

# Theoretical aspects of unparticle physics

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We review some selected topics on theoretical aspects of unparticle physics. The main emphasis is on the distinction between conformal invariance and scale invariance, SUSY unparticle with conformal sequestering, and applications of AdS/CFT to unparticle physics.

## 1 Introduction

The concept of “unparticle” was introduced by H. Georgi [1] and it has attracted a lot of attention in the last years. The unparticle can be most contrasted with ordinary particles; we all know that the concept of the particle has played a fundamental role in high energy theoretical physics. Indeed, if you take a look at the famous textbook on the quantum field theory by S. Weinberg [2], you see that relativistic quantum mechanics for particles is the basis of the quantum field theory: a consistent realization of particle interaction is only possible by quantum field theories.

However, Hilbert space of quantum field theories can be much larger than ordinary particle states, and one particular example is the state realized in scale invariant (or probably conformally invariant) field theories. H. Georgi proposed a possibility that such a state might exist in our real nature, and he named it “unparticle”. In his definition, unparticle is “(approximate) scale invariant field theory that weakly couples with the standard model sector”. In this proceeding, we review some theoretical aspects (and constraints) of unparticle physics, mainly focusing on the debate over scale invariance and conformal invariance. We also discuss applications of AdS-CFT correspondence to unparticle physics.

The rest is organized in the following way. In section 2, we briefly summarize the properties of unparticle, and in section 3, we discuss theoretical constraints on unparticle. In section 4, we study AdS-unparticle correspondence, and we conclude in section 5. I would like to thank C. M. Ho for collaboration and fruitful discussion about the contents presented in section 4.

## 2 Properties of Unparticle

Unparticle defined by “(approximate) scale invariant field theory that weakly couples with the standard model sector” has very peculiar properties compared with ordinary particles. The most important properties of the unparticle is, of course, its scale invariance.<sup>1</sup> The scale invariance should be imposed around the electro-weak energy scale  $E_{EW}$ , where we hope to find new physics

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<sup>1</sup>The distinction between scale invariance and conformal invariance is actually very important, and it would lead to a crucial physical consequence. We come back to this point later, but for a time being we do not make a clear distinction in this section.

in near-future experiments. The approximate scale invariance means that the scale invariance might (or might not) be broken at much higher (or lower) energy scale than we would like to observe the unparticle.

For a simple example of unparticle sector, H. Georgi considered Bank-Zaks type conformal field theory [3], which is defined by QCD with many massless fundamental fermions. Below the dynamical scale  $\Lambda_U$  of QCD, the theory is approximately conformal. If we introduced the mass for the fermions, the CFT would be broken at lower energy than the mass scale  $\Lambda_\psi$ . The approximate scale invariance demands the inequality  $\Lambda_\psi \ll E_{EW} \ll \Lambda_U$ . Another important scale in the unparticle physics is the UV scale  $M_U$  at which an unparticle operator  $O_{UV}$  couples with a standard model (SM) operator  $O_{SM}$  as  $\frac{O_{UV}O_{UV}}{M_U^k}$ . Below the conformal scale  $\Lambda_{CFT}$ , it becomes the effective coupling between the scale invariant field theory and the SM sector as  $\frac{c\Lambda_U^{d_{UV}-d_U}}{M_U^k}O_{SM}O_U$ , where  $d_{UV}$  and  $d_U$  are scaling dimension of unparticle operator at UV and at scale invariant fixed point respectively.

We assume that the coupling between the unparticle sector and the standard model is weak (typically non-renormalizable) so that the corrections to the standard model amplitude may be computed perturbatively: at the second order, we obtain

$$\begin{aligned} \delta T &= \langle SM \text{ in} | SM \text{ out} \rangle \\ &\sim \int d^4x d^4y \langle SM \text{ in} | O_{SM}^\dagger(x) O_{SM}(y) | SM \text{ out} \rangle_{SM} \langle 0 | O_U^\dagger(x) O_U(y) | 0 \rangle_h, \end{aligned} \quad (1)$$

where we have introduced the so-called unparticle propagator:

$$\langle 0 | O_U^\dagger(x) O_U(y) | 0 \rangle_h = \frac{C}{(x-y)^{2d}}. \quad (2)$$

The scale invariance determines the shape of the (scalar) two-point function up to a normalization factor  $C$  and scaling dimension  $d$ .<sup>2</sup>

To further investigate the unparticle propagator, we perform the spectral decomposition:

$$\langle 0 | O_U^\dagger(x) O_U(0) | 0 \rangle_h = \int dp e^{-ipx} |\langle 0 | O_U(0) | p \rangle|^2 \rho(P^2) \frac{dp^2}{(2\pi)^4}, \quad (3)$$

where the unparticle density of states is fixed by the scale invariance as

$$|\langle 0 | O_U(0) | p \rangle|^2 \rho(p^2) = A_d \theta(p^2) (p^2)^{d-2}. \quad (4)$$

It was noted in [1] that the density of states can be related to the formal analytic continuation ( $n \rightarrow d$ ) of the  $n$ -particle phase space. It is also convenient to introduce the momentum space

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<sup>2</sup>For higher spin two-point function, the scale invariance is not sufficient to fix the tensor structure while the conformal invariance does fix it.

propagator

$$\Delta_d(p) = \frac{iA_d}{2 \sin \pi d} (-p^2)^{d-2} . \quad (5)$$

Note that if  $p^2 > 0$ , there is a peculiar non-trivial phase in the propagator:  $|p|^{d-2} e^{-id\pi}$ . This imaginary part in the T-matrix is directly related to the unparticle emission from the optical theorem.

The unparticle propagator and its peculiar imaginary part is the basis of the unparticle phenomenology. Although there are many sources of experimental bounds e.g. constraints from flavor changing process or CP violation, we would like to focus on more theoretical aspects of constraints on unparticle physics, which we will discuss in the next section.

### 3 Theoretical Constraint on Unparticle

#### 3.1 Unitarity from Conformal Algebra

Scaling dimension of unparticle operators is very important because when  $d$  is large, the interaction may be too weak to be observed in nature. However, if you assume the conformal invariance in the unparticle sector, there is a severe unitarity bound for the scaling (=conformal) dimension of primary operators [4]:

$$d \geq j_1 + j_2 + 2 - \delta_{j_1 j_2, 0} , \quad (6)$$

where  $j_1$  and  $j_2$  are Lorentz spin of the operator. As first pointed out in [5] by the present author (see also [6]), this unitarity bound is neglected by many authors in the study of vector unparticles (including Georgi's original work).

One loophole, which they state as a refute, is that the unparticle physics is based not on conformal field theories, but only on merely scale invariant field theories. From the group theoretical viewpoint, one may say that since the scale invariance is just a subgroup of the conformal group, the conformal invariance is an extra huge assumption.

On the contrary to this naive group theoretical expectation, there is actually no known examples of scale invariant but non-conformal (unitary) field theories in four-dimension (actually the space-time dimension greater than two). If one dares to break the unitarity, there are some examples, however. The simplest one is a massless vector field theory with the most general kinetic term

$$L = \alpha \partial_\mu S^\nu \partial_\mu S^\nu + \beta (\partial_\mu S^\mu)^2 . \quad (7)$$

In general, this model breaks both gauge invariance (unitarity) and conformal invariance. It is also rather trivial to see that the model possesses the scale invariance. Thus, this is an (non-unitary) example of scale invariant but non-conformal field theories. It would be interesting

to note that when the Lagrangian precisely coincides with the Maxwell theory, the conformal symmetry and the gauge symmetry are both recovered. Another class of non-trivially interacting (but again non-unitary) field theory that breaks conformal invariance while preserving the scale invariance has been proposed in [7].

In two-dimension, the proof of the equivalence between the scale invariant field theory and the conformal field theory was given in [8]. The basic argument is because of the huge symmetry in two-dimension, when the scale invariance existst, it is always possible to improve the energy-momentum tensor so that the trace vanishes.<sup>3</sup>

In four-dimension, there is no proof or counterexample, but given the circumstances, it would be very surprising if one finds the vector unparticle whose scaling dimension violates the conformal unitarity bound: it would suggest the field theory no one has come up with yet!

### 3.2 CFT breaking

Another constraint on the unparticle coupling comes from the breaking of the scale invariance. As discussed before, the scale invariance should be intact at least around the electro-weak energy scale if we would like to observe unparticles in near future experiments. However, the coupling to the Higgs field is the most dangerous source in this scenario [9, 10]. Recall that the lowest dimensional gauge invariant operator in the standard model is the Higgs mass term  $\phi^\dagger\phi$ . If it couples with the unparticle operator as  $\phi^\dagger\phi O_u$ , it should be the leading order coupling between the SM sector and unparticle sector. The problem here is that Higgs will develop VEV  $\langle\phi\rangle = v$  and it generates the tadpole term for the unparticle operator  $O_u$  as  $|v|^2 O_u$ .

This tadpole term eventually leads to the breaking of the scale invariance of the unparticle sector as a back-reaction. The breaking scale can be estimated [9] as

$$\Lambda_\psi = \left[ \left( \frac{v}{\Lambda_U} \right)^2 \left( \frac{\Lambda_U}{M_U} \right)^{d_{UV}-2} \right]^{\frac{1}{4-d}} \Lambda_U . \quad (8)$$

If this breaking scale is around or above TeV, the unparticle scenario will be spoiled completely. An extra symmetry might be needed to suppress this coupling.

Of course, they are still unparticles above  $\Lambda_\psi$ , but the motivation to find them at near future experiments are completely lost. Similarly, a kind of conformal SUSY breaking scenario predicts unparticles at much higher scale (around the messenger scale), but again there would be no direct effects on collider experiments. We would like to emphasize, however, the energy scale is actually not so important in the study of theoretical nature of unparticles, and the physics of unparticles is relevant for the precise understanding of the conformal SUSY breaking scenario,

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<sup>3</sup>We recall that the scale invariant field theory is characterized by the energy-momentum tensor  $T^\mu_\mu = \partial_\mu J^\mu$ . When the Virial current  $J^\mu$  is a derivative of something  $J^\mu = \partial_\nu A^{\mu\nu}$ , one can improve the energy-momentum tensor so that it becomes traceless.

(see e.g. [11]). Other CFT related ideas to solve hierarchy problem (or any problem you can name) should be benefited from such a theoretical study of unparticle physics.

### 3.3 SUSY unparticle

There are at least two good reasons to introduce supersymmetry (SUSY) in the unparticle physics. First of all, if we would like to find SUSY in nature — whose benefit seems enormous e.g. as a solution to the hierarchy problem, the grand unification, dark matter to name a few — the unparticle sector should be supersymmetrized as well. Furthermore, even purely theoretically, the introduction of SUSY in the unparticle sector enables us to compute some crucial quantities in the unparticle physics, like conformal dimension, exactly.

The supersymmetric extension of the Banks-Zaks theory is given by the supersymmetric QCD (SQCD) with  $SU(N_c)$  gauge group coupled with  $N_f$  fundamental matter multiplets (denoted by  $Q_i$ ). The theory is conformal invariant when  $\frac{3}{2}N_c < N_f < 3N_c$ .

The conformal dimension of various gauge invariant scalar operators can be computed by using the supersymmetric field theory technique:

$$\begin{aligned} d(Q_i \bar{Q}_j) &= 3 \frac{N_c - N_f}{N_f} < 2 \\ d(Q_i^\dagger Q_j) &= 2 \\ d(Q_i^\dagger Q_i + \bar{Q}_i^\dagger \bar{Q}_i) &= 2 + \beta'(\alpha_*) > 2, \end{aligned} \tag{9}$$

where  $\beta'(\alpha_*)$  is the slope of the beta function at the conformal fixed point.

However, the SUSY is broken in nature, and the SUSY breaking effects will introduce soft mass to these superconformal unparticle sector just as we acquire soft mass for SUSY partners in the SM sector. The amount crucially depends on how the SUSY breaking will be mediated to the SM sector and the hidden unparticle sector. For example, when the SUSY breaking is mediated to the SM sector by gravity mediation. The gravitino mass is around 1 TeV, and the SUSY breaking mass effects to unparticle sector is of the same order if the gravity mediation also occurs in the unparticle sector (thus its observability is marginally difficult). If the gauge mediation occurs in the unparticle sector, the soft mass will be much larger and the unparticle is excluded, while anomaly mediation to the unparticle sector is idealistic situation (see table 1 and [5] for other scenarios).

As can be seen in the table, the anomaly mediation to the unparticle sector is the most promising way to realize the unparticle scenario in SUSY context. Usually, the problem of the anomaly mediation is how to suppress the apparent gravity mediation effect, and the one way to do is to use the conformal dynamics of the hidden sector (conformal sequestering [12]). Actually, it is rather easy to incorporate the effect of conformal sequestering in the unparticle scenario because the unparticle also depends on the conformal dynamics. See [5] for detailed construction

SM, Hidden	Gravity	Gauge	Anomaly
Gravity	?	NG	OK
Gauge	OK	?	OK
Anomaly	NG	NG	?

Table 1: The possibility of SUSY unparticle. OK combinations are promising theories where CFT breaking is far below the electro-weak scale. ? means they have around the same energy scale. The NG theories do not give rise to unparticles at electro-weak scale.

of conformal sequestered unparticle in order to avoid the conformal breaking in the unparticle sector due to the SUSY breaking.

### 3.4 Contact terms

Unparticle interaction  $\frac{O_{SM}O_{UV}}{M_U^k}$  might not be the dominant contribution to the standard model process in new physics. In particular, the contact term interaction  $\frac{O_{SM}^2}{M_U^{k'}}$  introduced at the same UV scale  $M_U$  could be the dominant piece. Indeed, in [6], they have shown that such an interaction should result from the renormalization group flow of the unparticle operators. Denoting the new interaction as  $C_2 \frac{O_{SM}O_{UV}}{M_U^k} + C_1 \frac{O_{SM}^2}{M_U^{k'}}$ , they have shown that the Callan-Symanzik equation gives

$$\left( \frac{\partial}{\partial \log \mu} + \beta(g) \frac{\partial}{\partial g} \right) C_i = \gamma_{ij} C_j, \quad (10)$$

where  $\gamma_{ij}$  are anomalous dimension matrix.

The solution of the renormalization group equation can be obtained as

$$C_1 = C' + C'' \frac{\gamma_{12}(g_*)}{\gamma_{11}(g_*)} \left( \left( \frac{\mu}{\Lambda} \right)^{\gamma_{11}(g_*)} - 1 \right). \quad (11)$$

Later in section 4, we reproduce this result from AdS-CFT correspondence.

Note that as discussed in [6], the ratio between the contribution from the contact term and the unparticle exchange can be computed as

$$\frac{A_{un}}{A_{con}} = \frac{C_2^2}{C_1} \left( \frac{E}{M_U} \right) \left( \frac{E}{\Lambda_U} \right)^{2(d-3)}. \quad (12)$$

In particular, for vector unparticle with dimension  $d > 3$  (which comes from unitarity. See section 3.1), the unparticle exchange is naturally suppressed.

## 4 AdS-Unparticle

One interesting theoretical approach to unparticle physics is to use AdS-CFT correspondence [13, 14, 15, 16].<sup>4</sup> The basic statement of the AdS-CFT correspondence is that a strongly coupled conformal field theory can be analysed by a weakly coupled gravitational theory on AdS space. Although there is no known way to represent the gravity dual for SQCD (or Banks-Zaks theory), many other non-trivial SCFTs can be analysed from gravity.

It is rather trivial to see that the both theories possess the same symmetry: on the CFT side, we have conformal  $SO(2, 4)$  symmetry while the AdS space has an isometry given by  $SO(2, 4)$ . In particular, under this correspondence, the AdS global energy (Hamiltonian) corresponds to the conformal dimension of CFT operators. In the following, we mainly consider the AdS space in the Poincare coordinate

$$ds^2 = \frac{dz^2 + dx^\mu dx_\mu}{z^2}, \quad (13)$$

where the radial direction  $z$  corresponds to the energy scale of the conformal field theory.

In addition to this kinematical correspondence, AdS/CFT predicts a dynamical relation (known as GKPW relation [17][18]) between the generating functions of the CFT correlation functions and the path integral for the gravitational theory with fixed boundary condition:

$$Z_{AdS}[A_{0,n}] = \int_{A_n=A_{0,n}|_{bound}} \mathcal{D}A_\mu \exp(-I[A_\mu]) \equiv Z_{CFT}[A_{0,n}] = \langle \exp(\int d^4x J_n A_{0,n}) \rangle, \quad (14)$$

where  $A_n$  is 5-dimensional vector field and  $J_n$  is the corresponding source current in the CFT. Later, we will use this relation to compute the unparticle propagator.

The unparticle hidden sector is not an idealistic CFT, however. At least we need (non-conformal) coupling between the hidden sector and the SM sector. We may also want to introduce IR cut-off (or relevant deformation) below the electro-weak scale. In the AdS-CFT language, this field theory cutoff can be understood as a modification of the geometry at UV (or IR). We can introduce UV brane at  $z = z_{UV} = \frac{1}{M_U}$  to mimic the coupling to the SM sector. The IR cut-off can be also introduced by capping off the geometry at  $z = z_{IR} = \frac{1}{\Lambda_\psi}$ . The construction is much like Randall-Sundrum scenario; and it is known as ‘‘unparticle deconstruction’’ [13].

Here, we would like to show the appearance of the unitarity bound and the renormalization group generation of the contact terms from AdS-CFT viewpoint. The action for the 5-

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<sup>4</sup>In this section, we would like to assume conformal invariance rather than mere scale invariance. The geometric description with only scale invariance is an interesting direction but it is not well understood. Maybe there is a geometrical way to prove or disprove the equivalence between conformal invariance and scale invariance in higher dimension.

dimensional massive vector (5d Proca action) is given by

$$I = \int d^5x \sqrt{g} \left( \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \frac{m^2}{2} A_\mu A^\mu \right), \quad (15)$$

where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ .

The action can be evaluated [19] by the boundary data  $\tilde{A}_{\epsilon,n}(k)$  at  $z = \epsilon$  (Fourier transformed in the brane directions) as

$$\begin{aligned} I = & \frac{1}{2} \left( \tilde{\alpha} + 1 - \frac{d}{2} \right) \int \frac{d^4k}{(2\pi)^4} \epsilon^{-d} \tilde{A}_{\epsilon,n} \tilde{A}_{\epsilon,n} \\ & + \frac{\Gamma(\tilde{\alpha} - 1)}{\Gamma(\tilde{\alpha})} \int \frac{d^4k}{(2\pi)^4} \epsilon^{2-d} \tilde{A}_{\epsilon,n} \left( -\frac{1}{2} \delta_{mn} k^2 + k_n k_m \right) \tilde{A}_{\epsilon,m} \\ & - \left( \frac{\epsilon}{2} \right)^{2\tilde{\alpha}} \frac{\Gamma(1 - \tilde{\alpha})}{\Gamma(\tilde{\alpha})} \int \frac{d^4k}{(2\pi)^4} \epsilon^{-d} \tilde{A}_{\epsilon,n} \left( -k^{2\tilde{\alpha}} \delta_{mn} + \frac{2\tilde{\alpha}}{\tilde{\alpha} + 1} k^{2\tilde{\alpha}-2} k_n k_m \right) \tilde{A}_{\epsilon,m} + \dots, \quad (16) \end{aligned}$$

where higher derivative terms with higher order  $\epsilon$  is neglected. For later purposes, however, we have incorporated the contact terms neglected in [19].

The mass  $m$  in the 5d-bulk space is related to the slope  $\tilde{\alpha}$  and the conformal dimension  $d$  of the dual operator as  $\tilde{\alpha} = d - 2 = \sqrt{1 + m^2}$ . Generalizing the discussion in [18], one can easily see that for the vector particle, the stability bound is  $m^2 \geq 0$ , corresponding to the unitarity bound for the vector unparticle  $d \geq 3$ .

The third line in (16), which is in general non-analytic, will reproduce the CFT two-point function from the AdS-CFT prescription [17][18]

$$Z_{AdS}[A_{0,n}] = \int_{A_n=A_{0,n}|_{bound}} \mathcal{D}A_\mu \exp(-I[A_\mu]) \equiv Z_{CFT}[A_{0,n}] = \langle \exp\left(\int d^4x J_n A_{0,n}\right) \rangle \quad (17)$$

with suitable analytic continuation in Fourier integral. Here, we specify the boundary data  $\tilde{A}_{0,n} = \lim_{\epsilon \rightarrow 0} \tilde{A}_{\epsilon,n}$  by the normalized field  $\tilde{\mathcal{A}}_{\epsilon,n} \equiv \epsilon^{\tilde{\alpha}-2} \tilde{A}_{\epsilon,n}$

In contrast, the first line and the second line in (16) are not dictated by the conformal invariance but they give contact terms [16]. To obtain the conformal invariance, at a given  $\epsilon$ , one can always eliminate such contact terms by adding the boundary counter terms as

$$\begin{aligned} \delta S_{bound} = & \int \frac{d^4k}{(2\pi)^4} C_0 \tilde{\mathcal{A}}_{\epsilon,n} \tilde{\mathcal{A}}_{\epsilon,n} + \tilde{\mathcal{A}}_{\epsilon,m} (C_1 k^2 \delta_{mn} + C_2 k_m k_n) \tilde{\mathcal{A}}_{\epsilon,n} + \dots \\ = & \epsilon^{4-2\tilde{\alpha}} \int \frac{d^4k}{(2\pi)^4} C_0 \tilde{\mathcal{A}}_{\epsilon,n} \tilde{\mathcal{A}}_{\epsilon,n} + \tilde{\mathcal{A}}_{\epsilon,m} (C_1 k^2 \delta_{mn} + C_2 k_m k_n) \tilde{\mathcal{A}}_{\epsilon,n} + \dots, \quad (18) \end{aligned}$$

which are localized on the UV-brane.



We find that it is natural to introduce the cut-off dependence on the boundary counter term by parameterizing  $C_i = \tilde{C}_i \epsilon^{4-2d_0}$ , where we have introduced the “native dimension”  $d_0$  of the current operator under consideration. With different cut-off, we have the relation [16]:

$$\begin{aligned} C_0(\epsilon) &= \tilde{C}_0 \epsilon^{4-2d_0} \left( 1 - \left( \frac{\tilde{\epsilon}_0}{\epsilon} \right)^\gamma \right) \\ C_1(\epsilon) &= \tilde{C}_1 \epsilon^{6-2d_0} \left( 1 - \left( \frac{\tilde{\epsilon}_1}{\epsilon} \right)^\gamma \right) \\ C_2(\epsilon) &= \tilde{C}_2 \epsilon^{6-2d_0} \left( 1 - \left( \frac{\tilde{\epsilon}_2}{\epsilon} \right)^\gamma \right) + \dots, \end{aligned} \tag{19}$$

where we have introduced the anomalous dimension  $\gamma = 2(d - d_0)$ .  $\epsilon_i$  denotes the scale where the boundary counter term cancels the bulk contribution, and they can be different for different  $i$  in principle. It is easy to see that they reproduce the Callan-Symanzik equation [6] reviewed in section 4.4.<sup>5</sup> In this way, we have shown how AdS-CFT correspondence also predicts the appearance of the contact terms and their evolution.

## 5 Conclusion

In this proceeding, I have reviewed theoretical aspects of unparticle physics. In more phenomenological perspective, the central question would be whether we can find it at LHC. I do not have a good answer since I am not a prophet. Besides, the unparticle does not seem to have as good motivation as, say, SUSY. If, however, it were discovered at LHC, it would be very surprising because, as we saw, there are many theoretical constraints on the unparticle sector. Why is the coupling between Higgs and unparticle suppressed? Why is the contact interaction suppressed? What about the conformal unitarity bound?

When muon was first discovered in nature, a famous theoretical physicist exclaimed “who ordered muon?”. The necessity of the muon might lie in cosmology: e.g. the origin of the baryon asymmetry could be due to the existence of three generations. Similarly, one might find a deeper reason of the unparticle, if any, once we observe it in nature and think hard about its *raison d’être*.

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<sup>5</sup>Note that unlike the claim in [6],  $C_0$  and  $C_1$  are not a-priori related while we could always relate them as a boundary condition at the cut-off. This is due to the fact that they implicitly assumed the simplest weakly coupled messengers that propagate between the unparticle sector and the SM sector. In more general strongly coupled mediation, the condition will be generically violated.

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