$$

$$\xi_n = \frac{2\pi}{\beta}n, n = 0, \pm 1, \pm 2,,,$$

Matsubara-frequencies

Abrikosov-Gorkov-Dzyalosinski-Fradkin Theorem

$$\hat{K}_\beta(\xi_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\Lambda(\omega)}{\omega - i\xi_n} = iK_\beta(i\xi_n)$$



On the lattice, we measure Temperature Green function at $\omega = \xi_n$

We must reconstruct Advance or Retarded Green function.

Transport Coefficients of QGP

We measure Correlations of Energy-Momentum tensors

$$< T_{\mu\nu}(0) T_{\mu\nu}(\tau) >$$

↓

Convert them (Matsubara Green Functions) to Retarded ones (real time).

↓

Transport Coefficients (Shear Viscosity, Bulk Viscosity and Heat Conductivity)

Ansatz for the Spectral Functions

We measure Matsubara Green Function on Lattice (in coordinate space).

$$< T_{\mu\upsilon}(t,\vec{x}) T_{\mu\upsilon}(0) > = G_\beta(t,\vec{x}) = F.T.G_\beta(\omega_n,\vec{p})$$

$$G_\beta(\vec{p}, i\omega_n) = \int d\omega \frac{\rho(\vec{p},\omega)}{i\omega_n - \omega}$$

We assume (Karsch-Wyld)

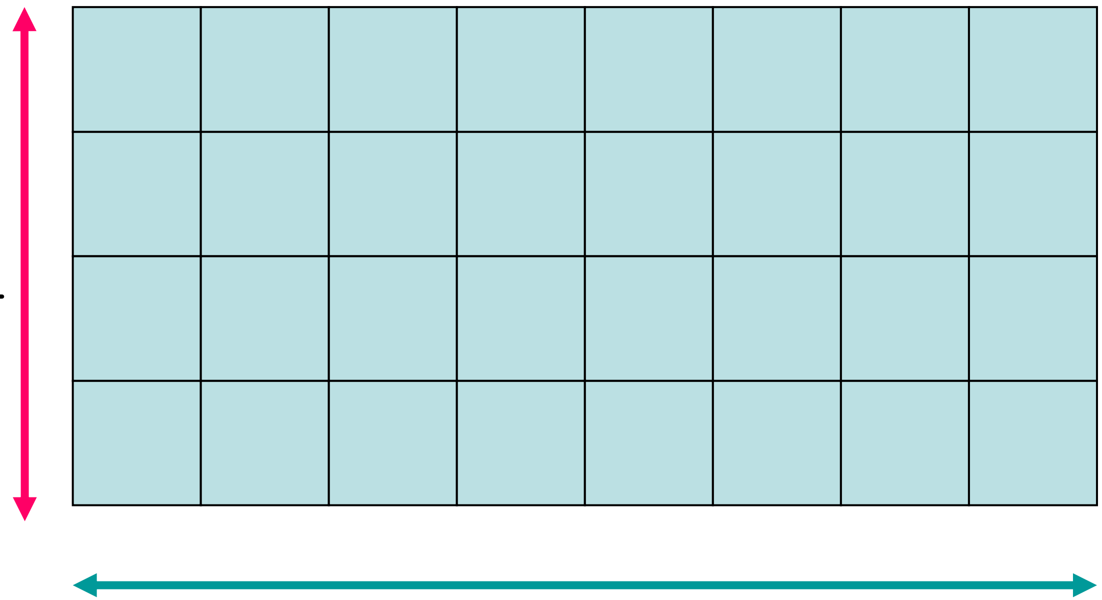
$$\rho = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2+\gamma^2} + \frac{\gamma}{(m+\omega)^2+\gamma^2} \right)$$

and determine three parameters,
A, m, γ.

We need large Nt !

Some Special Features of Lattice QCD at Finite Temperature

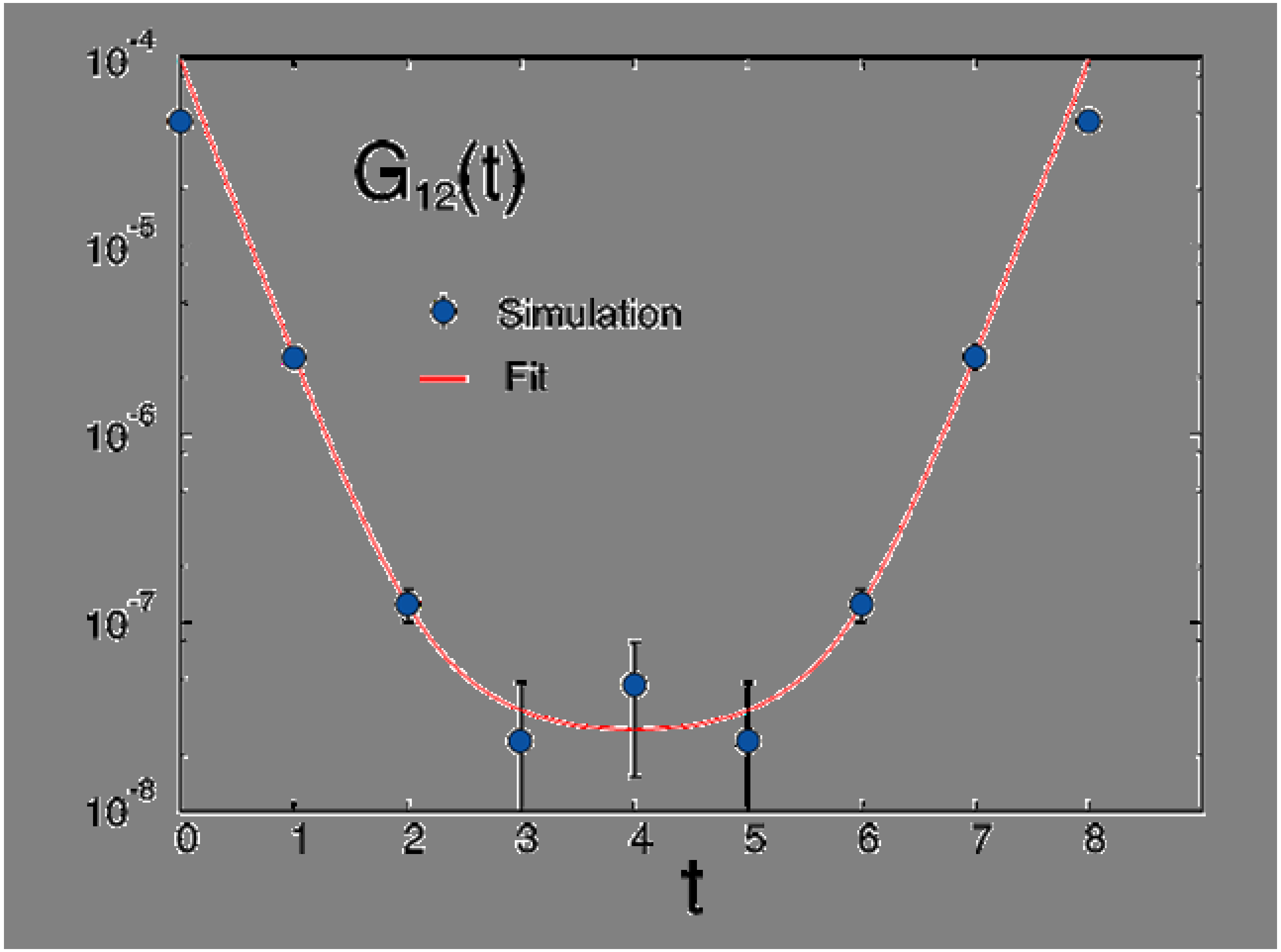
$$N_t a_t = \frac{1}{kT}$$



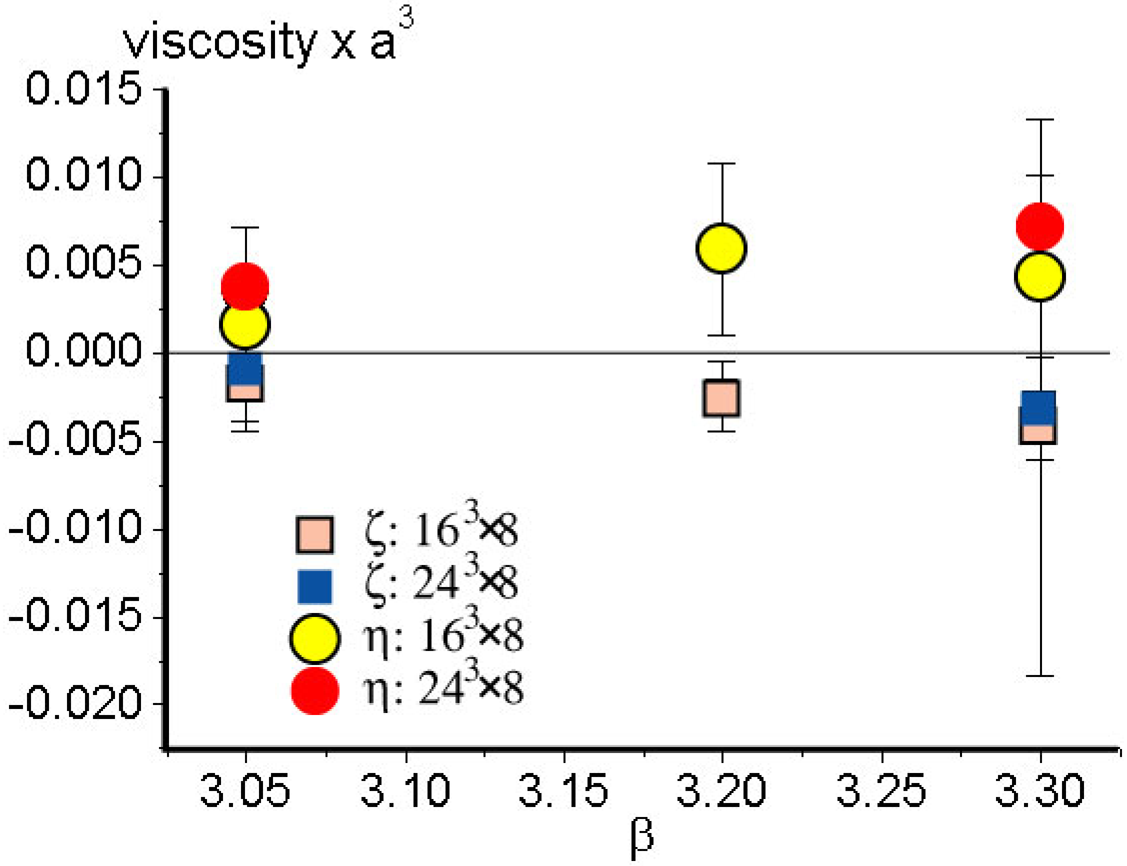
$$N_s a_s$$

High Temperature ⇒ $N_t a_t$: small

Nt=8

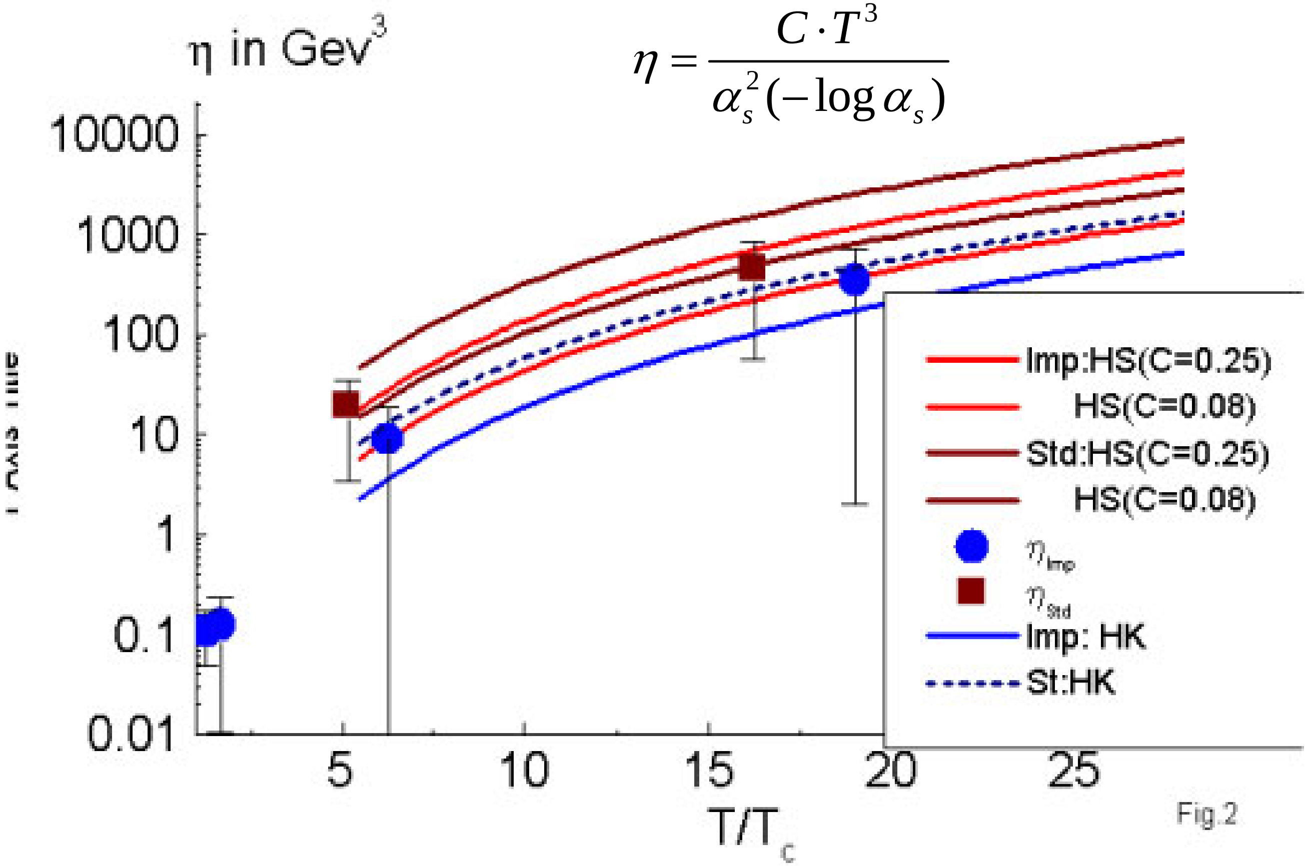


Results: Shear and Bulk Viscosities



Comparison with Pertubative Calculations

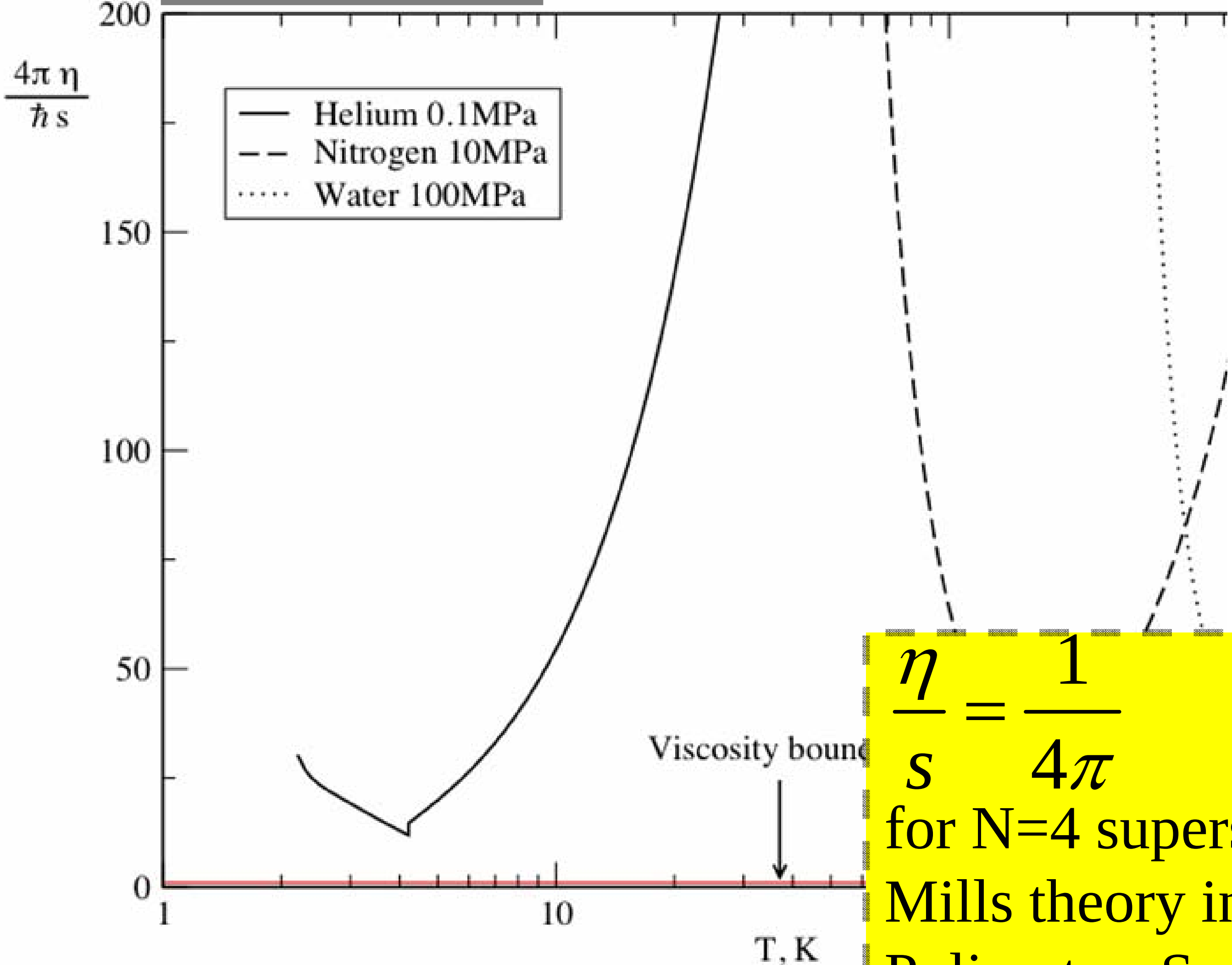
$$\eta = \frac{C \cdot T^3}{\alpha_s^2(-\log \alpha_s)}$$



Good for T/Tc>5

$$\frac{\eta}{s} \geq \frac{1}{4\pi} \ !$$

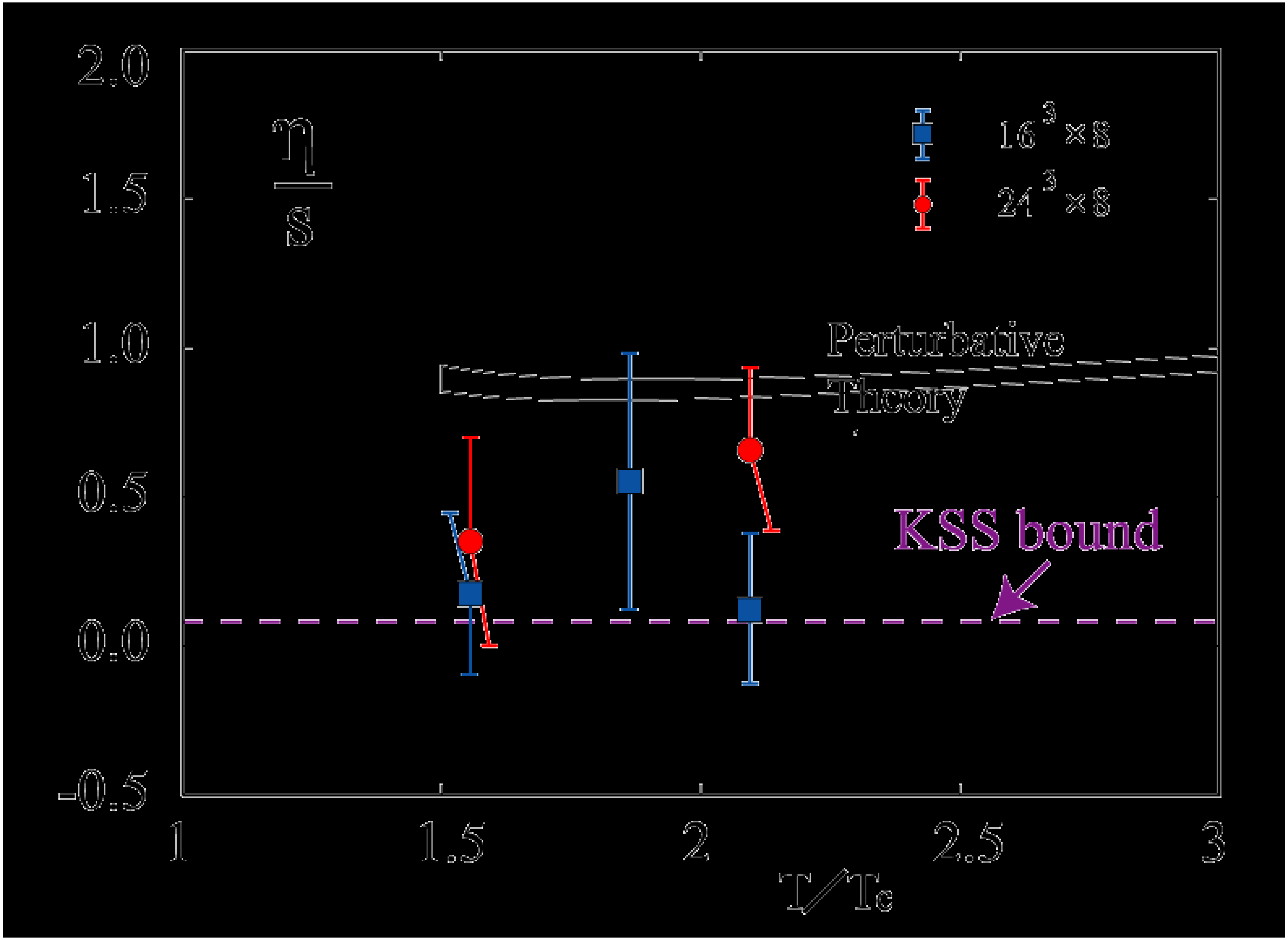
Kovtun, Son and Starinets, hep-th/0405231



$$\frac{\eta}{s} = \frac{1}{4\pi}$$

for N=4 supersymmetric Yang-Mills theory in the large N.

Policastro, Son and Starinets, Phys. Rev. Lett. 87 (2001) 081601



η/s
16³×8
24³×8
Perturbative
Theory
KSS bound
2.0
1.5
1.0
0.5
0.0
-0.5
1
1.5
2
2.5
3
T/Tc

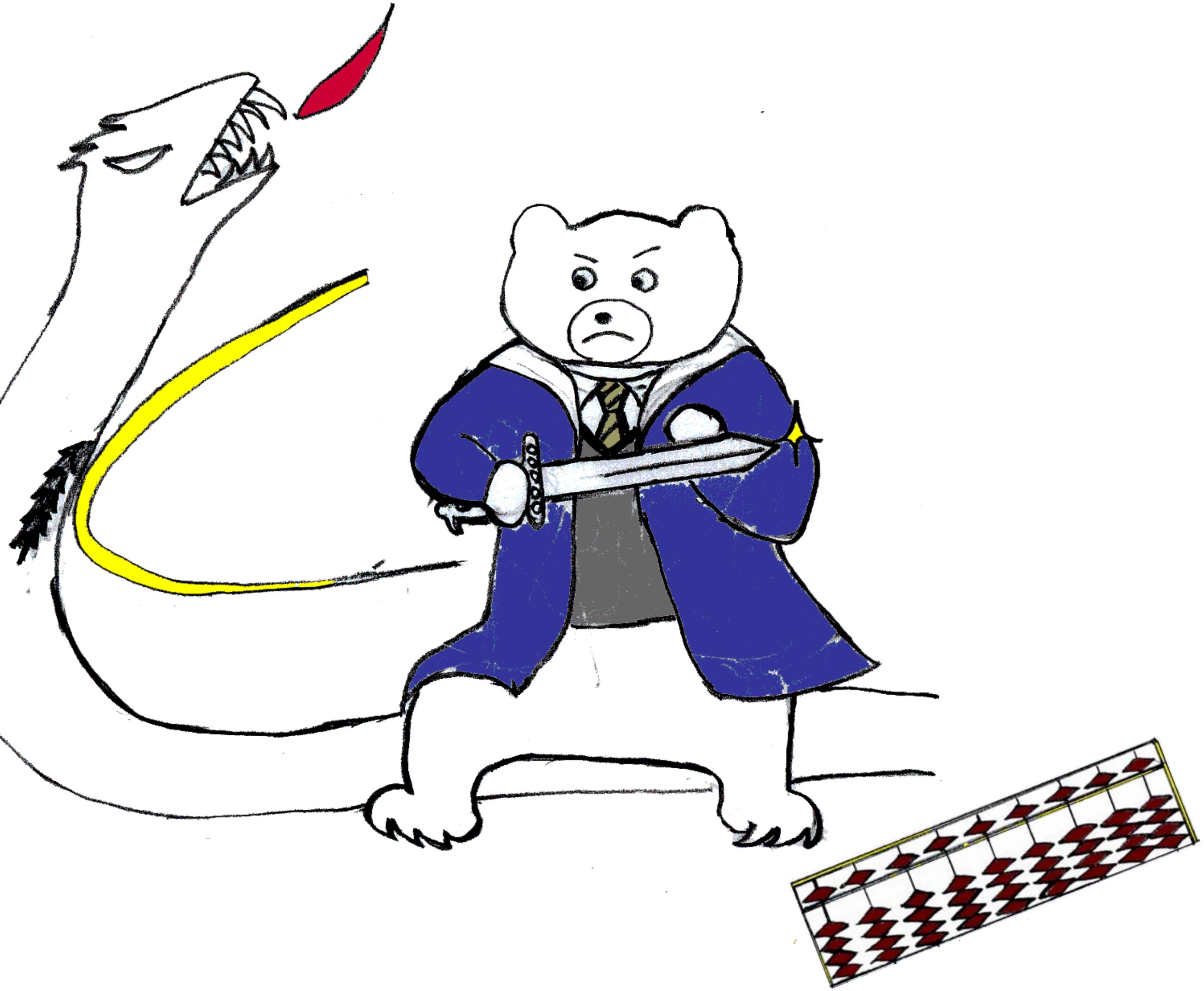
$\frac{\eta}{s}$ can have the lower limit ?

- Counter Example by Prof. Baym
  - We heat up Billiard Balls which have inter-structure. Then Entropy increases. If the surface of the balls does not change, the Viscosity should be the same.

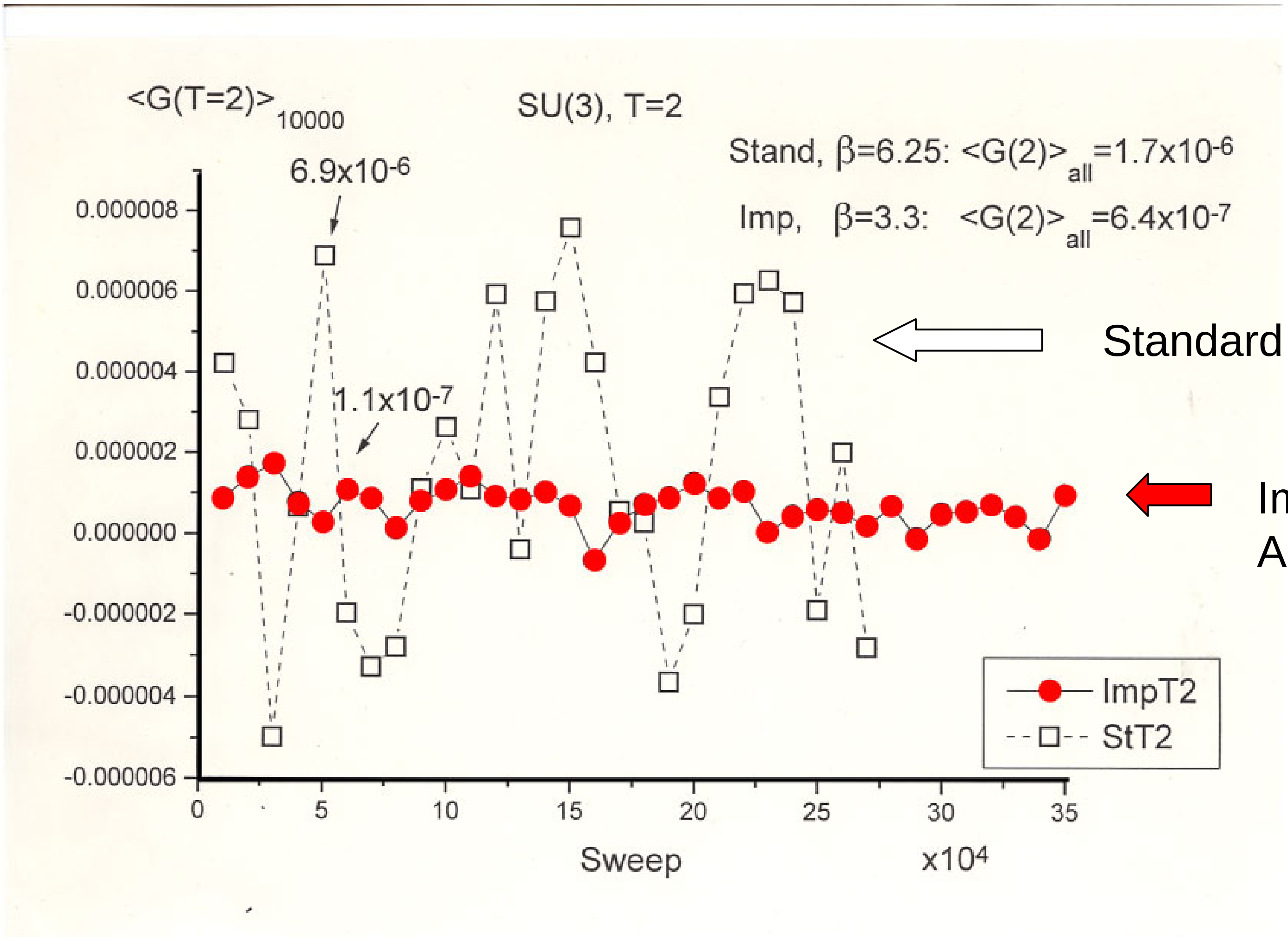
$$\Rightarrow \quad \frac{\eta}{s} \to 0$$

- We may give Counter-Argument ?

Fighting against Noise



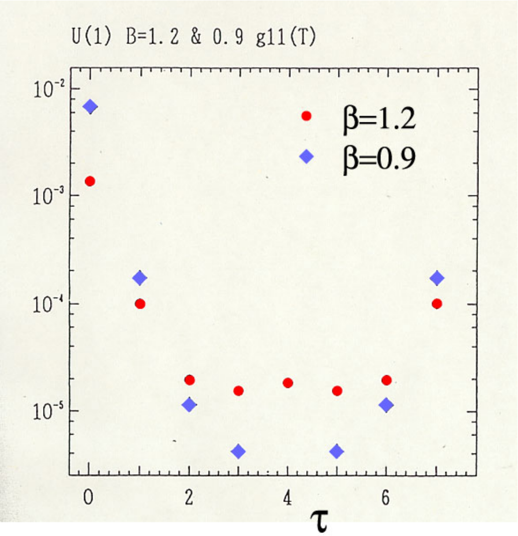
Fluctuations in MC sweeps



Standard Action

Improved Action

Correlators



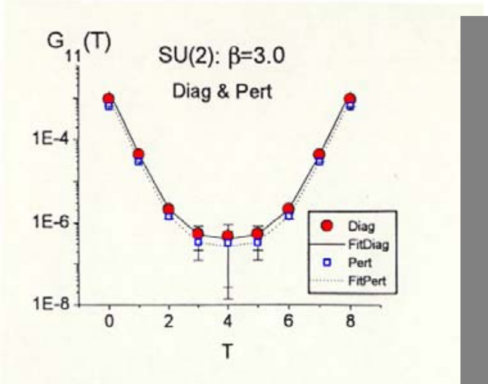
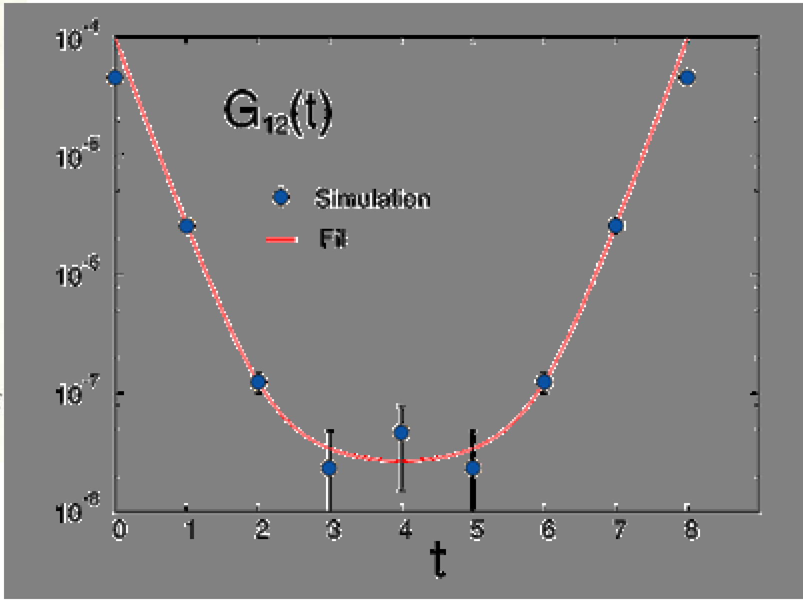


Figure 3. Fit of $G_{11}(T)$ by the parameters of spectral function for SU(2) at $\beta = 3.0$



U(1)
Coulomb and Confinement Phases

SU(2)

Two Definitions:

F=log U

F=U-1

SU(3)

Improved Action

Errors in U(1), SU(2), SU(3) standard and SU(3) improved

1995 U(1)

1997 SU(2)

1998 SU(3) preliminary

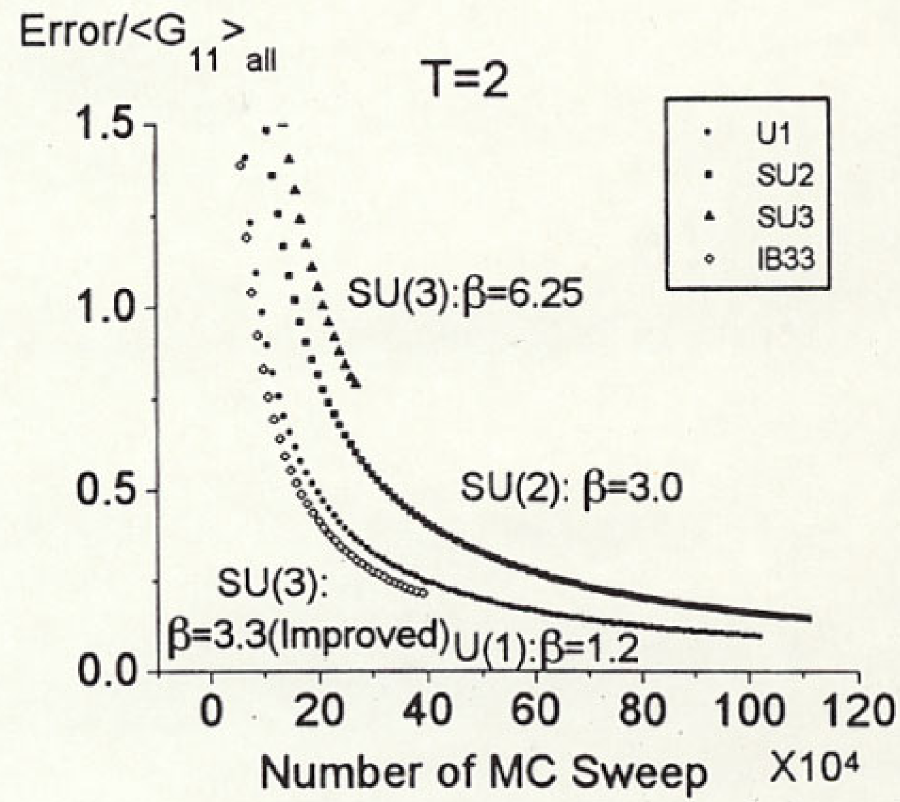


Figure 2. Error as a function of number of MC sweeps at $T = 2$ for U(1) $\beta = 1.2$, SU(2) $\beta = 3.0$, SU(3) $\beta = 6.25$ and improved action for SU(3) $\beta = 3.3$.

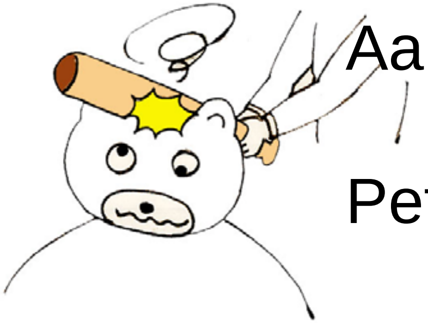
Low Frequency Region in Spectral Function $\rho(\omega)$ is Important

$$\eta = \pi \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$ Horsley and Shoenmaker

$(\varepsilon \longrightarrow 0)$ after the Thermo-Dynamics Limit

Long Range in τ of Thermal Green Function $< T_{\mu\nu}(0) T_{\mu\nu}(\tau) >$ on the Lattice should be precisely determined.

⇨ The finite volume scaling will be required.



Aarts and Martinez-Resco, JHEP0204 (2002)053

Criticism against the Spectrum Function Ansatz.

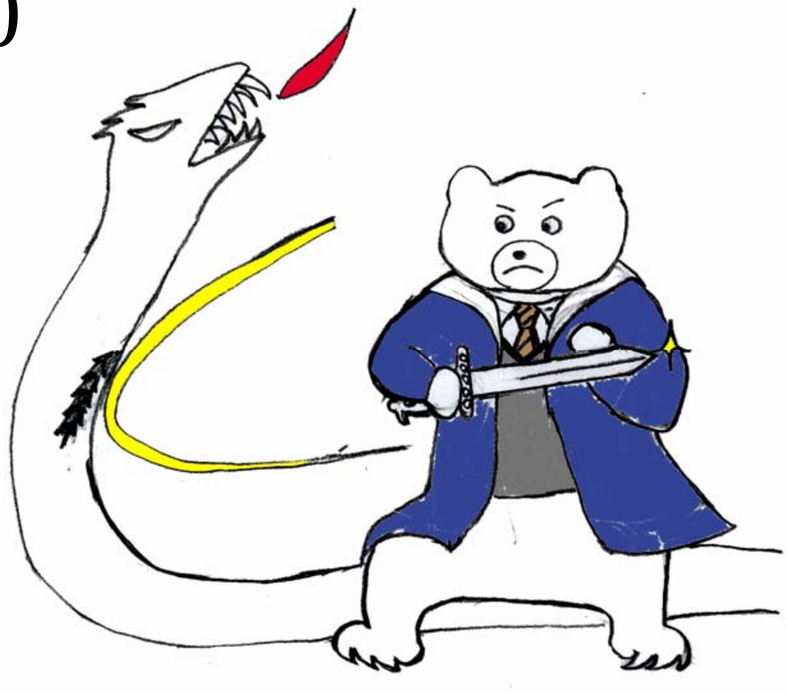
Petreczky and Teaney, hep-ph/0507318

Impossible to determine Heavy Quark Transport coefficient

Note that

Non-Equilibrium Calculations are in general subtle.

- Important Regions : $\omega \sim 0$
  - Physics is in Infra-Red i.e., Themodynamical Limit
- But this is Challenge of Lattice Simulation !



Summary

- We have calculated Transport Coefficients on Nt=8 Lattice. The limitations are
  - Quench Approximation
  - In order to convert Matsubara Green Function to Retarded one, we use Ansatz for Spectral Function with fitting parameters:

$$\rho = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2+\gamma^2}+\frac{\gamma}{(m+\omega)^2+\gamma^2}\right)$$

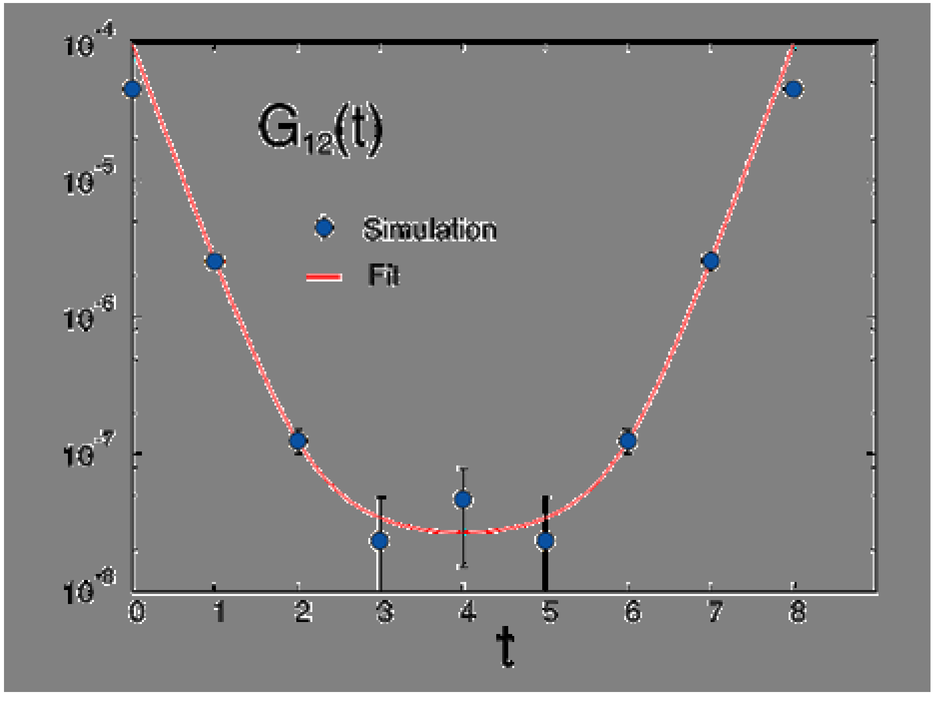
- Shear Viscosity
  - Positive $\eta / s \sim 0.1$
- Bulk Viscosity ~ 0
- Improved Action helps us a lot to get good Signal/Noise ratio.

Future direction ?

- If we can extract the Spectral Density $\rho(\omega)$ we can get the Transport Coefficients.
  - Maximum Entropy Method by Asakawa, Nakahara and Hatsuda
- We need (probably)
  - Anisotropic Lattice
  - Finite size scaling analysis
- Full QCD ?
  or
  with Quark Sector even in quench ?

We need data at large τ (small ω) with $O\left(\frac{1}{10}\right)$ Errors

- Brute Force ?
  - Not so crazy because the next Super-Computer is Peta-Flops Order.
- Good Operator
  - Extended
  - Renormalized



今日の話を要約すると

- 超高エネルギーで重い原子核同士を衝突させたら、温度が上がり閉じ込め/非閉じ込め相転移温度をおそらく超えた

実験の概要
イメージはつかめたでしょうか？

- そこで作られたQCD物質は、クォークとグルーオンが自由に飛んでいるガス状のものではなかった

- 完全流体に近いもの？

同意してくれる？

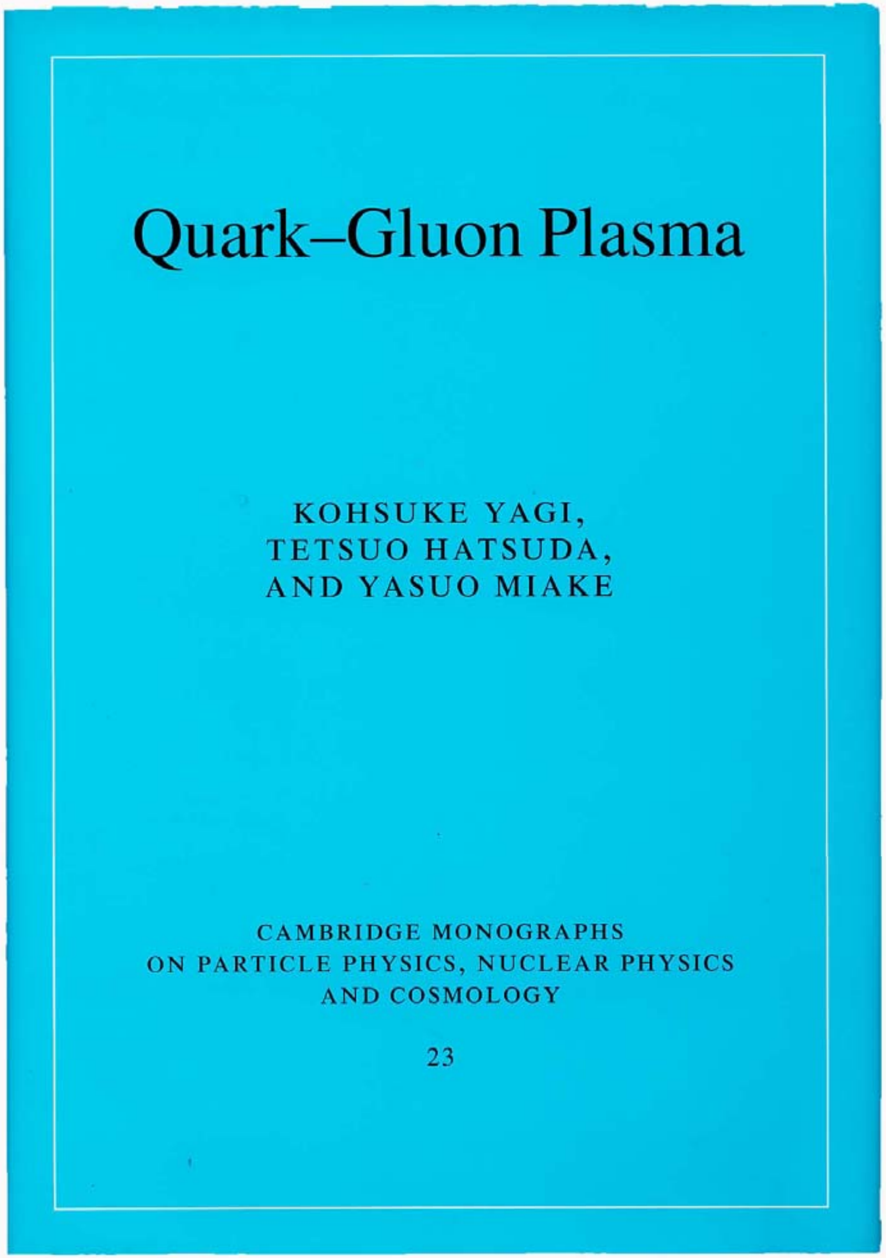
- 格子QCDで輸送係数の計算をいろいろ苦労しながら試みている

面白かった？

We are the poorest group among Lattice Society
But Interesting QGP Physics motivates us go further as possible as we can !
Anyone is welcome to join !

Backup Slides

頼まれてもいないのにコマーシャル



1. What is the quark-gluon plasma ?

   Part I Basic Concept of Quark-Gluon Plasma
2. Introduction to QCD
3. Physics of the quark-hadron phase transition
4. Field theory at finite temperature
5. Lattice gauge approach to QCD phase transition

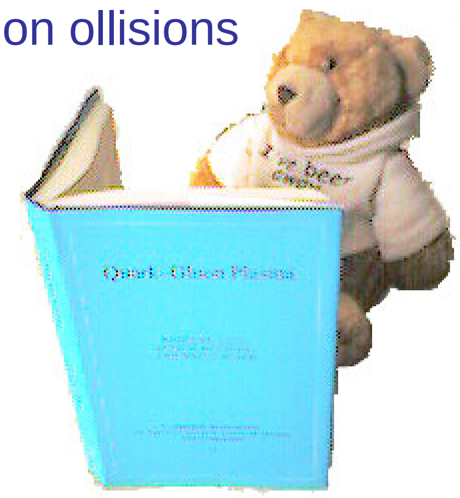
   ...

   Part II Quark-Gluon Plasma in Astrophysics

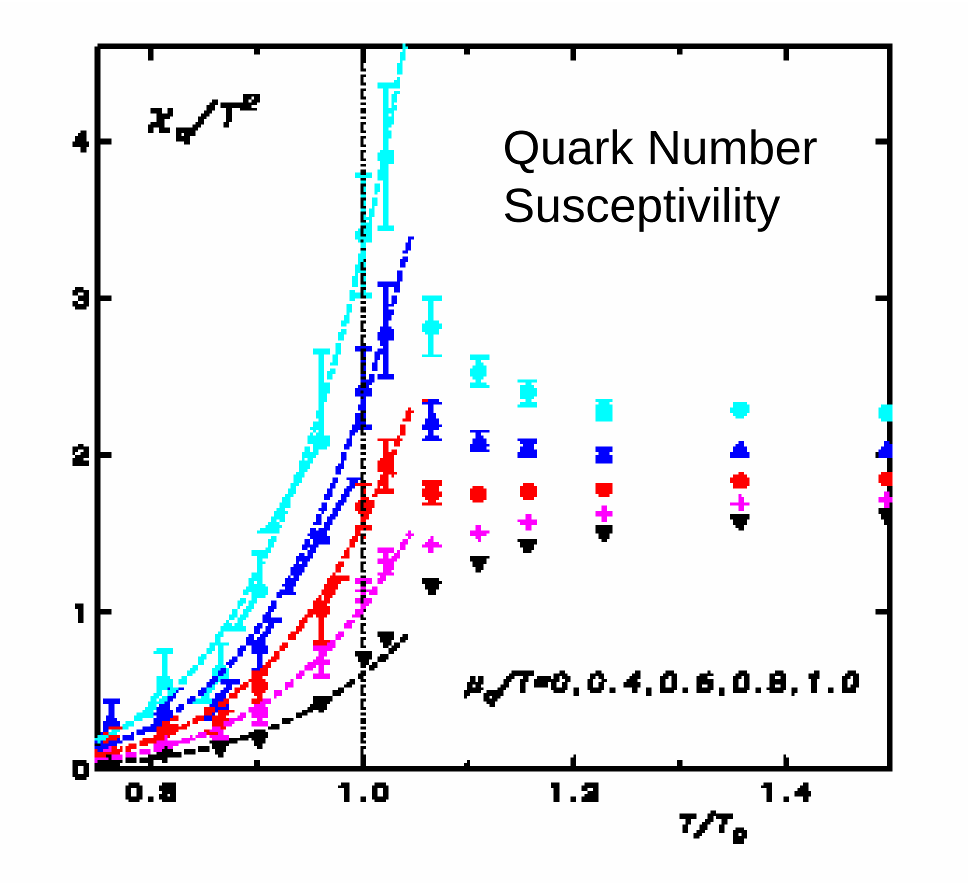
   ...

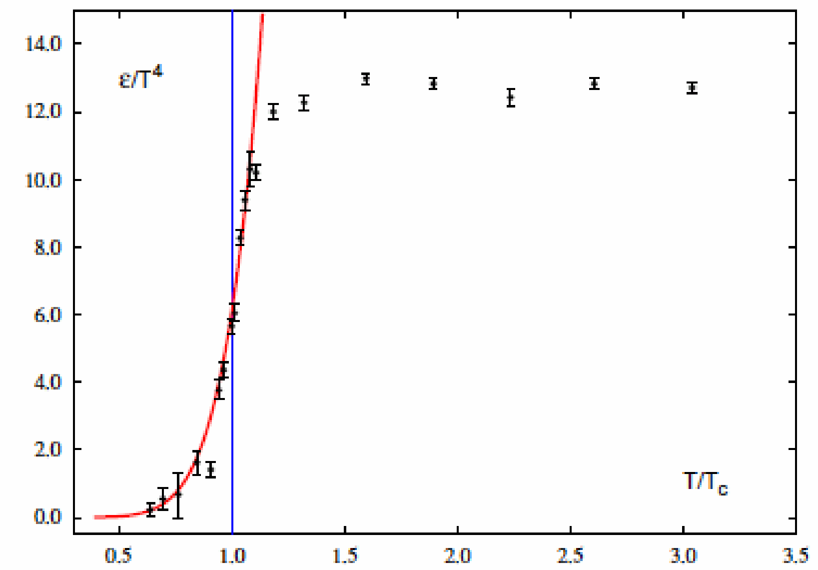
   Part III Quark-Gluon Plasma in Relativistic Heavy Ion ollisions

   ...



Comparison of Lattice with Resonance Gas Model





Karsch, Redlich and Tawfik

Phys.Lett. B571 (2003) 67

Masses in the model are modified to fit Lattice data.

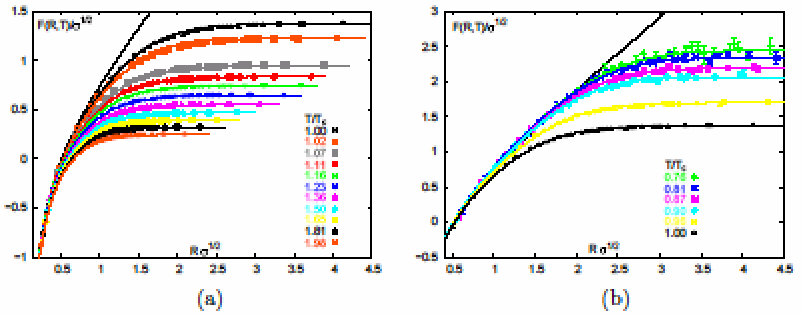
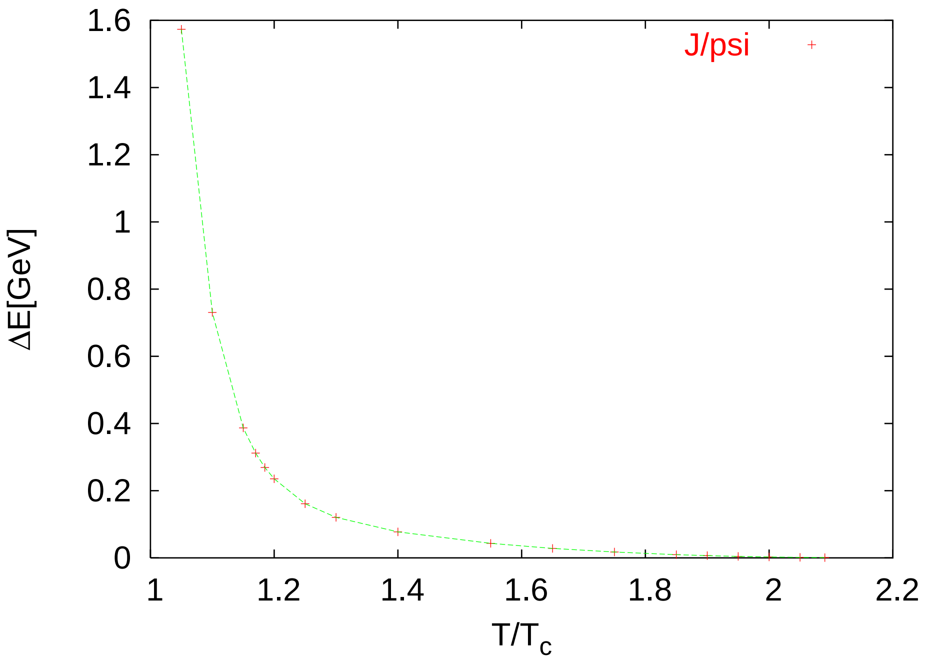


Figure 16: Screening fits to the $Q\bar{Q}$ free energy $F(r,T)$ for $T \geq T_c$ (left) and $T \leq T_c$ (right) [28]

Potential *V*

Schroedinger Eq.



T-dependence of binding energy for J/Psi.

H.Satz, hep-ph/0512217

Very high Temperature

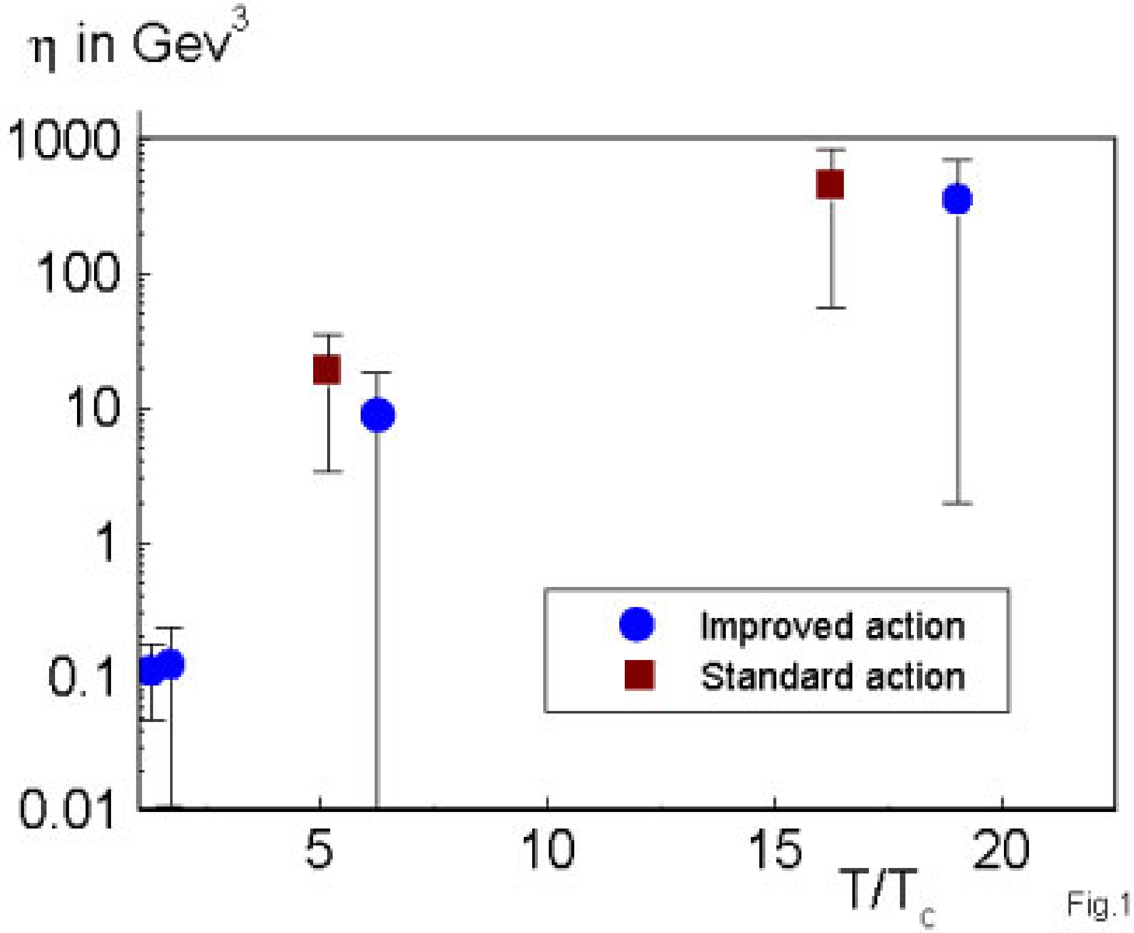


Fig.1

Entropy Density

$$F = fV$$

$$f = -p$$

$$U - TS = -T \log Z = F$$

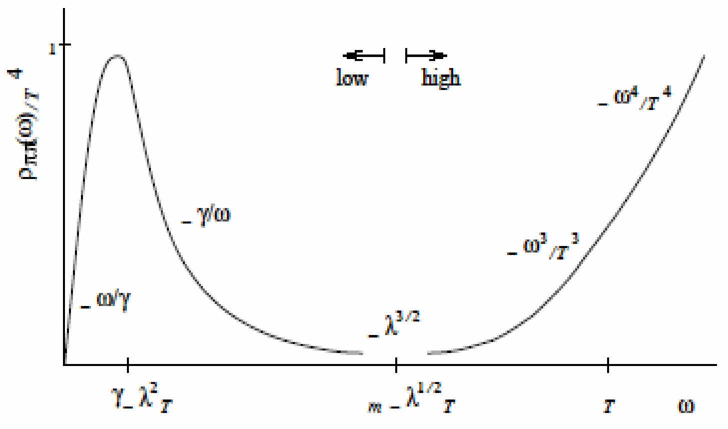
$$\Rightarrow \quad s = \frac{S}{V} = \frac{\varepsilon + p}{T}$$

$$\left. \frac{p}{T^4} \right|_{\beta_0}^{\beta} = \int_{\beta_0}^{\beta} d\beta' \frac{d}{d\beta'} \frac{p}{T^4}$$

We reconstruct *p* from Raw-Data by CP-PACS
(Okamoto et al., Phys.Rev.D (1999) 094510)

Spectral Function by Aarts and Resco

$$\rho(\omega) = \rho^{low}(\omega) + \rho^{high}(\omega)$$



$$\frac{\rho^{low}(\omega)}{T^4} = x\frac{b_1 + b_2 x^2 + \cdots}{1 + c_1 x^2 + c_2 x^4 + \cdots}$$

$$x \equiv \frac{\omega}{T}$$

$$\rho^{high}(\omega) = \theta(\omega - 2m_{th})\frac{(N_c{}^2 - 1)(\omega^2 - 4m_{th}{}^2)^{5/2}}{80\pi^2\omega}[n(\omega) + 0.5]$$

Fitting with three parameters, $b_1 \;\; c_1 \;\; m$

$c_1 < 0$?

Effect of High-Frequency part

$$\rho = \rho^{BW} + \rho^{high}$$

$$\frac{\rho^{low}(\omega)}{T^4} = x\frac{b_1 + b_2 x^2 + \cdots}{1 + c_1 x^2 + c_2 x^4 + \cdots} \qquad x \equiv \frac{\omega}{T}$$

$$\rho^{BW} = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2 + \gamma^2} + \frac{\gamma}{(m+\omega)^2 + \gamma^2}\right)$$

β=3.3

ηa^3	m_{th}
0.00225(201)	∞
0.00223(191)	5.0
0.00194(194)	3.0
0.00126(204)	2.0

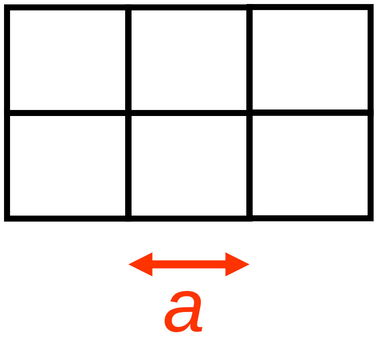
$m_{th} = 1.8$

ρ^{high} contribution is larger than

ρ^{BW} at t=1.

Why they are so noisy ?

- RG improved action helps lot.
  - Noise from Lattice Artifact ? (Finite *a* correction ?)
  - Once we checked that there is not so much difference between



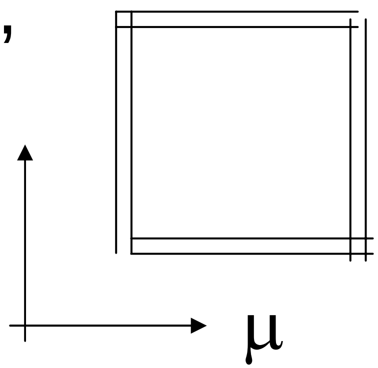
$$F_{\mu\nu} = \left(U_{\mu\nu} - U_{\mu\nu}{}^{\dagger}\right)/2i \text{ and } F_{\mu\nu} = \log U_{\mu\nu}/i$$

for SU(2). But we should check it again.

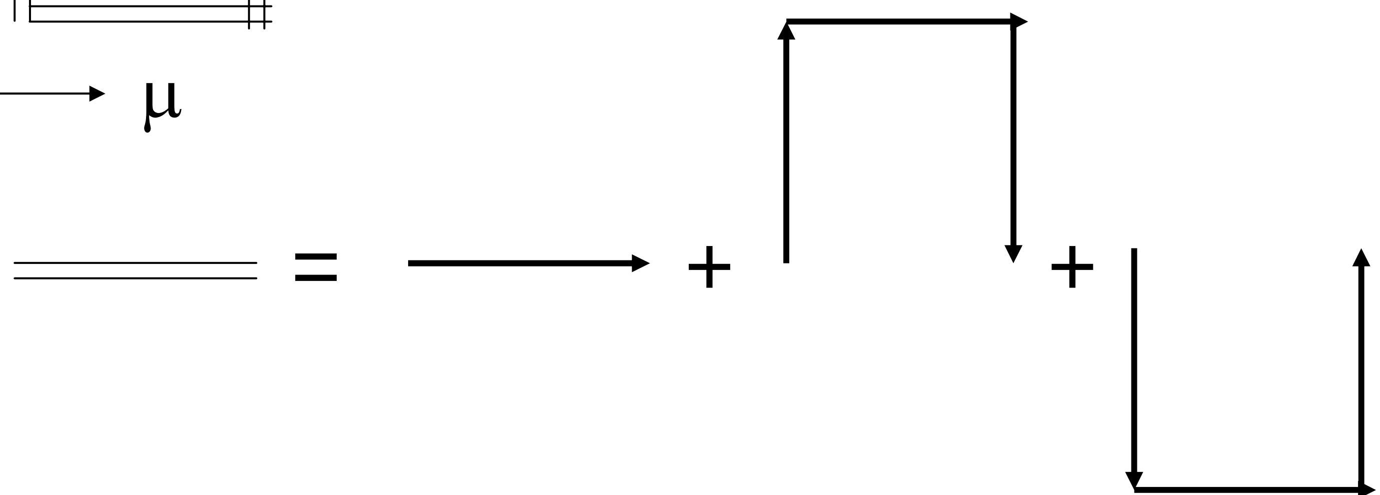
The situation reminds us Glue-Ball Case. (I thank Ph.deForcrand for discussions on this point.)

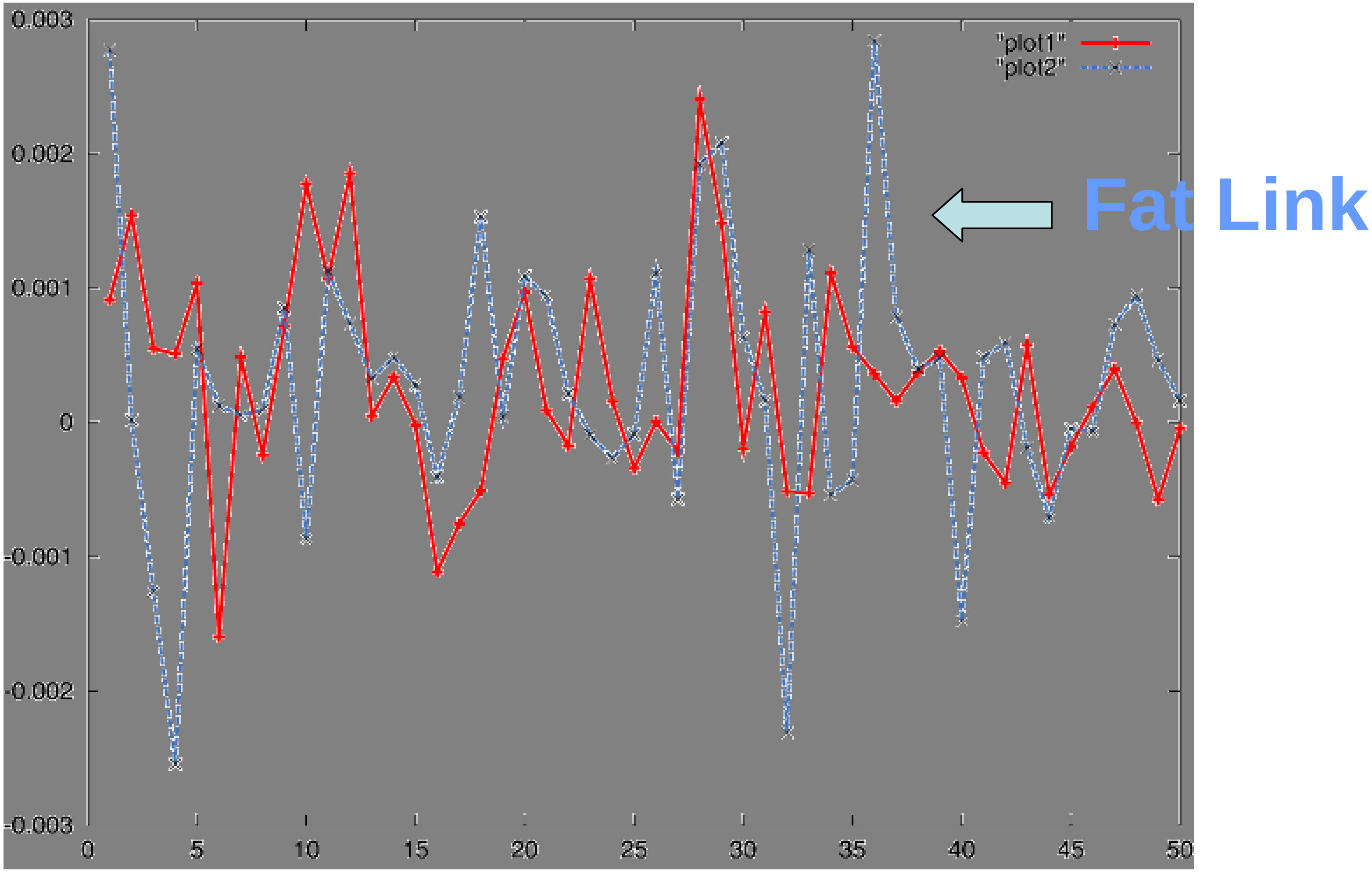
- Glue-Ball Correlators = $\left\langle \Box(\tau)\Box(0) \right\rangle$

- Large (extended) Operators work better, e.g.,



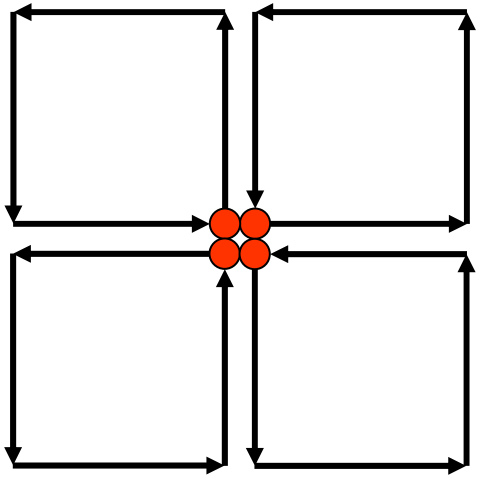
where





- Mmmm… not works …

Another Extended $F_{\mu\nu}$



A Crazy method
Source method + Langevin (Parisi)

Source Method

$$Z(J) = \int D\phi e^{-S+J\phi}$$

$$\langle \phi(x)\phi(y) \rangle = \frac{\delta}{\delta J(x)} \frac{\delta}{\delta J(y)} \log Z(J)$$

Langevin Update

$$\frac{d\phi(x)}{dt} = -\frac{\partial S}{\partial \phi(x)} + \eta$$

Deterministic
No Accept-
Reject step

t : Langevin time,

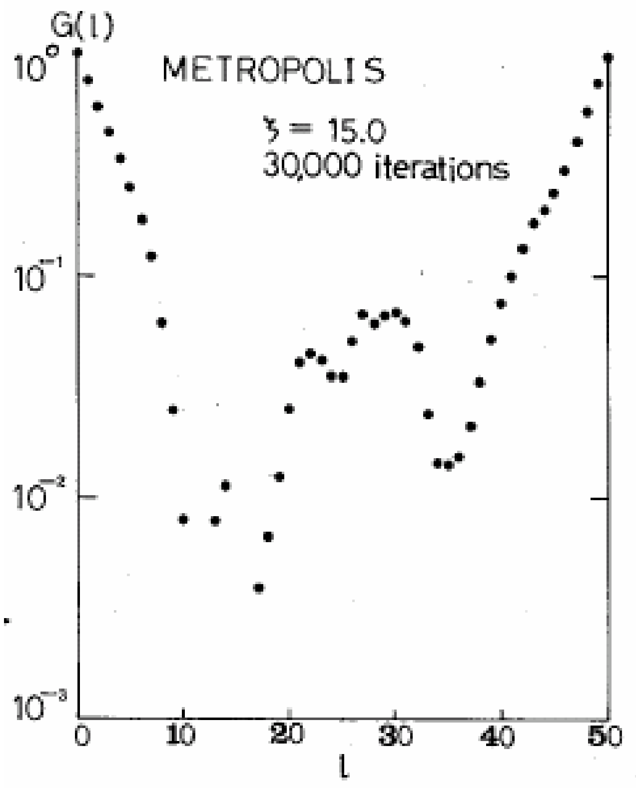
η : Gaussian Random Numbers

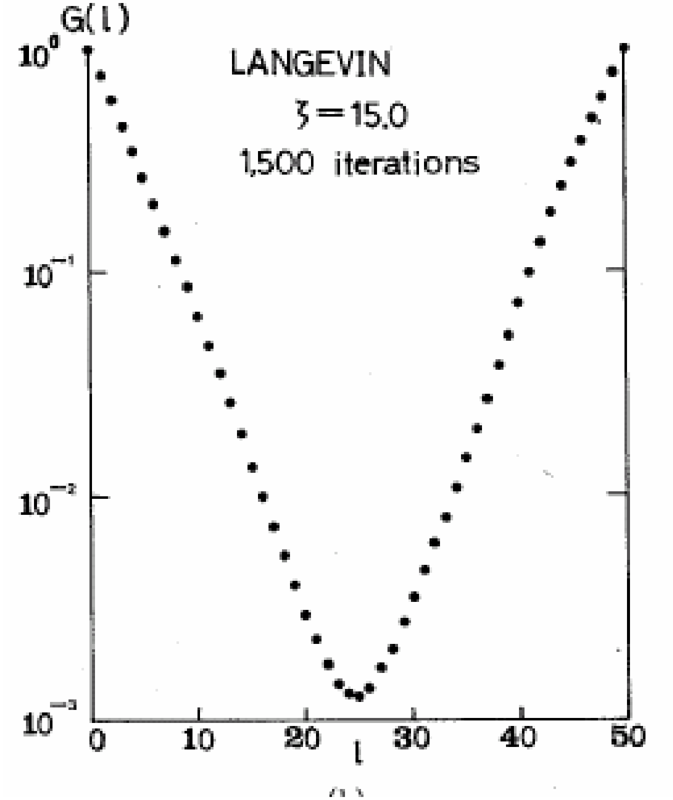
$$\left\langle \phi(x)\phi(y) \right\rangle = \frac{\delta}{\delta J(x)} \left\langle \phi(y) \right\rangle_J = \frac{\left\langle \phi(y) \right\rangle_{\varepsilon J} - \left\langle \phi(y) \right\rangle_0}{\varepsilon}$$

$$\left\langle \phi(y) \right\rangle_{\varepsilon J}$$

$$\left\langle \phi(y) \right\rangle_0$$

Calculate by Langevin
by the same Random Numbers

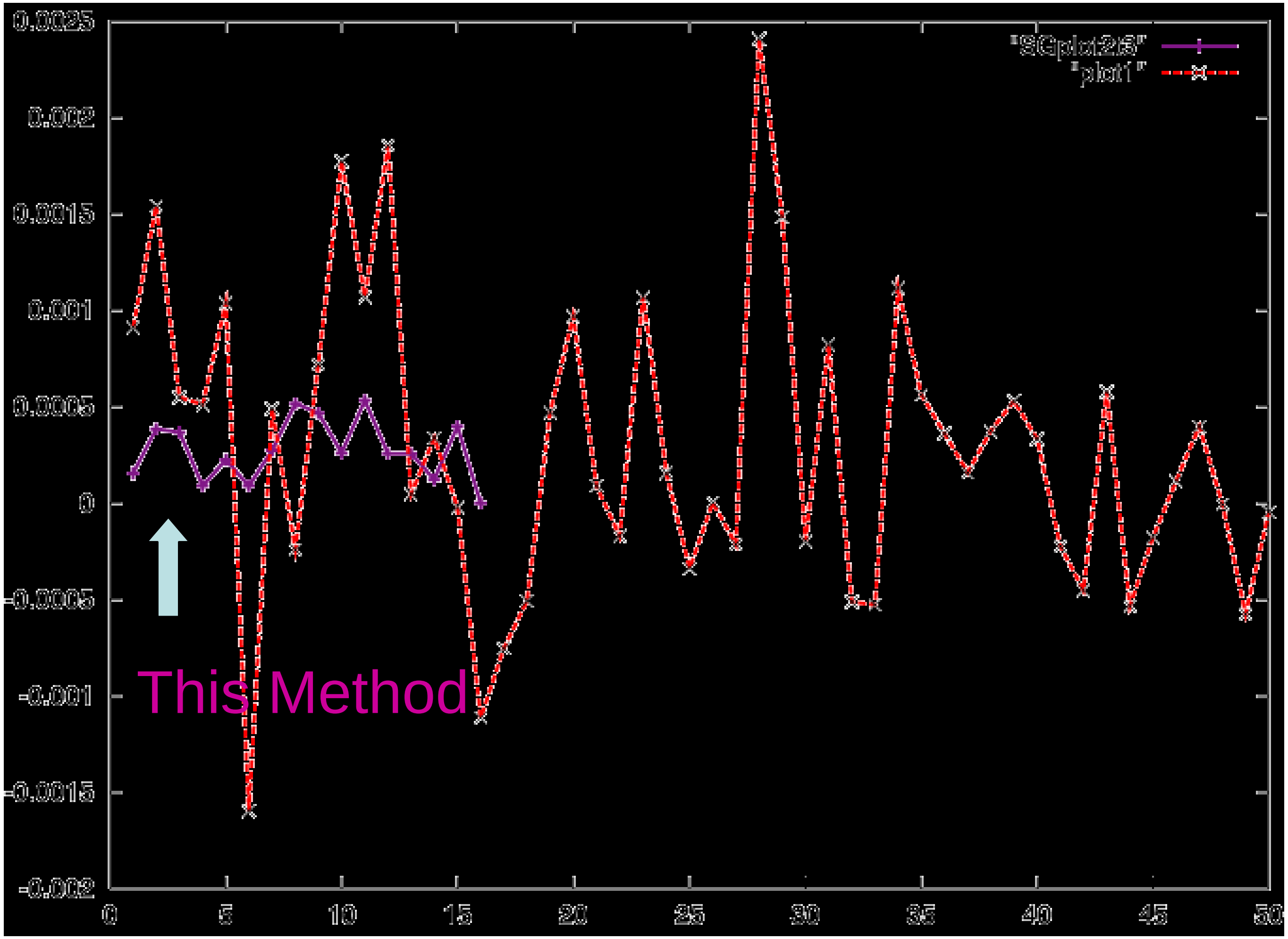




Namiki et al., Prog.Theor.Phys. 76 (1986) 501

O(3) Non-linear σ-model

In our case, …
(Very very preliminary)



Anisotropic Lattice ?

- Anisotropic lattice has matured and will help us to get more data points to determine the spectral function.