Fatten up identity based solution in string field theory

Syoji Zeze 瀨瀨將吏
National Taiwan University 國立臺灣大學

Collaboration with I. Kishimoto

6 Aug., QFT 07 @Kinki University
Schnabl’s analytic method in SFT

- Powerful method for explicit calculations of open cubic SFT
- Crucial feature: use of special conformal coordinate called “sliver frame”
- Star product is simplified, but not too simplified
Marginal solutions

- Insertion of $J(z_1)J(z_2)...J(z_n)$ in the solution collide each other

- Regular OPE case [Schnabl, KORZ, Kishimoto-Michishita, Okawa, Erler]

- Singular OPE case ... regularization and counter term needed
  - Up to 3rd order [KROZ]
  - Full order for $\partial X$ [Fuchs-Kroyter-Potting, Fuchs-Kroyter]
  - Full order, in general [Kiermaier-Okawa]
Our claim

- New solution of cubic open SFT which corresponds to marginal deformation
- The solution is obtained by "fattening up" Talahashi-Tanimoto (identity based) solution on the zero width cylinder
- Valid for singular OPE case
- Real (not complex)
Solution generation

if \( Q_B \phi \) and \( \phi \ast \phi = 0 \)

\[
\Psi = P_\alpha \ast \frac{1}{1 + \phi \ast A^{(\alpha+\beta)} \ast \phi \ast P_\beta}
\]

\[Q_B \Psi + \Psi \ast \Psi = 0\]

satisfies EOM

\[\phi \text{ has zero width for known solutions}\]

“fattening up” zero width solution

[Erler, ORZ, Kishimoto-Michishita]

[Kishimoto-S.Z]
\[ \frac{1}{1 + \phi \ast A} \ast \phi = \phi - \phi \ast A \ast \phi + \phi \ast A \ast \phi \ast A \ast \phi + \cdots \]

\[ \int_{0}^{\alpha + \beta} \int_{0}^{\alpha + \beta} \int_{0}^{\alpha + \beta} \, dx_1 dx_2 dx_3 \]
Takahashi-Tanimoto marginal solution

- Marginal solution with zero width
- Available for singular OPE case
- Before fattening it up, let us study an expression in sliver frame
TT in sliver frame

\[ \Psi_{m}^{TT} = - \lim_{\epsilon \to 0} U_{1+\epsilon} \int_{-\infty}^{\infty} \frac{dt}{2\pi} \left( \lambda_{a} f(t) c \left( it + \frac{\pi}{4} \epsilon \right) J^{a} \left( it + \frac{\pi}{4} \epsilon \right) + \frac{1}{2} g^{ab} \lambda_{a} \lambda_{b} f(t)^2 c \left( it + \frac{\pi}{4} \epsilon \right) \right) |0\rangle \]

\[ f(t) = \frac{4}{\cosh^2(2t)} \]

\[ J^{a}(z)J^{b}(w) \sim \frac{-g^{ab}}{\sin^2(z-w)} + \frac{f^{ab}c}{\sin(z-w)} J^{c}(w) \]
Equation of motion

\[ \langle \eta, Q_B \Psi_{m}^{TT} \rangle = - \int_{-\infty}^{\infty} \frac{dt}{2\pi} \frac{1}{2} (f(t))^2 \langle \eta \left( \frac{\pi}{4} \right) c \partial c(\text{i}t) \rangle \]

\[ \langle \eta, \Psi_{m}^{TT} * \Psi_{m}^{TT} \rangle = \frac{1}{2} \int_{\gamma_1} \frac{dz_1}{2\pi i} \int_{\gamma_2} \frac{dz_2}{2\pi i} \lambda_a \lambda_b \langle \eta \left( \frac{\pi}{4} \right) J^a(z_1) J^b(z_2) c(z_1) c(z_2) \rangle \]

\[ = + g^{ab} \lambda_a \lambda_b \int_{-\infty}^{\infty} \frac{dt}{2\pi} \frac{1}{2} (f(t))^2 \langle \eta \left( \frac{\pi}{4} \right) c \partial c(\text{i}t) \rangle \]

\[ Q_B \Psi_{m}^{TT} + \Psi_{m}^{TT} * \Psi_{m}^{TT} = 0 \]
Fatten up TT solution

\[ \Psi = P_\alpha \times \frac{1}{1 + \Psi_{TT}^{TT} \times A(\alpha+\beta) \times \Psi_{TT}^{m} \times P_\beta} \]

- Formally satisfies the E.O.M.
- Nontrivial issue: “regular” solution?
\[
(\Psi_m^T \star A^{\alpha+\beta})^k \star \Psi_m^T = (\mathcal{B}_0 + \mathcal{B}_0^{\dagger}) \times \]

"smeared" currents

\[
\mathcal{J}(x) = -\int_{-\infty}^{\infty} \frac{dt}{2\pi} \left( \lambda_a f(t) J^a(x + it) + \frac{1}{2} g^{ab} \lambda_a \lambda_b f(t)^2 \right)
\]

\[
\mathcal{C}\mathcal{J}(x) = -\int_{-\infty}^{\infty} \frac{dt}{2\pi} c(x + it) \left( \lambda_a f(t) J^a(x + it) + \frac{1}{2} g^{ab} \lambda_a \lambda_b f(t)^2 \right)
\]
\[
\mathcal{C} \mathcal{I}(r) \mathcal{C} \mathcal{I}(0), \quad \mathcal{C} \mathcal{I}(r) \mathcal{I}(0), \quad \mathcal{I}(r) \mathcal{I}(0)
\]

\[
\Rightarrow \quad I_{p,q}(r) = 2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dt}{2\pi} \frac{ds}{2\pi} f(s)f(s-t)(is)^p(-it)^q 
\times \frac{1 - \cosh(2t) \cos \pi r + i \sinh(2t) \sin \pi r}{(\cosh(2t) - \cos \pi r)^2}
\]

contribution from small \(t\) is dominant

finite!
• A possible divergence comes from a region where two smeared currents collide.

• We showed that

\[ CJ(x)CJ(0), \quad CJ(x)\mathcal{I}(0), \quad \mathcal{I}(x)\mathcal{I}(0) \]

are finite at \( x \sim 0 \).

• This ensures that our solution has finite coefficients each order of the coupling \( \lambda \).

• But this is not a whole story.
• In our solution,
  \(( \lambda \text{ expansion } \neq \text{ mode expansion w.r.t. } \mathcal{L}_0 )\)

• For example, a coefficient of a “width \(x\)” contribution

\[\hat{U}_{1+x}\hat{c}_1 |0\rangle\]

includes sum of all \((c \ J \ J \ J \ J \ldots J)\) contractions so includes infinite powers of \(\lambda\)

• Evaluation of such coefficients is very complicated. But if evaluated, it would give “effective coupling” \(\beta(\lambda)\)
• This story looks quite similar to “renormalization of boundary state” [Callen-Klevbanov-Ludwig-Maldacena, Kogetsu-Teraguchi]

• Anyway, the validity of our solution should be further explored by

  • Explicit examples of some marginal currents

  • Estimating infinite sum

  • Evaluating classical action (it should vanish)