ON THE DECONFINING LIMIT IN (2+1)-DIMENSIONAL YANG-MILLS THEORY

YASUHIRO ABE

Cereja Technology Co., Ltd.

Kyoto, July 28, 2008

Based on ArXiv:0804.3125v2 (Y.A.)

INTRODUCTION

1. DEVELOPMENTS OF (2+1)-DIM YANG-MILLS THEORY IN A HAMILTONIAN APPROACH INITIATED BY KARABALI AND NAIR (OVER 10 YEARS)

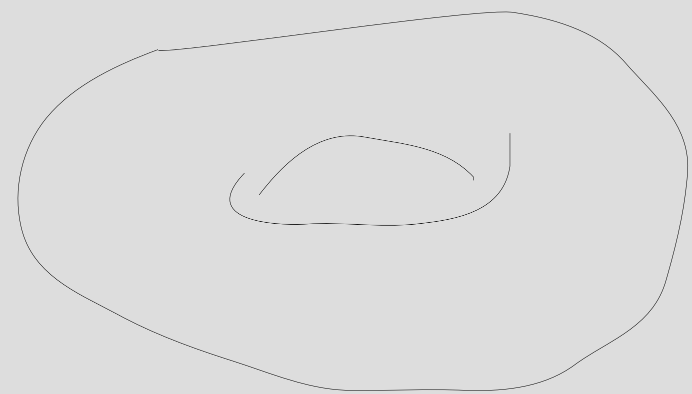
2. WANT TO SAY SOMETHING ABOUT DECONFINING LIMIT IN THE HAMILTONIAN APPROACH

INTRODUCTION

3. CONSIDER THE EUCLIDEAN THOERY ON

$$S^1 \times S^1 \times R$$

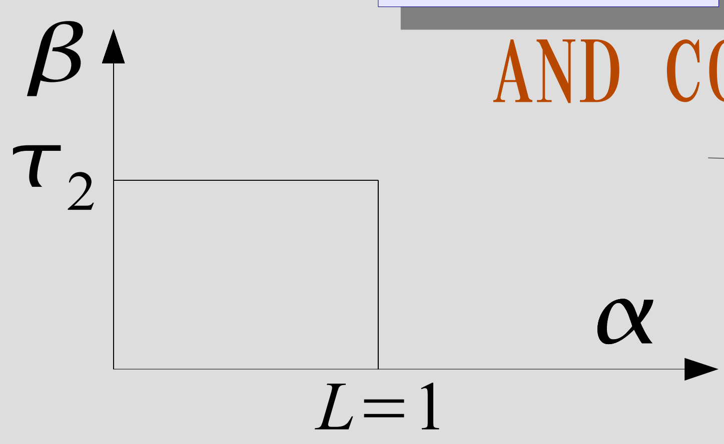
REGARD AS TIME DIRECTION

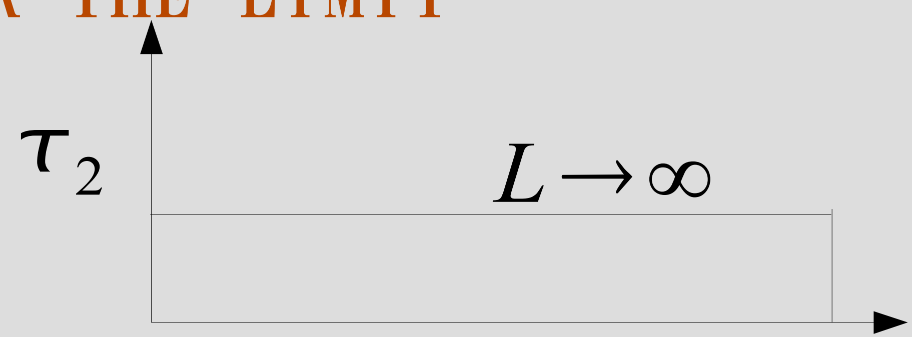


CHOOSE MODULAR PARAMETER

$$\tau = i\tau_2$$

AND CONSIDER THE LIMIT





INTRODUCTION

4. CALCULATE THE VEV OF WILSON LOOP AS IN THE PLANAR CASE AND CONSIDER A DECONFINING LIMIT IN TERMS OF τ_2

$$\langle \Psi_0 | W_C | \Psi_0 \rangle = \int d\mu(A/G_*) \; W_C \quad e^{-S_{S^1 \times S^1 \times R}} \bar{\Psi}_0 \Psi_0$$

$$S^1_\beta \times S^1_L \times R \;\rightarrow\; S^1_\beta \times R^2$$

(2+1)-DIM THEORY AT FINITE TEMP.

W_C : *Wilson loop operator of a loop C*

Ψ_0 : *Vacuum state wave functional*

A/G_* : *gauge−invariant configuration space*

TO-DO LIST: CONSIDER TORIC VERSIONS OF

- MATRIX PARAMETRIZATION
- CALCULATION OF GAUGE INVARIANT MEASURE
- EVALUATION OF Ψ_0
- CALCULATION OF $\langle W_C \rangle$
- READING OFF AREA LAW OR POSITIVE STRING TENSION FROM $\langle W_C \rangle$

IN THE FRAMEWORK OF KARABALI-KIM-NAIR APPROACH

RESUME OF KKN APPROACH

GAUGE POTENTIAL:

$$A_i = -i\, t^a A_i^a \qquad (i=1,2,3) \qquad t^a \in SU(N); Mat_{(N\times N)} \qquad Tr(t^a t^b) = \frac{1}{2}\delta^{ab}$$

MATRIX PARAMETRIZATION:

$$\boxed{\begin{aligned} A_z &= -\partial_z M\ M^{-1} \\ A_{\bar z} &= M^{\dagger -1}\partial_{\bar z} M^\dagger \end{aligned}}$$

$$z = x_1 - i\,x_2\,, \qquad \bar z = x_1 + i\,x_2$$

$$A_z = \frac{1}{2}(A_1 + i\,A_2), \quad A_{\bar z} = \frac{1}{2}(A_1 - iA_2)$$

$$\boxed{M\,, M^\dagger \in SL(N\,,\boldsymbol{C})}$$

(If these are unitary, the potentials are in pure gauge.)

GAUGE TRANSFORMATION:

$$A_i \to A_i^g = g^{-1} A_i\, g + g^{-1}\partial_i\, g \qquad g(x) \in SU(N)$$

IS REALIZED BY $\boxed{M \to g^{-1} M\,, \quad M^\dagger \to M^\dagger g}$

RESUME OF KKN APPROACH

CALCULATION OF GAUGE INVARIANT MEASURE:

$$\boxed{d\mu(C) = d\mu(H)\exp\left(2c_A S_{WZW}(H)\right)}$$

$C = A/G_*$: *gauge invariant configuration space*

$\boxed{H = M^\dagger M}$:GAUGE-INVARIANT VARIABLE

$S_{WZW}(H)$: $SL(N,\mathbf{C})/SU(N)-WZW$ *action*

c_A: *quadratic Casimir of* $SU(N)$, $c_A = N$

RESUME OF KKN APPROACH

$$S_{WZW}(H)=\frac{1}{2\pi}\int d^2 z\, Tr(\partial_z H\, \partial_{\bar{z}} H^{-1})$$

$$+\frac{i}{12\pi}\int d^3 x\, \epsilon^{\mu\nu\alpha}\, Tr(H^{-1}\partial_\mu H\ H^{-1}\partial_\nu H\ H^{-1}\partial_\alpha H)$$

INNER PRODUCT:

$$\boxed{\langle 1|2\rangle=\int d\mu(H)\, e^{2c_A S_{WZW}(H)}\, \Psi_1^*(H)\, \Psi_2(H)}$$

FOR VACUUM: $\Psi(H)=1$

THIS WAVE FUNCTION CAN BE INTERPRETED AS A HOLOMORPHIC WAVE FUNCTION OF CHERN-SIMONS THEORY.

RESUME OF KKN APPROACH

WILSON LOOP OPERATOR:

$$W(C) = Tr\ P\ \exp\left(-\oint (A_z dz + A_{\bar z} d\bar z)\right) = Tr\ P\ \exp\left(\frac{\pi}{c_A}\oint J\right)$$

$J = \frac{c_A}{\pi}\partial_z H\ H^{-1}$: CURRENT OF THE HERMITIAN WZW ACTION

FOR VACUUM STATES AND FOR STRONG COUPLING LIMIT, THE VEV OF THE WILSON LOOP IN THE THE FUNDAMENTAL REP. IS EVALUATED AS

$$\langle W(C)\rangle_0 \approx \exp[-\tilde\sigma (Area)_C]$$

RESUME OF KKN APPROACH

$$\langle W(C)\rangle_0 \approx \exp[-\tilde{\sigma}(Area)_C]$$

$(Area)_C$: *the area of the loop* C

$$\sigma = \frac{e^4}{4\pi} c_A c_F$$

quadratic Casimir for $SU(N)$:

$$c_F = \frac{(N-1)(N+1)}{2\text{N}} \quad \textit{for fundamental rep.}$$

$$c_A = N \quad \textit{for adjoint rep.}$$

$$\longrightarrow \quad \sqrt{\sigma} = e^2 \sqrt{\frac{N^2-1}{8\pi}}$$

LESS THAN 3% DIFFERENCE FROM LATTICE SIMULATIONS

THIS IS ONE OF THE IMPORTANT RESULTS IN THE KKN APPROACH, WHICH WE SHALL USE LATER.

MATRIX PARAMETRIZATION

COMPLEX COORDINATES OF TORUS

$$\boxed{z = \xi_1 + \tau\,\xi_2}$$

ξ_i: *real variables* $(\tau = i\,\tau_2)$

$\xi_i \to \xi_i$ + *integer*

HOLONOMIES OF TORUS

$$\boxed{\oint_{\alpha_i} \omega_j = \tau_2 \epsilon_{ij}}$$

$$\omega_1 = (d\,\bar{z} - dz)/2\mathrm{i} = -\tau_2\, d\,\xi_2$$

$$\omega_2 = (\tau\, d\,\bar{z} - \bar{\tau}\, dz)/2\mathrm{i} = \tau_2\, d\,\xi_1$$

α_i : cycles corresponding to the periodicity of ξ_i

MATRIX PARAMETRIZATION

INTRODUCE ZERO-MODE VARIABLES OF TORUS

$$\boxed{a_1 = \bar{a} - a, \quad a_2 = \tau \bar{a} - \bar{\tau} a} \qquad (a \in \mathbb{C})$$

WITH PERIODICITY $\boxed{a \to a + m + n\tau}$ $(m, n \in \mathbb{Z})$

The integral part of $\Re\tau = \tau_1$ *can be absorbed into* m.

OUR ASSUMPTION $(\tau_1 = 0)$ IS EQUIVALENT TO τ_1 BEING AN INTEGER, A LARGE INTEGER.

MATRIX PARAMETRIZATION

UNDER $a \rightarrow a + m + n\tau$,

$$\boxed{\delta a_1 \rightarrow (-2i\tau_2)n, \quad \delta a_2 \rightarrow (2i\tau_2)m}$$

THUS,

$$\boxed{\exp\left(\oint_{\alpha_2} \frac{\pi\omega_1}{\tau_2}\frac{a_2}{\tau_2}\right) = e^{-i2\pi m}, \quad \exp\left(\oint_{\alpha_1} \frac{\pi\omega_2}{\tau_2}\frac{a_1}{\tau_2}\right) = e^{-i2\pi n}}$$

HERE a_1, a_2 TAKE MATRIX-VALUED QUANTITIES DEFINED

WITH $\boxed{a = a_j t_j^{diag}}$ $(j = 1, 2, \ldots, N-1)$

t_j^{diag} *: diagonal generators of* $SU(N)$ *gauge group ,*

$-$ *Cartan subalgebra of* $SU(N)$

MATRIX PARAMETRIZATION

KKN MATRIX PARAMETRIZATION ON TORUS

$$\tilde{A}_{\xi_1} = -\partial_{\xi_1} M \; M^{-1} + M \left(\frac{\pi \omega_2}{\tau_2} \frac{a_1}{\tau_2} \right) M^{-1}$$

$$\tilde{A}_{\xi_2} = M^{\dagger -1} \partial_{\xi_2} M^{\dagger} + M^{\dagger -1} \left(\frac{\pi \omega_1}{\tau_2} \frac{a_2}{\tau_2} \right) M^{\dagger}$$

$M(z,\bar{z}), M^{\dagger}(z,\bar{z})$ *are the elements of* $SL(N, \boldsymbol{c})$.

GAUGE TRANSFORMATIONS ARE REALIZED BY

$$M \rightarrow g M, \quad M^{\dagger} \rightarrow M^{\dagger} g^{-1}$$

MATRIX PARAMETRIZATION

IN TERMS OF $(z,\bar{z})$-COORDINATES:

$$\tilde{A}_z = -\partial_z \tilde{M}\ \tilde{M}^{-1}, \quad \tilde{M} = M \exp\left(\frac{-\pi}{\tau_2}\int^z \omega\,\bar{a}\right) \equiv M\,\tilde{\gamma}_z$$

$$\tilde{A}_{\bar{z}} = \tilde{M}^{\dagger -1}\partial_{\bar{z}}\tilde{M}^{\dagger}, \quad \tilde{M}^{\dagger} = M^{\dagger}\exp\left(\frac{\pi}{\tau_2}\int^{\bar{z}} \bar{\omega}\,a\right) \equiv \tilde{\gamma}_{\bar{z}} M^{\dagger}$$

$\tilde{\gamma}_z, \tilde{\gamma}_{\bar{z}}$ *encode the zero modes of torus.*

GAUGE INVARIANT MEASURE

PLANAR CASE: $d\mu(C) = d\mu(H)\exp\left(2c_A S_{WZW}(H)\right)$

$\boxed{H = M^\dagger M}$:GAUGE-INVARIANT VARIABLE

$S_{WZW}(H)$: $SL(N,\mathbf{C})/SU(N) - WZW$ action

c_A: quadratic Casimir of $SU(N)$, $c_A = N$

$C = A/G_*$: gauge invariant configuration space

TORIC CASE: $d\mu(\tilde{C}) = d\mu(\tilde{H})\exp\left(2c_A S_{WZW}(\tilde{H})\right)$

$\boxed{\tilde{H} = \tilde{M}^\dagger \tilde{M} = \tilde{\gamma}_{\bar{z}} H \tilde{\gamma}_z}$

VACUUM-STATE WAVE FUNCTIONAL

KKN PLANAR CASE:

$$\Psi[A_{\bar{z}}] = e^{-\frac{K}{2}} \exp\left[k\, S_{WZW}(M^{\dagger})\right]$$

THIS CORRESPONDS TO (ANTI) HOLOMORPHIC WAVE FUNCTIONALS OF CHERN-SIMONS THEORY.

$$k : \text{ Level number of the Chern-Simons theory}$$

$$K = -\frac{k}{\pi}\int_{\Sigma} Tr(A_{\bar{z}} A_z) : \text{ Kähler potential for}$$

$$\text{the phase space of CS theory (with } A_0 = 0 \text{ gauge)}$$

VACUUM-STATE WAVE FUNCTIONAL

$S_{WZW}(M^{\dagger})$ ARISES FROM THE FLATNESS CONDITION

$$\boxed{F_{z\bar{z}} = \partial_z A_{\bar{z}} - \partial_{\bar{z}} A_z + [A_z, A_{\bar{z}}] = 0}$$

ACTED ON THE HOLOMORPHIC WAVE FUNCTION.

THE FLATNESS CONDITION IS REQUIRED AS
AN EQN. OF MOTION FOR A_0.

(OR THE GAUSS LAW CONSTRAINT OF CS THEORY)

VACUUM-STATE WAVE FUNCTIONAL

NARASHIMHAN-SEHSADRI THEOREM:
THERE EXIST CURVATURE-FLAT CONNECTIONS FOR ANY COMPACT 2-DIM SPACES WITH COMPLEX STRUCTURE.

WE CAN CONSTRUCT A VACUUM WAVE FUNCTIONAL FOR THE TORIC THEORY IN THE SAME WAY.

$$\Psi[\tilde{A}_{\bar{z}}] = e^{-\frac{\tilde{K}}{2}} \exp(\tilde{k}\, S_{WZW}(\tilde{M}^{\dagger}))\, Y(a)$$

VACUUM-STATE WAVE FUNCTIONAL

$\Upsilon(a)$: *function of Cartan subalgebra valued a on torus*

$\overline{\Upsilon(a)}$: *complex conjugate of* $\Upsilon(a)$

$\tilde{k}$: *toric version of the level number k*

$$\tilde{K} = -\frac{\tilde{k}}{\pi} \int_{\Sigma = S^1 \times S^1} Tr(\tilde{A}_{\bar{z}} \tilde{A}_z)$$

INNER PRODUCTS:

$$\langle 1|2 \rangle = \int_{S^1 \times S^1} d\,\mu(\tilde{H})\, e^{(2c_A + \tilde{k}) S_{WZW}(\tilde{H})} \overline{\Upsilon_1(a)}\, \Upsilon_2(a)$$

$$\langle 1|2 \rangle = \int_{\boldsymbol{R}^2} d\,\mu(H)\, e^{(2c_A + k) S_{WZW}(H)} \quad with \quad k \to 0$$

DIMENSIONAL DISCUSSION

WE ASSUME $\tilde{k} \to 0$ AS WELL FOR THE TORIC CASE BUT (FROM A GAUGE INV. ARGUMENT) $\tilde{k}$ CAN BE RELATED TO ZERO-MODE LEVEL NUMBER FOR U(1) CHERN-SIMONS THEORY.
SO FOR NON-TRIVIAL ZERO-MODE CONTRIBUTIONS, WE WILL HAVE NONTRIVIAL $\tilde{k}$.

MASS DIMENSION $$\boxed{[e^2] = \left[\frac{1}{\tau_2}\right] = 1}$$

$\longrightarrow$ WE CONSIDER $\tilde{k}$ AS $\tilde{k}\dfrac{\pi}{\tau_2 e^2}$

THE FACTOR OF π COMES FROM MATRIX PARAMETRIZATION.

DECONFINING LIMIT

THE VEV OF WILSON LOOP OPERATOR

$$\langle W(C)\rangle_0 = \int d\mu(\tilde{H})\, e^{(2c_A+\tilde{k})S_{WZW}(\tilde{H})}\, e^{S(\tilde{H})}\,\overline{\Upsilon(a)}\,\Upsilon(a)\,W(C)$$

$$W(C) = Tr\ P\ \exp\left(-\oint(\tilde{A}_z dz + \tilde{A}_{\bar{z}} d\bar{z})\right) = Tr\ P\ \exp\left(\frac{\pi}{c_A}\oint \tilde{J}\right)$$

$$\tilde{J} = \frac{c_A}{\pi}\partial_z \tilde{H}\ \tilde{H}^{-1}$$

$S(\tilde{H})$:CONTRIBUTION FROM POTENTIAL ENERGY

IN A CONTINUUM STRONG COUPLING LIMIT (FOR MODES OF LOW MOMENTA), WE CAN USE THE RESULT OF THE PLANAR CASE BY SETTING $\Upsilon(a)=1$.

DECONFINING LIMIT

$$\longrightarrow \qquad \langle W(C)\rangle_0 \approx \exp[-\tilde{\sigma}(Area)_C]$$

WITH STRING TENTION ON TORUS

$$\boxed{\tilde{\sigma} = \frac{e^4}{4\pi}\left(c_A + \frac{\tilde{k}}{2}\right)c_F}$$

$c_F = \dfrac{(N-1)(N+1)}{2N}$:QUADRATIC CASIMIR OF SU(N) IN THE FUNDAMENTAL REP.

$c_A = N$:FOR ADJOINT REP.

DECONFINING LIMIT

NOW FROM A MANIFESTLY GAUGE INVARIANT EXPRESSION OF AN INNER PRODUCTS FOR TORIC THEORY, WHICH WE HAVE NOT DISCUSSED HERE, WE FIND

$$\boxed{\tilde{k} = 2 k_{a\bar{a}}}$$

WHERE $k_{a\bar{a}}$ IS THE LEVEL NUMBER FOR THE ABELIAN CS THEORY ON TORUS. $(k_{a\bar{a}} \in 2\mathbb{Z})$

SO WE CAN SUBSTITUTE

$$\boxed{\tilde{k} = -\frac{2\pi k_{a\bar{a}}}{\tau_2 e^2}} \quad (k_{a\bar{a}} = 2l, \quad l = 1, 2, \ldots)$$

INTO $\tilde{\sigma}$.

DECONFINING LIMIT

IDENTIFYING l WITH n AND CHOOSING $n=1$,
WE HAVE VANISHING STRING TENTION AT $\left(\frac{1}{\tau_2}\right)_c = \frac{e^2 N}{2\pi}$

THEN WE HAVE A DECONFINEMENT TEMPERATURE:

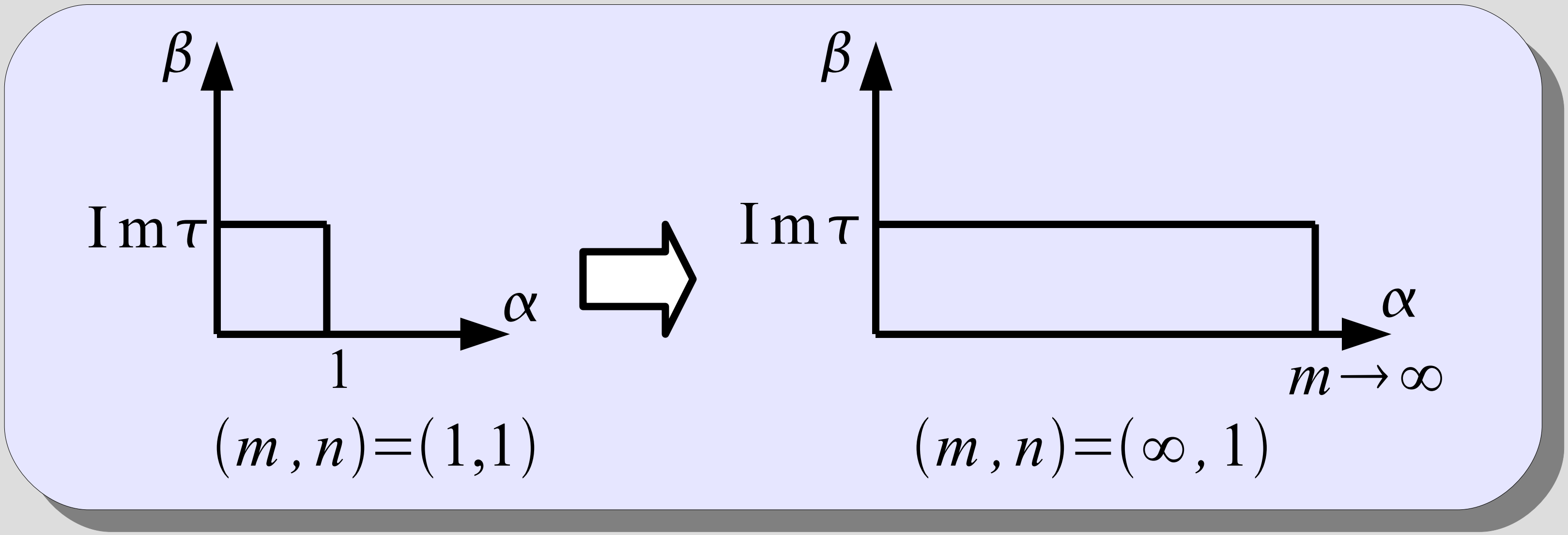
$$\boxed{T_c = \frac{e^2 N}{2\pi}}$$

NOTE:

THE CHOICE OF $n=1$ FOR T_c IS NOT CLEAR; SINCE PHYSICALY WE WOULD REACH T_c FROM LOW TEMP., NEGLECTING $n>1$ IS NOT PHYSICALY CLEAR.
A MATHEMATICAL REASON FOR $n=1$ IS ALSO LACKING.

DECONFINING LIMIT

WE CONSIDER THIS IS BECAUSE OF OUR CHOICE OF TORUS DEFORMATION IN THE BEGINNING.



THE VALUE OF $T_c = \frac{e^2 N}{2\pi}$ SEEMS TO BE PLAUSIBLE SINCE THIS IS THE SAME AS A PROPAGATOR MASS FOR (NON-PERTURBATIVE) GLUONS GIVEN BY KKN.

DECONFINING LIMIT

NOTE THAT STRING TENTION IN THE PLANAR THEORY IS

$$\sigma = e^4 \left(\frac{N^2 - 1}{8\pi} \right)$$

$$\longrightarrow \quad \boxed{\frac{T_c}{\sqrt{\sigma}} = \sqrt{\frac{2}{\pi}} \sqrt{\frac{N^2}{N^2 - 1}} = 0.798 \sqrt{\frac{N^2}{N^2 - 1}}}$$

LATTICE SIMULATIONS FOR THIS VALUE ARE 0.865, 0.903 (LIDDLE & TEPPER) AND 0.86(7) (NARAYANAN & OTHERS) AT $N \to \infty$.

(ABOUT 10% AGREEMENT TO THE NUMERICAL DATA)

CONCLUSIONS

(2+1)-DIM YANG-MILLS ON PLANE
IN THE KKN HAMILTONIAN APPROACH

- NARASHIMHAN-SESHADRI THEOREM
- (GAUGE-INVARIANCE OF VACUUM-STATE INNER PRODUCTS)
- DIMENSIONAL ANALYSIS OF THE LEVEL NUMBER OF VACUUM STATE WAVE-FUNCTION

PREDICTION FOR DECONFINEMENT TEMP.