Constructing Non-Abelian Vortices with Arbitrary Gauge Groups

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Recently there has been a significant progress in the understanding of non-Abelian vortices in $SU(N) \times U(1)$ gauge theories. Although many interesting features have been extensively explored, most studies have been restricted to the gauge group $SU(N) \times U(1)$. We proposed a simple framework for writing the most general non-Abelian BPS vortex solutions in theories with an arbitrary gauge group of the type $G = G' \times U(1)$ [1]. Our model is a $(3+1)$-dimensional $\mathcal{N} = 2$ supersymmetric gauge theory coupled to $N_{\rm F}$ Higgs fields H in a representation R of a rank r simple Lie group G' and charged under $U(1)$. If the VEV of the Higgs fields completely breaks the gauge symmetry, the topological charges of the vortex configurations are classified by $\pi_1((U(1) \times G')/Z)$, where Z is the center of G'. The boundary condition for the Higgs field is $H \to e^{i\alpha(\theta)} g(\theta) \langle H \rangle,\ e^{i\alpha} \in U(1),\ g \in G'$, where $\langle H \rangle$ is a VEV of the Higgs fields and θ is the angular coordinate parameterizing large S^1 at spatial infinity. In general, the elements of the gauge groups $e^{i\alpha}$ and g can be written as $e^{i\alpha} g = \exp[i(k/n_0 + \vec{\nu} \cdot \vec{H})\theta]$, where k is the vortex number, n_0 is the order of the center Z and $\vec{H}$ is an r-vector of the generators of the Cartan subalgebra of G'. The r-vector $\vec{\nu}$ should satisfy the condition $k/n_0 + \vec{\nu} \cdot \vec{\mu} \in \mathbb{Z}$ with $\vec{\mu}$ being the weight vectors of the representation R. That is, $\vec{\nu}$ should be an element of the coweight lattice, which is identified with the weight lattice of the GNO (or Langlands) dual group ${}^L G'$.

The BPS equations for the gauge fields and the Higgs fields H can be rewritten in terms of new variables $S_e(z, \bar{z})$, $S'(z, \bar{z})$ and $H_0(z)$, where S_e and S' are elements of the complexified gauge groups $U(1)^{\mathbb{C}}$ and $G'^{\mathbb{C}}$ respectively and H_0 is related to H as $H_0 = S_e S' H$. From one of those equations, we find that H_0 should be holomorphic in the complex coordinate z parameterizing the plane perpendicular to the vortex string. Once $H_0(z)$ is given, the other equations are specified and S_e and S' can be determined by solving those equations. Then, the BPS solution can be obtained by the inverse transformation to the original fields. The complexified transformations $V_e(z) \in U(1)^{\mathbb{C}}$ and $V'(z) \in G'^{\mathbb{C}}$ which act on S_e, S', $H_0(z)$ do not change the original fields. Therefore, sufficient and necessary information for specifying the BPS vortex configuration is the equivalence class of $H_0(z)$ and the parameters contained in $H_0(z)$ parameterize the moduli space of vortices. Our method gives a powerful tool to study the moduli space of vortices.

References

[1] M. Eto, T. Fujimori, S. B. Gudnason, K. Konishi, M. Nitta, K. Ohashi and W. Vinci, arXiv:0802.1020 [hep-th].