Observables and Correlation Functions in OSp String Field Theory

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§1. Introduction

▶ What we would like to do in this work

In the OSp invariant string field theory (SFT) for bosonic closed strings

(i) define BRST invariant observables corresponding to particle modes

(ii) evaluate correlation functions among them

(iii) derive S-matrix elements derived from the above
and show that they coincide with those of the light-cone gauge SFT

▶ Why OSp invariant SFT?

D-brane state in the second-quantization is proposed in this SFT

(Baba-Ishibashi-KM) [→ *cf.* Baba-kun's talk]

However

- An extra time variable t (as a 26 dimensional space-time theory) is contained.
- Expanded in terms of component fields, the action looks very different from that of the usual field theory.

$\left(\begin{array}{l}\text{The } OSp \text{ invariant SFT should be considered as something like stochastic or} \\ \text{Parisi-Sourlas type formulation of field theory.}\end{array}\right)$

A priori, it is not clear how the closed string particle modes are realized in this SFT

⇒ We would like to clarify this point.

Plan of the talk

§1. Introduction

§2. OSp Invariant SFT

§3. BRST Cohomology and Observables

§4. Correlation Functions and S-matrix Elements

§5. Summary

§2. OSp Invariant SFT

▶ OSp extension $\Leftarrow$ Procedure for covariantizing the LC gauge SFT (Siegel)

LC gauge SFT		OSp invariant SFT
$O(25,1)\left\{\begin{array}{c} t = X^+ \\ \alpha = p^+ \\ \hline O(24):\quad X^i \end{array}\right.$	OSp extension $\Longrightarrow$	$\left.\begin{array}{c} t = X^+ \\ \alpha = p^+ \\ \hline OSp(26\|2): X^M = \begin{pmatrix} X^\mu \\ C \\ \bar{C} \end{pmatrix} \end{array}\right\} OSp(27,1\|2)$

α: string length, $\qquad X^\mu = (X^i, X^{25}, X^{26})$

- metric for $OSp(26|2)$ "transverse directions": $\quad \delta_{ij} \quad \longrightarrow \quad \eta_{MN} = $

$$\begin{array}{cc} & \begin{array}{cc} \quad\quad\quad\quad & C \quad \bar{C} \end{array} \\ \eta_{MN} = & \left(\begin{array}{c|cc} \delta_{\mu\nu} & & \\ \hline & 0 & -i \\ & i & 0 \end{array}\right)\begin{array}{c} \\ C \\ \bar{C} \end{array} \end{array}$$

- mode expansion:

$$X^M(\tau,\sigma) = x^M - 2ip^M\tau + i\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^M e^{-n(\tau+i\sigma)} + \tilde{\alpha}_n^M e^{-n(\tau-i\sigma)}\right)$$

with $[x^N, p^M\} = i\eta^{NM}, \quad [\alpha_n^N, \alpha_m^M\} = n\eta^{NM}\delta_{n+m,0}, \quad [\tilde{\alpha}_n^N, \tilde{\alpha}_m^M\} = n\eta^{NM}\delta_{n+m,0}$

- notations: $\alpha_n^M = (\alpha_n^\mu, -\gamma_n, \bar{\gamma}_n), \quad \alpha_0^M = \tilde{\alpha}_0^M = p^M = (p^\mu, -\pi_0, \bar{\pi}_0), \quad x^M = (x^\mu, C_0, \bar{C}_0)$

▶ Action

$$S = \int dt \Bigg[\frac{1}{2} \int d1d2 \; \langle R(1,2)\,|\Phi\rangle_1 \left(i\frac{\partial}{\partial t} - \frac{L_0^{(2)} + \tilde{L}_0^{(2)} - 2}{\alpha_2} \right) |\Phi\rangle_2$$

$$+ \frac{2g}{3} \int d1d2d3 \; \left\langle V_3^0(1,2,3) \right| \Phi\rangle_1 |\Phi\rangle_2 |\Phi\rangle_3 \Bigg]$$

- reflector: $\langle R(1,2)| = \delta(1,2)\; {}_{12}\langle 0|\, e^{-\sum_{n=1}^{\infty}\frac{1}{n}\left(\alpha_n^{M(1)}\alpha_n^{N(2)}+\tilde{\alpha}_n^{M(1)}\tilde{\alpha}_n^{N(2)}\right)\eta_{MN}}\; \dfrac{1}{\alpha_1}$
- three string vertex: $\langle V_3^0(1,2,3)| = \delta(1,2,3)\dfrac{|\mu(1,2,3)|^2}{\alpha_1\alpha_2\alpha_3}\; {}_{123}\langle 0|e^{E(1,2,3)}\mathcal{P}_{123}$

$$E(1,2,3) = \frac{1}{2}\sum_{n,m\geq 0}\sum_{r,s}\bar{N}^{rs}_{nm}\left(\alpha_n^{N(r)}\alpha_m^{M(s)} + \tilde{\alpha}_n^{N(r)}\tilde{\alpha}_m^{M(s)}\right)\eta_{NM} \;,$$

$${}_{12\cdots N}\langle 0| = {}_1\langle 0|\,{}_2\langle 0|\cdots\, {}_N\langle 0| \;, \quad \mathcal{P}_{123} = \mathcal{P}_1\mathcal{P}_2\mathcal{P}_3 \;, \quad \mathcal{P}_r = \int_0^{2\pi}\frac{d\theta}{2\pi}e^{i\theta\left(L_0^{(r)}-\tilde{L}_0^{(r)}\right)} \;,$$

$$\delta(1,2,\ldots,N) = (2\pi)^{26}\,\delta^{26}\left(\sum_{r=1}^{N}p_r\right)\; 2\delta\left(\sum_{s=1}^{N}\alpha_s\right)\; i\left(\sum_{r'=1}^{N}\bar{\pi}_0^{(r')}\right)\left(\sum_{s'=1}^{N}\pi_0^{(s')}\right) \;,$$

$$\mu(1,2,3) = \exp\left(-\hat{\tau}_0\sum_{r=1}^{3}\frac{1}{\alpha_r}\right) \;, \quad \hat{\tau}_0 = \sum_{r=1}^{3}\alpha_r\ln|\alpha_r| \;, \quad dr = \frac{\alpha_r d\alpha_r}{2}\frac{d^{26}p_r}{(2\pi)^{26}}id\bar{\pi}_0^{(r)}d\pi_0^{(r)}$$

(Note: The zero modes are expressed by wave functions in the momentum representation)

▶ commutation relation

$$|\Phi\rangle = \begin{cases} |\bar{\psi}\rangle & \alpha < 0 \quad \text{creation} \\ |\psi\rangle & \alpha > 0 \quad \text{annihilation} \end{cases} \qquad [\,|\psi\rangle_r\,,\,|\bar{\psi}\rangle_s\,] = |R(r,s)\rangle$$

$|0\rangle\rangle$: vacuum in the 2nd-quantization $\Leftarrow$ defined by $|\psi\rangle|0\rangle\rangle = 0$

▶ Relationship between S-matrices in LC gauge SFT and OSp invariant SFT

- S-matrix symmetry in the OSp SFT: $OSp(27,1|2)$
- Due to the Parisi-Sourlas mechanism,

For on-shell S-matrix elements,

the "longitudinal directions" $X^{\pm}$ (i.e. (t,α))	cancel out each other $\Longleftrightarrow$	the ghost directions $(C,\bar{C})$

- The resulting $SO(26)$ symmetric S-matrices
  $\Rightarrow$ S-matrices of the LC SFT (in the Euclidean signature)

By using this (intuitively obvious) fact, we will provide a prescription for obtaining the S-matrix elements of the LC gauge SFT from the correlation functions in the OSp invariant SFT.

▶ The action of the OSp invariant SFT is BRST invariant

BRST symmetry = non-linearly realized J^{-C} symmetry of the OSp$(27,1|2)$

(Siegel-Zwiebach, Bengtsson-Linden)

$$\delta_{\mathrm{B}}\Phi = Q_{\mathrm{B}}\Phi + g\Phi * \Phi$$

- BRST charge

$$Q_{\mathrm{B}} = \frac{C_0}{2\alpha}(L_0 + \tilde{L}_0 - 2) - i\pi_0\frac{\partial}{\partial\alpha} + \frac{i}{\alpha}\sum_{n=1}^{\infty}\left(\frac{\gamma_{-n}L_n - L_{-n}\gamma_n}{n} + \frac{\tilde{\gamma}_{-n}\tilde{L}_n - \tilde{L}_{-n}\tilde{\gamma}_n}{n}\right)$$

L_n, $\tilde{L}_n$: Virasoro generators

$$L_n \equiv \frac{1}{2}\sum_m {}^{\circ}_{\circ}\alpha^M_{n+m}\alpha^N_{-m}\eta_{MN}{}^{\circ}_{\circ}\ , \quad \tilde{L}_n \equiv \frac{1}{2}\sum_m {}^{\circ}_{\circ}\tilde{\alpha}^M_{n+m}\tilde{\alpha}^N_{-m}\eta_{MN}{}^{\circ}_{\circ}$$

- $*$-product: $|\Phi * \Psi\rangle_4 = \int d1d2d3\ \langle V_3(1,2,3)\,|\Phi\rangle_1\,|\Psi\rangle_2\,|R(3,4)\rangle$

$$\langle V_3(1,2,3)| = \delta(1,2,3)_{123}\langle 0|e^{E(1,2,3)}C(\sigma_I)\mathcal{P}_{123}\frac{|\mu(1,2,3)|^2}{\alpha_1\alpha_2\alpha_3} \quad \Big(\sim \langle V^0(1,2,3)|C(\sigma_I)\Big)$$

§3. BRST Cohomology and Observables

▶ on-shell physical states

on-shell: $\left(i\frac{\partial}{\partial t} - \frac{L_0 + \tilde{L}_0 - 2}{\alpha} \right) |\ \ \rangle = 0$, physical: $Q_{\mathrm{B}} |\ \ \rangle = 0$

Hamiltonian $\frac{L_0+\tilde{L}_0-2}{\alpha}$ is Q_{B}-exact $\Rightarrow$ We may set $t = 0$ in the Q_{B}-cohomology

▶ Relationship between Q_{B} in OSp invariant theory and Kato-Ogawa's $Q_{\mathrm{B}}^{\mathrm{KO}}$

- identification

$$C_0 = 2\alpha c_0^{+} \qquad \bar{\pi}_0 = \frac{1}{2\alpha} b_0^{+}$$

$$\gamma_n = in\alpha c_n \qquad \bar{\gamma}_n = \frac{1}{\alpha} b_n$$

$$\tilde{\gamma}_n = in\alpha \tilde{c}_n \qquad \bar{\tilde{\gamma}}_n = \frac{1}{\alpha} \tilde{b}_n$$

no counter part in (b, c) system

$\bar{C}_0$, $\underline{\pi_0 \ , \quad \alpha}$

($\nwarrow$ contained in Q_{B})

$$\Longrightarrow \qquad Q_{\mathrm{B}} = Q_{\mathrm{B}}^{\mathrm{KO}} - ic\left(\alpha \frac{\partial}{\partial \alpha} + 1 \right) , \qquad c \equiv \frac{\pi_0}{\alpha}$$

▶ Q_{B} cohomology: Solve $Q_{\mathrm{B}} |\ \ \rangle = 0$

$|\mathrm{phys}\rangle$: $Q_{\mathrm{B}}^{\mathrm{KO}}$-physical ,

$$\Rightarrow |\ \ \rangle = \frac{1}{\alpha} |\mathrm{phys}\rangle + h(\alpha)\, c\, |\mathrm{phys}\rangle$$

$h(\alpha)$: $\forall$ function of α

For $|\mathrm{phys}\rangle$, we can choose $\quad |0\rangle_{b,c} \otimes |\mathrm{primary}; k\rangle_X \quad$ or $\quad b_0^{+} |0\rangle_{b,c} \otimes |\mathrm{primary}; k\rangle_X$

▶ boundary condition for α direction ($C, \bar{C}$-representation)

For the bra or ket states, $0 < |\alpha| < \infty$ $\Rightarrow$ introduce ω s.t. $\alpha = e^{\omega}$, then $-\infty < \omega < \infty$

$\Rightarrow$ integration measure: $\int_0^\infty \alpha d\alpha = \int_{-\infty}^{\infty} d\omega \underline{e^{2\omega}}$

$\Rightarrow$ The wave functions should be expand with respect to $\exp\left(-\omega + i\omega x_\omega\right)$

$$\left(\Longrightarrow \left\{ \begin{array}{l} \bullet \text{ the wave functions are delta-function normalizable} \\ \bullet\ Q_{\rm B} \text{ is hermitian} \\ \qquad\qquad\qquad \text{etc...} \end{array} \right. \right)$$

This yields
$$|\ \rangle = \underbrace{\frac{1}{\alpha}|\text{phys}\rangle}_{\text{auxiliary fields}} + \underbrace{\frac{1}{\alpha}\pi_0\bar{\pi}_0|\text{phys}\rangle}_{\text{physical external states}} + Q_{\rm B}|*\rangle$$

▶ Observable associated with $|\text{primary}; k\rangle_X = |\overline{\text{primary}}\rangle_X (2\pi)^{26}\delta^{26}(p_\mu - k_\mu)$

$$\mathcal{O}(t,k) = \int dr \, \frac{1}{\alpha_r}{}_r\Big({}_{C\bar{C}}\langle 0| \otimes {}_X\langle \text{primary}; k|\Big)|\Phi(t)\rangle_r$$

with ${}_X\langle\overline{\text{primary}}|\overline{\text{primary}}\rangle_X = 1$

§4. Correlation Functions and S-matrix Elements

▶ Two-point correlation function for $\mathcal{O}_r(t_r)$ $(r=1,2)$ with $|\text{primary}_1\rangle_X = |\text{primary}_2\rangle_X$,

mass M: $(L_0+\tilde{L}_0-2)|\text{primary};k\rangle_X \otimes |0\rangle_{C,\bar{C}} = (k^2+2i\pi_0\bar{\pi}_0+M^2)|\text{primary};k\rangle_X \otimes |0\rangle_{C,\bar{C}}$

$$\left\langle\!\!\left\langle \tilde{\mathcal{O}}_1(E_1)\tilde{\mathcal{O}}_2(E_2)\right\rangle\!\!\right\rangle \equiv \int dt_1 dt_2 e^{iE_1t_1+iE_2t_2} \langle\!\langle 0|\mathrm{T}\mathcal{O}_1(t_1)\mathcal{O}_2(t_2)|0\rangle\!\rangle$$

$$= \prod_{r=1}^{2}\left(\frac{i}{2}\int d\alpha_r d\bar{\pi}_0^{(r)} d\pi_0^{(r)}\right)\frac{i\delta(1,2)2\pi\delta(E_1+E_2)}{\alpha_1E_1 - p_1^2 - M^2 - 2i\pi_0^{(1)}\bar{\pi}_0^{(1)} + i\epsilon}$$

$$\vdots \qquad \delta(1,2) \equiv 2\delta(\alpha_1+\alpha_2)(2\pi)^{26}\delta^{26}(p_1+p_2)i(\bar{\pi}_0^{(1)}+\bar{\pi}_0^{(2)})(\pi_0^{(1)}+\pi_0^{(2)})$$

$$= \frac{2\pi\delta(E_1)\,2\pi\delta(E_2)}{p_1^2+M^2}(2\pi)^{26}\delta^{26}(p_1+p_2)$$

- $2\pi\delta(E) \Leftarrow \mathcal{O}(t+\delta t,p)$ and $\mathcal{O}(t,p)$ are BRST equivalent

  Thus here we choose $\quad \varphi(p) \equiv \displaystyle\int \frac{dE}{2\pi}\tilde{\mathcal{O}}(E,p) = \mathcal{O}(t=0,p)$

$$\langle\!\langle \varphi_1(p_1)\varphi(p_2)\rangle\!\rangle_{\text{free}} = \frac{1}{p_1^2+M^2}(2\pi)^{26}\delta^{26}(p_1+p_2)$$

$\Leftarrow$ Euclidean propagator for a particle of mass M with correct normalization

▶ N-point correlation functions

$$
\begin{aligned}
\left\langle\!\!\left\langle \prod_{r=1}^{N} \tilde{\mathcal{O}}_r(E_r) \right\rangle\!\!\right\rangle &\equiv \prod_{r=1}^{N} \left(\int dt_r e^{iE_r t_r} \right) \langle\!\langle 0 | \mathrm{T} \prod_{r=1}^{N} \mathcal{O}_r(t_r) | 0 \rangle\!\rangle \\
&\equiv \prod_{r=1}^{N} \left(\frac{i}{2} \int d\alpha_r d\bar{\pi}_0^{(r)} d\pi_0^{(r)} \frac{i}{\alpha_r E_r - p_r^2 - M_r^2 - 2i\pi_0^{(r)}\bar{\pi}_0^{(r)}} \right) \\
&\qquad \times \delta^{OSp}\left(\sum_{s=1}^{N} p_s^{OSp} \right) \; G_{\text{amputated}}(p_1^{OSp}, \ldots, p_N^{OSp})
\end{aligned}
$$

where $p^{OSp} = (E,\, \alpha,\, p^\mu,\, \pi_0,\, \bar{\pi}_0)$

- Look for singular behaviors at $p_r^2 + M_r^2 = 0$.

$$
\left.\begin{array}{l}
\text{time evolution op } e^{-i\frac{t}{\alpha}(p^2 + 2i\pi_0\bar{\pi}_0 + M^2)} \\
\text{interaction vertex } \langle V_3^0(1,2,3)|
\end{array}\right\} \Leftarrow \text{regular at } p_r^2 + M_r^2 = 0.
$$

$\Rightarrow$ For generic p_r^μ, such singularities come from the integration over α_r

$\Rightarrow$ one can find that the singular behavior $\Leftarrow$ behavior around $\alpha_r \sim 0$

- Studying the behavior of the integrand around $\alpha_r \sim 0$ yields

$$\left\langle\!\!\left\langle \prod_{r=1}^{N} \tilde{\mathcal{O}}_r(E_r) \right\rangle\!\!\right\rangle \sim -i \left(\prod_{r=1}^{N} \frac{2\pi\delta(E_r)}{p_r^2 + M_r^2} \right) (2\pi)^{26} \delta^{26}\left(\sum_{r=1}^{N} p_r \right) \; G_{\text{amputated}}(p_r^{OSp}) \Big|_0$$

$$\left(\quad \Big|_0 \quad \Leftrightarrow \quad \text{set } p_r^2 + M_r^2 = E_r = \alpha_r = \pi_0^{(r)} = \bar{\pi}_0^{(r)} = 0 \right)$$

Hence

$$\left\langle\!\!\left\langle \prod_{r=1}^{N} \varphi_r(p_r) \right\rangle\!\!\right\rangle \sim -i \left(\prod_{r=1}^{N} \frac{1}{p_r^2 + M_r^2} \right) (2\pi)^{26} \delta^{26}\left(\sum_{r=1}^{N} p_r \right) \; G_{\text{amputated}}(p_r^{OSp}) \Big|_0$$

$\Leftarrow$ 26 dim. Euclidean correlation function

Concerning the S-matrix elements S_{OSp} for the OSp invariant SFT for the external states with the vanishing polarization in the $X^{\pm}$, C and $\bar{C}$ directions,

$$G_{\text{amputated}}(p_r^{OSp})\Big|_0 = S_{OSp}(p^{OSp_r})\Big|_0 \underset{\substack{\uparrow \\ \text{Parisi-Sourlas mechanism}}}{=} S_{\text{LC}}^{(\text{E})}(p_{\mu,r})$$

$\Rightarrow$ LC SFT's S-matrix elements in the Euclidean signature

$\Rightarrow$ the LC SFT's S-matrix elements are reproduced after the wick rotation !!!

§5. Summary

▶ What we did

☐ We considered the on-shell asymptotic states for closed string particles.

- BRST invariant observables
- correlation functions
- S-matrix elements

The kinetic term of the action for the OSp invariant SFT is unusual.

$\rightarrow$ difficult to fix the normalizations of the external states

$\Rightarrow$ We could fix them by evaluating the two-point correlation functions

☐ We showed the S-matrix elements reproduce the usual results.

▶ D-brane states (⇒ Baba-kun's talk)

off-shell

$\Rightarrow$ nonlinear terms in the BRST transformation should be taken into account