Extended Supersymmetric σ-Model Based on the SO(2N+1) Lie Algebra of the Fermion Operators -Optimization of a Reduced Scalar Potential and a Proper Solution-

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Extended supersymmetric σ-model is given standing on the $SO(2N+1)$ Lie algebra of fermion operators composed of annihilation-creation operators and pair operators. Canonical transformation, the extension of the $SO(2N)$ Bogoliubov transformation to the $SO(2N+1)$ group, is introduced. Embedding the $SO(2N+1)$ group into an $SO(2N+2)$ group and using $\frac{SO(2N+2)}{U(N+1)}$ coset variables, we investigate a new aspect of the supersymmetric σ-model on the Kähler manifold of the symmetric space $\frac{SO(2N+2)}{U(N+1)}$. We construct a Killing potential which is just the extension of the Killing potential in the $\frac{SO(2N)}{U(N)}$ coset space given by van Holten et al. to that in the $\frac{SO(2N+2)}{U(N+1)}$ coset space. The Killing potential is equivalent with the generalized density matrix. Its diagonal-block matrix is related to a reduced scalar potential with a Fayet-Ilipoulos term. The reduced scalar potential is optimized in order to see the behaviour of the vacuum expectation value of the σ-model fields and a proper solution for one of the $SO(2N+1)$ group parameters is obtained. (For details see Ref. [1]).

The above simple solution and the proper solution, however, are only an extremely little part of whole solutions which should be obtained from anomaly-free supersymmetric coset models. As already shown by van Holten et al. [2], if we construct some quantum field theories based on pure coset models, we meet with a serious problem of anomalies in a holonomy group which particularly occur in pure supersymmetric coset models due to their chiral fermions [3]. This is also the cases for our orthogonal cosets, $\frac{SO(2N)}{U(N)}$ and its extended $\frac{SO(2N+2)}{U(N+1)}$, though each spinor representation of $SO(2N)$ group and its extended $SO(2N+2)$ group is anomaly free. A compact form of the anomaly cancellation condition was first given by Georgi and Glashow [4]. To construct the **consistent** supersymmetric coset models, we must embed each coset coordinate in each anomaly-free spinor representation of $SO(2N)$ group and $SO(2N+2)$ group and give a corresponding Kähler potential and then a Killing potential for each anomaly-free $\frac{SO(2N)}{U(N)}$ and its extended $\frac{SO(2N+2)}{U(N+1)}$ model based on each positive chiral spinor representation. To achieve such a subject on the case of the $SO(2N)$ group/algebra, van Holten et al. have proposed a method of constructing the Kähler potential and then the Killing potential. This idea is considered to be very suggestive and useful for our present purpose of constructing the corresponding Kähler potential and then the Killing potential for the case of the $SO(2N+2)$ group/algebra.

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