超対称性の自発的破れの格子シミュレーションによる測定

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RIKEN NiSHiNA CENTER

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based on: I.K.-Suzuki-Sugino Phys.Rev.D77 091502(2008)
I.K.-Sugino-Suzuki Prog.Theo.Phys. 119 (2008)
and work in progress

Introduction

Recent development of lattice SUSY: any practical usage?

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$$\Downarrow$$

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non-perturbative

Lattice simulation gives non-perturbative result (cf. QCD)

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Lattice simulation gives non-perturbative result (cf. QCD)

- not broken in tree level $\Rightarrow$ not broken in all orders
- Witten index: *Not* available in some models
  - 2-dim $\mathcal{N} = (2, 2)$ pure SYM (maybe broken? Hori-Tong)

Plan

1. Introduction
2. Lattice formulation: Sugino model
3. Method for observing SUSY breaking:
   Hamiltonian as the order parameter
4. Simulation result: SQM and SYM
5. Conclusion

Lattice Formulation

Scalar Q for $\mathcal{N} \geq 2$ on lattice

$\boxed{\text{SUSY on lattice: impossible?}(\because\ Q \sim \sqrt{P})}$

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(Models for SYM)
Cohen-Kaplan-Katz-Ünsal, Sugino, Catterall,
D'Adda-I.K.-Kawamoto-Nagata,
Suzuki-Taniguchi,...

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- topological twist $\Rightarrow$ scalar Q on a site
- simulation (check of the formulation)

Catterall, Suzuki, Fukaya-I.K.-Suzuki-Takimi

Sugino model

Sugino, JHEP 01(2004)067

target: 2-dim $\mathcal{N} = (2,2)$ SYM

nilpotent Q (Twisted) SUSY Algebra, continuum

$$Q^2 = \delta_{\phi}^{(\mathrm{gauge})} \quad Q_0^2 = \delta_{\overline{\phi}}^{(\mathrm{gauge})} \quad \{Q, Q_0\} = 2i\partial_0 + 2\delta_{A_0}^{(\mathrm{gauge})}$$

Q-exact Lagrangian (continuum)

$$\mathcal{L} = Q(\dots) = \frac{1}{g^2}\mathrm{tr}\Big\{\frac{1}{4}F_{01}^2 + D_\mu\phi D_\mu\overline{\phi} + \frac{1}{4}[\phi, \overline{\phi}]^2 + i\psi_\mu D_\mu \eta + \cdots - \frac{1}{4}\eta[\phi, \eta] + \cdots\Big\}$$

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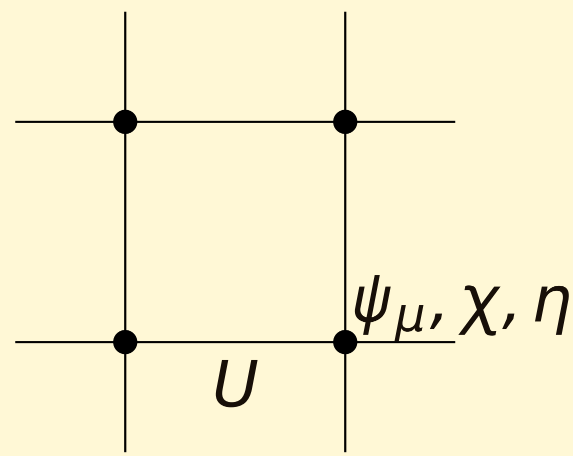
nilpotent Q Lattice version

$$Q^2 = \delta_{\phi}^{(\text{gauge})}$$

Q-exact Lagrangian (lattice)

$$\mathcal{L} = Q(\dots) = \mathcal{L}[U(x,\mu), \phi(x), \overline{\phi}(x), H(x) \qquad \text{bosons}$$
$$\eta(x), \chi(x), \psi_0(x), \psi_1(x)] \qquad \text{fermions}$$

$$QU(x,\mu) = i\psi_\mu(x)U(x,\mu)$$
$$Q\psi_\mu(x) = i\psi_\mu(x)\psi_\mu(x) - i(\phi(x) - U(x,\mu)\phi(x+\hat{\mu})U(x,\mu)^{-1})$$
$$Q\phi = 0$$
$$\vdots$$



Model

Lattice formulation with

- Nilpotent Q
- Q-exact action (= Q-invariant)

Method for Observing ~~SUSY~~

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Advantage

- 1 point function, easy to measure

How to make Hamiltonian?

Order parameter for SUSY breaking: Hamiltonian

$\mathcal{H} = 0$: SUSY $\mathcal{H} > 0$: ~~SUSY~~

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Use the algebra! $\{Q, Q_0\} = 2i\partial_0$

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$\boxed{\text{Use the algebra!}}$ $\{Q, Q_0\} = 2i\partial_0$

$Q\mathcal{J}_0^{(0)} = 2\mathcal{H}$

Discretized "Noether current" for Q_0:

$$\mathcal{J}_0^{(0)}(x) = \frac{1}{a^4 g^2}\,\mathrm{tr}\Big\{\eta(x)[\phi(x),\overline{\phi}(x)]^2 + 2\chi(x)H(x)$$
$$- 2i\psi_0(x)\big(\overline{\phi}(x) - U(x,0)\overline{\phi}(x+a\hat{0})U(x,0)^{-1}\big)$$
$$+ 2i\psi_1(x)\big(\overline{\phi}(x) - U(x,1)\overline{\phi}(x+a\hat{1})U(x,1)^{-1}\big)\Big\}$$

Anti-periodic boundary condition

The conjugate applied field: temperature $\quad Z = \mathrm{tr}\, e^{-\beta H}$

$\Rightarrow$ *anti*-periodic boundary condition for fermions

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- ~~SUSY~~ by the boundary condition(or temperature)
- We need $\beta \to \infty$

We should measure

the ground state energy:

$$\mathcal{E}_0 \equiv \lim_{\beta\to\infty} \langle \mathcal{H} \rangle_{\text{αPBC}} = \lim_{\beta\to\infty} \langle Q \mathcal{J}_0^{(0)}/2 \rangle_{\text{αPBC}} \begin{cases} = 0 & \text{SUSY} \\ > 0 & \cancel{\text{SUSY}} \end{cases}$$

Using

- Nilpotent Q
- Q-exact action (= Q-invariant)

Simulation Result

Example: Supersymmetric Quantum Mechanics

(known): form of the potential $\Rightarrow$ broken or not

lattice SQM: a model by Catterall

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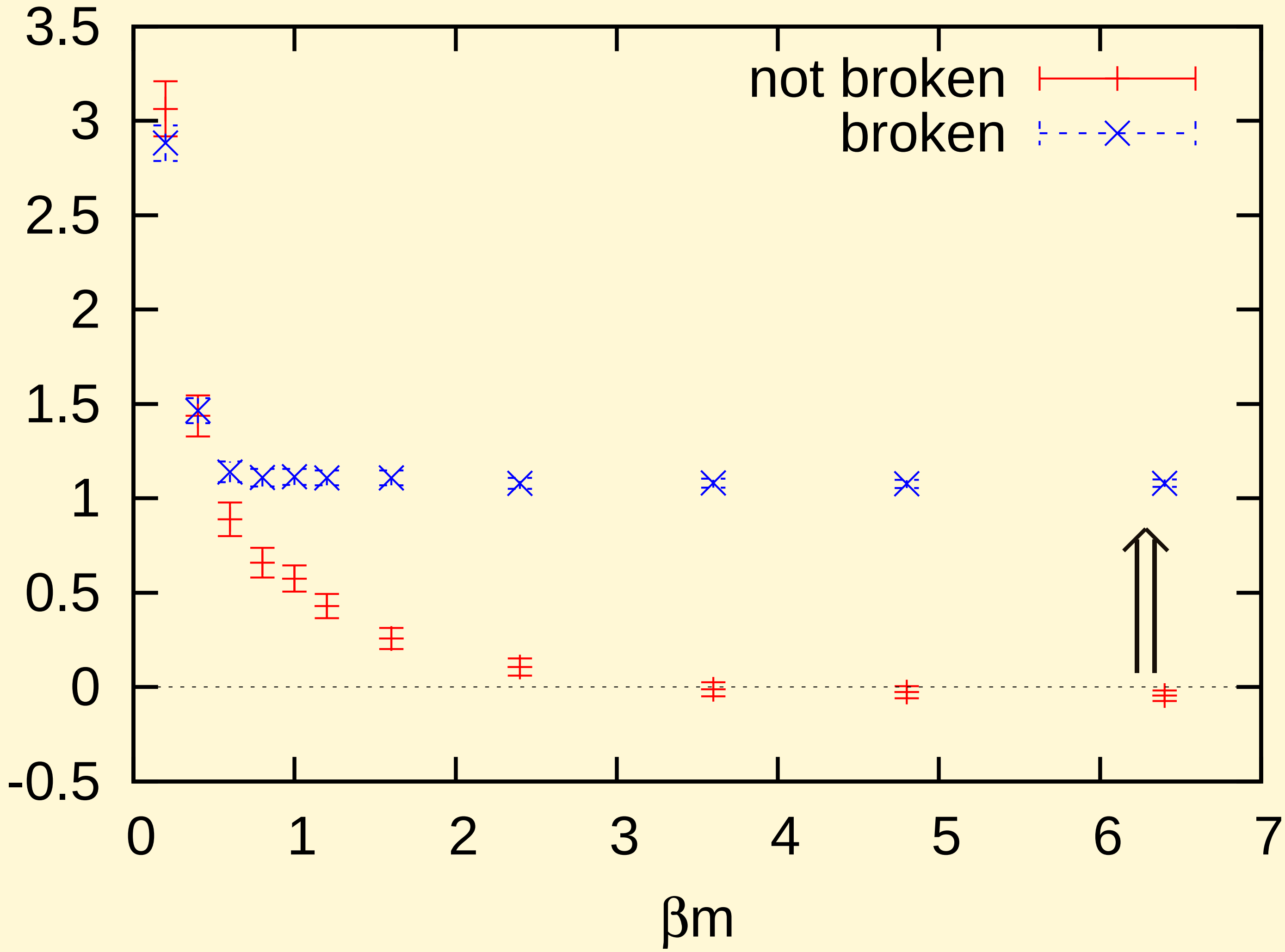
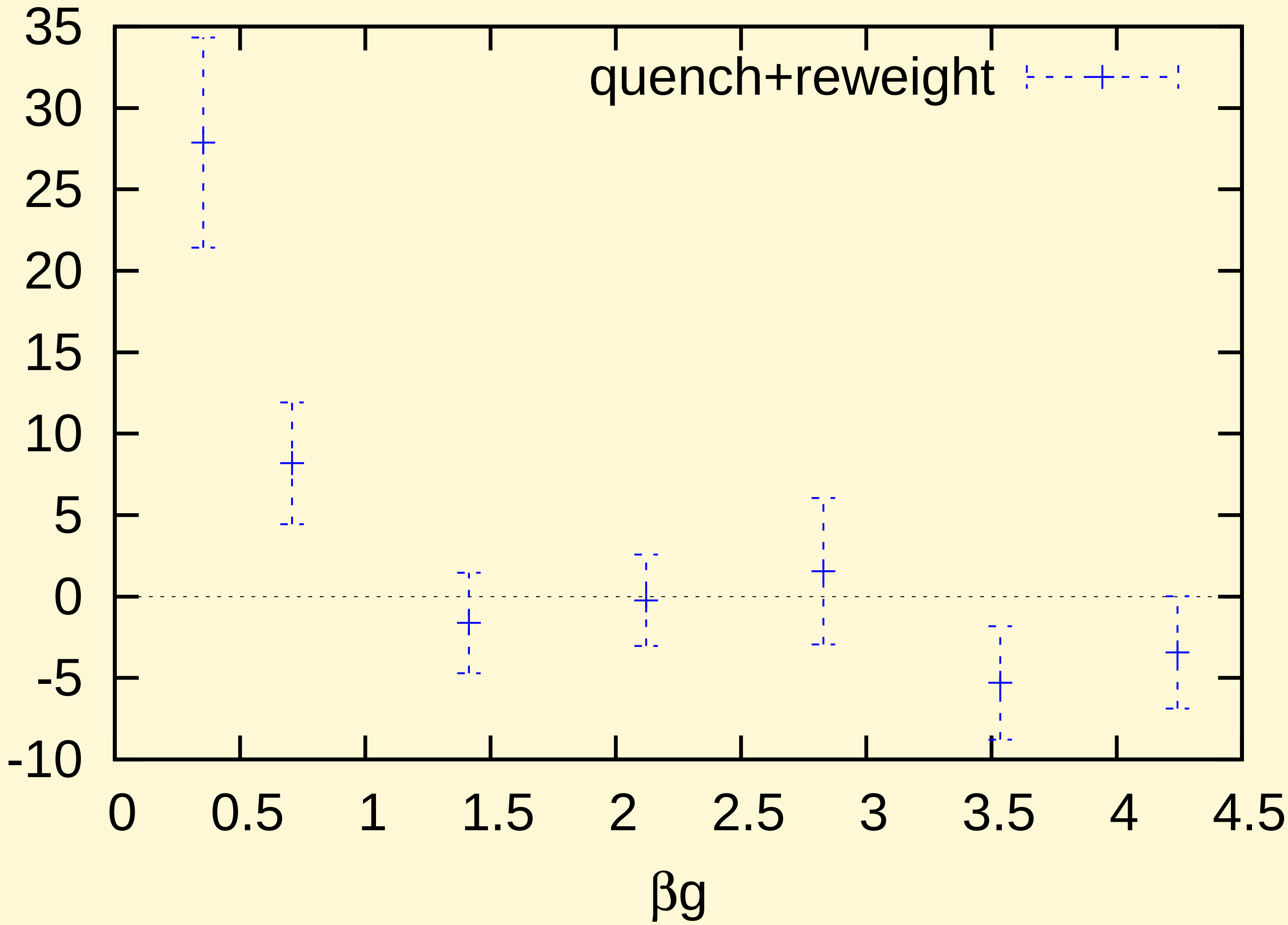


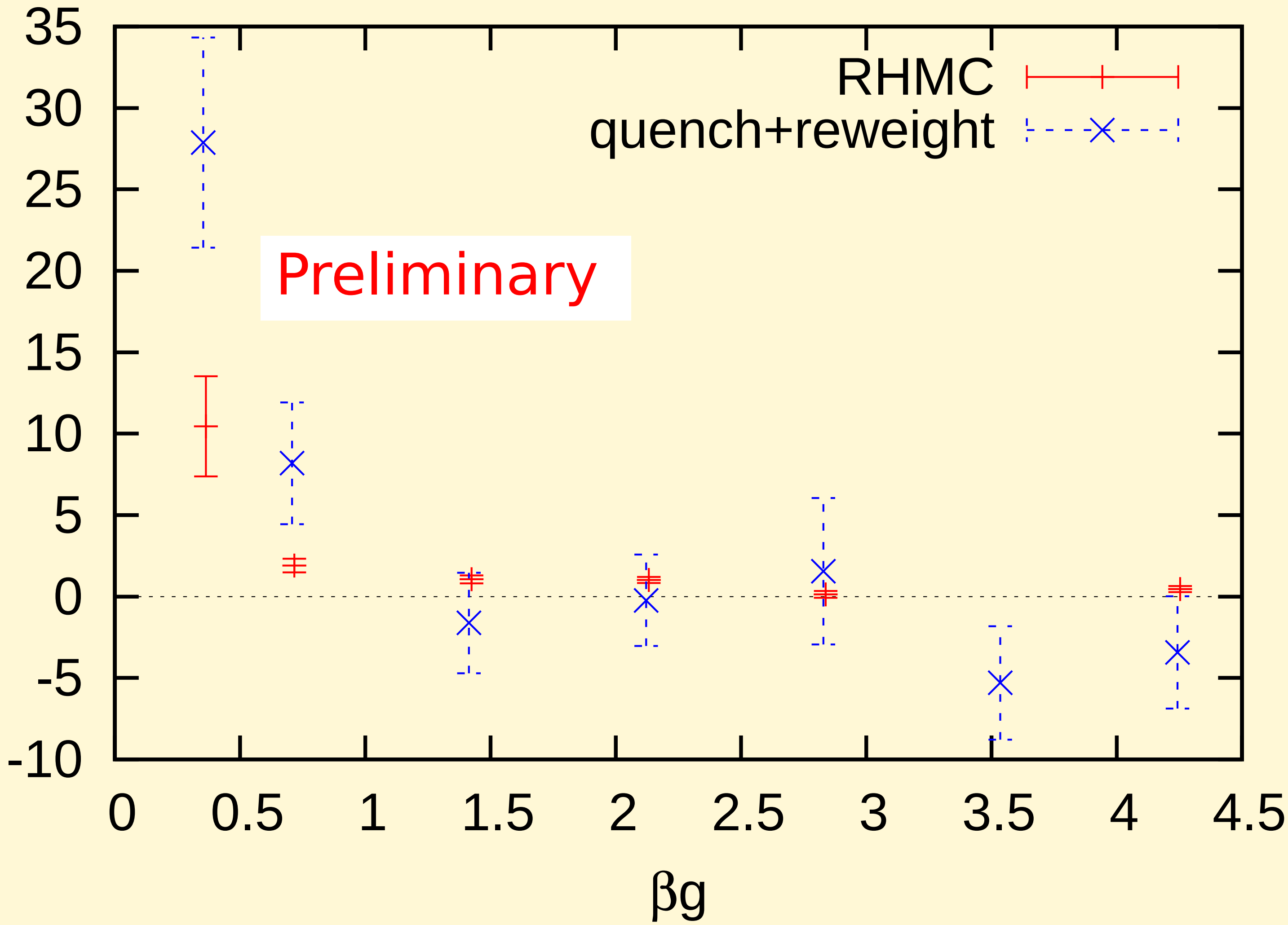
Result for 2-dim $\mathcal{N} = (2,2)$ SYM

$SU(2)$, ground state energy, old result



Result for 2-dim $\mathcal{N} = (2,2)$ SYM

$SU(2)$, ground state energy seems small



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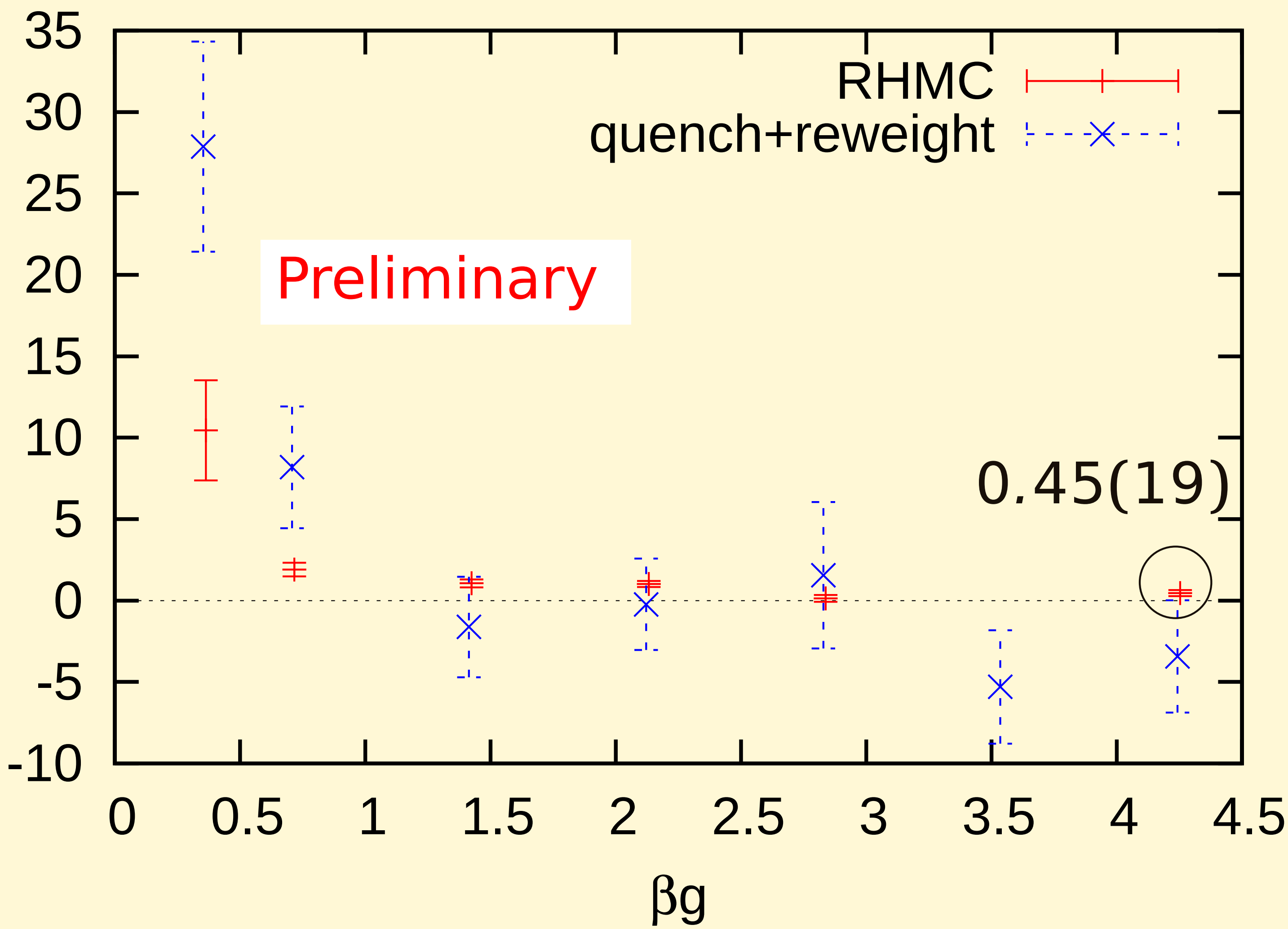
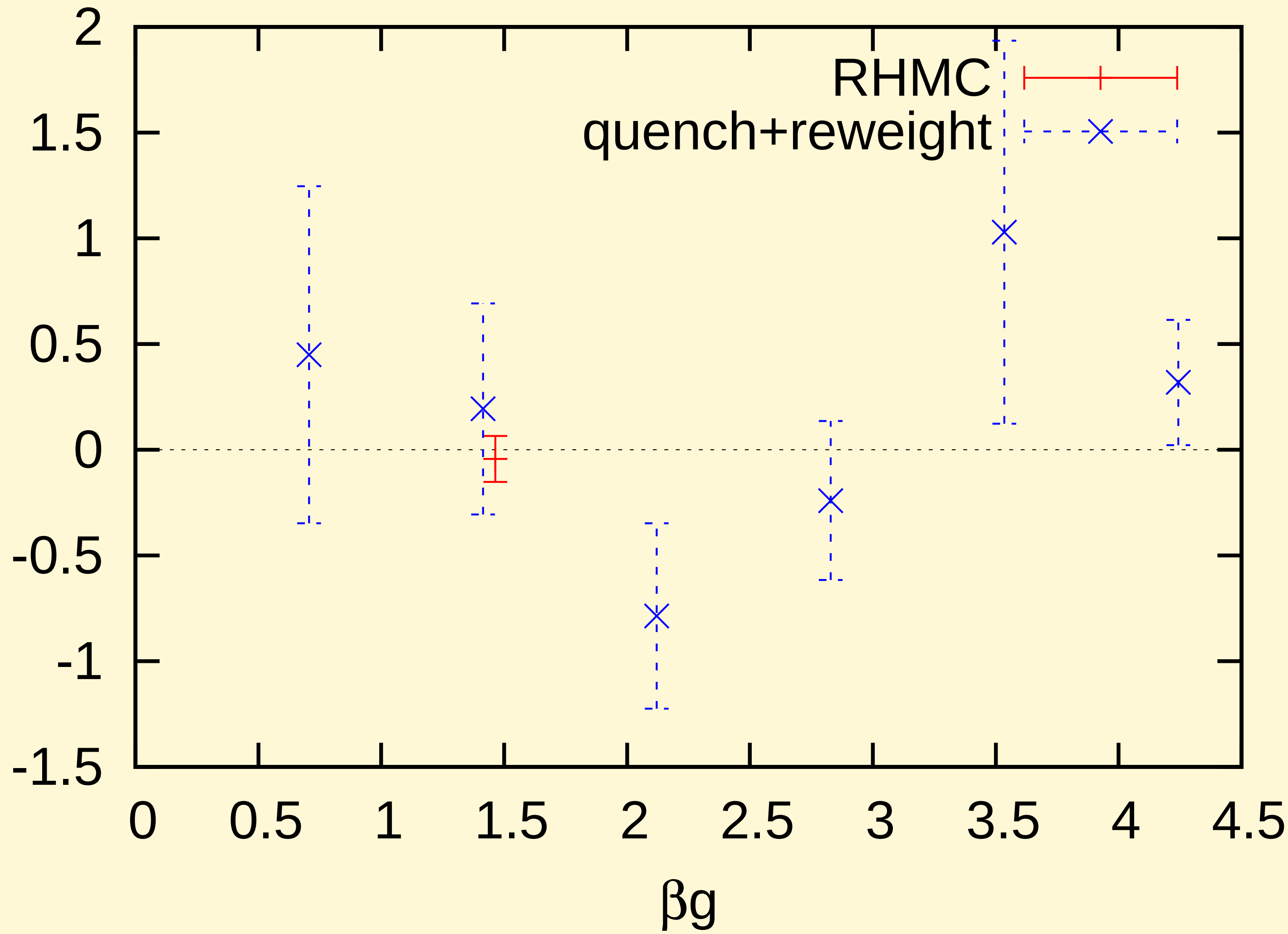


$SU(2)$, Periodic boundary condition $ag = 0.2357$
Ground state energy (Q-exact Hamiltonian) should be zero



Simulation detail(SYM)

- Computer: RIKEN Super Combined Cluster
- fixed spatial physical length: $gL = 1.414$
- lattice size: 3×6 – 36×12
- lattice spacing: ag = 0.2357 – 0.0707
- independent configurations for each parameter: 9900–99900(old)/20–1700(new)

Simulation detail(SYM)

- (old:)
  quench + reweight: fermion effect as a part of the observable
  $S = S_b + S_f, \quad Z = \int \mathcal{D}f\,\mathcal{D}b\,e^{-S_b - S_f} = \int \mathcal{D}b\,\mathrm{Pf}(D)e^{-S_b}$
  $$\langle \mathcal{O} \rangle_S = \frac{\langle \mathcal{O}\mathrm{Pf}(D) \rangle_{S_b}}{\langle \mathrm{Pf}(D) \rangle_{S_b}}$$

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  $$\langle \mathcal{O} \rangle_S = \frac{\langle \mathcal{O}\mathrm{Pf}(D) \rangle_{S_b}}{\langle \mathrm{Pf}(D) \rangle_{S_b}}$$
- (new:)
  Rational Hybrid Monte Carlo(RHMC): dynamical fermion

$$\mathrm{Pf}(D) = \int \mathcal{D}F \exp\left\{-F^\dagger (D^\dagger D)^{-1/4} F\right\} \mathrm{sign}(\mathrm{Pf}(D))$$
$$= \int \mathcal{D}F \exp\left\{-F^\dagger \left[a_0 + \sum_{i=1}^{n} \frac{a_i}{D^\dagger D + b_i}\right] F\right\} \mathrm{sign}(\mathrm{Pf}(D))$$

Conclusion

Application of the lattice SUSY: $\boxed{\text{available}}$

- model and simulation detail
  [Two-dim. $N=(2,2)$ SYM (Sugino Model)]
- application:

observing dynamical SUSY breaking measurement of the ground state energy

  Application to other system is straightforward
  cf. Sugino's talk (SQCD)

outlook

- [in progress]
  simulation with dynamical fermion: check of the correct target theory in the continuum, now we can check it
- [future] behavior in $N_C \to \infty$ (AdS/CFT)

Thank you.

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Under the periodic condition: simulation does NOT work

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$$(\text{Witten index}) = Z_{\text{PBC}} = (\operatorname{tr} e^{-\beta H})_{\text{PBC}}$$

Fujikawa Z.Phys.C15(1982)275

$$\langle H \rangle_{\text{PBC}} = \frac{-\frac{\partial}{\partial\beta}(\text{Witten index})}{Z_{\text{PBC}}} = \frac{0}{Z_{\text{PBC}}} \xrightarrow{\text{simulation}} 0$$

	SUSY	~~SUSY~~
correct $\langle H \rangle_{\text{PBC}}$	$= 0$	> 0
simulation of "$\langle H \rangle_{\text{PBC}}$"	$= 0$	$= 0$

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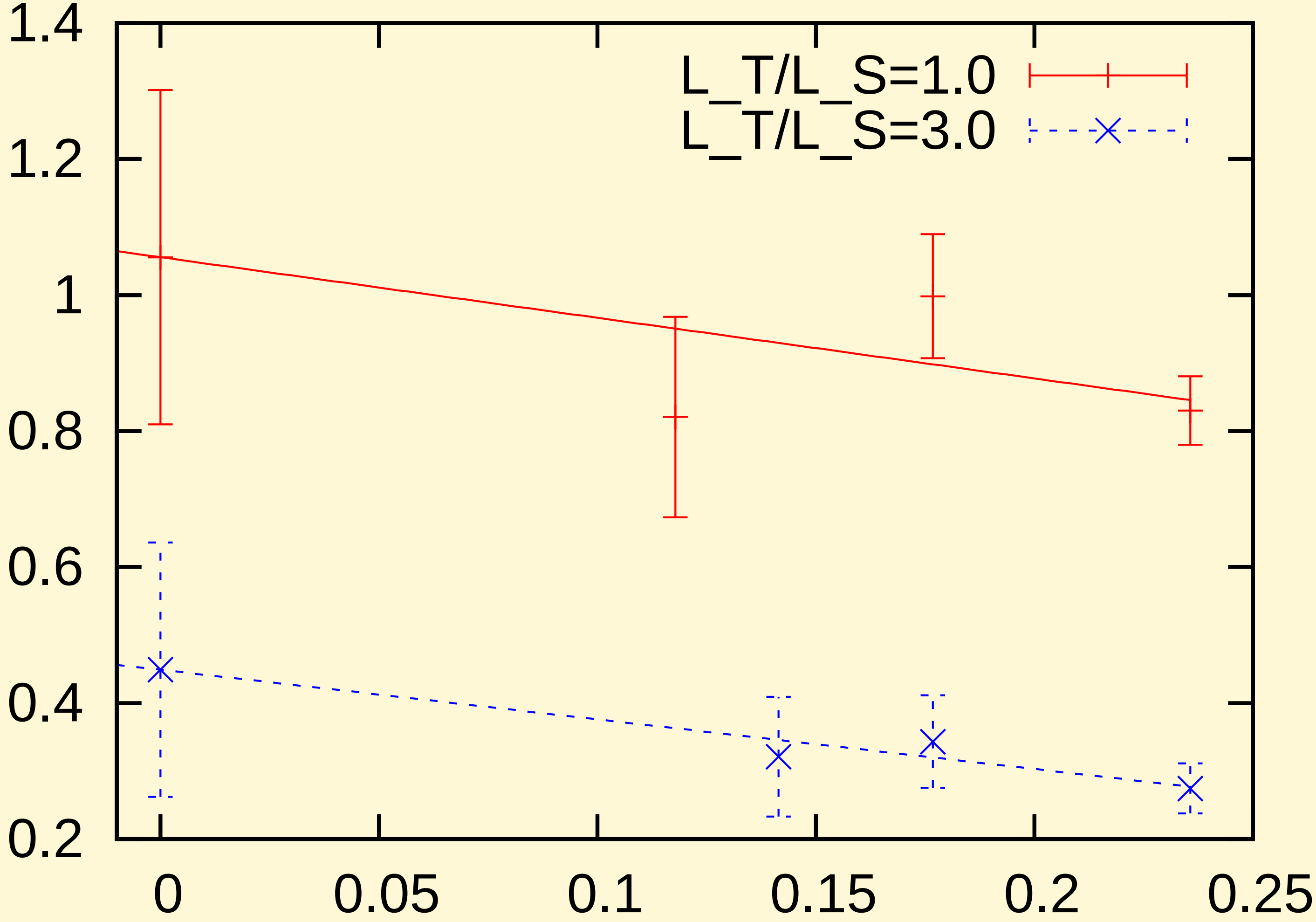
Our hamiltonian: $\langle Q\text{-exact} \rangle_{\text{PBC}} = 0 \propto -\frac{\partial}{\partial\beta}(\text{Witten index})$

Taking the continuum limit



Taking the continuum limit(SQM, broken)