Deformation of Dijkgraaf-Vafa Relation via Spontaneously Broken $\mathcal{N}=2$ Supersymmetry

H.I. with K. Maruyoshi
arXiv:0704.1060 [hep-th]
Phys.Lett.**B**650(2007)298-303
Osaka City Univ.

c.f.

· [hep-th/0409060] Prog.Theor.Phys.113(2005)429-455

· [hep-th/0503113] Nucl.Phys.**B**723(2005)33-52

· [hep-th/0510255] Nucl.Phys.**B**740(2006)58-78

with K. Fujiwara & M. Sakaguchi

- The model
- Spontaneous Partial Breaking of $\mathcal{N}=2$ Supersymmetry
- Mass Spectrum $\cdots$
- Computation & Structure of W_{eff}

The Model and Limiting Cases

$\mathcal{N}=1$ large	interpolates by e, m, ξ	small $\mathcal{N}=2$
$S_W^{\mathcal{N}=1}$	$S_{\rm FIS}^{\mathcal{N}=2}$	$S_{\rm SYM}^{\mathcal{N}=2}$

μ_0

μ $\quad \mathcal{F}^{(\rm eff)}_{\rm m.m.}(S_i)$, $W_{\rm eff}$

DV relation

R.S. & matrix model description

DV, CIV, CDSW, H.I.-Morozov '02

$\cdots$

$\mu_{\rm IR}$ $\quad \mathcal{F}^{\rm eff}_{\rm SW}(\phi_i)$ SW

R.S. description

Gorsky et.al., Martinec-Warner, H.I.-Morozov '95 $\cdots$

$$S_{\rm FIS}^{\mathcal{N}=2} = \int d^4x d^4\theta \left[-\frac{i}{2} {\rm Tr} \left(\bar{\Phi} e^{adV} \frac{\partial \mathcal{F}(\Phi)}{\partial \Phi} - h.c. \right) + \xi V^0 \right] + \left[\int d^4x d^2\theta \left(-\frac{i}{4} \frac{\partial^2 \mathcal{F}(\Phi)}{\partial \Phi^a \partial \Phi^b} \mathcal{W}^a \mathcal{W}^b + e\Phi^0 + m \frac{\partial \mathcal{F}(\Phi)}{\partial \Phi^0} \right) + h.c. \right]$$

cf. $\quad S_W^{\mathcal{N}=1} = \int d^4x d^4\theta {\rm Tr} \bar{\Phi} e^{adV} \Phi + \left[\int d^4x d^2\theta {\rm Tr} \left(i\tau \mathcal{W}\mathcal{W} + W(\Phi) \right) + h.c. \right]$

$\mathcal{N}=2$ Supersymmetry with (Bare) Superpotential

- Strategy to get $\mathcal{N}=2$:

$$\boldsymbol{\lambda}_i^a = \begin{pmatrix} \lambda^a \\ \psi^a \end{pmatrix} \to \boldsymbol{\lambda}^{ia} = \begin{pmatrix} \psi^a \\ -\lambda^a \end{pmatrix} = R\boldsymbol{\lambda}_i^a R^{-1}$$

$R\delta^{(1,\xi)}_{\eta_1=\theta}R^{-1} \equiv \delta^{(2,-\xi)}_{\eta_2=\theta}$ so that $0=\delta^{(2,\xi)}_{\eta_2=\theta}S(\xi)$ follows from $R\delta^{(1,\xi)}_{\eta_1=\theta}S(\xi)R^{-1}=0$

- Take a generic superpotential and a gauge kinetic function and impose R invariance:

The solution $W = eA^0 + m\mathcal{F}_0, \quad \tau_{ab} = \mathcal{F}_{ab}$

- Transformation laws:

$$\boldsymbol{\delta\lambda}_J{}^a = i(\boldsymbol{\tau}\cdot\boldsymbol{D}^a)_J{}^K\boldsymbol{\eta}_K + \cdots \ .$$

$$\boldsymbol{D}^a \quad = \quad \hat{\boldsymbol{D}}^a - \sqrt{2}g^{ab^*}\partial_{b^*}\left(\boldsymbol{\mathcal{E}}A^{*0} + \boldsymbol{\mathcal{M}}\mathcal{F}_0^*\right).$$

fermion bilinears

$$\boldsymbol{\mathcal{E}} \quad = \quad (0,\ -e,\ \xi)\ , \quad \boldsymbol{\mathcal{M}} = (0,\ -m,\ 0),$$

Spontaneous Partial Breaking of $\mathcal{N}=2$ Supersymmetry

• basic mechanism: $\left\{\bar{Q}^j_{\dot{\alpha}}, \mathcal{S}^m_{\alpha i}(x)\right\} = 2(\sigma^n)_{\alpha\dot{\alpha}}\delta_i{}^j T^m_n(x) + (\sigma^m)_{\alpha\dot{\alpha}} C_i{}^j$

· $C_i{}^j$: not a VEV but follows simply from the algebra.

The model predicts:

• $C_i{}^j = 4m\xi\tau_1 \overset{90^{\rm o}{\rm rot.}}{\longrightarrow} 4m\xi\tau_3$ The scalar ptl VEV $\langle\langle\mathcal{V}\rangle\rangle = \mp 2m\xi = 2|m\xi|$

• ∴ Half of the supercharges annihilates the vacuum while the remaining half takes $\infty \sim |m\xi| \int d^4x$ matrix elements.

∴ Partial Breaking of Extended SUSY is a Reality.

A Few Tree Properties

$$\langle\!\langle \mathcal{F}_{\underline{jj}} \rangle\!\rangle = -2(\frac{e}{m} \mp i\frac{\xi}{m}) = -2\zeta \qquad \text{; the vac. condition}$$

$$\langle\!\langle g_{\underline{jj}} \rangle\!\rangle = \mp 2\frac{\xi}{m},$$

$$\langle\!\langle \boldsymbol{D}^{\underline{j}} \rangle\!\rangle = \frac{m}{\sqrt{N}} \begin{pmatrix} 0 \\ -i \\ \pm 1 \end{pmatrix}$$

- NG fermion $\frac{1}{\sqrt{2}}(\lambda^o + \psi^0)$ resides in the overall $U(1)$ part but not decoupled

$$\frac{\partial^2 \mathcal{F}(\Phi)}{\partial \Phi^a \partial \Phi^b} \mathcal{W}^a \mathcal{W}^b = \left\langle \frac{\partial^3 \tilde{\mathcal{F}}(\tilde{\Phi})}{\partial \tilde{\Phi}^0 \partial \tilde{\Phi}^{\hat{a}} \partial \tilde{\Phi}^{\hat{b}}} \right\rangle \underbrace{\mathcal{W}^0}_{\uparrow\ \text{overall U(1)}} \underbrace{\mathcal{W}^{\hat{a}} \tilde{\Phi}^{\hat{b}}}_{\nwarrow\ \text{SU}(N)} + \cdots$$

- Breaking pattern of gauge symmetry: $\deg \mathcal{F} = n + 2$

$$U(N) \to \prod_{i=1}^{n} U(N_i) \quad \text{with} \quad \sum_{i=1}^{n} N_i = N$$

cf. partition of N eigenvalues

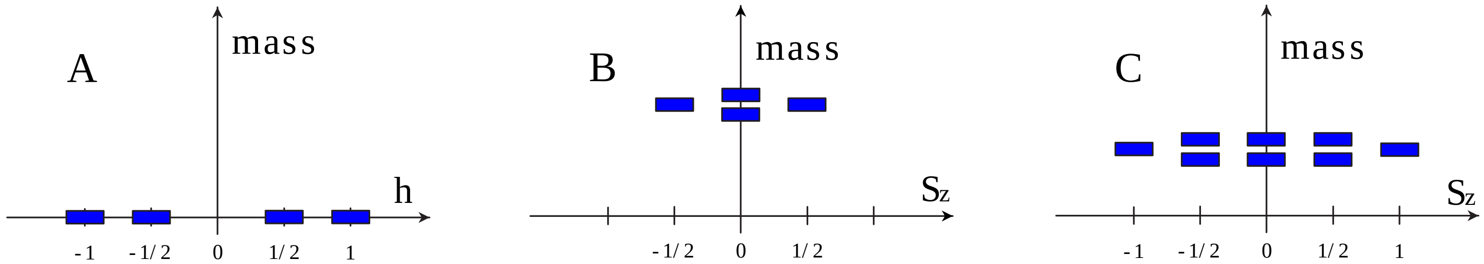
Mass Spectrum

index labelling $a, b, \cdots = \begin{cases} \alpha, \beta, \cdots \text{ for unbroken generators} \\ \mu, \nu, \cdots \text{ for broken generators} \end{cases}$

- the table

field	mass	label	# of polarization states
v_m^{α}	0	A	$2d_u (d_u \equiv \dim \prod_i U(N_i))$
v_m^{μ}	$\lvert\frac{1}{\sqrt{2}} f^{\nu}_{\mu\underline{i}} \lambda^{\underline{i}}\rvert$	C	$3(N^2 - d_u)$
$\frac{1}{\sqrt{2}}(\lambda^{\alpha} \pm \psi^{\alpha})$	0	A	$2d_u$
$\frac{1}{\sqrt{2}}(\lambda^{\alpha} \mp \psi^{\alpha})$	$\lvert m \langle\langle g^{\alpha\alpha} \rangle\rangle \langle\langle \mathcal{F}_{0\alpha\alpha} \rangle\rangle \rvert$	B	$2d_u$
$\boldsymbol{\lambda}_I^{\mu}$	$\lvert\frac{1}{\sqrt{2}} f^{\nu}_{\mu\underline{i}} \lambda^{\underline{i}}\rvert$	C	$4(N^2 - d_u)$
A^{α}	$\lvert m \langle\langle g^{\alpha\alpha} \rangle\rangle \langle\langle \mathcal{F}_{0\alpha\alpha} \rangle\rangle \rvert$	B	$2d_u$
$\mathcal{P}_{\mu}^{\tilde{\mu}} A^{\mu}$	$\lvert\frac{1}{\sqrt{2}} f^{\nu}_{\mu\underline{i}} \lambda^{\underline{i}}\rvert$	C	$N^2 - d_u$

- $\mathcal{N} = 1$ supermultiplet



Fermionic Shift Symmetry of $S_W^{\mathcal{N}=1}$ and $W_{\rm eff}$

$$S \equiv -\frac{1}{32\pi^2}{\rm Tr}\mathcal{W}^\alpha\mathcal{W}_\alpha \quad \ni {\rm Tr}\lambda^\alpha\lambda_\alpha \qquad \text{gluino condensate variables} \qquad \text{DV, CDSW}$$

$$w^\alpha \equiv \frac{1}{4\pi}{\rm Tr}\mathcal{W}^\alpha \qquad\qquad U(N) \text{ unbroken for simplicity}$$

- Introduce "grassmann coordinates" ψ^α

$$\begin{aligned}\hat{\mathcal{S}} &= -\frac{1}{2}{\rm Tr}\left(\frac{1}{4\pi}\mathcal{W}^\alpha - \psi^\alpha\mathbf{1}\right)\left(\frac{1}{4\pi}\mathcal{W}_\alpha - \psi_\alpha\mathbf{1}\right)\\ &= S + \psi w - \frac{1}{2}\psi\psi N\end{aligned}$$

- The fermionic shift symmetry $\quad \rightsquigarrow \quad$ decoupling of overall $U(1)$

  acts as $\qquad \delta\hat{\mathcal{S}} = \epsilon\frac{d}{d\psi}\hat{\mathcal{S}}$

- ${}^{\exists}\mathcal{F}$ s.t.

$$W_{\rm eff} = \int d^2\psi\mathcal{F}(\hat{\mathcal{S}}) = N\frac{\partial\mathcal{F}(S)}{\partial S} + \frac{\partial^2\mathcal{F}(S)}{\partial S^2}ww \qquad \text{DV relation}$$

- Remnant of the 2nd supersymmetry of $S_{\rm FIS}^{\mathcal{N}=2}$

$W_{\rm eff}$ of $S^{\mathcal{N}=2}_{\rm FIS}$; Deformation of DV Formula

So far the matter induced part only

- summary of our understanding;

$$W^{(h-1)}_{\rm eff} = N\frac{\partial F^{(h-1)}}{\partial S} + \frac{\partial^2 F^{(h-1)}}{\partial S^2} w^\alpha w_\alpha - \frac{16\pi^2 i m g_3}{m g_2}\left(\frac{\partial F^{(h-1)}}{\partial S}\right)\frac{S}{m} + W^{(h-1)}_2 + O\left(\frac{1}{m^2}\right)$$

h : # of index loops
$F^{(h-1)}$; the $(h-1)$ loop contribution to the planar free energy of the matrix model
$W^{(h-1)}_2$; replace one coupling constant mg_ℓ by $\frac{16\pi^2 i g_{\ell+1} S}{Nh}$, for $\ell \geq 3$ in the 1st term

- basis of our argument;
  - · integrate $\bar{\Phi}$ out
  - · propagator

$$\Delta(p,\pi) = \int_0^\infty ds e^{-s(p^2 + m' + \frac{1}{2} ad\mathcal{W}^\alpha \pi_\alpha - i g_3' M)}$$

$$M_{abcd} = (\mathcal{W}\mathcal{W})_{da}\delta_{bc} + (\mathcal{W}\mathcal{W})_{bc}\delta_{da} + \mathcal{W}_{da}\mathcal{W}_{bc}$$

cf. Grisaru et. al.

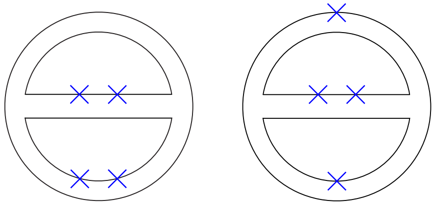
  - · vertices

type I. $\quad m\frac{g_k a^k}{k!}{\rm Tr}\Phi^k, \quad k = 3, \ldots, n+1$

type II. $\quad -\frac{i}{4}\sum_{s=0}^{k-1}\frac{g_k a^{k-1}}{k!}{\rm Tr}(\mathcal{W}\Phi^s\mathcal{W}\Phi^{k-1-s}),$
$k = 4, \ldots, n+1$

- outline of our argument;

e.g. $h-1=2$



- · universal to every $(h-1)$-loop planar diagram up to c.c. & symmetric factors

- · π^α momentum integration must be saturated:

  $\Rightarrow$ only the planar diagrams contribute

- · suppose that our finding were absent:

  $\Rightarrow$ up to the factors mentioned, we get

$$\left(\prod_{i=1}^{h}\int ds_i\right) e^{-(\sum s_i)m'}\frac{1}{4^{h-1}}\{NhS^{h-1}+{}_hC_2 2S^{h-2}w^\alpha w_\alpha\}$$

$$\equiv \left(\prod_{i=1}^{h}\int ds_i\right) e^{-(\sum s_i)m'}\mathcal{A}_0^{(h-1)}$$

$\star$ $^{\exists}$ two types of corrections to $\mathcal{A}_0^{(h-1)}$:

- $\mathcal{A}_1^{(h-1)}$; propagator correction insert two more $\mathcal{W}$, namely, $\times\times$

$$\left(\prod_{i=1}^{h}\int ds_i\right) e^{-(\sum s_i)m'}(\mathcal{A}_0^{(h-1)} + \mathcal{A}_1^{(h-1)}(s_i)) = \frac{h}{m'}\left(\frac{S}{4m'}\right)^{h-1}\left(N - \frac{16\pi^2 i g_3 S}{m g_2}\right) + \frac{{}_hC_2}{2m'^2}\left(\frac{S}{4m'}\right)^{h-2} w^\alpha w_\alpha.$$

- $\mathcal{A}_2^{(h-1)}$; the correction obtained by replacing $\mathrm{Tr}\Phi^l$ by

$$\mathrm{Tr}(2\mathcal{W}\mathcal{W}\Phi^\ell + \mathcal{W}\Phi\mathcal{W}\Phi^{\ell-1} + \ldots + \mathcal{W}\Phi^{\ell-1}\mathcal{W}\Phi)$$

$\Rightarrow$ can use only once and exhaust all possibilities

$$mg_\ell \to \frac{16\pi^2 i g_{\ell+1} S}{Nh}, \qquad \text{for} \quad \ell \geq 3$$

- explicit computation for $h-1=2$ loop

$$W_{\rm eff}^{(2)} = -\frac{(mg_3)^2}{32(mg_2)^3}NS^2 - \frac{(mg_3)^2}{16(mg_2)^3}Sw^\alpha w_\alpha + \frac{\pi^2 i(mg_3)^3}{2(mg_2)^4}\frac{S^3}{m} - \frac{\pi^2 i(mg_3)(mg_4)}{2(mg_2)^3}\frac{S^3}{m}$$

Generalized Konishi Anomaly Equation

$$R(z) = -\frac{1}{64\pi^2}\left\langle \mathrm{Tr}\mathcal{W}^\alpha \mathcal{W}_\alpha \frac{1}{z-\Phi}\right\rangle_\Phi, \qquad T(z) = \left\langle \mathrm{Tr}\frac{1}{z-\Phi}\right\rangle_\Phi$$

$$R(z)^2 = W'(z)R(z) + \frac{1}{4}f(z),$$

$$2R(z)T(z) = W'(z)T(z) + \frac{1}{4}c(z) + 16\pi^2 i\mathcal{F}'''(z)R(z) + \frac{1}{4}\tilde{c}(z)$$

$f(z)$ and $c(z)$ are polynomials of degree $n-1$ in z and $\tilde{c}(z)$ is a polynomial of degree $n-2$.