Y-Formalism and Curved Beta-Gamma Systems

Department of Physics, Faculty of Science, University of the Ryukyus,
Nishihara, Okinawa 903-0213, JAPAN
Ichiro Oda
E-mail: ioda@phys.u-ryukyu.ac.jp

In this talk, we explained the Y-formalism to study $\beta - \gamma$ systems on hypersurfaces. We computed the operator product expansions of gauge-invariant currents and we discussed some applications of the Y-formalism to model on Calabi-Yau spaces.

The technique of the Y-formalism developed in a series of our papers has been used to derive the complete operator products for the composite operat ors involved in the pure spinor string theory approach. The Y-formalism is very useful to compute the contact terms and the anomaly terms in the OPE's. It is based on the observation that, in general to derive those terms only the local structure of the theory is needed and not the globa l information on the space.

Curved $\beta-\gamma$ systems are an example of systems whose non-trivial information is encoded into an algebraic curve on a hypersurface. One can regard these systems as free field systems excepting that the fields are constrained to live on a hypersurface. The analysis of the $\beta-\gamma$ systems on hypersurfaces has been discussed in some mathematical papers until now. These constructions already encode some of the conformal field theory analysis for $\beta - \gamma$ systems. However, they never explicitly compute the CFT algebra in the presence of hypersurface constraints except some simple cases when the constraints are solvable in a simple way.

The aim of our study is to develop the Y-formalism for computation of generic $\beta-\gamma$ systems. Thus the main issue of this talk is the quantization of a system in the presence of constraints. This is a well-known problem in quantum field theory and it has been discussed in the literature since the advent of quantum mechanics and quantum field theory. Nonetheless, in the case of 2d field theories such as string theories and sigma models, one has the advantage of performing the computations in an explicit and exact way using the radial quantization technique and using the conformal field theory methods. In this regard, one would like to maintain such a strong feature even in the presence of constraints. Therefore, one has to treat the constraints in a radically different way by imposing them at each step of computation without actually solving them.

This procedure is encoded in the Y-formalism which gives a systematical way to compute all possible OPE's among different CFT operators , in particular among gauge-invariant currents (which are not sensible to the details of the "gauge-fixing" procedure).

Though the formalism in the present form works only for quadratic or partly quadratic constraints, there is a vast number of applicatio ns which can be done even by using the present status of the formalism. The detail of the contents of the present talk can be found in the paper arXiv:0808.1394 [hep-th].