2007/08/10 @ Kinki Univ.

Two-dimensional $\mathcal{N} = (2, 2)$ super Yang-Mills theory on computer

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arXiv:0706.1392 [hep-lat]

It will be very exciting if non-perturbative questions in SUSY gauge theories can be studied numerically at one's will !

- **spontaneous SUSY breaking**
- **string/gauge correspondence**
- **test of various "solutions" (e.g., Seiberg-Witten)**

SUSY vs lattice !

$$\{Q, Q^{\dagger}\} \sim P$$

SUSY restores only in the continuum limit !

Present status:

- For 4d $\mathcal{N} = 1$ SYM (gaugino condensation, degenerate vacua, Veneziano-Yankielowicz effective action, etc.), numerically promising formulation exists

- Even in this "simplest realistic" model, no conclusive evidence of SUSY has been observed

- Investigation of low-dimensional SUSY gauge theories (simpler UV structure) would thus be useful to test various ideas

- Kaplan et. al., Sugino, Catterall, Sapporo group. . .

- SUSY QM (16 SUSY charges!) $\Leftarrow$ Takeuchi-kun

In this work, we carry out a (very preliminary) Monte Carlo study of Sugino's lattice formulation of 2d $\mathcal{N} = (2, 2)$ SYM (4 SUSY charges)

F. Sugino, JHEP 03 (2004) 067 [hep-lat/0401017]

Two-dimensional square lattice (size L)

$$\Lambda = \left\{ x \in a\mathbb{Z}^2 \mid 0 \leq x_\mu < L \right\}$$

The lattice action

$$S = Q a^2 \sum_{x \in \Lambda} \left(\mathcal{O}_1(x) + \mathcal{O}_2(x) + \mathcal{O}_3(x) + \frac{1}{a^4 g^2} \mathrm{tr} \left\{ \chi(x) H(x) \right\} \right),$$

where

$$\mathcal{O}_1(x) = \frac{1}{a^4 g^2} \mathrm{tr} \left\{ \frac{1}{4} \eta(x) [\phi(x), \overline{\phi}(x)] \right\}$$

$$\mathcal{O}_2(x) = \frac{1}{a^4 g^2} \mathrm{tr} \left\{ -i \chi(x) \hat{\Phi}_{\mathbf{TL}}(x) \right\}$$

$$\mathcal{O}_3(x) = \frac{1}{a^4 g^2} \mathrm{tr} \left\{ i \sum_{\mu=0}^{1} \psi_\mu(x) \left(\overline{\phi}(x) - U(x,\mu) \overline{\phi}(x + a\hat{\mu}) U(x,\mu)^{-1} \right) \right\}$$

A lattice counterpart of the BRST-like transformation Q

$$QU(x,\mu) = i\psi_\mu(x)U(x,\mu)$$
$$Q\psi_\mu(x) = i\psi_\mu(x)\psi_\mu(x) - i\left(\phi(x) - U(x,\mu)\phi(x+a\widehat{\mu})U(x,\mu)^{-1}\right)$$
$$Q\phi(x) = 0$$
$$Q\chi(x) = H(x) \qquad QH(x) = [\phi(x), \chi(x)]$$
$$Q\overline{\phi}(x) = \eta(x) \qquad Q\eta(x) = [\phi(x), \overline{\phi}(x)]$$

$Q^2 = 0$ on gauge invariant quantities

From this nilpotency, the lattice action is manifestly invariant under one of four super-transformations, Q.

More explicitly

$$S = a^2 \sum_{x\in\Lambda} \left(\sum_{i=1}^{3} \mathcal{L}_{\mathbf{B}i}(x) + \sum_{i=1}^{6} \mathcal{L}_{\mathbf{F}i}(x) + \frac{1}{a^4 g^2} \mathrm{tr} \left\{ H(x) - \frac{1}{2} i \hat{\Phi}_{\mathbf{TL}}(x) \right\}^2 \right)$$

where

$$\mathcal{L}_{\mathbf{B}1}(x) = \frac{1}{a^4 g^2} \mathrm{tr} \left\{ \frac{1}{4} [\phi(x), \overline{\phi}(x)]^2 \right\}$$

$$\mathcal{L}_{\mathbf{B}2}(x) = \frac{1}{a^4 g^2} \mathrm{tr} \left\{ \frac{1}{4} \hat{\Phi}_{\mathbf{TL}}(x)^2 \right\}$$

$$\mathcal{L}_{\mathbf{B}3}(x) = \frac{1}{a^4 g^2} \mathrm{tr} \left\{ \sum_{\mu=0}^{1} \left(\phi(x) - U(x,\mu)\phi(x+a\hat{\mu})U(x,\mu)^{-1} \right) \right.$$
$$\left. \times \left(\overline{\phi}(x) - U(x,\mu)\overline{\phi}(x+a\hat{\mu})U(x,\mu)^{-1} \right) \right\}$$

and

$$\mathcal{L}_{\mathbf{F}1}(x) = \frac{1}{a^4g^2}\,\mathrm{tr}\left\{-\frac{1}{4}\eta(x)[\phi(x),\eta(x)]\right\}$$

$$\mathcal{L}_{\mathbf{F}2}(x) = \frac{1}{a^4g^2}\,\mathrm{tr}\left\{-\chi(x)[\phi(x),\chi(x)]\right\}$$

$$\mathcal{L}_{\mathbf{F}3}(x) = \frac{1}{a^4g^2}\,\mathrm{tr}\left\{-\psi_0(x)\psi_0(x)\left(\overline{\phi}(x)+U(x,0)\overline{\phi}(x+a\hat{0})U(x,0)^{-1}\right)\right\}$$

$$\mathcal{L}_{\mathbf{F}4}(x) = \frac{1}{a^4g^2}\,\mathrm{tr}\left\{-\psi_1(x)\psi_1(x)\left(\overline{\phi}(x)+U(x,1)\overline{\phi}(x+a\hat{1})U(x,1)^{-1}\right)\right\}$$

$$\mathcal{L}_{\mathbf{F}5}(x) = \frac{1}{a^4g^2}\,\mathrm{tr}\left\{i\chi(x)Q\hat{\Phi}(x)\right\}$$

$$\mathcal{L}_{\mathbf{F}6}(x) = \frac{1}{a^4g^2}\,\mathrm{tr}\left\{-i\sum_{\mu=0}^{1}\psi_\mu(x)\left(\eta(x)-U(x,\mu)\eta(x+a\hat{\mu})U(x,\mu)^{-1}\right)\right\}$$

Advantage of this formulation

• Q-invariance (a part of the supersymmetry) is manifest even with finite lattice spacings and volume (probably, so far the unique formulation?)

• global $\mathrm{U}(1)_R$ symmetry (this is a chiral symmetry!)

$$
\begin{aligned}
&U(x,\mu) \to U(x,\mu) && \psi_\mu(x) \to e^{i\alpha}\psi_\mu(x) \\
&\phi(x) \to e^{2i\alpha}\phi(x) && \\
&\chi(x) \to e^{-i\alpha}\chi(x) && H(x) \to H(x) \\
&\overline{\phi}(x) \to e^{-2i\alpha}\overline{\phi}(x) && \eta(x) \to e^{-i\alpha}\eta(x)
\end{aligned}
$$

is also manifest

Possible disadvantage of the formulation

- **The pfaffian $\mathrm{Pf}\{iD\}$ resulting from the integration of fermionic variables is generally a complex number (lattice artifact)**

- **would imply the sign (or phase) problem in Monte Carlo simulation**

cf. H.S. and Taniguchi, JHEP 10 (2005) 082 [hep-lat/0507019]

Continuum limit:

$$a \to 0, \text{ while } g \text{ and } L \text{ are kept fixed}$$

It can be argued that the full SUSY of the 1PI effective action for elementary fields is restored in this limit

- Power counting

- scalar mass terms are the only source of SUSY breaking
  $\Leftarrow$ super-renormalizability

- exact Q-invariance forbids the mass terms

Monte Carlo study ($SU(2)$ only)

For SUSY, quantum effect of fermions is vital !

Quenched approximation ($S_{\mathbf{B}}$ bosonic action)

$$\langle \mathcal{O} \rangle = \frac{\int \mathrm{d}\mu_{\mathbf{B}}\, \mathcal{O}\, e^{-S_{\mathbf{B}}}}{\int \mathrm{d}\mu_{\mathbf{B}}\, e^{-S_{\mathbf{B}}}}$$

is meaningless, though it provides a useful standard

Here we adopt the re-weighting method

$$\langle\!\langle \mathcal{O} \rangle\!\rangle = \frac{\int \mathrm{d}\mu\, \mathcal{O}\, e^{-S}}{\int \mathrm{d}\mu\, e^{-S}} = \frac{\langle \mathcal{O}\, \mathrm{Pf}\{iD\} \rangle}{\langle \mathrm{Pf}\{iD\} \rangle}$$

(potential overlap problem)

We developed a hybrid Monte Carlo algorithm code for the action $S_{\mathbf{B}}$ by using a C++ library, FermiQCD/MDP

For each configuration, we compute the inverse (i.e., fermion propagator) and the determinant of the lattice Dirac operator iD by using the LU decomposition

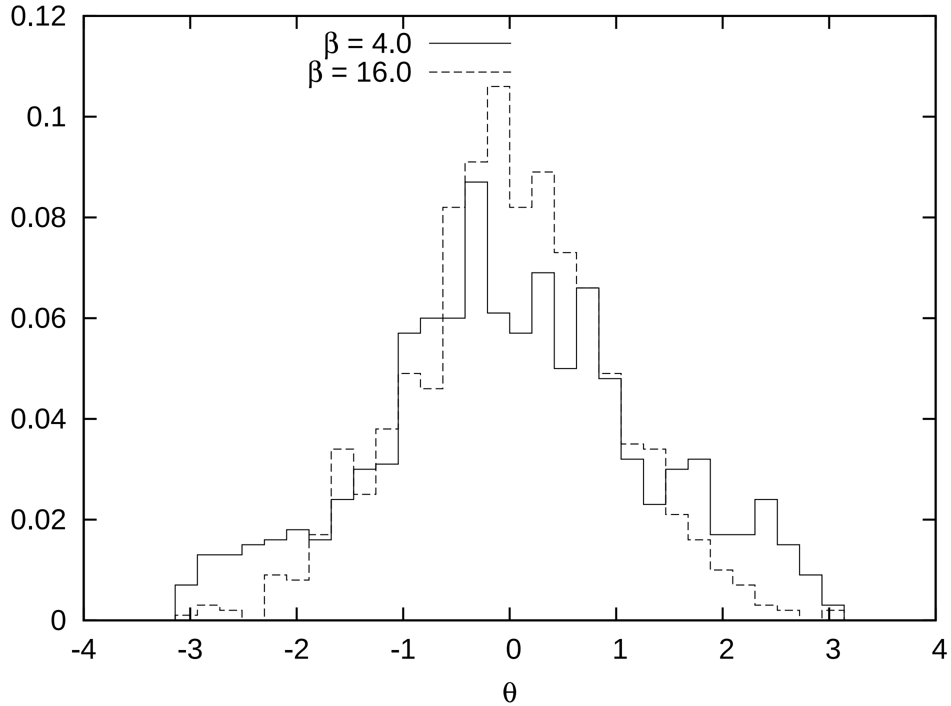
Expressing the determinant of the Dirac operator as

$$\mathrm{det}\{iD\} = re^{i\theta}, \qquad -\pi < \theta \leq \pi$$

(the complex phase is lattice artifact) we define

$$\mathrm{Pf}\{iD\} = \sqrt{r}e^{i\theta/2}, \qquad \because (\mathrm{Pf}\{iD\})^2 = \mathrm{det}\{iD\}$$

However, with this prescription, the sign may be wrong



To estimate the systematic error introduced with this, we compute also the phase-quenched average

$$\langle\langle \mathcal{O} \rangle\rangle_{\textbf{phase-quenched}} = \frac{\langle \mathcal{O}\, |\mathrm{Pf}\{iD\}| \rangle}{\langle |\mathrm{Pf}\{iD\}| \rangle}$$

Parameters in our Monte Carlo study ($\beta = 2N_c/(a^2g^2)$)

N	8	7	6	5	4
β	16.0	12.25	9.0	6.25	4.0
N_{conf}	1000	10000	10000	10000	10000
ag	0.5	0.571428	0.666666	0.8	1.0

This sequence corresponds to the fixed physical lattice size $Lg = 4.0$

For each value of β, we stored 1000–10000 independent configurations extracted from 10^6 trajectories of the molecular dynamics

Statistical error is estimated by the jackknife analysis

(The constant ϵ for the admissibility is fixed to be $\epsilon = 2.6$)

One-point SUSY Ward-Takahashi identities

Since the action is Q-exact, we have $\langle\langle S \rangle\rangle = 0$, or

$$\sum_{i=1}^{3} \langle\langle \mathcal{L}_{\mathbf{B}i}(x) \rangle\rangle + \sum_{i=1}^{6} \langle\langle \mathcal{L}_{\mathbf{F}i}(x) \rangle\rangle + \frac{1}{a^4 g^2} \left\langle\!\!\left\langle \mathrm{tr} \left\{ H(x) - \frac{1}{2} i \widehat{\Phi}_{\mathbf{TL}}(x) \right\}^2 \right\rangle\!\!\right\rangle = 0$$

but

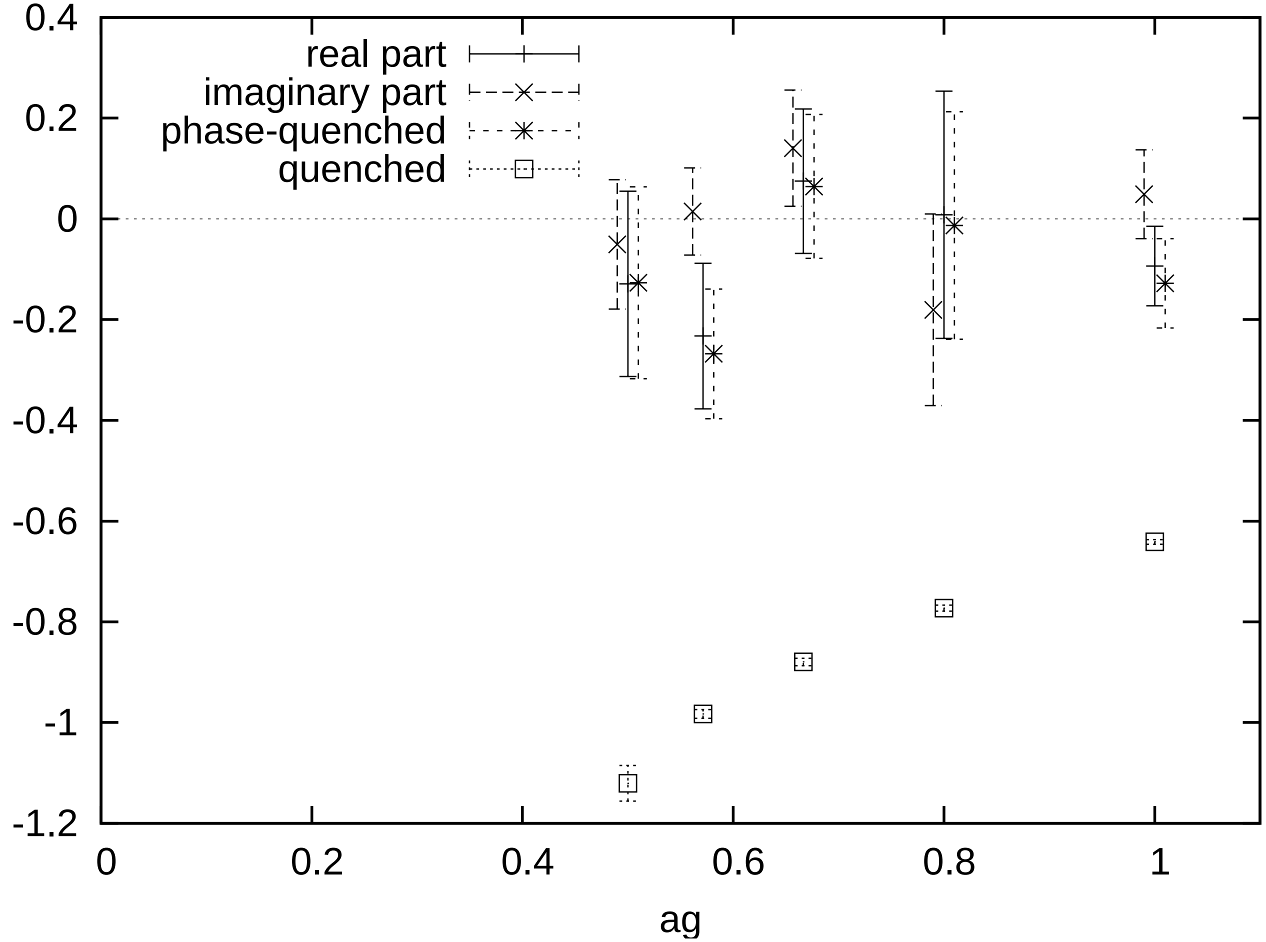
$$\sum_{i=1}^{6} \langle\langle \mathcal{L}_{\mathbf{F}i}(x) \rangle\rangle = -2(N_c^2 - 1)\frac{1}{a^2}$$

and

$$\frac{1}{a^4 g^2} \left\langle\!\!\left\langle \mathrm{tr} \left\{ H(x) - \frac{1}{2} i \widehat{\Phi}_{\mathbf{TL}}(x) \right\}^2 \right\rangle\!\!\right\rangle = \frac{1}{2}(N_c^2 - 1)\frac{1}{a^2}$$

Thus

$$\sum_{i=1}^{3} \langle\langle \mathcal{L}_{\mathbf{B}i}(x) \rangle\rangle - \frac{3}{2}(N_c^2 - 1)\frac{1}{a^2} = 0$$



Expectation values of $\sum_{i=1}^{3} \mathcal{L}_{\mathbf{B}i}(x) - \frac{3}{2}(N_c^2 - 1)\frac{1}{a^2}$

- **The real part is consistent with the expected identity within 1.5σ ($\Rightarrow$ strongly supports the correctness of our code/algorithm)**

- **The imaginary part is consistent with zero**

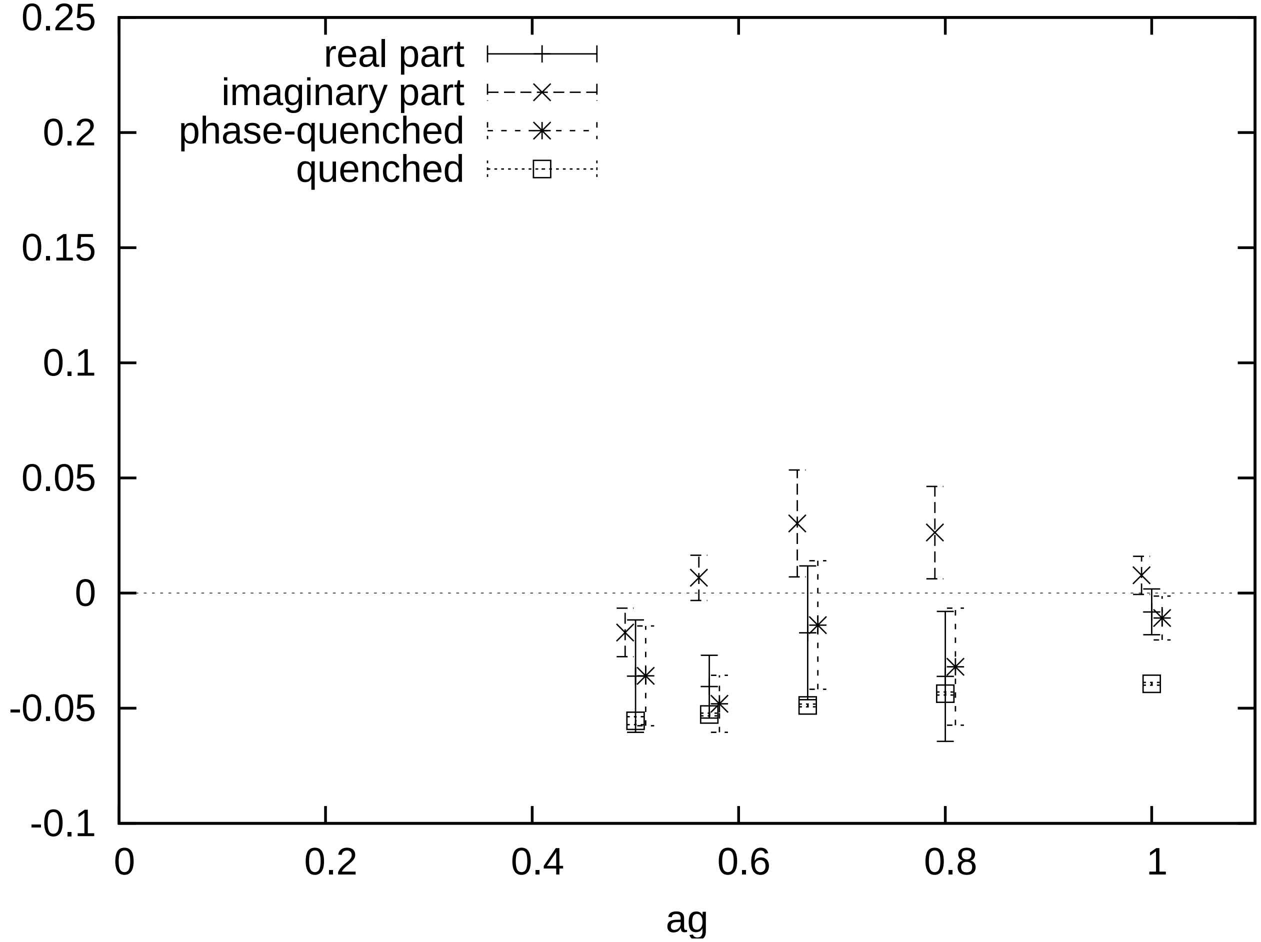
- **No notable difference of the phase-quenched average ($\Rightarrow$ systematic error due to wrong-sign determination is negligible)**

- **Clear distinction from the quenched average ($\Rightarrow$ effect of dynamical fermions is properly included)**

- **Effect of quenching starts at 2-loop $\sim g^2 \ln(a/L)$**

Another exact relation

$$\langle\!\langle Q\mathcal{O}_1(x)\rangle\!\rangle = \langle\!\langle \mathcal{L}_{\mathbf{B}1}(x)\rangle\!\rangle + \langle\!\langle \mathcal{L}_{\mathbf{F}1}(x)\rangle\!\rangle = 0$$



Expectation values of $\mathcal{L}_{\mathbf{B}1}(x) + \mathcal{L}_{\mathbf{F}1}(x)$

- **The relation is confirmed within** 2σ **(note the difference in scale of vertical axis compared to the previous figure)**

- **The quenched average is certainly inconsistent with the SUSY relation**

- **No clear separation between the re-weighted average and the quenched one (**$\Leftarrow$ **The effect of quenching starts at 3-loop** $\sim a^2 g^4 \ln(a/L)$**)**

Another relation

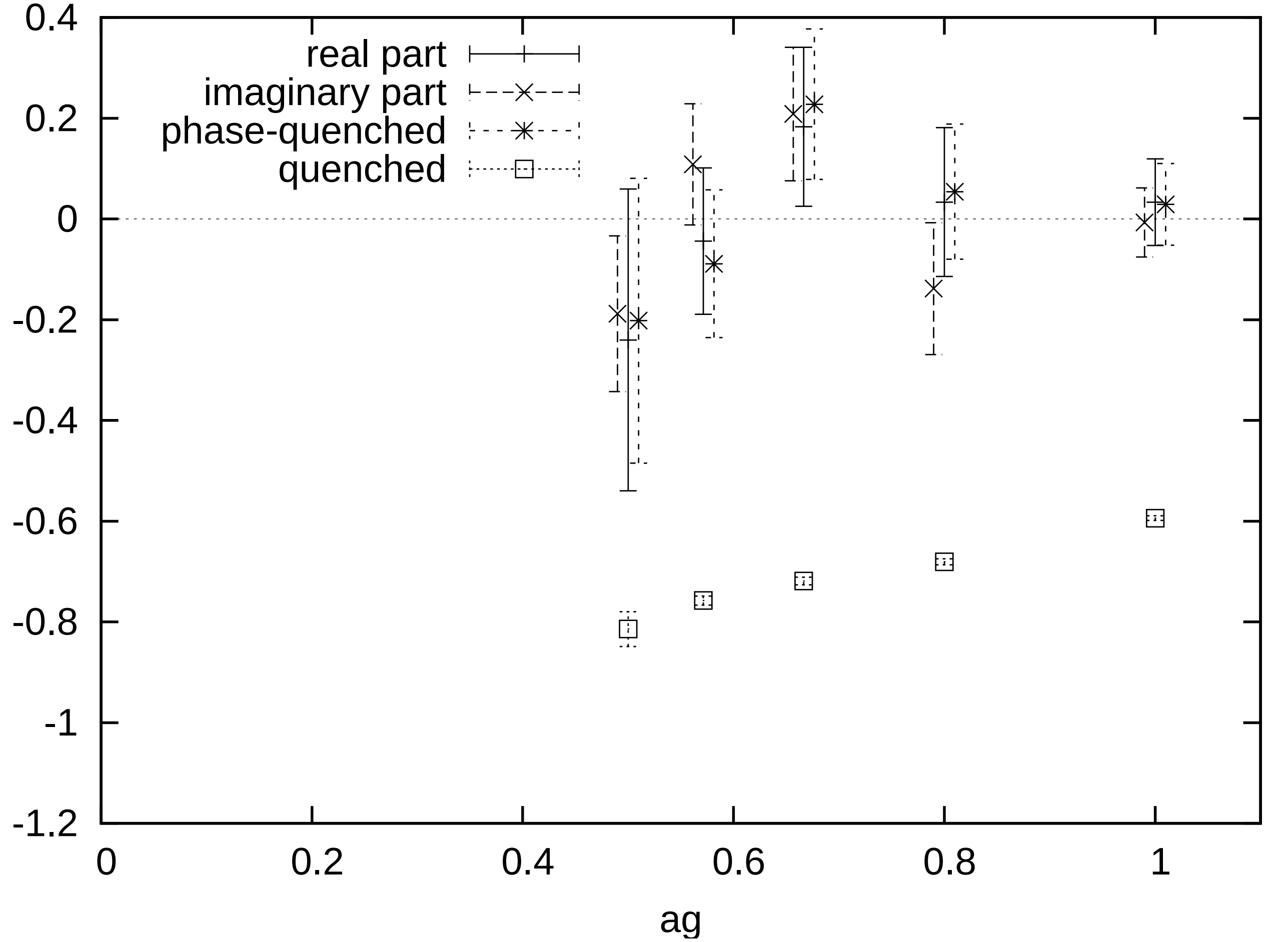
$$\langle\!\langle Q\mathcal{O}_2(x)\rangle\!\rangle = \frac{1}{a^4 g^2}\left\langle\!\!\left\langle \mathrm{tr}\left\{-iH(x)\hat{\Phi}_{\mathbf{TL}}(x)\right\}\right\rangle\!\!\right\rangle + \langle\!\langle \mathcal{L}_{\mathbf{F}5}(x)\rangle\!\rangle = 0$$

but

$$H(x) = \frac{1}{2} i\hat{\Phi}_{\mathbf{TL}}(x)$$

and thus

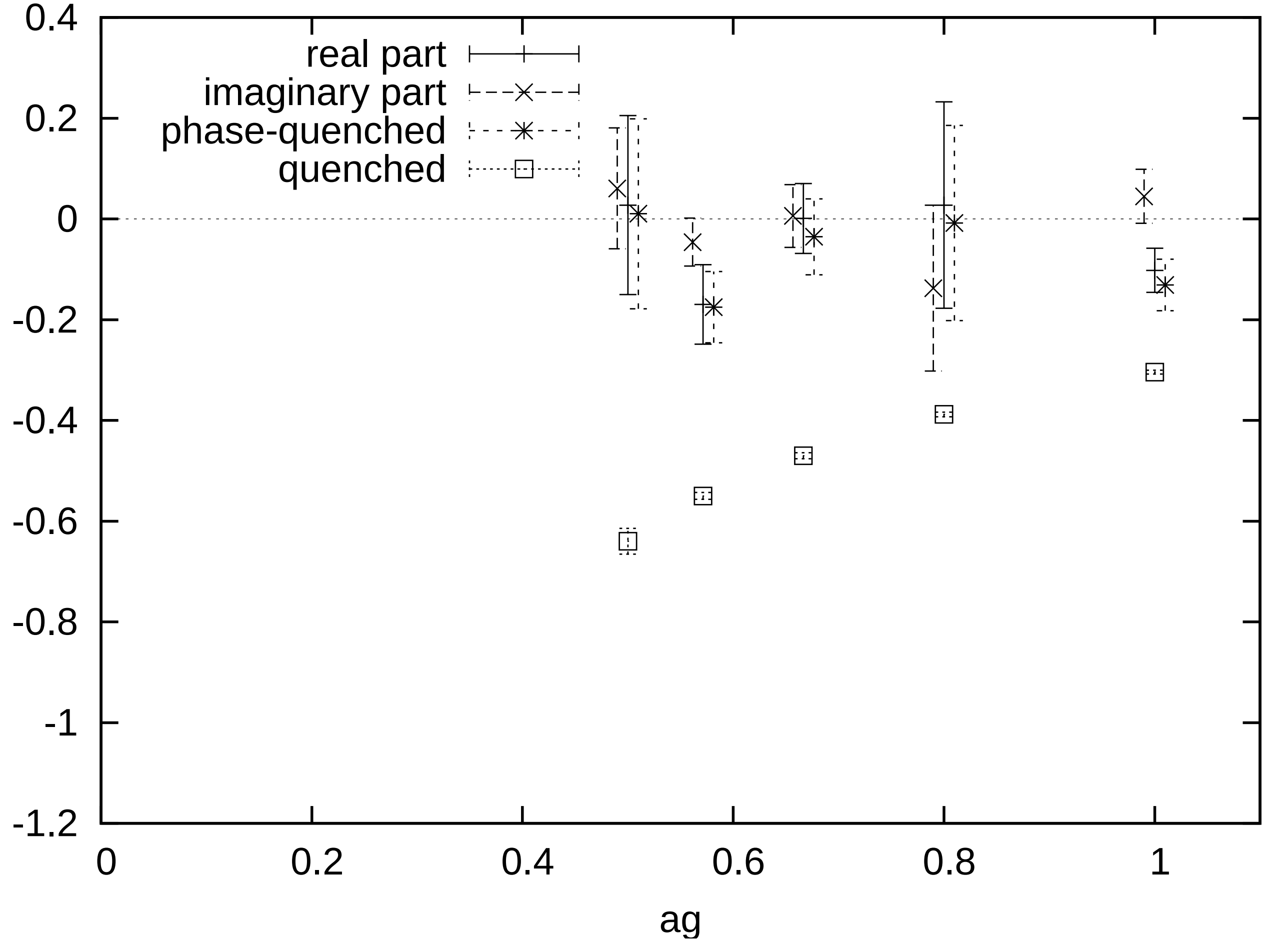
$$2\,\langle\!\langle \mathcal{L}_{\mathbf{B}2}(x)\rangle\!\rangle + \langle\!\langle \mathcal{L}_{\mathbf{F}5}(x)\rangle\!\rangle = 0$$



Expectation values of $2\mathcal{L}_{\mathbf{B}2}(x) + \mathcal{L}_{\mathbf{F}5}(x)$

The situation is again similar with the last piece of the relation

$$\langle\!\langle Q\mathcal{O}_3(x)\rangle\!\rangle = \langle\!\langle \mathcal{L}_{\mathbf{B}3}(x)\rangle\!\rangle + \langle\!\langle \mathcal{L}_{\mathbf{F}3}(x)\rangle\!\rangle + \langle\!\langle \mathcal{L}_{\mathbf{F}4}(x)\rangle\!\rangle + \langle\!\langle \mathcal{L}_{\mathbf{F}6}(x)\rangle\!\rangle = 0$$



Expectation values of $\mathcal{L}_{\mathbf{B}3}(x) + \mathcal{L}_{\mathbf{F}3}(x) + \mathcal{L}_{\mathbf{F}4}(x) + \mathcal{L}_{\mathbf{F}6}(x)$

So far, we have observed WT identities implied by the exact Q-symmetry of the lattice action

The continuum theory is invariant also under other fermionic transformations, Q_{01}, Q_0 and Q_1

$$Q_{01}A_\mu = -\epsilon_{\mu\nu}\psi_\mu \qquad Q_{01}\psi_\mu = i\epsilon_{\mu\nu}D_\nu\phi$$

$$Q_{01}\phi = 0$$

$$Q_{01}\eta = 2H \qquad Q_{01}H = \frac{1}{2}[\phi, \eta]$$

$$Q_{01}\overline{\phi} = -2\chi \qquad Q_{01}\chi = -\frac{1}{2}[\phi, \overline{\phi}]$$

$$
\begin{aligned}
Q_0 A_0 &= \frac{1}{2}\eta & Q_0 \eta &= -2iD_0\overline{\phi} \\
Q_0 A_1 &= -\chi & Q_0 \chi &= iD_1\overline{\phi} \\
Q_0 \overline{\phi} &= 0 & & \\
Q_0 \psi_1 &= -H & Q_0 H &= [\overline{\phi}, \psi_1] \\
Q_0 \phi &= -2\psi_0 & Q_0 \psi_0 &= \frac{1}{2}[\overline{\phi}, \phi]
\end{aligned}
$$

Another fermionic symmetry Q_1 is obtained by further exchange $\psi_0 \leftrightarrow \psi_1$

Invariance under these transformations is expected to be restored only in the continuum limit

In the supersymmetric continuum theory

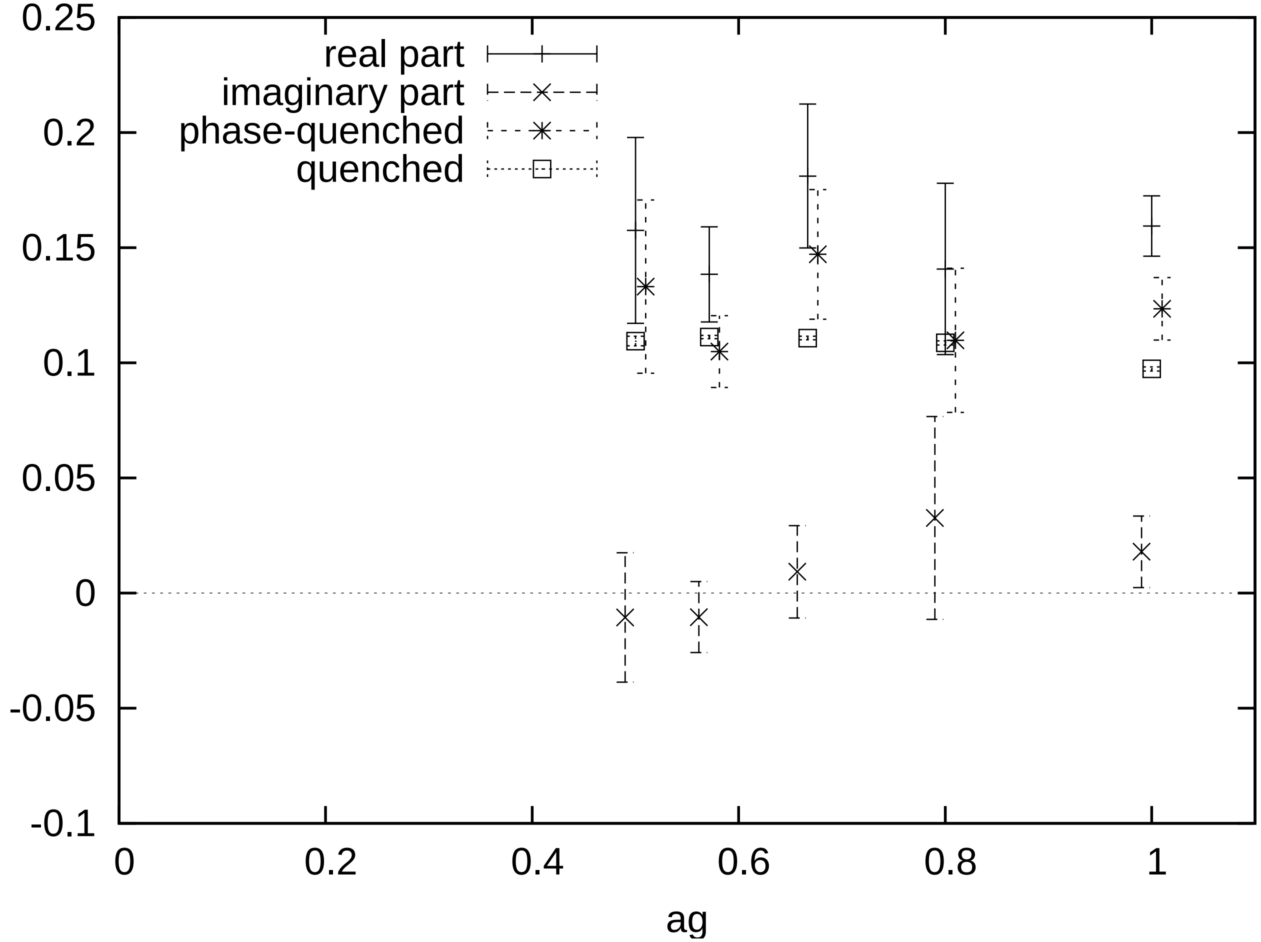
$$\left\langle\left\langle Q_{01}\frac{1}{g^2}\,\mathrm{tr}\left\{-\frac{1}{2}\chi[\phi,\overline{\phi}]\right\}\right\rangle\right\rangle_{\text{continuum}}$$

$$=\frac{1}{g^2}\left\langle\left\langle\mathrm{tr}\left\{\frac{1}{4}[\phi,\overline{\phi}]^2\right\}\right\rangle\right\rangle_{\text{continuum}}+\frac{1}{g^2}\langle\langle\mathrm{tr}\,\{-\chi[\phi,\chi]\}\rangle\rangle_{\text{continuum}}=0$$

Corresponding to this relation, one might expect

$$\langle\langle\mathcal{L}_{\mathbf{B}1}(x)\rangle\rangle+\langle\langle\mathcal{L}_{\mathbf{F}2}(x)\rangle\rangle\to 0?$$

holds in the continuum limit $a\to 0$



Expectation values of $\mathcal{L}_{\mathbf{B}1}(x) + \mathcal{L}_{\mathbf{F}2}(x)$

• It appears that the average approaches a non-zero number around 0.15 (not zero)

• This does not contradict with SUSY restoration. The argument of SUSY restoration is not applied to correlation functions containing composite operators

• Composite operators $\mathcal{L}_{\mathbf{B}1}(x)$ and $\mathcal{L}_{\mathbf{F}2}(x)$ induce logarithmic UV divergence at 2-loop level. If SUSY of the 1PI effective action is restored, this 2-loop level divergence should be the only source of UV divergence

• Moreover, that remaining 2-loop level divergence is cancelled out in the sum $\langle\!\langle \mathcal{L}_{\mathbf{B}1}(x) \rangle\!\rangle + \langle\!\langle \mathcal{L}_{\mathbf{F}2}(x) \rangle\!\rangle$

• **This argument indicates that, if SUSY in the 1PI effective action restores, $\langle\!\langle \mathcal{L}_{\mathbf{B}1}(x) \rangle\!\rangle + \langle\!\langle \mathcal{L}_{\mathbf{F}2}(x) \rangle\!\rangle$ approaches a constant (but not necessarily zero) as $ag \to 0$**

• **The behavior is consistent with this picture based on a restoration of SUSY**

• **Within almost 1σ the re-weighted average and the quenched average are degenerate and this also appears consistent with a perturbative picture ($\Leftarrow$ The effect of quenching starts at 3-loop $\sim a^2 g^4 \ln(a/L)$)**

• **So, the figure is consistent with the scenario of SUSY restoration, but, it may be dangerous to conclude the restoration of SUSY from the above result alone.**

Another example:

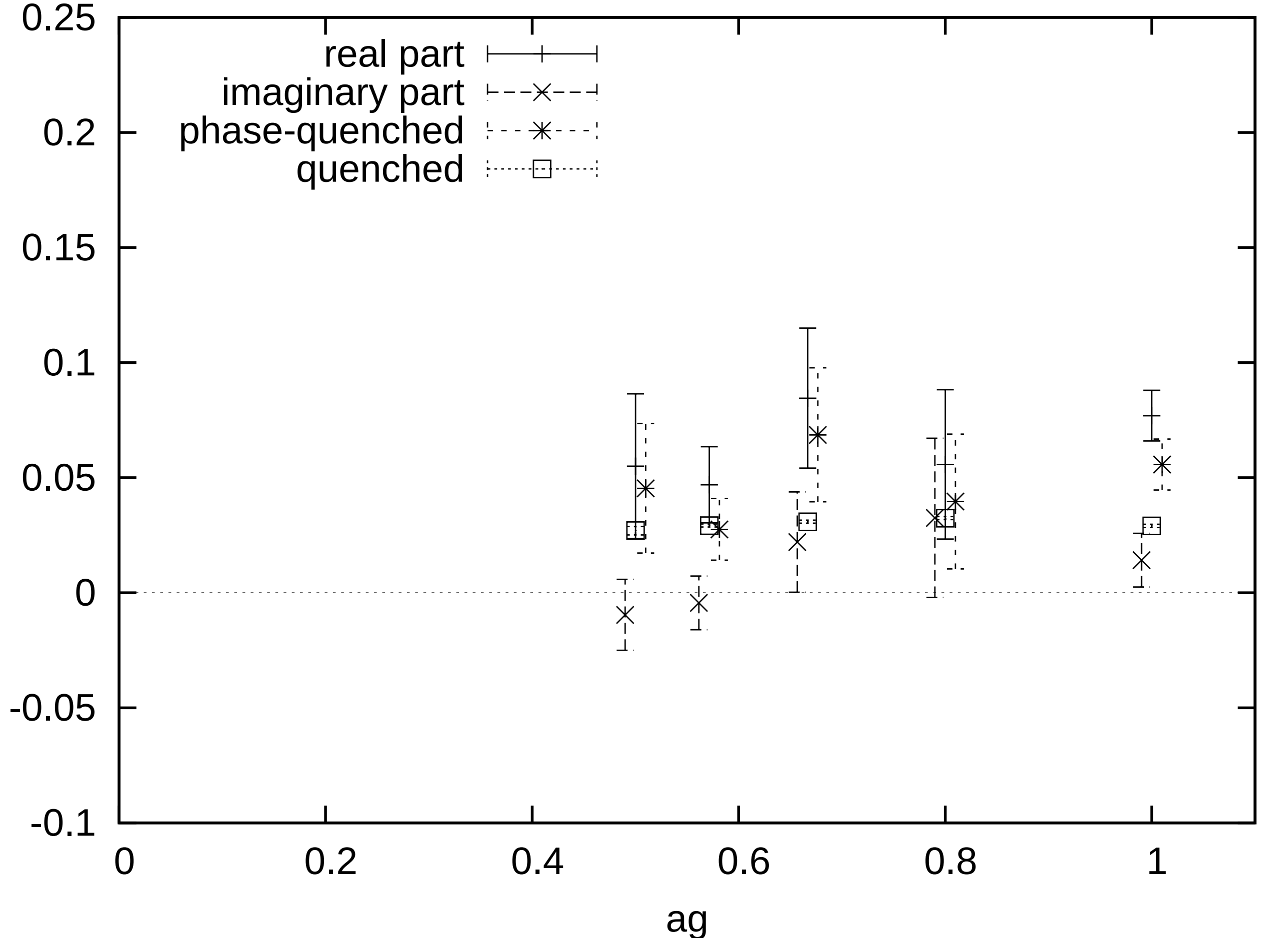
$$\left\langle\!\!\left\langle Q_0 \frac{1}{g^2} \,\mathrm{tr}\left\{ -\frac{1}{2}\psi_0[\phi,\overline{\phi}] \right\} \right\rangle\!\!\right\rangle_{\textbf{continuum}}$$

$$= \frac{1}{g^2}\left\langle\!\!\left\langle \mathrm{tr}\left\{ \frac{1}{4}[\phi,\overline{\phi}]^2 \right\} \right\rangle\!\!\right\rangle_{\textbf{continuum}} + \frac{1}{g^2}\left\langle\!\!\left\langle \mathrm{tr}\left\{ -\psi_0[\psi_0,\overline{\phi}] \right\} \right\rangle\!\!\right\rangle_{\textbf{continuum}} = 0$$

and one might expect

$$\langle\!\langle \mathcal{L}_{\mathbf{B}1}(x) \rangle\!\rangle + \langle\!\langle \mathcal{L}_{\mathbf{F}3}(x) \rangle\!\rangle \to 0?$$

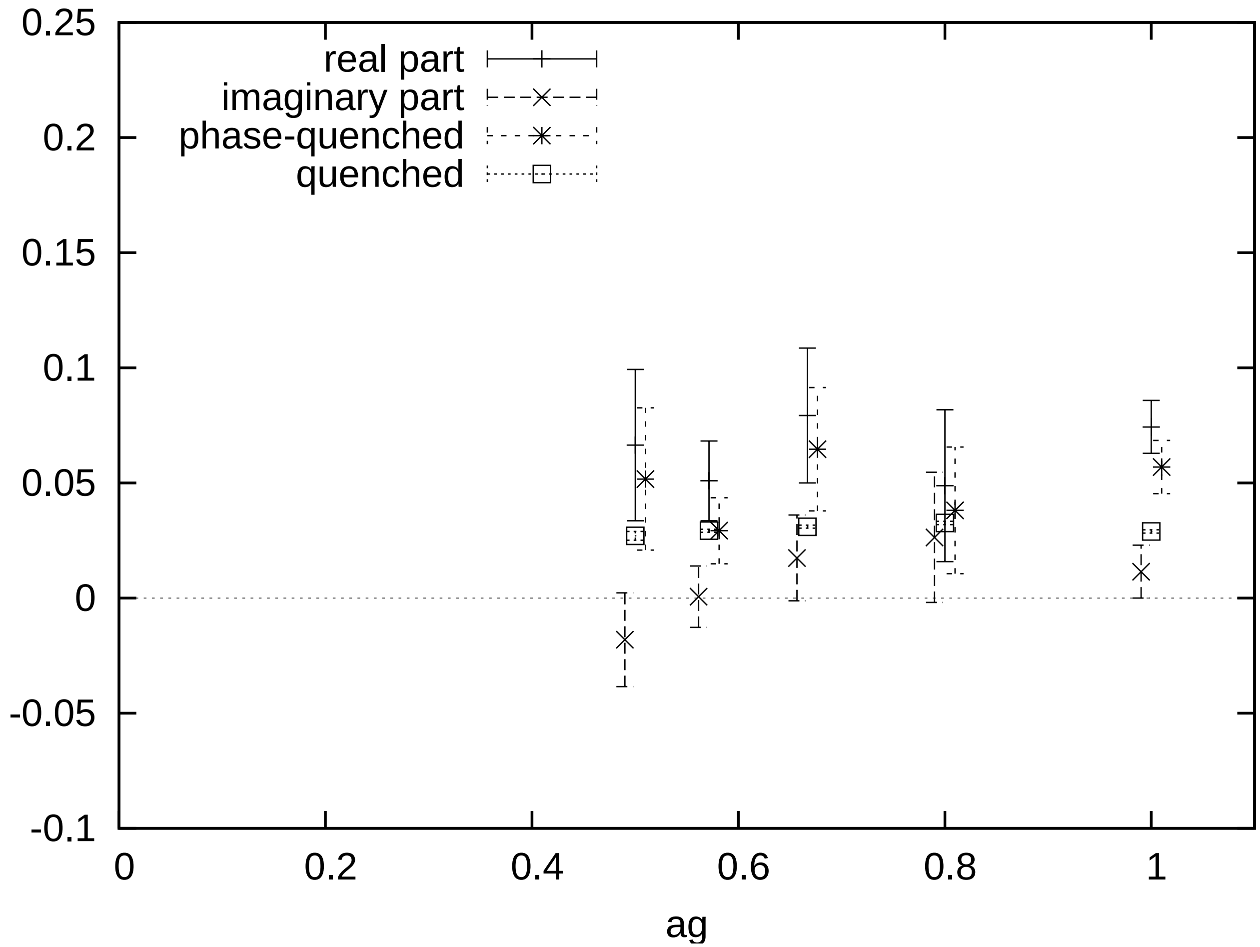
in the continuum limit $a \to 0$



Expectation values of $\mathcal{L}_{\mathbf{B}1}(x) + \mathcal{L}_{\mathbf{F}3}(x)$

Yet another:

$$\langle\!\langle \mathcal{L}_{\mathbf{B}1}(x) \rangle\!\rangle + \langle\!\langle \mathcal{L}_{\mathbf{F}4}(x) \rangle\!\rangle \to 0?$$



Expectation values of $\mathcal{L}_{\mathbf{B}1}(x) + \mathcal{L}_{\mathbf{F}4}(x)$

Gauge invariant scalar bi-linear operators

Classical "moduli space"

$$[\phi, \overline{\phi}] = 0$$

This degeneracy is not lifted to all order of loop expansion (the so-called flat directions)

Gauge-invariant scalar bi-linear operators

$$a^{-2}\,\mathrm{tr}\{\phi(x)\overline{\phi}(x)\}$$
$$a^{-2}\,\mathrm{tr}\{\phi(x)\phi(x)\}$$

$a^{-2}\,\mathrm{tr}\{\phi(x)\overline{\phi}(x)\}$ **is invariant under the global** $\mathrm{U}(1)_R$ **transformation**

$$\phi(x) \to e^{2i\alpha}\phi(x) \qquad \overline{\phi}(x) \to e^{-2i\alpha}\overline{\phi}(x)$$

The continuum limit of this quantity itself is meaningless, because it is a bare quantity and suffers from UV divergence. Power counting shows that the over-all UV divergence comes from the simplest 1-loop diagram and $\sim \ln(a/L)g^2$

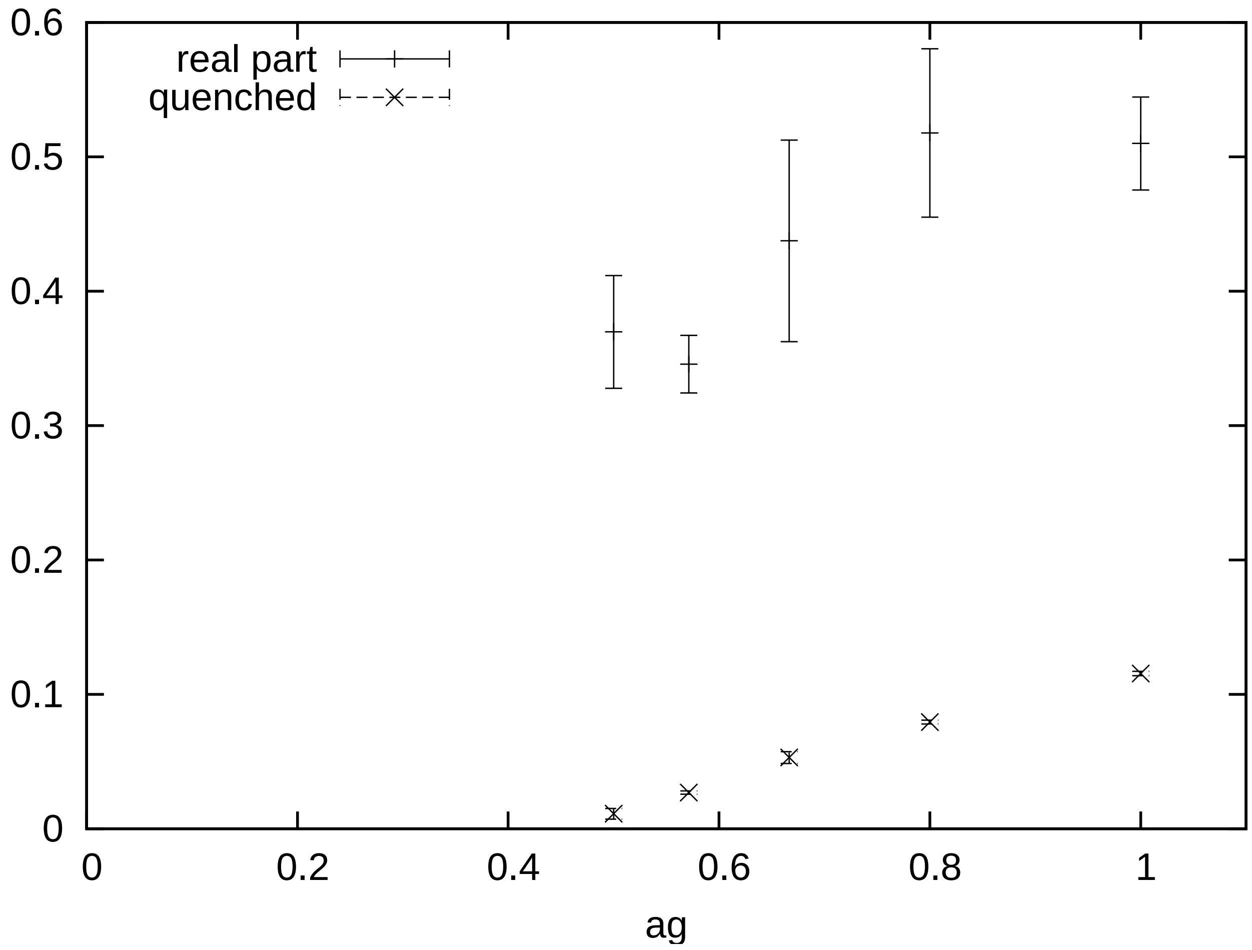
If SUSY of the 1PI effective action is restored in the continuum limit, this 1-loop divergence is the only source of UV divergence

So we define the renormalized operator (the normal product)

$$\mathcal{N}[a^{-2}\,\mathrm{tr}\{\phi(x)\overline{\phi}(x)\}] \equiv a^{-2}\,\mathrm{tr}\{\phi(x)\overline{\phi}(x)\} - (N_c^2 - 1)c(a/L)g^2$$

This subtraction must remove all the UV divergence of the composite operator

$$c(a/L = 1/N) = \frac{1}{2N^2} \sum_{n_0=0}^{N-1} \sum_{n_1=0}^{N-1} \frac{1}{\sum_{\mu=0}^{1} \left(1 - \cos\frac{2\pi}{N} n_\mu\right)}$$



Expectation values of $\mathcal{N}[a^{-2}\,\mathrm{tr}\{\phi(x)\overline{\phi}(x)\}]$

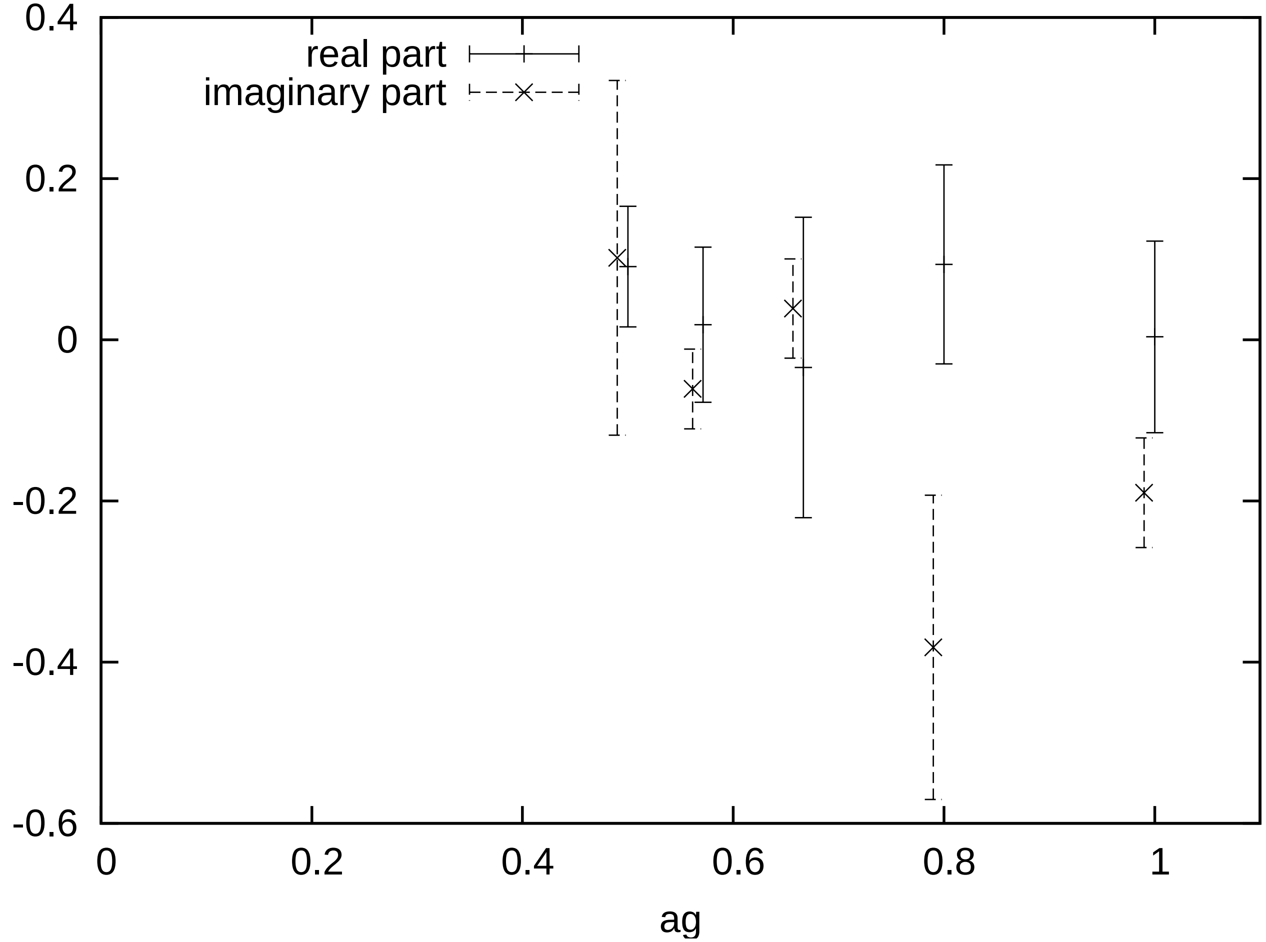
- **Clear separation between the re-weighted average and the quenched one (quantum effect of dynamical fermions)**

- **Fermions actually uplifts the expectation value !**

- **The expectation value appears to approach some finite number (in a unit of g^2) in the continuum limit after the renormalization**

- **Without the renormalization, there is a tendency that the expectation values grow as $ag \to 0$**

- **If SUSY is restored in the continuum limit, the expectation value is expected to become independent of ag as $a \to 0$. The behavior in figure is more or less consistent with this expectation (though we need much data to conclude this)**



Expectation values of $a^{-2}\,\mathrm{tr}\{\phi(x)\phi(x)\}$

Conclusion

- Preliminary numerical study of Sugino's lattice formulation of 2d $\mathcal{N} = (2,2)$ SYM

- WT identities associated with the Q-symmetry were confirmed in fair accuracy ($\Rightarrow$ re-weighting method is basically working)

- On the other hand, all results are consistent with the basic scenario of SUSY restoration (encouraging), though we could not conclude the restoration of full SUSY in a definite level

Prospects

- **Much larger lattice with (RIKEN) PC cluster**

- **Two-point functions (conservation of SUSY current, mass spectrum)**

- **Wilson loops (screening by adjoint fermions?)**

- **2d $\mathcal{N} = (4,4)$ SYM (and 2d $\mathcal{N} = (8,8)$ SYM)**

RIKEN Symposium

Quantum Field Theory and Symmetry

12/22 (Sat.) and 12/23 (Sun.)

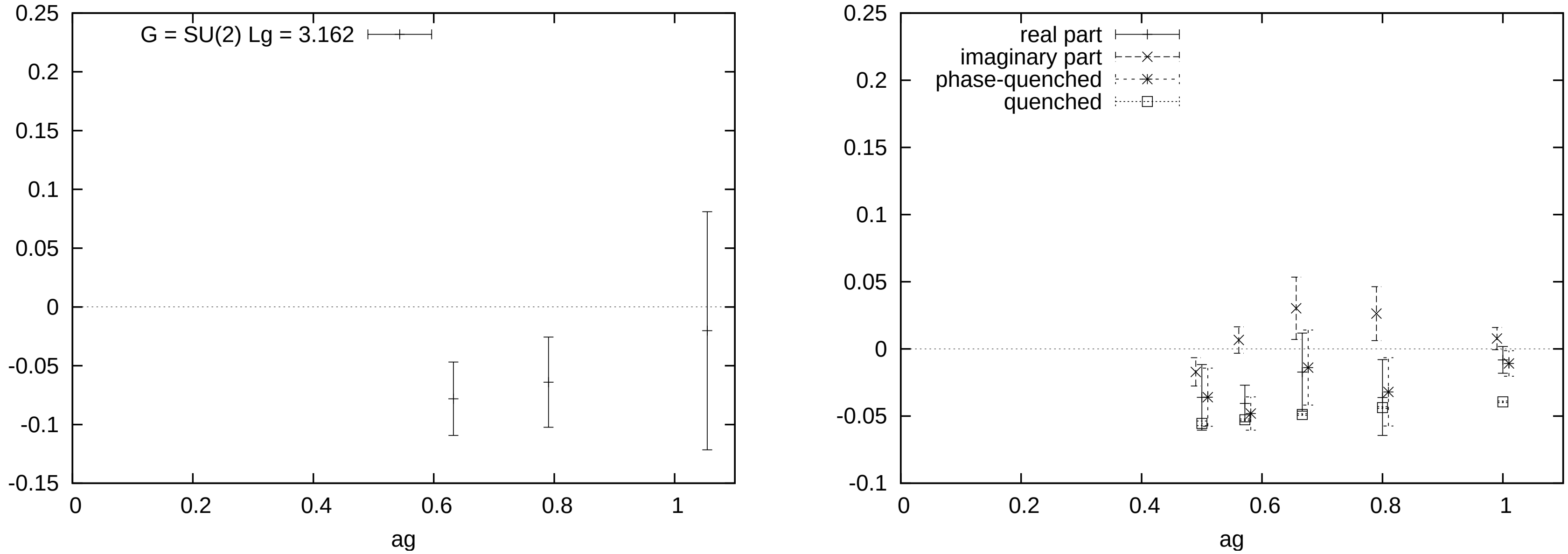
You Are Welcome !

To be announced in sg-l (hopefully) soon

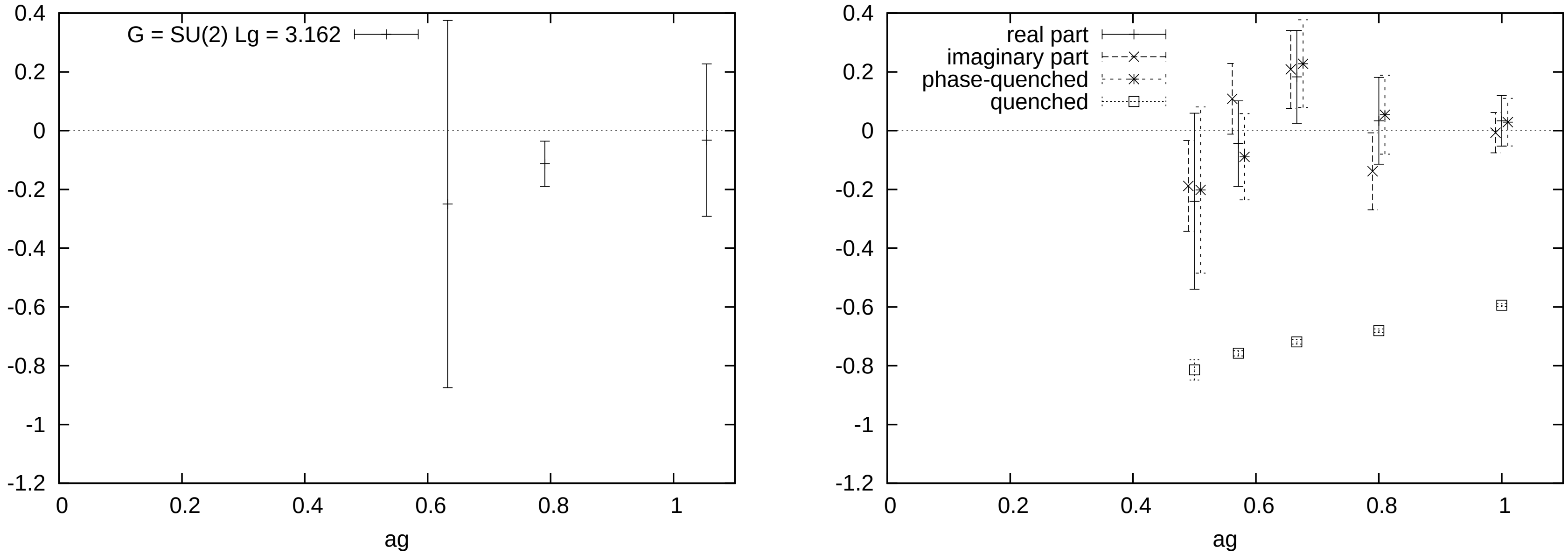
Appendix

Comparison with

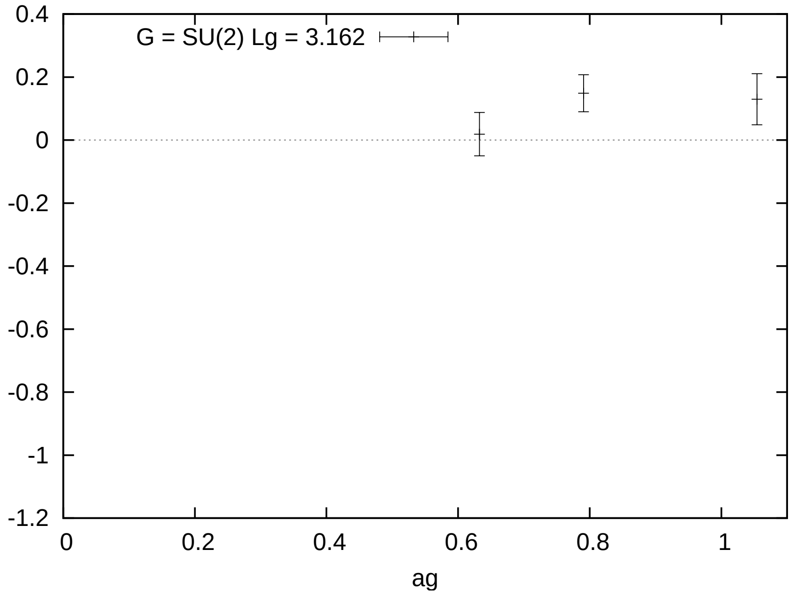
Catterall, JHEP 04 (2007) 015 [hep-lat/0612008]

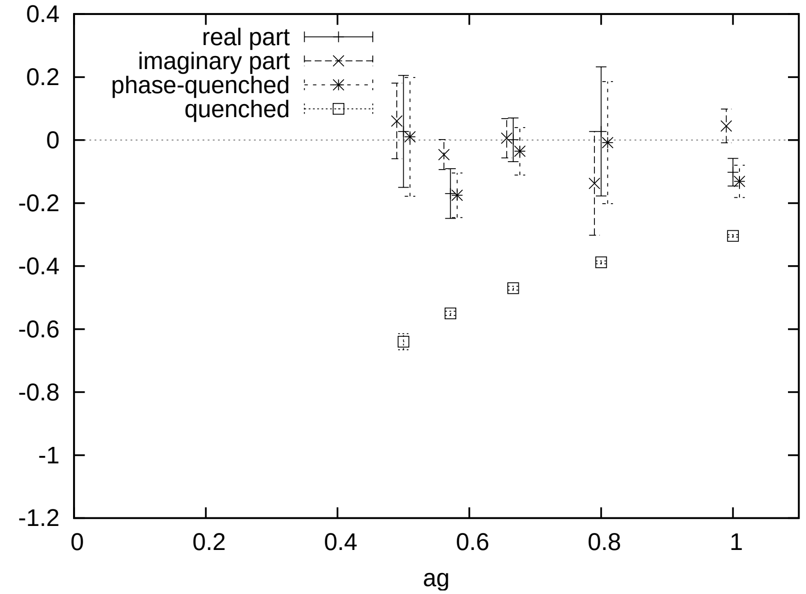


Expectation values of $\mathcal{L}_{\mathbf{B1}}(x) + \mathcal{L}_{\mathbf{F1}}(x)$

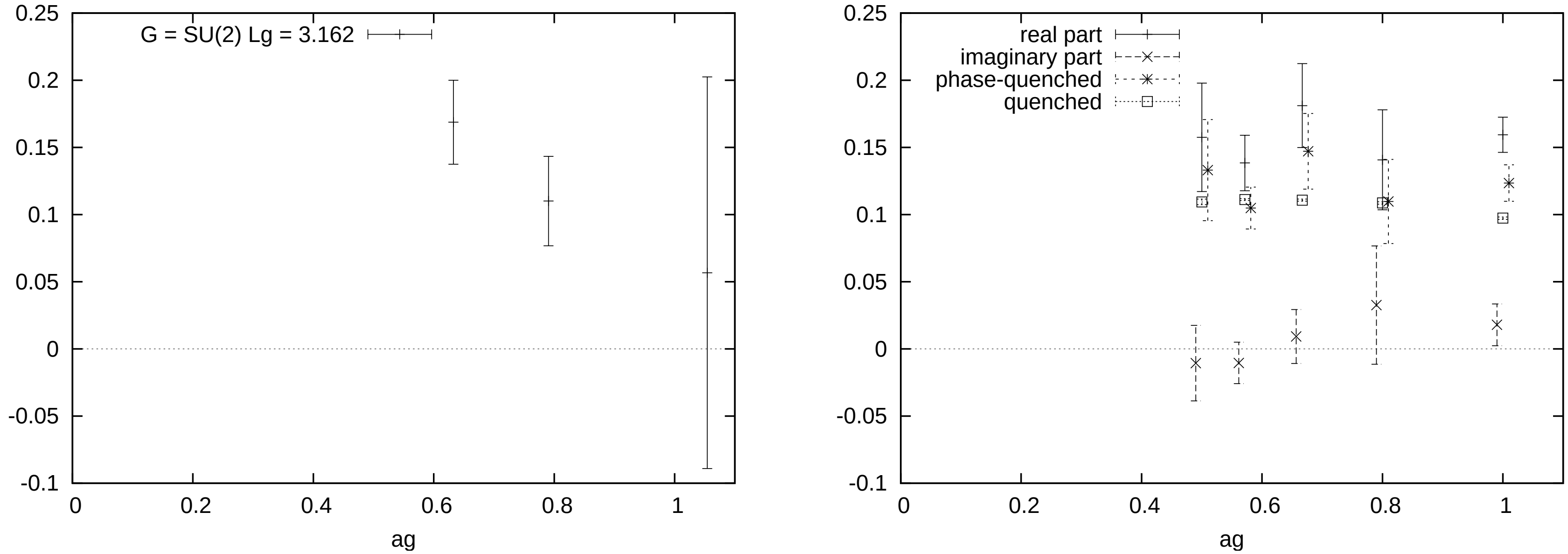


Expectation values of $2\mathcal{L}_{\mathbf{B}2}(x) + \mathcal{L}_{\mathbf{F}5}(x)$

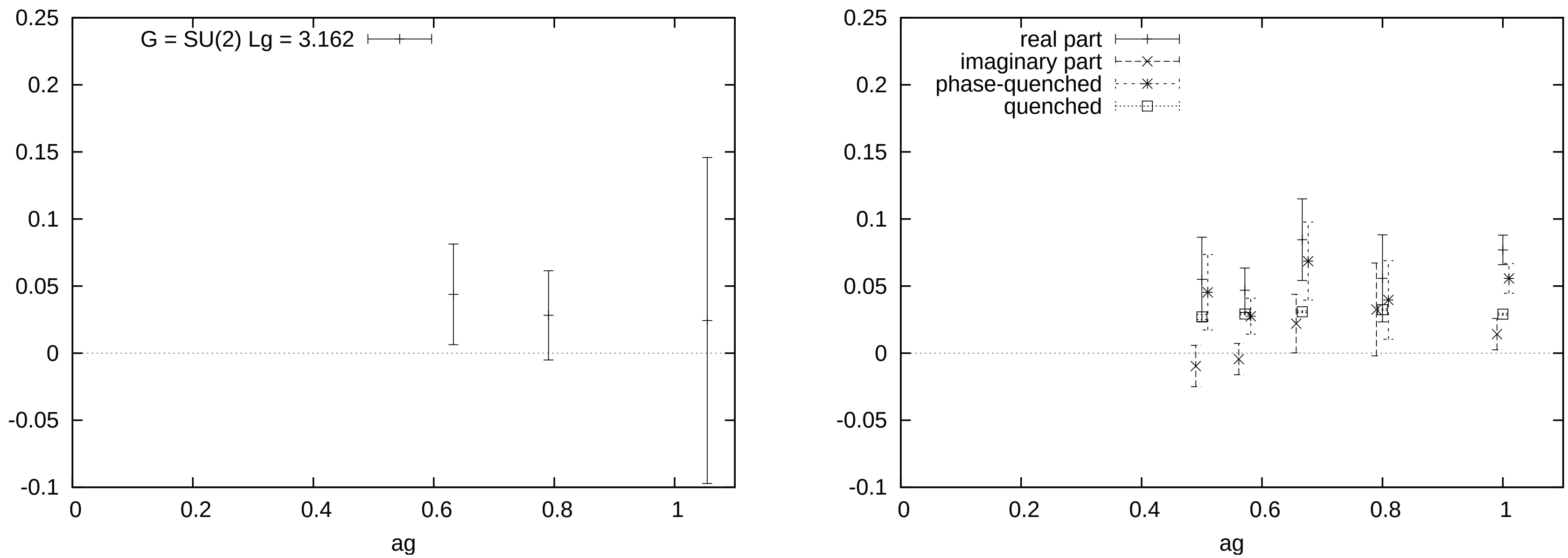




Expectation values of $\mathcal{L}_{\mathbf{B}3}(x) + \mathcal{L}_{\mathbf{F}3}(x) + \mathcal{L}_{\mathbf{F}4}(x) + \mathcal{L}_{\mathbf{F}6}(x)$



Expectation values of $\mathcal{L}_{\mathbf{B}1}(x) + \mathcal{L}_{\mathbf{F}2}(x)$



Expectation values of $\mathcal{L}_{\mathbf{B}1}(x) + \mathcal{L}_{\mathbf{F}3}(x)$