On gauge symmetry breaking via Euclidean time component of gauge fields

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contents

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I. Introduction ($\langle A_\tau \rangle$ as a source of gauge symmetry breaking)

Finite temperature quantum field theory; [Dolan and Jackiw,'74, Weinberg, '74]

Phase structures, symmetry breaking/restoration, $et.al.$

The Euclidean time direction is compactified on S^1_τ $\quad (R = 1/(2\pi T) = \beta/2\pi)$

The boundary condition of fields for the S^1_τ; $\begin{cases} \text{periodic b.c} \\ \text{antiperiodic b.c} \end{cases}$ (quantum statistics)

Gauge theories at finite temperature $(R^{D-1} \times S^1_\tau)$

$$A_{\hat{\mu}} = (A_i,\ A_D \equiv A_\tau) \qquad (i = 1, \cdots, D-1)$$

$A_\tau \cdots$ the Euclidean time component; Dynamical variable (cannot be gauged away)

The gauge transformation $U(x^i,\ \tau)$ satisfying $U(x^i, \tau + \beta) = U(x^i, \tau)$;

$$U(x^i, \tau) = \text{diag.}\left(\mathrm{e}^{i2\pi T m_1 \tau},\ \mathrm{e}^{i2\pi T m_2 \tau},\ \cdots,\ \mathrm{e}^{i2\pi T m_N \tau}\right) \quad \text{with} \quad \sum_{i=1}^{N} m_i = 0$$

$$A_\tau \quad \to \quad UA_\tau U^\dagger + \frac{2\pi T}{g}\,\text{diag.}\,(m_1,\ m_2,\ \cdots,\ m_N)$$

writing $\quad \langle A_\tau \rangle = \dfrac{T}{g}\,\text{diag.}(\varphi_1, \cdots, \varphi_N),$ then,

$$\to \quad \frac{T}{g}\,\text{diag.}\,(\varphi_1 + 2\pi m_1,\ \varphi_2 + 2\pi m_2,\ \cdots,\ \varphi_N + 2\pi m_N)$$

Physically distinct from $\langle A_\tau \rangle = 0$ for $\varphi_i \neq 2\pi k\ (k \in \mathbf{Z})$

A consequence;

$$\text{tr}(F_{\tau i})^2 \quad \Longrightarrow \quad (A_i^{(l)})_{ij}\ \cdots \quad (2\pi T)^2 \left(l + \frac{\varphi_i - \varphi_j}{2\pi} \right)^2, \qquad l \in \mathbf{Z}$$

A signal for gauge symmetry breaking through nontrivial values of φ_i

[N.B] Phase structures (Z_N symmetry) in pure QCD with massless adjoint fermion,
Color screening effects in QCD with massless adjoint and fundamental fermion
[Weiss,'81, Gross,$et.al.$,'81, Korthals and Laine,'01, Farakos and Pasipoularides,'05]

II. Possibilities to break gauge symmetry through $\langle A_\tau \rangle$

The effective potential for $\langle A_\tau \rangle$ in perturbation theory; $A_{\hat{\mu}} = \langle A_\tau \rangle \delta_{\tau\hat{\mu}} + \bar{A}_{\hat{\mu}}$

gauge $\cdots$ $$V^T_{gauge} = \frac{-\Gamma(D/2)}{\pi^{D/2}}(D-2)T^D \sum_{i,j=1}^{\infty} \frac{1}{n^D} \cos[n(\varphi_i - \varphi_j)]$$

fermion $\cdots$ $$V^{f,T}_{fd} = \frac{\Gamma(D/2)}{\pi^{D/2}} 2^{[D/2]} T^D \sum_{i=1}^{\infty} \frac{1}{n^D} \cos[n(\varphi_i - \pi)] \qquad (S^1_\tau \times R^{D-1})$$

The $SU(N)$ gauge theory with massless matter

$$V^T_{total} = V^T_{gauge} + N^f_{fd} V^{f,T}_{fd} + N^f_{adj} V^{f,T}_{adj} + N^b_{fd} V^{b,T}_{fd} + N^b_{adj} V^{b,T}_{adj}$$

The vacuum configuration goes like $(k = 0, 1, \cdots, N-1)$

$$\varphi_{i(=1,\cdots,N)} = \begin{cases} 2\pi k/N \ (\text{mod } 2\pi) & \text{for } \ N^f_{fd} = N^b_{fd} = 0, \\ 0 & \text{for } \ \text{otherwise} \end{cases}$$

The Polyakov loop,

$$W_p \equiv \mathcal{P}\text{exp}\left(ig \int_0^\beta d\tau \langle A_\tau \rangle \right) = \begin{cases} \mathrm{e}^{i\frac{2\pi k}{N}} \mathbf{1}_{N\times N} & \text{the center of } SU(N), \\ \mathbf{1}_{N\times N} & \end{cases}$$

No nontrivial values of $\langle A_\tau \rangle$,

No gauge symmetry breaking (massless mode for $A_i^{(l)}$).

The boundary condition for the τ direction is crucial.

[N.B] The results do not change for massive matter.

The effects of an extra dimension on $\langle A_\tau \rangle$

Gauge theories on $S_\tau \times R^{D-2} \times S^1$ (the circumference of S^1 (S^1_τ); $L = 2\pi R$ $(\frac{1}{T})$)

$$A_{\hat{\mu}} = (A_\tau,\ A_{k=1,\cdots,D-2},\ A_y) \quad \text{dynamical degrees of freedom} \ \ A_\tau, \ \ A_y$$

$$\left(\frac{g}{T} \langle A_\tau \rangle = \text{diag.}(\varphi_1, \cdots, \varphi_N), \quad gL\langle A_y \rangle = \text{diag.}(\theta_1, \cdots, \theta_N) \right)$$

The mode expansion for A_k gives

$$\left(A_k^{(\bar{n},\ n)} \right)_{ij} \quad \cdots \quad (2\pi T)^2 \left(\bar{n}^2 + \frac{\varphi_i - \varphi_j}{2\pi} \right)^2 + \frac{1}{R^2} \left(n^2 + \frac{\theta_i - \theta_j}{2\pi} \right)^2$$

$$(\bar{n},\ n \in \mathbf{Z})$$

Two sources, $\varphi_i,\ \theta_i$ for the gauge symmetry breaking.

The one-loop potential for φ_i and θ_i.

¶ Massless matter

$$
\begin{aligned}
\bar{F}_{massless} \quad &= \quad (-)^{f+1}\Bigg[\sum_{i,j=1}^{N}\sum_{m=1}^{\infty}\frac{1}{m^D}\cos\left[m(\theta_i-\theta_j-\beta)\right] \qquad \cdots T=0 \\
&+(LT)^D\sum_{i,j=1}^{N}\sum_{l=1}^{\infty}\frac{1}{l^D}\cos\left[l(\varphi_i-\varphi_j+2\pi\eta)\right] \qquad \cdots \text{purely } T\neq 0 \\
&+2(LT)^D\sum_{i,j}^{N}\sum_{l,m=1}^{\infty}\frac{\cos\left[m(\theta_i-\theta_j-\beta)\right]\cos\left[l(\varphi_i-\varphi_j+2\pi\eta)\right]}{\left[(mLT)^2+l^2\right]^{D/2}}\Bigg].
\end{aligned}
$$

$\beta =$ twisted b.c. for the S^1 direction, $\eta = 0$ $(\frac{1}{2})$ for bosons (fermions), $f = 0(1)$

$\varphi_i - \varphi_j \Rightarrow \varphi_i, \quad \theta_i - \theta_j \Rightarrow \theta_i, \quad \sum_{i,j} \Rightarrow \sum_i$ for the fundamental representation.

• $T = 0$; gauge symmetry breaking through θ_i (Hosotani mechanism)

matter content, b.c. for the S^1 direction

[e.g.] $SU(2)$ gauge + adjoint fermions (periodic b.c.); min. at $\theta = \pi/2$

$$A^{(n)1,2},\ \frac{\left(n+\frac{1}{2}\right)^2}{R^2}, \quad A^{(n=0)3},\ \text{massless},\ SU(2) \to U(1)$$

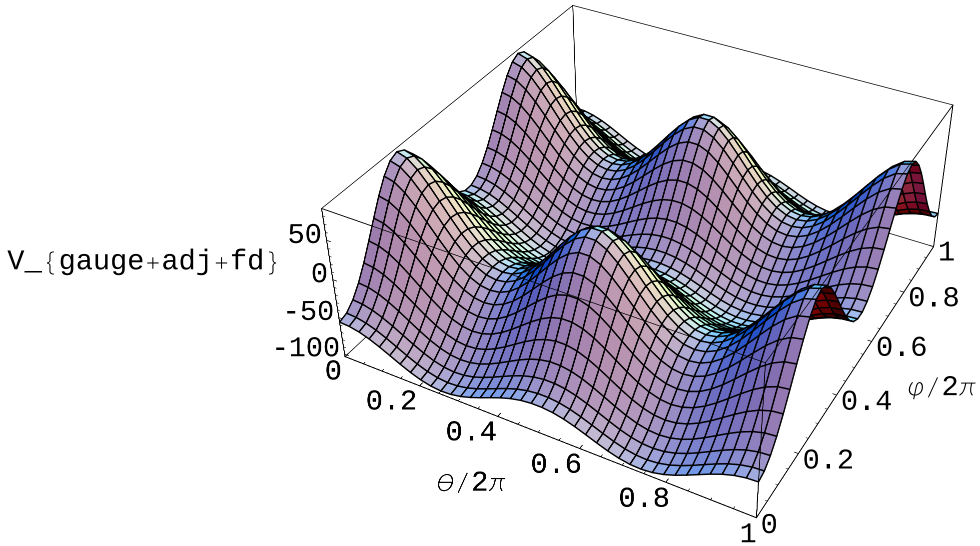
• High T limit ; the second term dominates in the potential

No nontrivial values for φ_i;

$$\varphi_{i(=1,\cdots,N)} = \begin{cases} 2\pi k/N \ (\text{mod } 2\pi) & \text{for } N^f_{fd} = N^b_{fd} = 0, \\ 0 & \text{for otherwise} \end{cases}$$

• $L^{-1} \sim T$

[E.G] Massless fermions, $(N_{adj},\ N_{fd}) = (1,\ 5)$ with $SU(2),\ LT = 1.0$



The vacuum configuration $(\varphi,\ \theta) = (0,\ 0.261 \times 2\pi)$ (mod 2π)

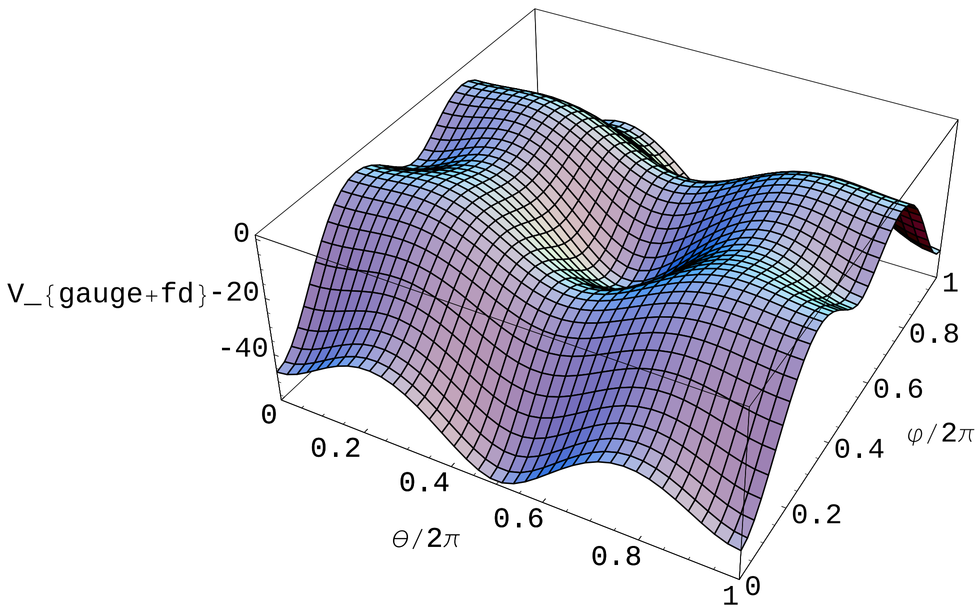
The gauge symmetry breaking $SU(2) \to U(1)$ only through the nontrivial values of θ [N.B] $(\varphi,\ \theta) = (0,\ 0.5 \times 2\pi)$ for $\beta/2\pi = 0.2$; no gauge symmetry breaking

¶ Massive matter $(D=5, \quad S_\tau \times R^3 \times S^1, \quad z \equiv ML,\ t \equiv LT)$

$$
\begin{aligned}
\bar{F}_{massive} \quad = \quad & (-1)^{f+1}\Bigg[\sum_{i=1}^{N}\sum_{m=1}^{\infty}\frac{1}{m^5}\left(1+mz+\frac{m^2z^2}{3}\right)\mathrm{e}^{-mz}\cos[m(\theta_i-\beta)] \\
& +t^5\sum_{i=1}^{N}\sum_{l=1}^{\infty}\frac{(-1)^l}{l^5}\left(1+\frac{lz}{t}+\frac{l^2z^2}{3\ t^2}\right)\mathrm{e}^{-lz/t}\cos(l(\varphi_i+2\pi\eta)) \\
& +2t^5\sum_{i=1}^{N}\sum_{l,m=1}^{\infty}\frac{(-1)^l}{[(mt)^2+l^2]^{5/2}} \\
& \times\left(1+\sqrt{(mz)^2+(lz/t)^2}+\frac{(mz)^2+(lz/t)^2}{3}\right)\mathrm{e}^{-\sqrt{(mz)^2+(lz/t)^2}} \\
& \times\cos[m(\theta_i-\beta)]\cos(l(\varphi_i+2\pi\eta))\Bigg],
\end{aligned}
$$

$\eta = 1/2\ (0)$ for fermions (bosons). One recovers the case for the massless matter for $z \longrightarrow 0$.

[E.G] Massive fundamental fermions, $N_{fd} = 1, SU(2),\ \beta = 0,\ (t,\ z) = (1.1,\ 0.8)$



The vacuum configuration $(\varphi,\ \theta) = (0,\ \pi)$ (mod $= 2\pi$).

No nontrivial values of φ is obtained.

The $SU(2)$ gauge symmetry is unbroken in this case.

III. Summary

- We have studied $\langle A_\tau \rangle$ from a point of view of gauge symmetry breaking.

(The effective potential for $\langle A_\tau \rangle = \frac{T}{g}\text{diag.}(\varphi_1, \cdots, \varphi_N)$ in one-loop approximation)

- Finite temperature without an extra dimension $(S^1_\tau \times R^{D-1})$.

$$\varphi_{i(=1,\cdots,N)} = \begin{cases} 2\pi k/N \ (\text{mod } 2\pi) & \text{for } \ N^f_{fd} = N^b_{fd} = 0, \\ 0 & \text{for } \ \text{otherwise}. \end{cases}$$

- Finite temperature with an extra dimension $(S_\tau \times R^{D-2} \times S^1)$.

Two kinds of the order parameters, $\langle A_\tau \rangle, \ gL\langle A_y \rangle = \text{diag.}(\theta_1, \cdots, \theta_N)$

No nontrivial values for $\langle A_\tau \rangle$ is obtained (valid for $SU(N)$).

The gauge symmetry can be broken only through the $\langle A_y \rangle$ (Hosotani mechanism).

It is crucial that the b.c. for the S_τ direction is fixed by the quantum statistics.

More interestingly, (details; coming soon, with M. Sakamoto)

$\star$ Symmetry restoration (dynamics of θ_i (Hosotani mechansim) at high T)

$$T=0\ ; \quad \begin{array}{l}\text{gauge symmetry breaking}\\ \text{fermions, bosons, representation}\\ \text{b.c.'s for the } S^1 \text{ direction}\end{array} \quad \overset{\text{high T}}{\Longrightarrow} \begin{cases}\text{restored (Higgs mechanism),}\\ \text{reduced,}\\ \text{unchanged}\end{cases}$$

$$F_{fermion}(\theta, \varphi=0) \simeq 0. \quad \text{(no massless Matsubara mode (antiperiodic b.c.))}$$

$$F_{boson}(\theta, \varphi=0) \simeq -\frac{T}{L^{D-1}} \frac{\Gamma(\frac{D-1}{2})}{\pi^{\frac{D-1}{2}}} \sum_{m=1}^{\infty} \frac{1}{m^{D-1}} \cos[m(\theta-\beta)].$$

(the massless Matsubara mode).

Hence, θ_i is determined by the bosonic degrees of freedom (gauge + scalars). The behavior of the Hosotani mechanism and that of the Higgs mechanism at high T are different.