On the general action of boundary (super)string field theory

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1. Introduction

Boundary string field theory (BSFT)

BSFT is formulated on the space of all boundary interactions in worldsheet sigma-models (specified by couplings λ's) based on Batalin-Vilkovisky (BV) formalism [Witten(1992)]

For bosonic string

$$dS = \frac{1}{2}\int_0^{2\pi} d\sigma d\sigma' \, \langle d\mathcal{O}(\sigma)\{Q_B, \mathcal{O}(\sigma')\}\rangle_\lambda$$

$\mathcal{O}$: boundary operator, $\mathcal{O} = \sum \lambda^i \mathcal{O}_i$
$\langle \cdots \rangle_\lambda$: correlator with the boundary perturbation defined by λ's

The action S is related to the disk partition function Z and β-functions

$$S(\lambda) = \left(-\beta^i(\lambda)\frac{\partial}{\partial\lambda^i} + 1\right) Z(\lambda), \qquad Z(\lambda) = \int DX \, e^{-S_0 - S_B(\lambda)}$$

Boundary superstring field theory (super BSFT)

The BSFT action is identified with the corresponding disk partition function: [Kutasov-Marino-Moore(2000)]

$$S(\lambda) = Z(\lambda)$$

Recently, bosonic BSFT was reformulated in terms of closed string Hilbert space and the action S itself was obtained without any assumption: [Teraguchi(2006)]

$$S = \frac{1}{4}\langle B|e^{2i\{b_0^-,O\}}c_0^- Q_B c_0^-|0\rangle - \frac{i}{2}\langle B|\mathrm{Sym}\left[e^{2i\{b_0^-,O\}};\{Q_B,O\}\right]c_0^-|0\rangle$$

Our work

- Reconstruct super BSFT via boundary states as in the bosonic case
- Obtain the general action S for superstrings without any assumption

Plan of Talk

1. Introduction

2. BV-formulation of super BSFT and its modification

3. General action of boundary superstring field theory

4. Revisiting the conjecture $S = Z$

5. Summary

2. BV-formulation of super BSFT and its modification

Construction of super BSFT

- Consider boundary perturbation: $S_{\rm bdy} = \int \frac{d\sigma d\theta}{2\pi} \mathcal{V}(\sigma, \theta)$, $\mathcal{V} = \sum_I \lambda^I \mathcal{V}_I$
- Boundary operator $\mathcal{O}$ is a basic object in BSFT, $\mathcal{V} = b_{-1}^{\rm BSFT} \mathcal{O}$
  $\rightarrow$ $\mathcal{O}$ has picture number 0 and ghost number 1.

Key ingredients of BV formulation

We need a fermionic vector V and a fermionic 2-form ω obeying

$$\left. \begin{array}{rl} V^2 = 0 : & \text{nilpotency} \\ d\omega = 0 : & \text{closedness} \\ d(i_V \omega) = 0 : & V\text{−invariance} \end{array} \right\} \rightarrow \begin{array}{l} \text{gauge invariant action } S \\ dS = i_V \omega \end{array}$$

These are proposed in [Marino, Niarchos-Prezas (2001)]

- V $\rightarrow$ the vector generated by the BRST operator

- 2-form ω (two inverse picture-changing operators are inserted to saturate picture number -2)

$$\omega = \frac{1}{2}\int d\sigma_1 d\sigma_2 d\theta_1 d\theta_2 \langle Y(\sigma_1) d\mathcal{O}(\sigma_1,\theta_1) Y(\sigma_2) d\mathcal{O}(\sigma_2,\theta_2)\rangle_\lambda$$

where $Y = c\partial\xi e^{-2\phi}$

- We propose a modified definition of the two-form ω:

$$\omega = \frac{1}{2}\int d\sigma_1 d\sigma_2 d\theta_1 d\theta_2 \langle Y\tilde{Y}(0) d\mathcal{O}(\sigma_1,\theta_1) d\mathcal{O}(\sigma_2,\theta_2)\rangle_\lambda.$$

$\rightarrow$ double-step inverse picture-changing operator at the center of the disk

cf.) modified cubic SFT

Remarks

- Positions of picture-changing operators cannot freely be changed for off-shell operators.
- Hence our super BSFT is, in principle, different from the original one.
- Under the new definition of the two-form, the proof of closedness and V-invariance are much more simplified.

3. General action of boundary superstring field theory

We reconstruct super BSFT in terms of boundary states.

- Insert the double step inverse picture changing operator

$$
\begin{aligned}
Y(0)\tilde{Y}(0) &\sim \lim_{z\to 0} Y(z)\tilde{Y}(\bar{z})|0\rangle \\
&= \lim_{z\to 0}\left(zc(z)\partial\xi(z)e^{-\phi(z)}e^{-\phi(0)}\right)\left(\bar{z}\tilde{c}(\bar{z})\bar{\partial}\tilde{\xi}(\bar{z})e^{-\tilde{\phi}(\bar{z})}e^{-\tilde{\phi}(0)}\right)|0\rangle \\
&= -\beta_{-1/2}\tilde{\beta}_{-1/2}|\Omega\rangle \equiv |Y\tilde{Y}\rangle
\end{aligned}
$$

$|\Omega\rangle$: Fock vacuum, $\beta = e^{-\phi}\partial\xi$, $\gamma = \eta e^{\phi}$ (bosonization)

- the operator b_{-1}^{BSFT}

$$
b_{-1}^{\mathrm{BSFT}} = -i(b_0 - \tilde{b}_0) = -2ib_0^-
$$

$|Y\tilde{Y}\rangle$ satisfies $b_0^-|Y\tilde{Y}\rangle = Q_B|Y\tilde{Y}\rangle = 0$. (cf. $b_0^-|0\rangle = Q_B|0\rangle = 0$)
$\rightarrow$ $|Y\tilde{Y}\rangle$ shares common properties with the $SL(2,C)$ vacuum $|0\rangle$ in bosonic string.

Definition of the vector V and fermionic 2-form

$$\delta_V O \equiv \{Q_B, O\},$$
$$\omega \equiv \frac{1}{2}\langle B|\mathrm{Sym}[e^{2i\{b_0^-, O\}}; dO, dO]|Y\tilde{Y}\rangle$$

where

$$O \equiv \int_0^{2\pi} \frac{d\sigma d\theta}{2\pi}\mathcal{O}(\sigma,\theta), \qquad O = \sum_I \lambda^I O_I$$

$$\mathrm{Sym}[e^{-V}; O_1, O_2, \cdots, O_n]$$
$$= \int_0^1 dt_1 \int_{t_1}^1 dt_2 \cdots \int_{t_{n-1}}^1 dt_n e^{-t_1 V} O_1 e^{-(t_2-t_1)V} O_2 \cdots O_n e^{-(1-t_n)V} \pm (\mathrm{perms}).$$

$\langle B|$: BRST invariant boundary state, $\langle B|b_0^- = \langle B|Q_B = 0$

Closedness:

$$\begin{aligned} d\omega &= i\langle B|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}}; \{b_0^-, dO\}, dO, dO\big]|Y\tilde{Y}\rangle \\ &= \frac{i}{3}\langle B|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}}; dO, dO, dO\big]b_0^-|Y\tilde{Y}\rangle = 0 \end{aligned}$$

V-invariance:

$$\begin{aligned} d(i_V\omega) &= 2i\langle B|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}}; \{b_0^-, dO\}, dO, \{Q_B, O\}\big]|Y\tilde{Y}\rangle \\ &\quad - \langle B|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}}; dO, \{Q_B, dO\}\big]|Y\tilde{Y}\rangle \\ &= -i\langle B|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}}; dO, dO, \big[b_0^-, \{Q_B, O\}\big]\big]|Y\tilde{Y}\rangle \\ &\quad - i\langle B|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}}; dO, dO, [Q_B, \{b_0^-, O\}]\big]|Y\tilde{Y}\rangle \\ &= -\frac{i}{2}\langle B|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}}; dO, dO, [L_0 - \tilde{L}_0, O]\big]|Y\tilde{Y}\rangle = 0 \end{aligned}$$

Then the gauge invariant action is given by

$$dS = i_V\omega = \langle B|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}}; dO, \{Q_B, O\}\big]|Y\tilde{Y}\rangle$$

dS can be integrated by a simple algebraic calculations: [Teraguchi (2006)]

$$
\begin{aligned}
dS &= \langle B|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}};dO,\{Q_B,O\}\big]|Y\tilde{Y}\rangle \\
&= \langle B|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}};dO,\{Q_B,O\}\big]b_0^-c_0^-|Y\tilde{Y}\rangle \\
&\vdots \\
&= d\left(\frac{1}{4}\langle B|e^{2i\{b_0^-,O\}}c_0^-Q_Bc_0^-|Y\tilde{Y}\rangle - \frac{i}{2}\langle B|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}};\{Q_B,O\}\big]c_0^-|Y\tilde{Y}\rangle\right)
\end{aligned}
$$

We obtain the general form of the action

$$
S = \frac{1}{4}\langle B|e^{2i\{b_0^-,O\}}c_0^-Q_Bc_0^-|Y\tilde{Y}\rangle - \frac{i}{2}\langle B|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}};\{Q_B,O\}\big]c_0^-|Y\tilde{Y}\rangle
$$

Comments

- bosonic BSFT ($|Y\tilde{Y}\rangle \to |0\rangle$)
- When one consider a boundary fermion, one should modify the BRST operator so that $\{b_0^-,Q_B\}$ generates the rotation in this sector.

Expansion form and gauge transformation

- Expand the general action

$$S = \frac{1}{4}\langle B|e^{2i\{b_0^-,O\}}c_0^- Q_B c_0^- |Y\tilde{Y}\rangle - \frac{i}{2}\langle B|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}}; \{Q_B, O\}\big]c_0^- |Y\tilde{Y}\rangle$$
$$= \frac{1}{4}\langle B|c_0^- Q_B c_0^- |0\rangle + \sum_{n=0}^{\infty} \frac{(2i)^n}{(n+2)!} \sum_{m=0}^{n} \langle B|(Ob_0^-)^{n-m} O Q_B O (b_0^- O)^m |0\rangle$$
$$= \frac{1}{4}\langle B|c_0^- Q_B c_0^- |0\rangle + \frac{1}{2}\langle B|O Q_B O|0\rangle + \frac{i}{3}\langle B|\big(O Q_B O b_0^- O + O b_0^- O Q_B O\big)|0\rangle + \cdots$$

- No linear term in $O \to O_I = 0$ is a solution.

- The action has gauge symmetry:

$$\delta_\Lambda O = [Q_B, \Lambda] + i\langle B|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}}; \{Q_B, O\}, [b_0^-, \Lambda], O_I\big]|0\rangle \omega^{IJ} O_J$$

4. Revisiting the conjecture $S = Z$

It is widely believed that $S = Z$ when matter and ghosts are decoupled ($\mathcal{O} = C\mathcal{V}$). We check this conjecture.

Consider D-brane boundary state

$$ {}_{\rm NS}\langle Dp| = {}_{\rm NS}\langle Bp,+| - {}_{\rm NS}\langle Bp,-| $$

Since $|Y\tilde{Y}\rangle = P_{\rm GSO}|Y\tilde{Y}\rangle$, GSO-invariant combination is automatically chosen.
$\longrightarrow$ We focus only on $\langle Bp,+|$.

$$ S = -\frac{i}{4}\langle Bp,+|e^{2i\{b_0^-,O\}}c_0^- Q_B c_0^-|Y\tilde{Y}\rangle - \frac{1}{2}\langle Bp,+|\mathrm{Sym}\left[e^{2i\{b_0^-,O\}};\{Q_B,O\}\right]c_0^-|Y\tilde{Y}\rangle $$

Suppose matter and ghosts are completely decoupled:

$$ O = \int \frac{d\sigma d\theta}{2\pi} C\mathcal{V}(\sigma,\theta) = \int_0^{2\pi}\frac{d\sigma}{2\pi}(\gamma\mathcal{V}^{(-1)}(\sigma) - c\mathcal{V}^{(0)}(\sigma)). $$

$$ C = c(\sigma) + \theta\gamma(\sigma), \quad \mathcal{V} = \mathcal{V}^{(-1)} + \theta\mathcal{V}^{(0)}. $$

The exponential does not depend on ghosts at all:

$$e^{2i\{b_0^-,O\}} = \exp\left[\int_0^{2\pi} \frac{d\sigma}{2\pi}\mathcal{V}^{(0)}\right].$$

First term $\rightarrow$ disk partition function

$$S_1 = -\langle Bp, +|e^{2i\{b_0^-,O\}}c_0^-|\Omega\rangle = Z(\lambda).$$

What about the second term? ($\beta_r^+ = \beta_r + i\tilde{\beta}_{-r}$, $G_r^+ = G_r + i\tilde{G}_{-r}$)

$$
\begin{aligned}
S_2 &= -\frac{i}{2}\langle Bp,+|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}};\{Q_B,O\}\big]\,c_0^-\beta_{1/2}^+\beta_{-1/2}^+|\Omega\rangle \\
&= -\frac{i}{2}\langle Bp,+|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}};\{G_{-1/2}^+,[O,\beta_{1/2}^+]\}\big]\,c_0^-|\Omega\rangle \\
&\quad -\frac{i}{2}\langle Bp,+|\mathrm{Sym}\big[e^{2i\{b_0^-,O\}};\{G_{1/2}^+,[O,\beta_{-1/2}^+]\}\big]\,c_0^-|\Omega\rangle \\
&\quad +\langle Bp,+|\mathrm{Sym}\Big[e^{2i\{b_0^-,O\}};\int_0^{2\pi}\frac{d\sigma}{2\pi}\mathcal{V}^{(0)}\Big]\,c_0^-|\Omega\rangle.
\end{aligned}
$$

Suppose $\mathcal{V}$ is superconformal primary

$$
T_F(z)\mathcal{V}^{(-1)}(0) \sim -\frac{1}{z}\mathcal{V}^{(0)}(0)
$$

Then

$$
\{G_{\mp 1/2}^+,[O,\beta_{\pm 1/2}^+]\} = -i\int\frac{d\sigma}{2\pi}\mathcal{V}^{(0)}
$$

$\rightarrow$ the terms in S_2 exactly cancels each other.

The second term S_2 does not vanish for generic operators.

Role of the second term

- For some non-primary massive operators, one-point functions do not vanish. [Frolov (2001), Hashimoto-Terashima (2004)]

- If we define the BSFT action by $S = Z$ $\rightarrow$ BSFT action has a tadpole
  $\rightarrow$ A trivial background is not a solution and it is unlikely.

- We have shown that the general action has no tadpoles.

- The second term S_2 is necessary to cancel a tadpole

5. Summary

- We have reconstructed super BSFT in closed string Hilbert space via boundary states.

- General form of super BSFT action

$$S = \frac{1}{4}\langle B| e^{2i\{b_0^-,O\}} c_0^- Q_B c_0^- |Y\tilde{Y}\rangle - \frac{i}{2}\langle B|\mathrm{Sym}\left[e^{2i\{b_0^-,O\}}; \{Q_B, O\}\right] c_0^- |Y\tilde{Y}\rangle$$

- This general action has a gauge symmetry

- $S = Z$ is not always true when matter and ghosts are decoupled. Our general action reduces to partition function when matter $\mathcal{V}$ is superconformal primary.

- The second term is necessary to cancel any tadpole for some massive nonprimary operators.