Apparent Horizons in Holographic Dual of Hydrodynamics

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arXiv:0807.3797

Motivation

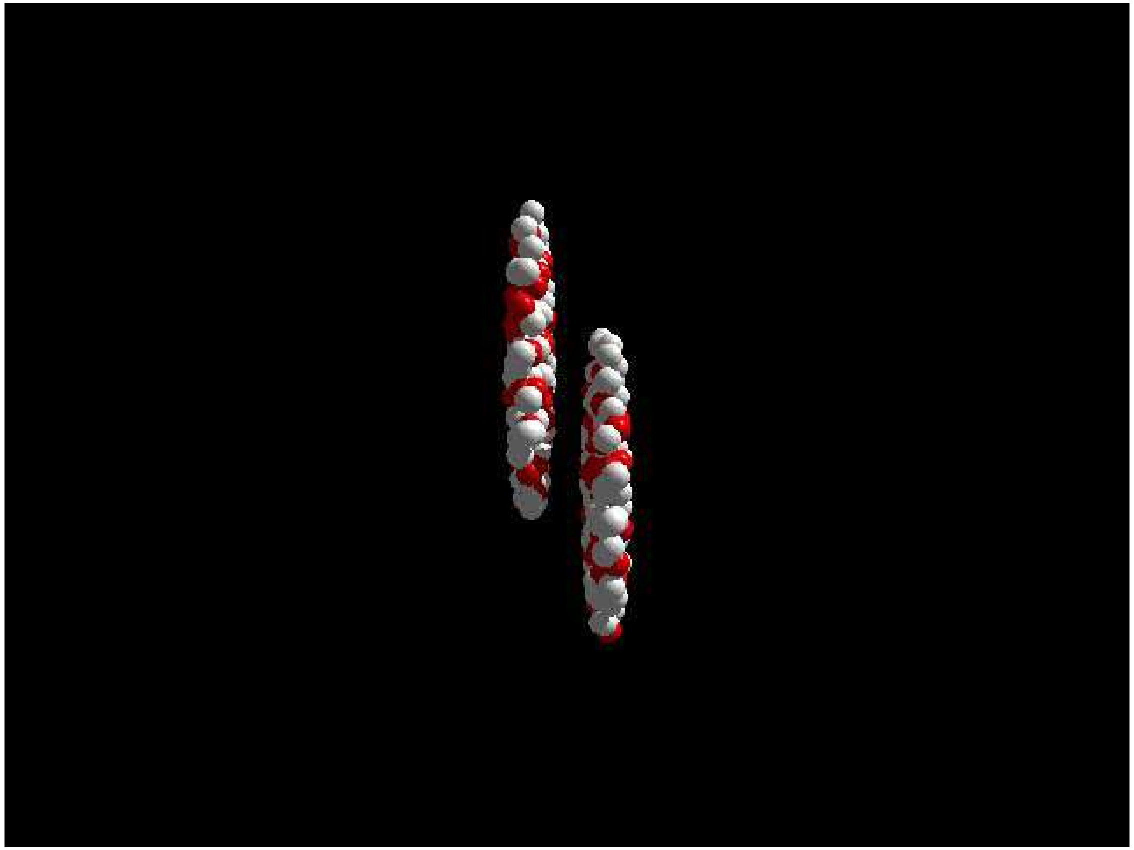
RHIC: Relativistic Heavy Ion Collider
(@ Brookhaven National Laboratory)

Heavy ion:
e.g. ^{197}Au
$\sqrt{S_{NN}}$ ~200GeV.

Similar exp. at
LHC: ALICE/CMS

http://www.bnl.gov/RHIC/inside_1.htm

Quark-gluon plasma (QGP) is created



For the movie: http://www.bnl.gov/RHIC/heavy_ion.htm

This is a time-dependent system

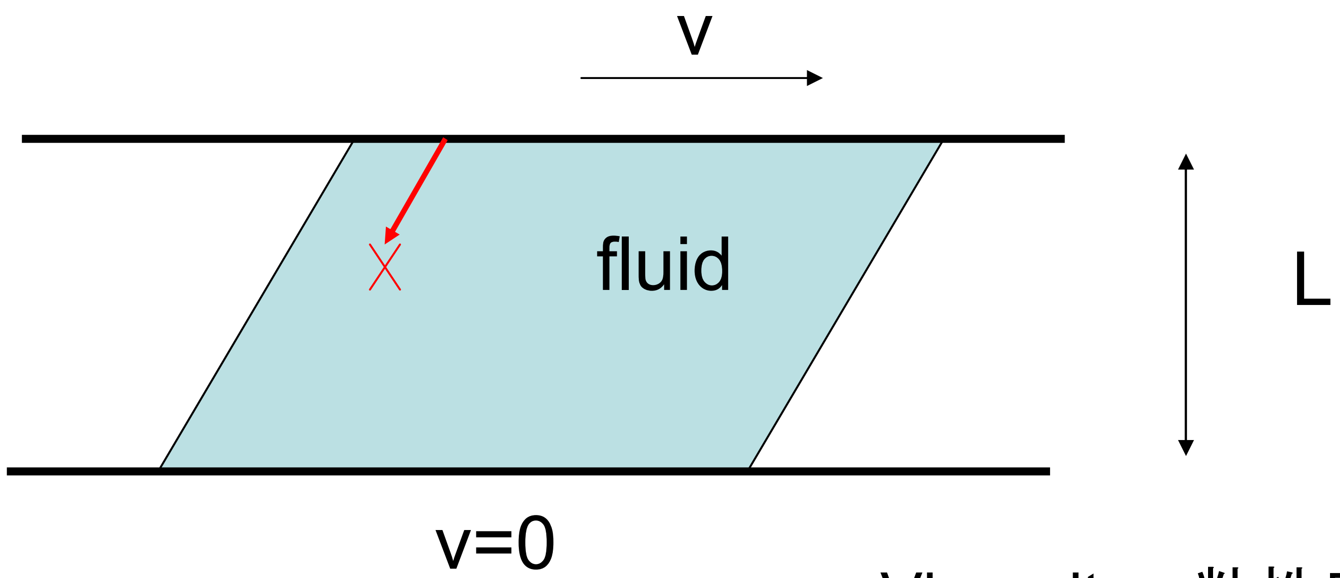
- Furthermore, QGP is known to be a strongly interacting system.

  (small viscosity)

$\Updownarrow$ AdS/CFT

A time-dependent 5d geometry.

Shear viscosity



$$\frac{F}{A} = \eta \frac{v}{L}$$

Viscosity: 粘性度

Shear: 剪断

Strong interaction ⟺ Small viscosity

Construction of time-dependent AdS/CFT itself is a challenge.

- In this talk, we consider the Bjorken flow of N=4 SYM theory.

A standard, simplest model of QGP

How to obtain the geometry?

Solution to the 5d Einstein equation with $\Lambda < 0$
with appropriate boundary conditions
and initial conditions.

⇕

- The 4d geometry where the YM theory lives.
- The initial stress tensor of the YM fluid.

⇓

Time-dependent solutions

General relativity as hydrodynamics

time-dependent 5d Geometry → time-dependent 4d Stress tensor

GKP-Witten prescription
(AdS/CFT dictionary)

Time evolution of (the long-range modes of) stress tensor

⇕

Described by hydrodynamics.

Hydrodynamics comes out of the gravity!

Summary of our work

Hydrodynamics | **Our model**

- hydrodynamic equation (energy-momentum conservation) ⟺ 5d Einstein's eq. at the vicinity of the boundary
- equation of state (conformal invariance)

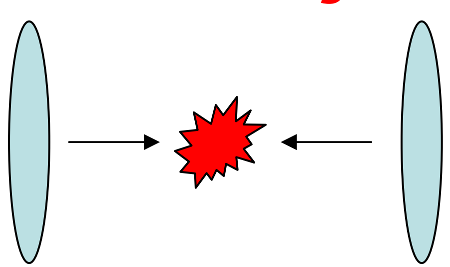
- transport coefficients (viscosity, relaxation time…) ⟺ Reguarity around the horizon

Related to thermal equilibrium

Physical parameters from Cosmic Censorship.

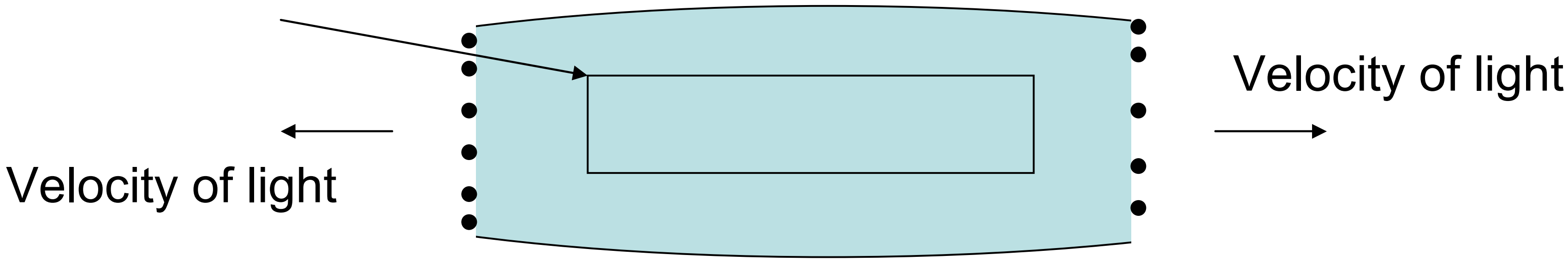
Our system: Bjorken flow

A standard model of QGP fluid



Relativistically accelerated heavy nuclei

Central Rapidity Region (CRR)



After collision

(Bjorken 1983)

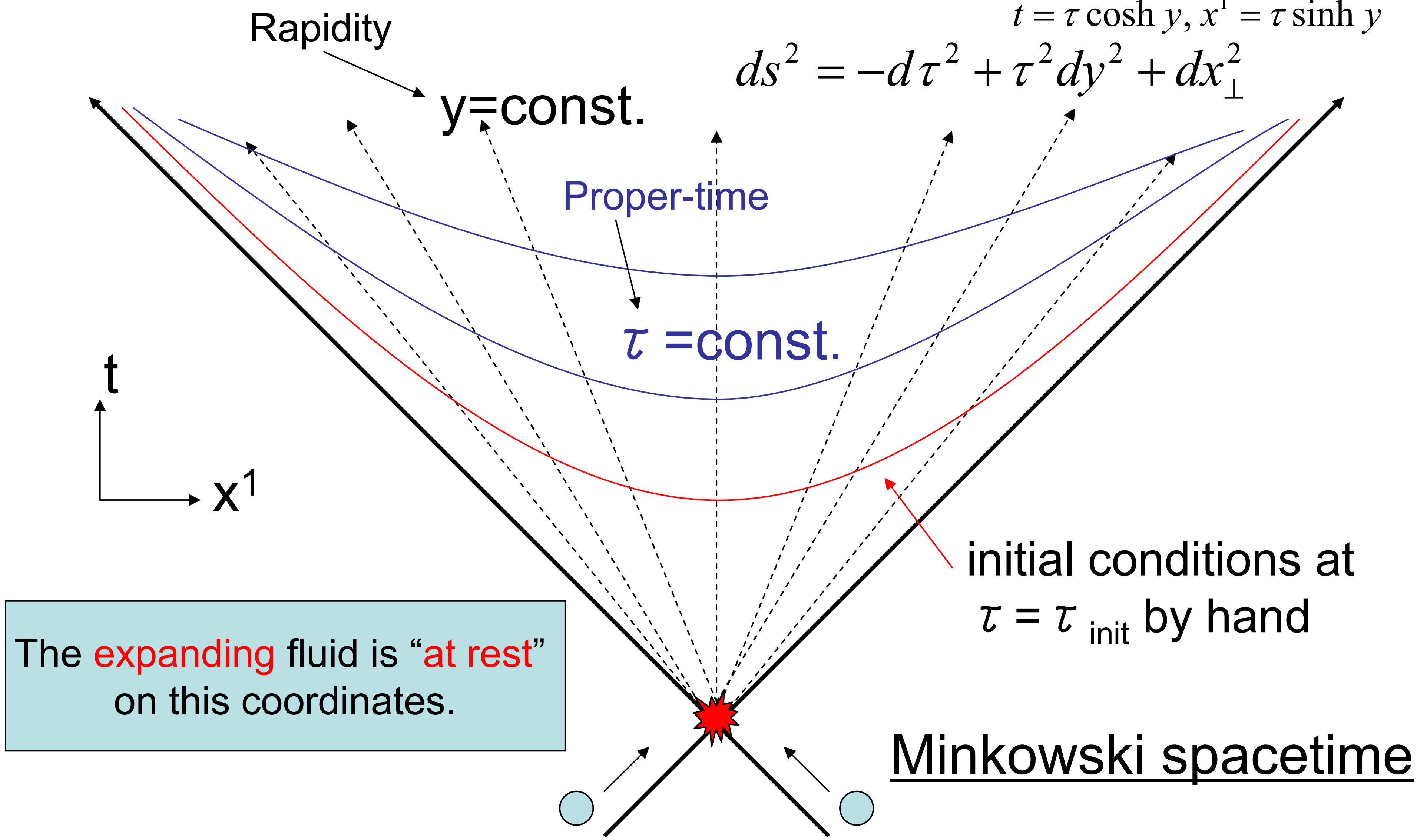
- (Almost) **one-dimensional expansion**.
- We have **boost symmetry** in the CRR.

⇨ Time dependence of the physical quantities are written by the **proper time**.

Local rest frame(LRF)

$$t = \tau \cosh y,\ x^1 = \tau \sinh y$$

$$ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2$$



Stress tensor on LRF

The stress tensor becomes diagonal:

energy density

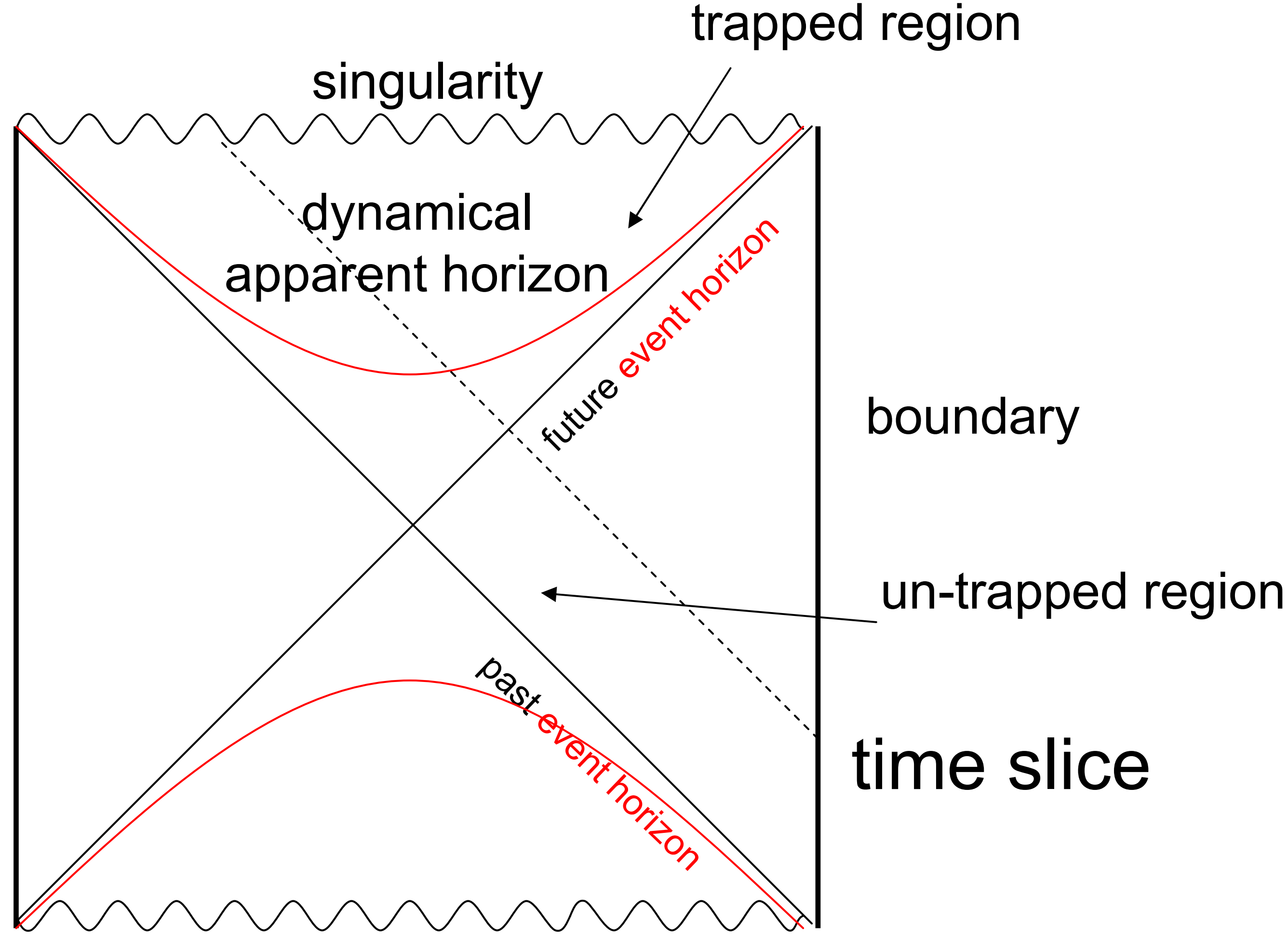
$$T_{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \tau^2 (p - \frac{4}{3}\frac{\eta}{\tau}) & 0 & 0 \\ 0 & 0 & p + \frac{2}{3}\frac{\eta}{\tau} & 0 \\ 0 & 0 & 0 & p + \frac{2}{3}\frac{\eta}{\tau} \end{pmatrix}$$

pressure shear viscosity

If the stress tensor is diagonal

⇨ you are on the flow.

Penrose diagram



Cf. Janik-Peschanski (2005)
on the Fefferman-Graham coordinates.

Eddingtong-Finkelstein
coordinates

Our dual geometry

- Parametrization (Eddington-Finkelstein type):

$$ds^2 = -r^2 a\, d\tau^2 + 2d\tau\, dr + r^2\tau^2 e^{2b-2c}\left(1+(r\tau)^{-1}\right)^2 dy^2 + r^2 e^c d\vec{x}_\perp^2$$

  - r is the radial (5th) direction, the boundary is at r = ∞.
  - the boundary condition (LRF):

    a→1, b→0, c→0, at r= ∞.

The Einstein's equation

⇩

Differential equations for a, b, c.

⇨ Difficult to get the exact solution.

Approximation:

Hydrodynamics: $\tau^{-2/3}$ expansion works for Bjorken flow.

Expansion around equilibrium.

Gravity dual: $\tau^{-2/3}$ expansion, with r fixed?

We found that
$\tau^{-2/3}$ expansion with $u = r\tau^{1/3}$ fixed works well.

$$ds^2 = -r^2 a\, d\tau^2 + 2d\tau\, dr + r^2\tau^2 e^{2b-2c}\left(1+(r\tau)^{-1}\right)^2 dy^2 + r^2 e^c d\vec{x}_\perp^2$$

$$a = a_0(u) + a_1(u)\tau^{-2/3} + a_2(u)\tau^{-4/3} + \ldots\ldots$$

Similar for b and c.

Solve the equation order by order.

The zeroth-order solution (at the leading order)

$$ds^2 = -r^2\left(1-\frac{w^4}{(r\tau^{1/3})^4}\right)d\tau^2 + 2d\tau dr + r^2(\tau^2 dy^2 + d\vec{x}_\perp{}^2)$$

This is u

$$w^4 = \varepsilon_0\left(\frac{3N_c^2}{8\pi^2}\right)$$

- This reproduces the correct zeroth-order stress tensor of the Bjorken flow.

$$T_{\tau\tau} = \varepsilon = \varepsilon_0\left(\frac{1}{\tau^{4/3}} + \dots\right)$$

- We have an apparent horizon.

$$e^F\theta_+\theta_- = -\frac{9}{2}\left(1-u^{-4}w^4\right)$$

trapped region if u<w.

Normalized product of expansions

The location of the apparent horizon: u=w+O($\tau^{-2/3}$)

The (event) horizon is necessary

$$\left(R_{\mu\nu\rho\lambda}\right)^2 = 8\left(5 + \frac{9w^8}{u^8}\right) + O(\tau^{-2/3})$$

We have a physical singularity at the origin.

However, this is hidden by the apparent horizon at u=w hence the event horizon (outside it).

⇒ Not a naked singularity.

The first-order solution
(at the order of $\tau^{-2/3}$)

$$ds^2 = -r^2 a\, d\tau^2 + 2d\tau\, dr + r^2\tau^2 e^{2b-2c}\left(1+(r\tau)^{-1}\right)^2 dy^2 + r^2 e^c d\vec{x}_\perp^{\,2}$$

$$a_1 = -\frac{2}{3}\frac{(1+\xi_1)u^4 + \xi_1 w^4 - 3\eta_0 u w^4}{u^5}$$

$$b_1 = -\frac{1+\xi_1}{u}$$

gauge degree of freedom

$$c_1 = \frac{1}{3w}\left\{\arctan(uw^{-1}) - \pi/2 + 1/2\log(u-w) - 1/2\log(u+w)\right\}$$

$$-\frac{1}{2}\eta_0 \log(1-w^4u^{-4}) - 2\xi_1/3u$$

The dual geometry is regular at the horizon, only when

$$(R_{\mu\nu\rho\lambda})^2 = \frac{4(9\eta_0^2 w^2 - 1)}{3(u-w)}\left(\frac{1}{u-w} - 2\right)\tau^{-2/3} + \text{regular} + O(\tau^{-4/3})$$

$$\boxed{\eta_0 = \frac{1}{3w}\,.}$$

What is η_0?

Read the stress tensor from the geometry

$\Downarrow$

η_0 is found to be proportional to the shear viscosity η.

$$\boxed{\eta_0 = \frac{1}{3w}} \longrightarrow \boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$

The famous ratio by Kovtun-Son-Starinets (2004)

HIgher-order results:

- From the regularity of the 2nd-order geometry, "relaxation time" is uniquely determined.

consistent with Heller-Janik, Baier et. al., and Bhattacharyya et. al.

Furthermore, we can show by using induction that the same mechanism holds to <u>arbitrary higher order</u>.

⇓

No un-removable singularity like
Benincasa-Buchel-heller-Janik, arXiv:0712.2025.

Our model is totally well-defined.

Non-staticity of the loal geometry

Projected Weyl tensor

$$C^{x^1x^2}{}_{x^1x^2} = \frac{w^4}{u^4} - \frac{4w^4}{3u^5}\tau^{-2/3} + \ldots.$$

$$C^{x^1y}{}_{x^1y} = \frac{w^4}{u^4} - \left(\frac{4w^4}{3u^5} + \frac{3\eta_0 w^4}{u^4}\right)\tau^{-2/3} + \ldots.$$

An-isotropy evolves in time.

The dual geometry is not static, if we include dissipation.

Area of the apparent hrizon

entropy creation due to disspation

$$A_{ap} = w^3 - \frac{3w^3\eta_0}{2}\tau^{-2/3} + \ldots.$$

Consistent with the time evolution of the entropy density to the first order.

What we have done

- Gravity dual of the Bjorken flow on an Eddington-Finkelstein type coordinates.
- Presence of the apparent horizon and the event horizon: a dynamical black hole.
- Transport coefficients from the absence of naked singularity.
- Our late-time expansion is consistent and the geometry can be regular for all orders.

Advertisement

- Our model provides the first consistent gravity dual of the Bjorken flow.
  (cf. Heller-Loganayagam-Spalinski-Surowka-Vazquez, arXiv:0805.3774)

- Our model is a concrete well-defined example of time-dependent AdS/CFT based on a well-controled approximation.

I hope that

I hope that the gravity dual provides a computational method beyond what we already know.

I hope that:

$$\text{Einstein} + \text{Penrose} \overset{?}{>} \text{Kubo} + \text{Landau}$$

Challenging problems:

Description of phenomena that appear only in time-dependent systems, such as

- anomalous viscosty
- turbulence
- ....

For the details

- Please take a look at our paper arXiv:0807.3797.
- Please feel free to make a contact with me.