Hierarchy of quantum mechanical supersymmetries and its application to compactified field theory[1]

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Quantum mechanical supersymmetry (SUSY) plays an essential role to generate massive Kaluza-Klein particles in higher-dimensional field theory. It has been shown that in 5d gravity two $\mathcal{N}=2$ quantum mechanical SUSYs are needed in order for the "Higgs" mechanism to generate massive spin-2 particles; one of the two quantum mechanical SUSYs ensures the degeneracy between spin-2 and spin-1 excitations and the other between spin-1 and spin-0 excitations. A crucial ingredient of this coexistence of two quantum mechanical SUSYs is the refactorization of Hamiltonians, which can be realized if and only if the background metric satisfies the Einstein equation. Thus it would be natural to guess that in a higher-dimensional spin-N field theory there would exist a level N hierarchical SUSYs in the 4d mass spectrum, whose typical structure must be as follows:

$$
\begin{array}{lllll}
H_0 & = Q_0^\dagger Q_0 + c_0 & & & \\
H_1 & = Q_0 Q_0^\dagger + c_0 & = Q_1^\dagger Q_1 + c_0 + c_1 & & \\
H_2 & & = Q_1 Q_1^\dagger + c_0 + c_1 & = Q_2^\dagger Q_2 + c_0 + c_1 + c_2 & \\
H_3 & & & = Q_2 Q_2^\dagger + c_0 + c_1 + c_2 & \\
\vdots & & & \vdots &
\end{array}
$$

where the n^{th} supercharge and its adjoint are assumed to be of the forms $Q_n = \mathrm{d}/\mathrm{d}x + W_n'(x)$, $Q_n^\dagger = -\mathrm{d}/\mathrm{d}x + W_n'(x)$ and c_n is a real constant. In the context of higher-dimensional field theory, W_n is proportional to the warp factor and c_n corresponds to the cosmological constant on 3-branes.

Motivated by this observation, we classify all the possible boundary conditions consistent with the hermiticity of each Hamiltonian in the above hierarchy in the framework of one-dimensional supersymmetric quantum mechanics on a bounded domain. We show that it is not possible to construct a quantum mechanical system with finite superpotentials on an interval beyond the level 3 hierarchical SUSYs, which suggests some no-go theorem of the "Higgs" mechanism for spin-$N(\geq 3)$ particles in the context of five-dimensional field theory with single extra dimension compactified on an interval. But this is an open question. Further studies are needed.

[1]This presentation is based on the work with T. Nagasawa, K. Sakamoto, M. Sakamoto and K. Sekiya.