Negative modes of Schwarzschild black hole in Einstein-Gauss-Bonnet theory

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This contribution is based on my recent paper with the same title [1]. I study non conformal negative modes of the black holes in the five dimensional Einstein-Gauss-Bonnet gravity theory with a negative cosmological constant Λ. The action has the Gauss-Bonnet term with the coefficient c in addition to the Einstein-Hilbert term. The black hole solution [2] (which smoothly becomes Schwarzschild black hole in $c \to 0$ limit) is characterized by the horizon radius r_h. I study time independent s-mode metric perturbations around the black hole metric, and whether the value of action increases or decreases. The perturbations should be normalizable modes otherwise those perturbations do not contribute to the path integral. This requirement gives a boundary condition at the horizon and the radius infinity. If I find normalizable negative mode(s), the action decreases and the black hole is quantum mechanically unstable. (Notice that negative mode(s) do not necessary indicate a classical instability.)

Here I only show the case with $(\Lambda, c) = (-1, 0.01)$ since in this case something interesting behavior happens. From the black hole thermodynamics [3], the heat capacity is positive for $r_h > 1.71$ and $0 < r_h < 0.143$, is negative for $0.143 < r_h < 1.71$. I found that there is no negative mode(s) in $r_h > 1.71$ and $r_h < 0.1$, there is one negative mode in $0.143 < r_h < 1.71$, and two negative modes in $0.1 < r_h < 0.143$. Therefore except for $0.1 < r_h < 0.143$, the thermodynamic stability and the stability against small perturbations agree with each other. In fact, in Einstein theory with matter fields for all the known black holes, which are uniquely determined by the conserved charges, these two stabilities agree. However I found for $0.1 < r_h < 0.143$ the black hole has the thermodynamic stability, but shows an instability against small perturbations. This indicates that the black hole thermodynamics does not capture the full black hole dynamics in quantum gravity. It would be interesting to develop more knowledge in this direction.

References

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