Multi-quark baryons and color screening at finite temperature

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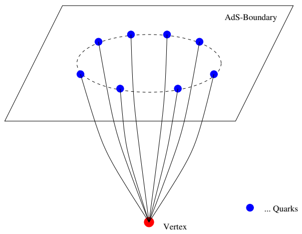
Abstract

- We study baryons in SU(N) gauge theories, according to the gauge/string correspondence based on IIB string theory.
- The baryon is constructed out of D5 brane and N fundamental strings to form a color singlet N-quark bound "baryons"
- In the confining phase, we find that quarks in the baryon feel the potential increasing linearly with the distance from the vertex.
- At finite temperature, we could find $k(< N)$-quark baryons.

Baryon and D5-brane



(A. Brandhuber, et al. JHEP 9807:020,1998)

Baryon in SU(N) gauge theory

$\cdots$ the strings stretched between the boundary of the AdS_5 and the D5-brane wrapped on the S_5

(E. Witten('98))

Background geometry and the D5-brane embedding

The supergravity background of N D3-branes with R-R scalar.

$$ds^2 = e^{\phi/2}\left(\frac{r^2}{R^2}\left(-dt^2+(dx^i)^2\right)+\frac{R^2}{r^2}dr^2+R^2d\Omega_5^2\right)$$

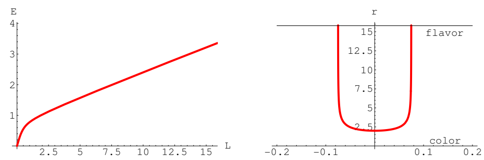
$$e^{\phi} = 1+\frac{q}{r^4} \quad R=(4\pi N)^{1/4}$$

$q \equiv \langle F^{\mu\nu}F_{\mu\nu}\rangle \cdots$ VEV of gauge fields condensate

(H. Liu and A.A. Tseytlin('99))

The dual gauge theory of this background is in the confinement phase.

The $q\bar{q}-$potential is given by the energy of the U-shaped string is linear for large L region.



D5-brane action

$$S = -T_5\int d^6\xi\, e^{-\Phi}\sqrt{-\det(g+F)}+T_5\int A_{(1)}\wedge G_{(5)} \ ,$$

where, G_5 is self-dual Ramond-Ramond field strength

$$G_{(5)} \equiv dC_{(4)} = 4R^4\left(\mathrm{vol}(S^5)d\theta_1\wedge\ldots\wedge d\theta_5-\frac{r^3}{R^8}dt\wedge\ldots\wedge dx_3\wedge dr\right)$$

where $\mathrm{vol}(S^5)\equiv\sin^4\theta_1\mathrm{vol}(S^4)\equiv\sin^4\theta_1\sin^3\theta_2\sin^2\theta_3\sin\theta_4$ and $F_{(2)}=dA_{(1)}$ is the U(1) worldvolume field strength.

We fix the world volume coordinates of the $D5$-brane as $\xi^a=(t,\theta,\theta_2,\ldots,\theta_5)$, and r and A_t depend only on θ. Then, the embedded configuration of the D5 brane is

$$S = T_5\Omega_4R^4\int dt d\theta\sin^4\theta(-\sqrt{e^{\Phi}(r^2+r'^2)-F_{\theta t}^2}+4A_t)$$

where, $\Omega_4=8\pi^2/3$ is the volume of the unit four sphere, and $'\equiv\partial/\partial\theta$.

We define the dimensionless displacement as $D=\frac{1}{T_5\Omega_4R^4}\frac{\delta S}{\delta F_{t\theta}}$. Then, the gauge field equation of motion is

$$\partial_\theta D = -4\sin^4\theta$$

The solution to this equation is

$$D\equiv D(\nu,\theta)=\left[\frac{3}{2}(\nu\pi-\theta)+\frac{3}{2}\sin\theta\cos\theta+\sin^3\theta\cos\theta\right]$$

We eliminate the gauge filed in favor of D.

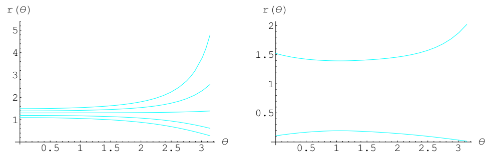
$$U_{D5} = \frac{N}{3\pi^2\alpha'}\int d\theta\, e^{\Phi/2}f\sqrt{r^2+r'^2+(r/R)^4}\sqrt{V_\nu(\theta)}$$

$$V_\nu(\theta) = D(\nu,\theta)^2+\sin^8\theta$$

where we used $T_5\Omega_4R^4=N/(3\pi^2\alpha')$.

Then, the equation of motion is given as

$$\partial_\theta\left(\frac{r'}{\sqrt{r^2+(r')^2}}\sqrt{V_\upsilon(\theta)}\right)-\frac{1-q/r^4}{1+q/r^4}\frac{r}{\sqrt{r^2+(r')^2}}\sqrt{V_\upsilon(\theta)}=0$$

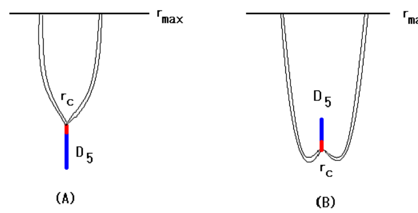


Family of solutions for $q=2.8$ and $r(0)=1.49,\ 1.39,\ 1.30, 1.09, 0.99$ for the curves from the above. The right hand side shows the solutions for $\nu=0.2$ and $r(\theta_c)=1.39$ (the upper) and 0.19 (the lower).

These solutions have cusps at
$\theta=\pi$ and $\theta=0$ for $\nu=0.2$
$\theta=\pi$ for $\nu=0$

Baryon configuration and no-force condition

We consider here the case of $\nu=0$.



the point vertex baryon
(A) $r_c > r_0$ $\quad r_0\equiv r(\theta)\quad r_c\equiv r(\theta=\pi)$
(B) $r_c < r_0$
the tension in the r-direction for the D5-brane at the cusp $r=r_c$

$$\frac{\partial U_{D5}}{\partial r_c} = NT_Fe^{\Phi/2}\frac{r_c'}{\sqrt{r_c'^2+r_c^2}} \ ,$$

where $r_c'=\partial_\theta r|_{\theta=\pi}$.

the energy of the F-string and its r-directed tension at the point r_c are obtained as

$$U_F = T_F\int dx\ e^{\Phi/2}\sqrt{r_x^2+(r/R)^4} \ ,$$

$$\frac{\partial U_F}{\partial r_c} = T_Fe^{\Phi/2}\frac{r_x}{\sqrt{r_x^2+(r_c/R)^4}} \ ,$$

the forces of F-strings should keep a valence with that of the $D5$-brane.

$$N\frac{\partial U_F}{\partial r_c} = \frac{\partial U_{D5}}{\partial r_c}$$

Then,

$$r_x^{(c)} = r_c'\frac{r_c}{R^2} \ , \quad \text{at } r=r_c$$

where $r_x^{(c)}$ denotes the value of r_x at $r=r_c$. the sign of $r_x^{(c)}$ and r_c' should be the same.

Baryon mass and distance between quark and vertex

We can set the following constant h from the action of the F-strings

$$e^{\Phi/2}\frac{r^4}{R^4\sqrt{r_x^2+(r/R)^4}} = h \ .$$

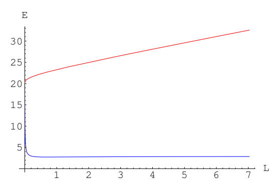
by eliminating r_x wiht a constant h, the distance L between the quark and the vertex and the string energy E is given as

$$L_{q-v} = R^2\int_{r_c}^{r_{\max}}dr\frac{1}{\sqrt{e^{\Phi}r^4/(h^2R^4)-1}}$$

$$E_F = T_F\int_{r_c}^{r_{\max}}dr\frac{e^{\Phi/2}}{\sqrt{1-h^2R^4/(e^{\Phi}r^4)}}$$

the total energy of the baryon is given as

$$E = NE_F+E_{D5}$$



E-L_{q-v} plots for $q=2.0$, $R=1$, $r_{\max}=20$. The upper (lower) curve shows the baryon energy E (the vertex energy E_{D5}).